# Pulse Forming Lines

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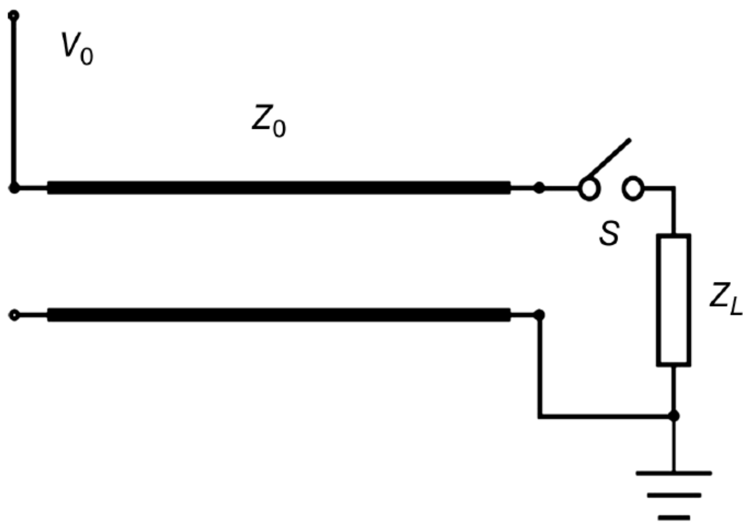
**Kyoung-Jae Chung**

***Department of Nuclear Engineering***

**Seoul National University**

# Pulse forming line (PFL)

- There are numerous applications in both physics and electrical engineering for short ($\sim 10\ ns < t_p < 100\ \mu s$) electrical pulses. These applications often require that the pulses have a "good" square shape.
- Although there are many ways for generating such pulses, the pulse-forming line (PFL) is one of the simplest techniques and can be used even at extremely high pulsed power levels.
- A transmission line of any geometry of length $l$ and characteristic impedance $Z_0$ makes a pulse forming line (PFL), which when combined with a closing switch $S$ makes the simple transmission line pulser.

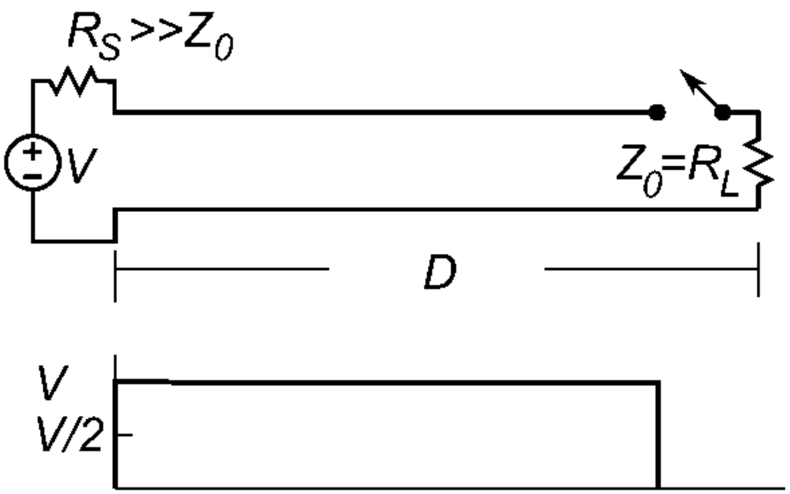


# Simple PFL

- When the switch closes, the incident wave $V_I$, with a peak voltage of $(1/2)V_0$, travels toward the load, while the reverse-going wave $V_R$, also with a peak voltage of $(1/2)V_0$, travels in the opposite direction.
- The incident wave $V_I$, then, supplies a voltage of $(1/2)V_0$ for a time determined by the electrical length of the transmission line $T_T$ to the load. The reverse-going wave $V_R$ travels along the transmission line for a duration $T_T$ and then reflects from the high impedance of the voltage source, and becomes a forward-going wave traveling toward the load with peak voltage $(1/2)V_0$ and duration $T_T$.
- The two waves add at the load to produce a pulse of amplitude $(1/2)V_0$ and pulse duration $T_p = 2T_T$.
- Matching condition: $R_L = Z_0$
- Pulse characteristics

$$V = \frac{V_0}{2}$$

$$T_p = 2T_T = \frac{2l}{v_p} \approx \frac{2l}{c}\sqrt{\epsilon_r}$$



# Coaxial PFL

- Basic parameters

$$L' = \frac{\mu}{2\pi}\ln\left(\frac{R_2}{R_1}\right) \qquad C' = \frac{2\pi\epsilon}{\ln(R_2/R_1)}$$

$$Z_0 = \sqrt{\frac{L'}{C'}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}}\ln\left(\frac{R_2}{R_1}\right) = 60\sqrt{\frac{\mu_r}{\epsilon_r}}\ln\left(\frac{R_2}{R_1}\right)$$

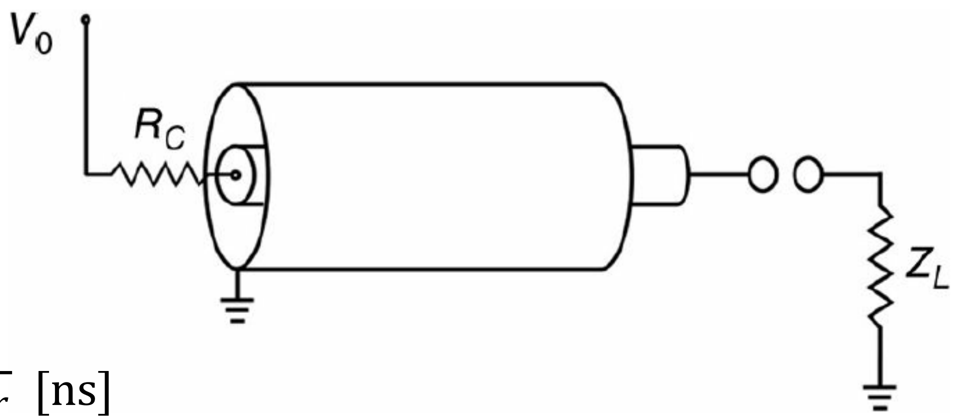
$$v_p = \frac{1}{T_T} = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r\epsilon_r}} \approx \frac{30}{\sqrt{\epsilon_r}}\left[\frac{\text{cm}}{\text{ns}}\right]$$

- Matching condition: $Z_L = Z_0$

- Pulse characteristics

$$V = \frac{V_0}{2}$$

$$T_p = 2T_T = \frac{2l}{v_p} \approx \frac{2l}{c}\sqrt{\epsilon_r} = \frac{l\ [\text{cm}]}{15}\sqrt{\epsilon_r}\ \ [\text{ns}]$$



# Coaxial PFL

- Electric field

$$E(r) = \frac{V_0}{r\ln(R_2/R_1)} \qquad E_{max}(r = R_1) = \frac{V_0}{R_1\ln(R_2/R_1)}$$

- Voltage at the maximum electric field

$$V_0 = E_{max}R_1\ln\left(\frac{R_2}{R_1}\right)$$

- The value of $R_2/R_1$ that optimizes the inner conductor voltage occurs when $dV_0/dR_1 = 0$, yielding

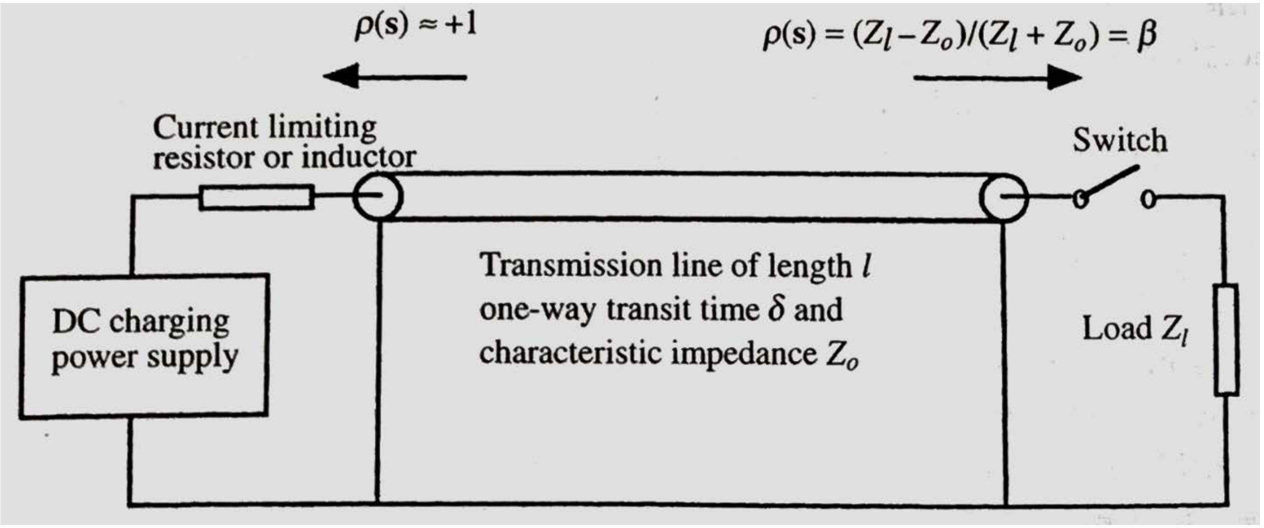
$$\ln\left(\frac{R_2}{R_1}\right) = 1$$

- Optimum impedance for maximum voltage

$$Z_{opt} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\epsilon}} = 60\sqrt{\frac{\mu_r}{\epsilon_r}} \approx \frac{60}{\sqrt{\epsilon_r}}$$

$$Z_{opt}^{water} = \frac{60}{\sqrt{81}} = 6.7\ \Omega \qquad Z_{opt}^{oil} = \frac{60}{\sqrt{2.4}} = 38.7\ \Omega$$

# Analysis of simple PFL




- On closure of the switch, the voltage on the load rises from zero to a value determined by

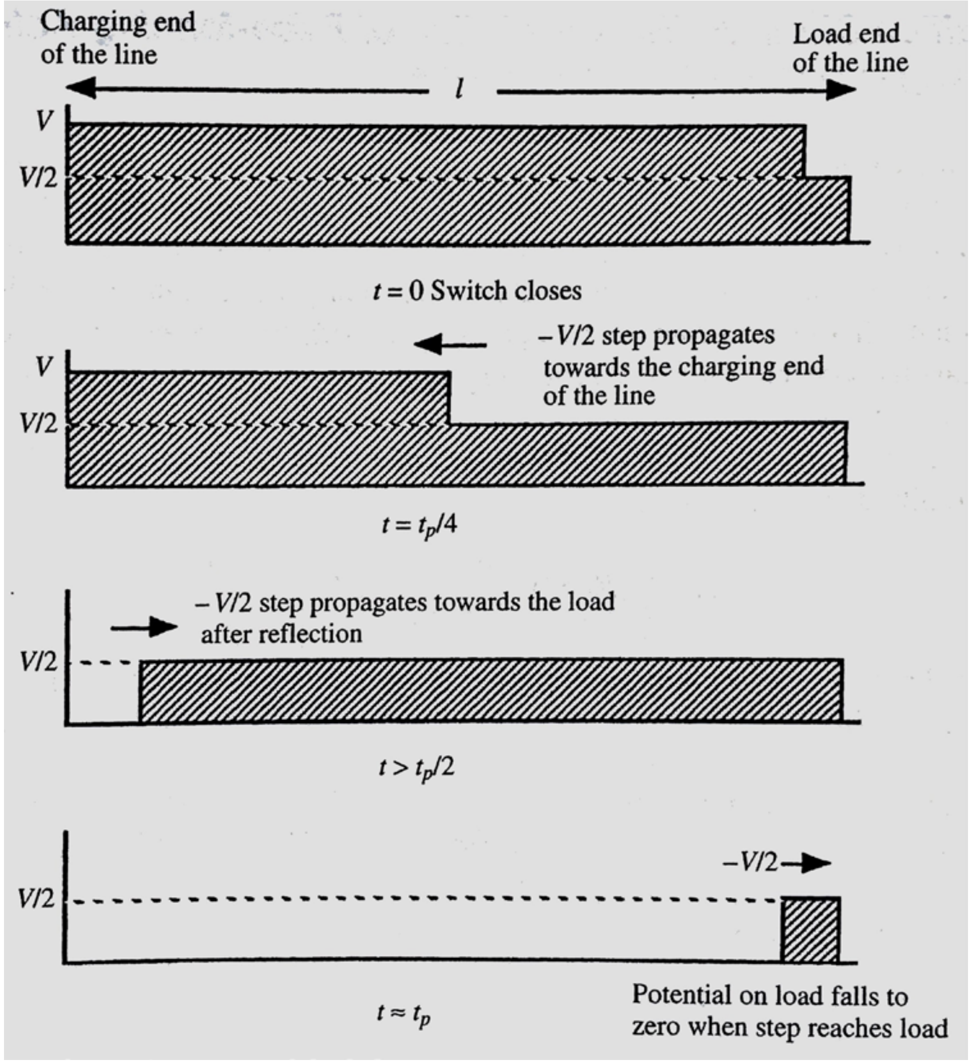
$$V_L = V\frac{Z_L}{Z_L + Z_0}$$ $$V_L = \frac{V}{2}$$ (matched)

- Simultaneously, a voltage step $V_s$ is propagated away from the load towards the charging end of the line. It takes $\delta = l/v_p$ for the wave to reach the charging end.

$$V_s = V_L - V = V\left(\frac{Z_L}{Z_L + Z_0} - 1\right) = V\left(\frac{-Z_0}{Z_L + Z_0}\right)$$ $$V_s = -\frac{V}{2}$$ (matched)

# Analysis of simple PFL

- Potential distribution (matched load)



# Lattice diagram representation of pulse-forming action

- On closure of the switch, the voltage on the load rises from zero to a value determined by

$$V_L = V\frac{Z_L}{Z_L + Z_0} = \alpha V$$

- The potential on the load is given by

$$V_L = \alpha V \qquad (0 < t < 2\delta)$$

$$V_L = \alpha V + (\alpha - 1)\gamma V \qquad (2\delta < t < 4\delta)$$

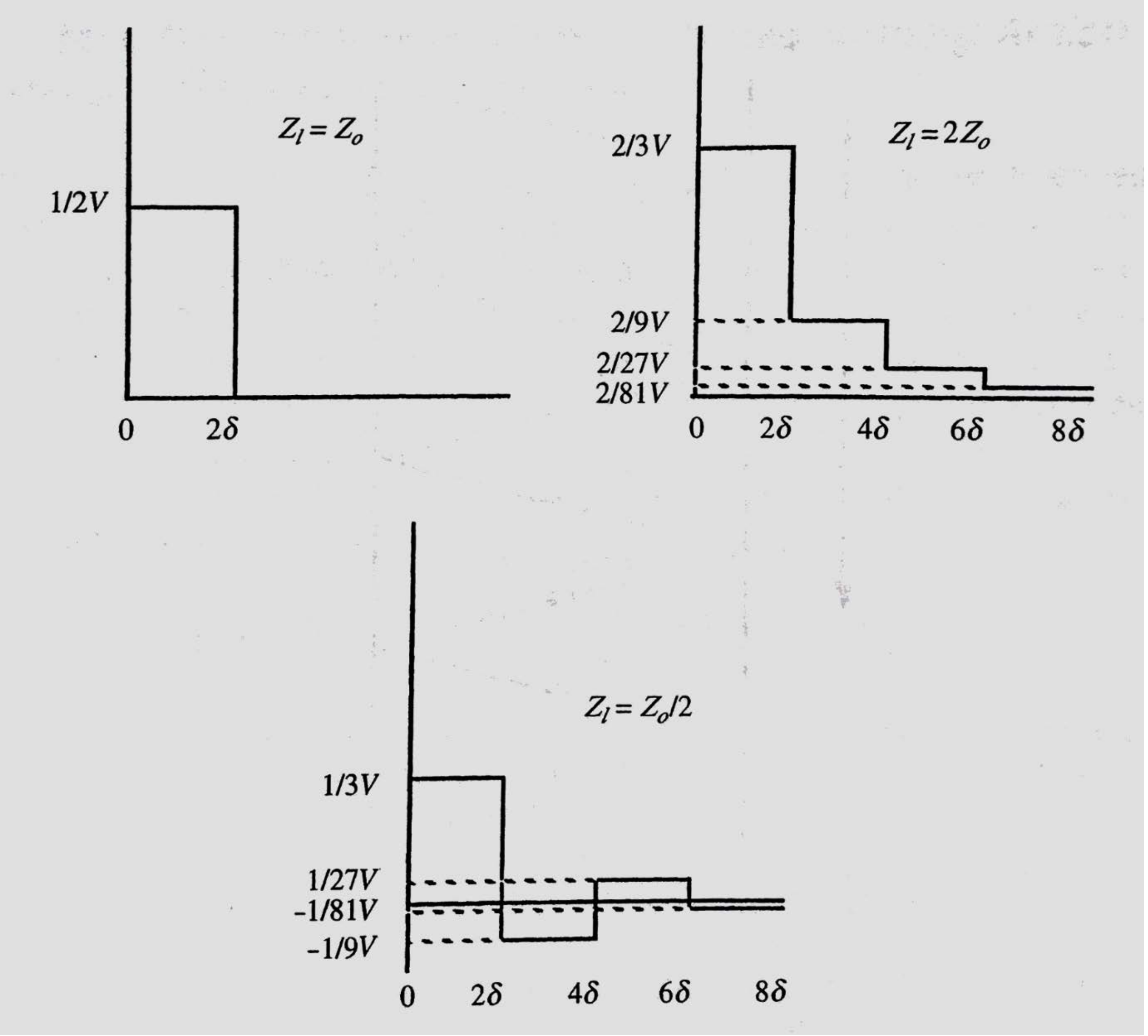
$$V_L = \alpha V + (\alpha - 1)\gamma V + \beta(\alpha - 1)\gamma V$$
$$(4\delta < t < 6\delta)$$

$$\gamma = \beta + 1$$

- Finally

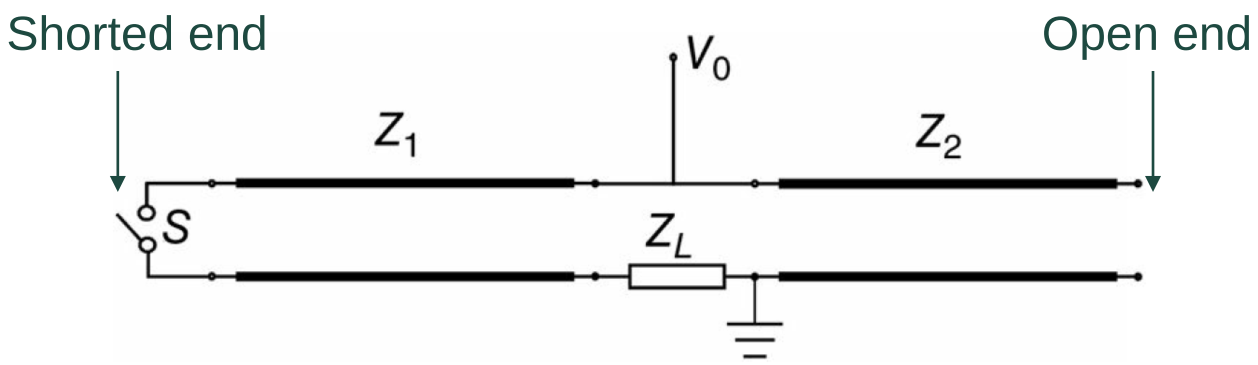
$$V_L = V[\alpha + (\alpha - 1)\gamma(1 + \beta + \beta^2 + \cdots)]$$

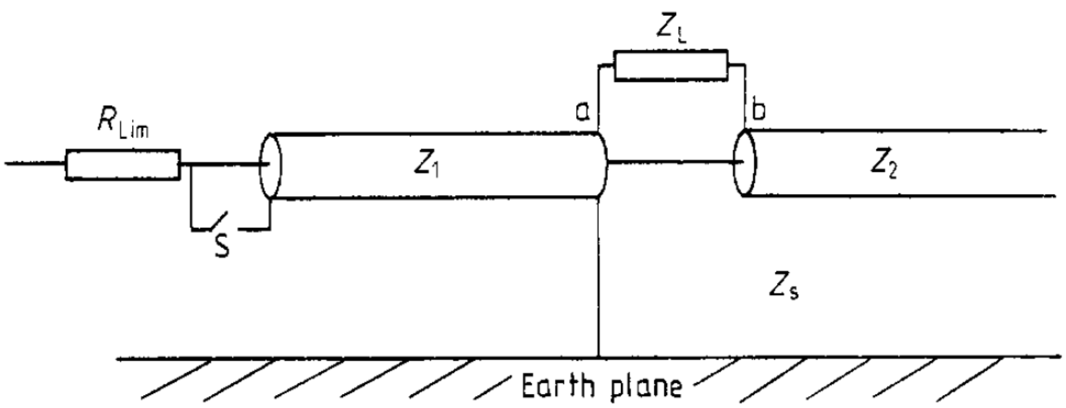
# Typical waveforms from PFL under matched and unmatched conditions



# Blumlein PFL

- An important disadvantage of the simple PFL is that the pulse generated into a matched load is only equal to $V_0/2$.
- This problem can be avoided using the Blumlein PFL invented by A. D. Blumlein.
- Two transmission lines and one switch is used to construct the generator.



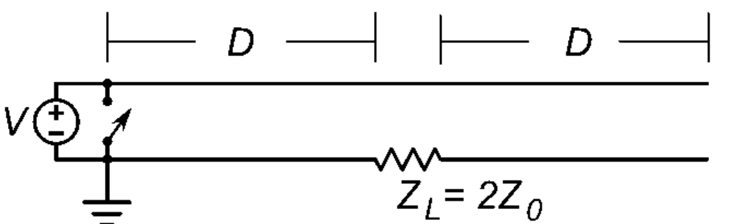



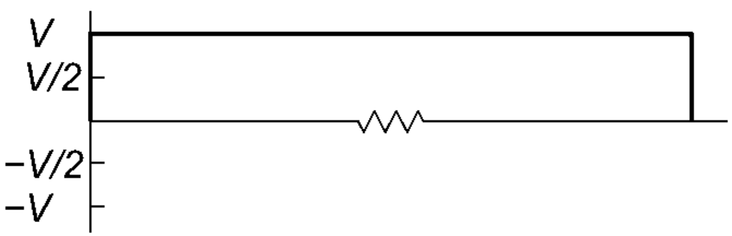
# Blumlein PFL

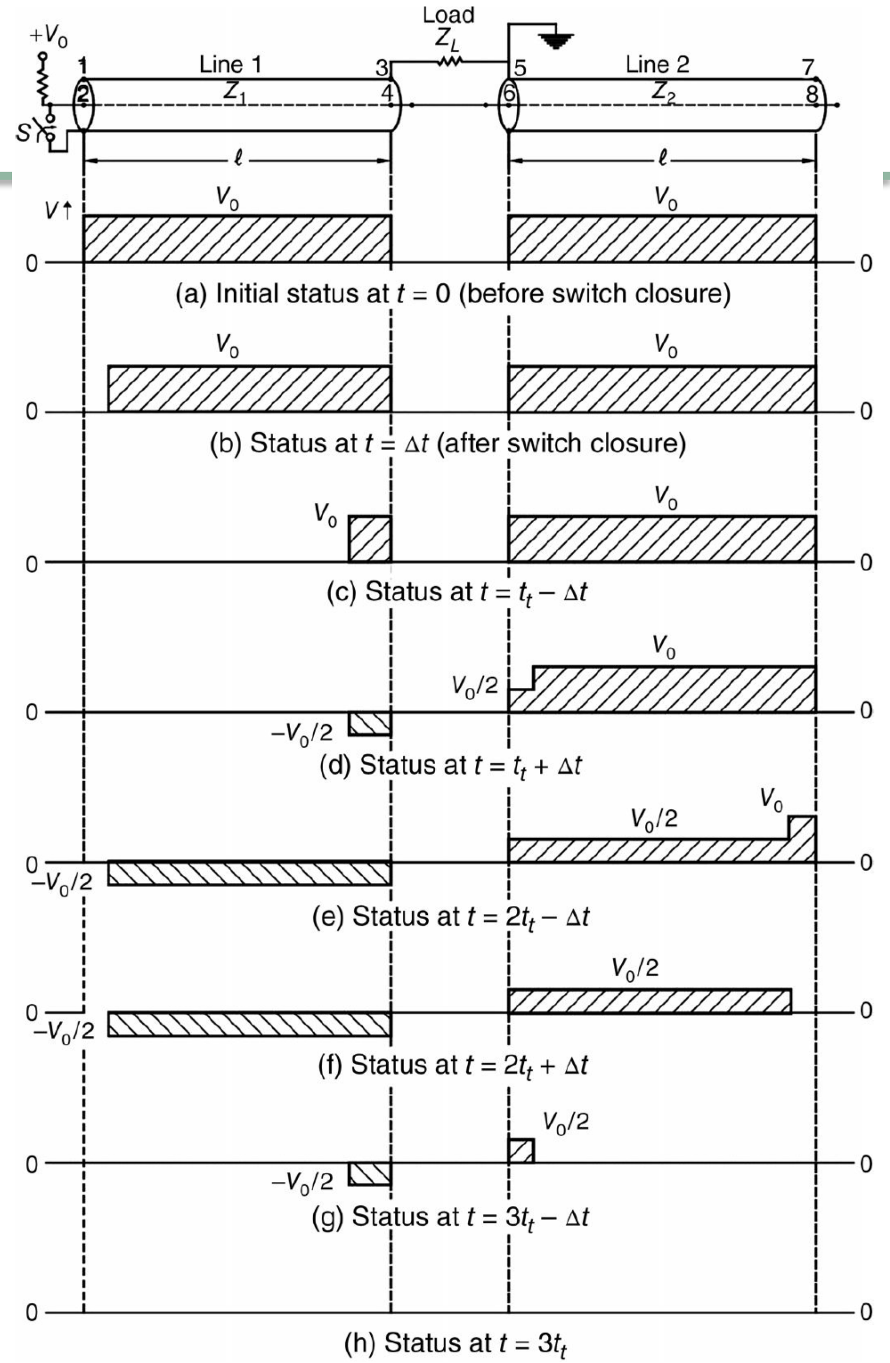
- After switch closure, the end of line 1 is effectively shorted; thus the reflection coefficient is +1.
- At the junction of line 1 and the load, the reflection coefficient is given by

$$\rho = \frac{(Z_L + Z_2) - Z_1}{(Z_L + Z_2) + Z_1} = \frac{Z_L}{Z_L + 2Z_0} = \frac{1}{2}$$

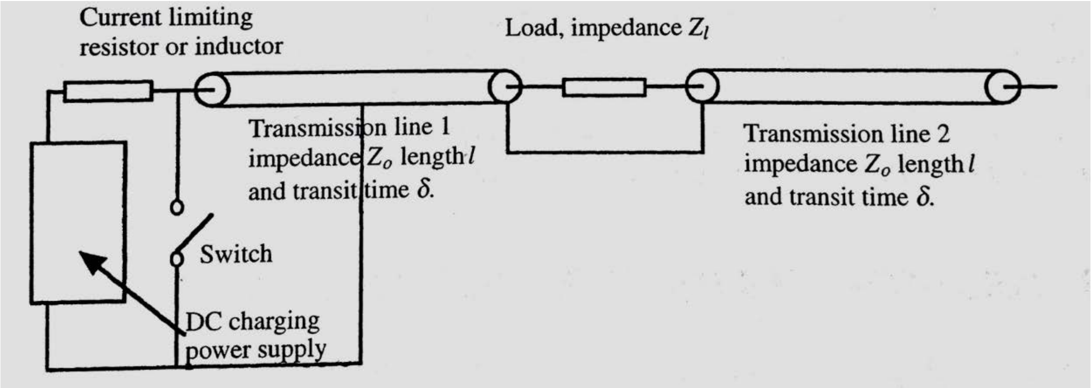
- Matching condition: $Z_L = 2Z_0$

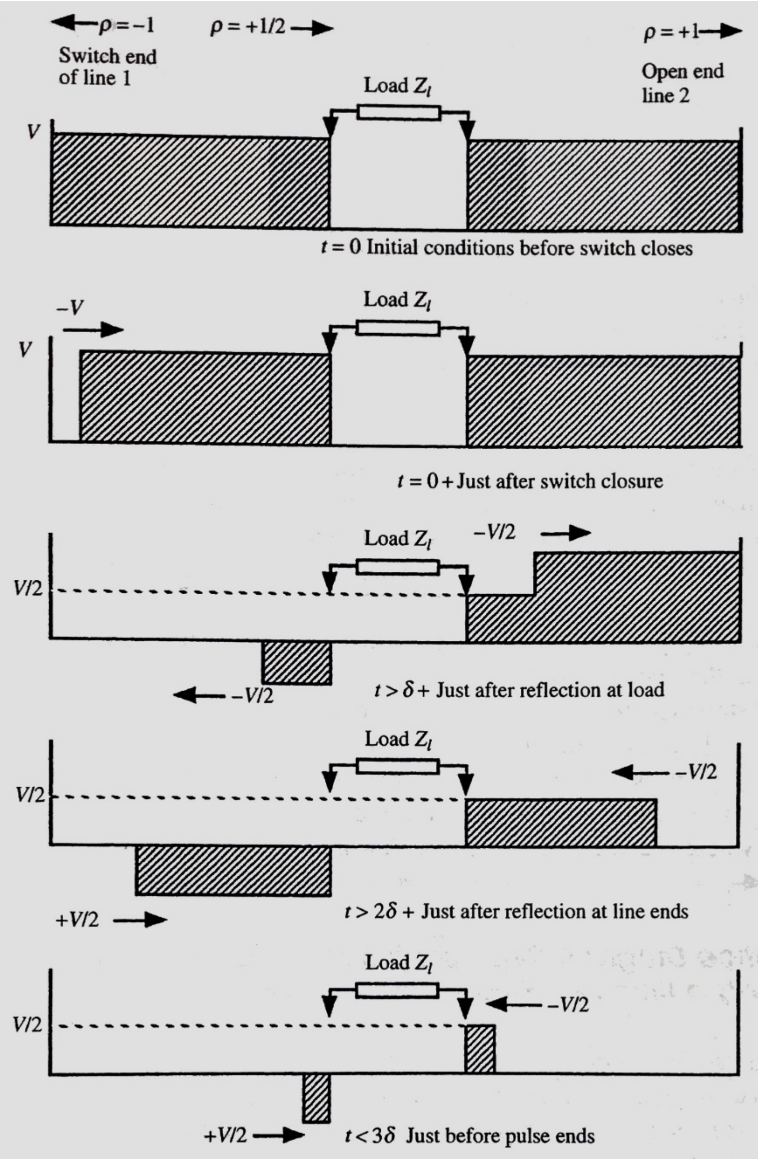


# Analysis of Blumlein PFL




- The reflection coefficient at the load

$$\rho = \frac{Z_L}{Z_L + 2Z_0}$$

- The reflected step at the load

$$V_- = \rho(-V) = -V\frac{Z_L}{Z_L + 2Z_0}$$

- The step $V_T$ transmitted to the load and the line 2

$$V_T = V_+ + V_- = -2V\frac{Z_L + Z_0}{Z_L + 2Z_0}$$

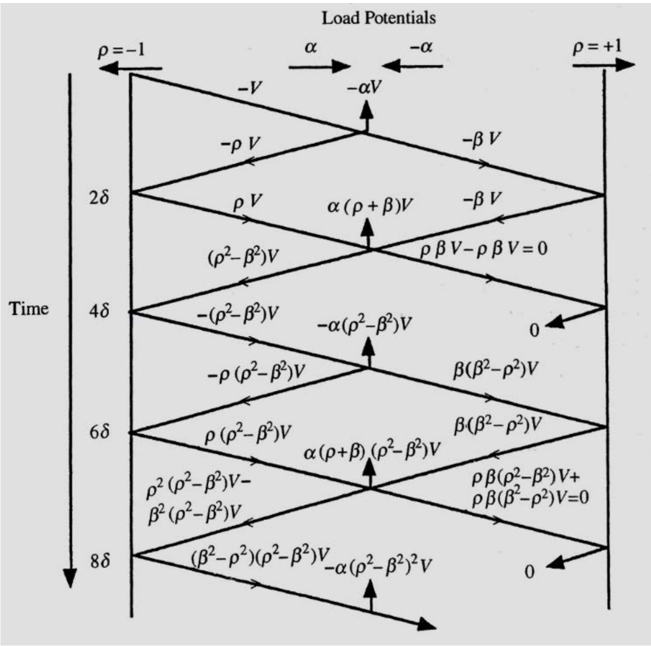
# Analysis of Blumlein PFL

- The fraction of the step to the load and the line 2

$$V_L = -2V\frac{Z_L + Z_0}{Z_L + 2Z_0} \times \frac{Z_L}{Z_L + Z_0} = -\alpha V$$

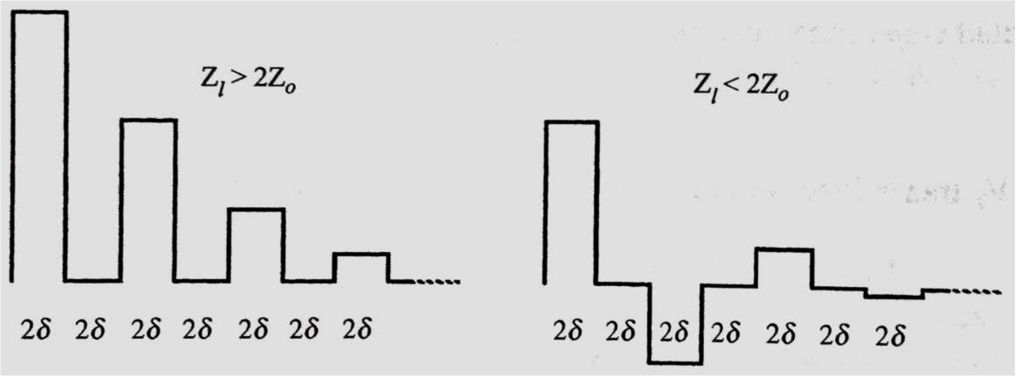
$$V_{2T} = -2V\frac{Z_L + Z_0}{Z_L + 2Z_0} \times \frac{Z_0}{Z_L + Z_0} = -\beta V$$

$$\alpha = \frac{2Z_L}{Z_L + 2Z_0} \qquad \beta = \frac{2Z_0}{Z_L + 2Z_0}$$




- Finally

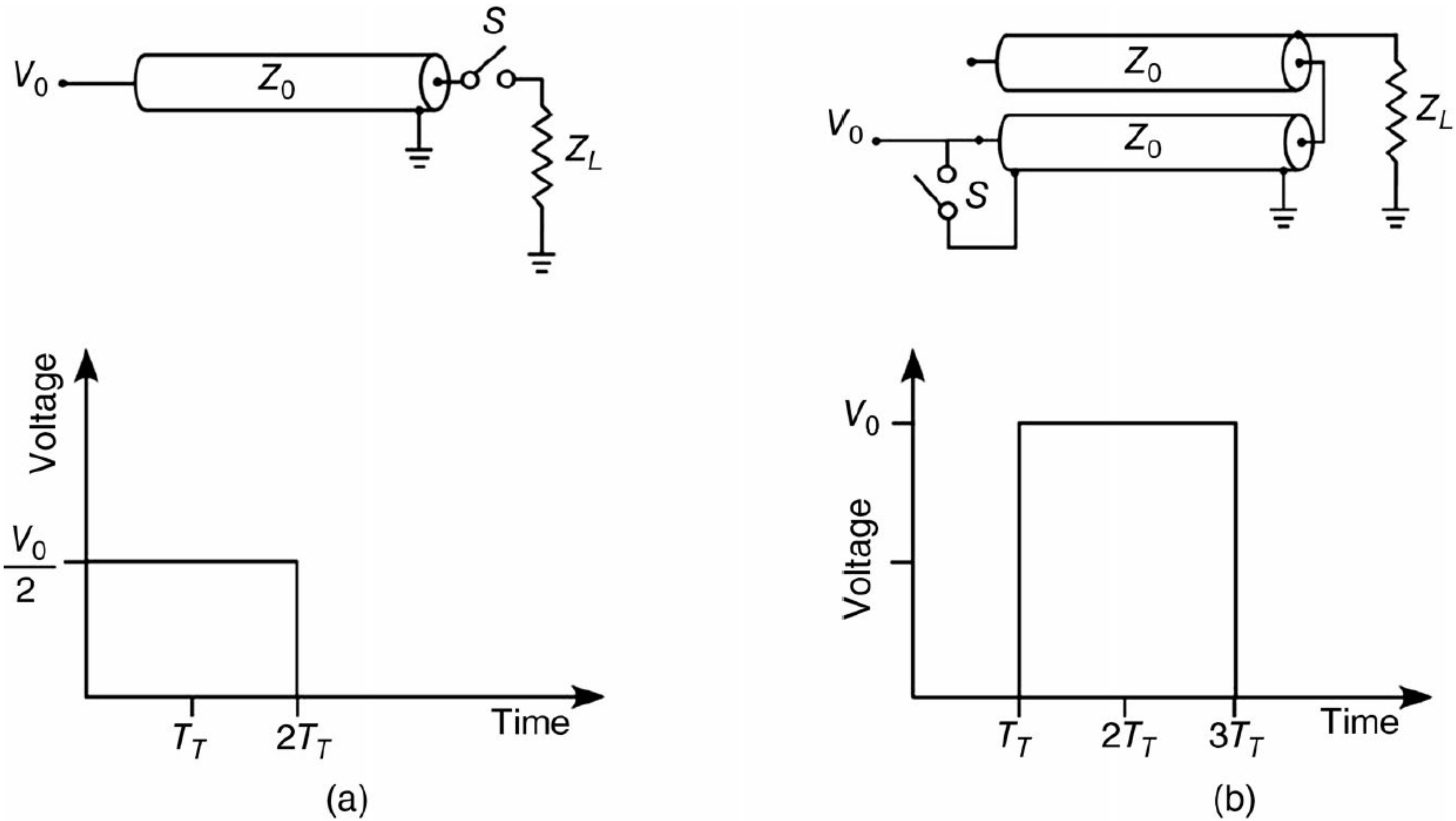
$$V_L = -\alpha V[1 - 1 + (\rho - \beta) - (\rho - \beta) + (\rho - \beta)^2 - (\rho - \beta)^2 + \cdots]$$



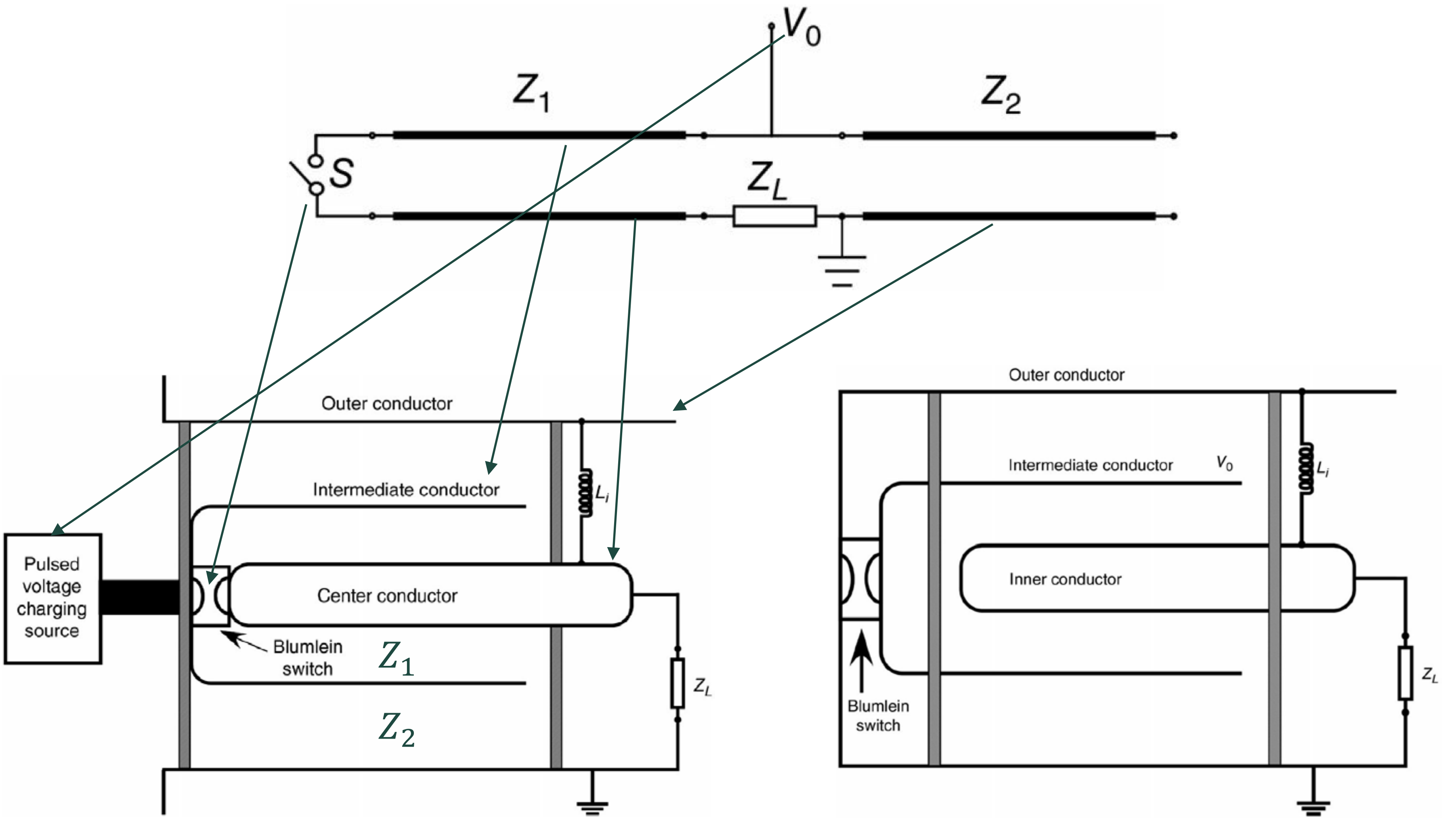
# Simple PFL vs. Blumlein PFL

Matching condition: $Z_L = Z_0$

Matching condition: $Z_L = 2Z_0$



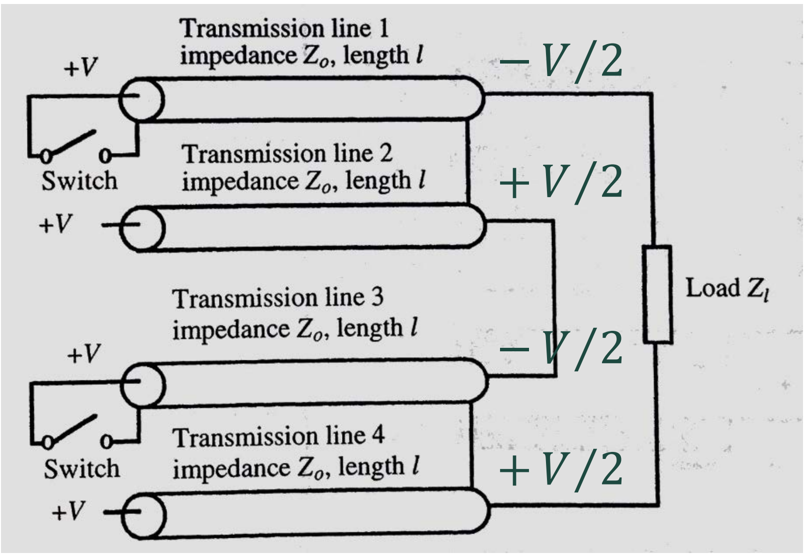
# Coaxial Blumleins

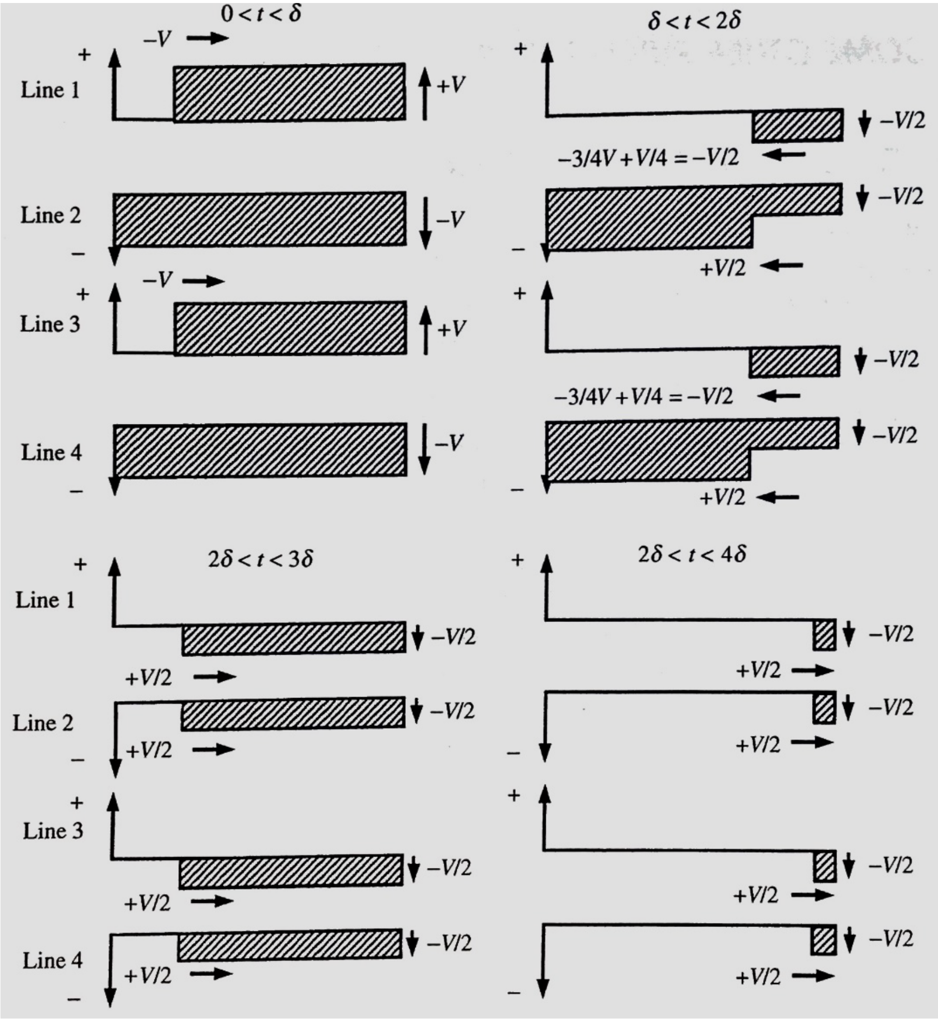



- Matching condition: $Z_L = Z_1 + Z_2$

# Two-stage stacked Blumlein

- Matching condition: $Z_L = 4Z_0$





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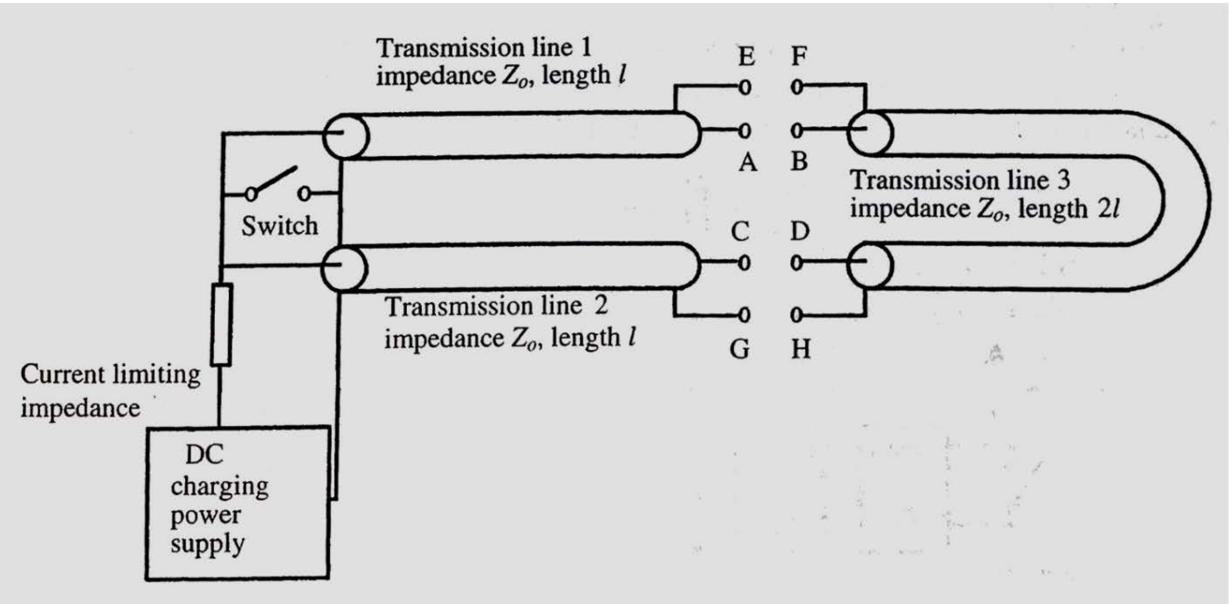
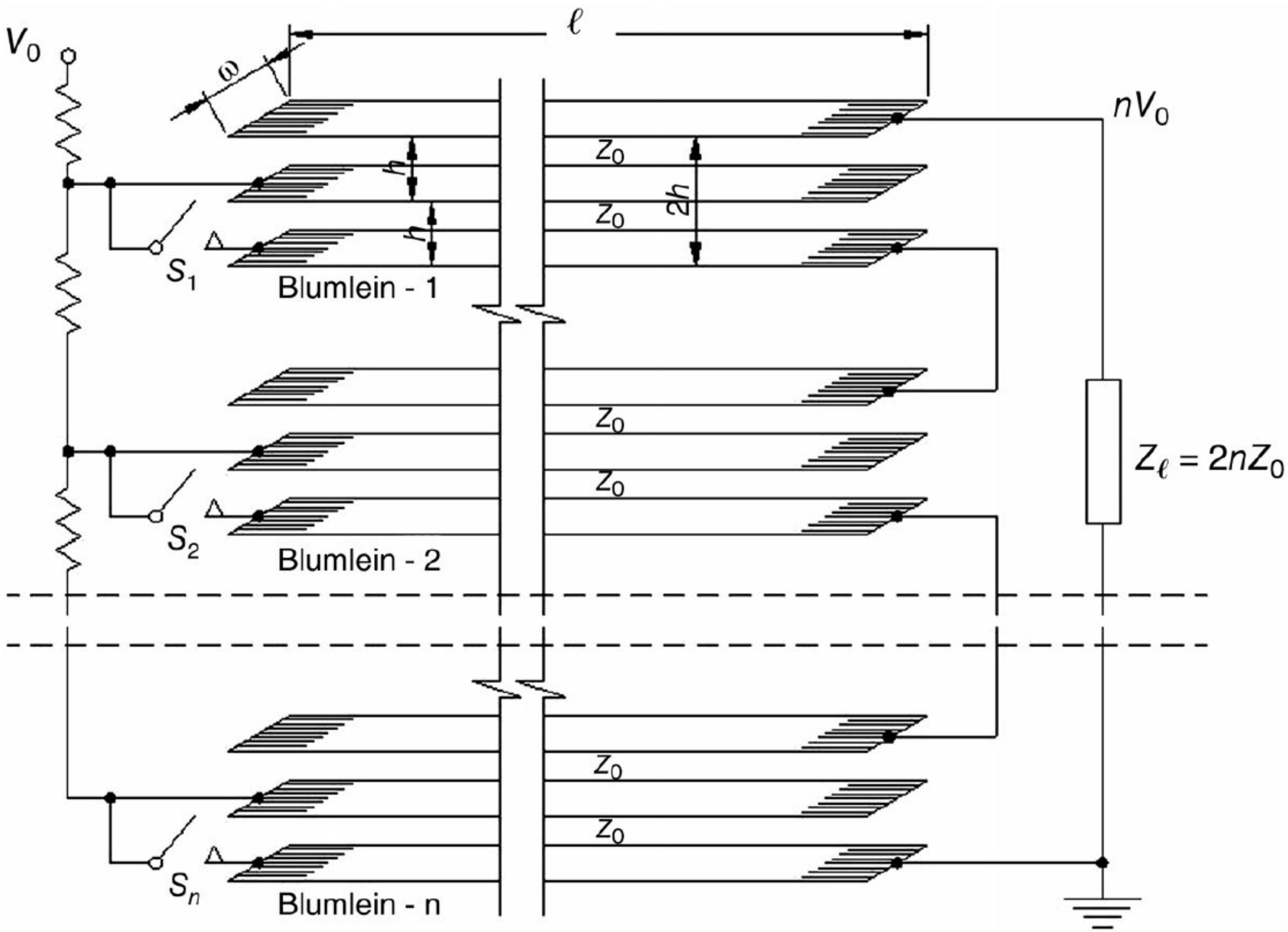
# Two-stage stacked Blumlein

- A more practical realization of two-stage Blumlein PFL using a single switch



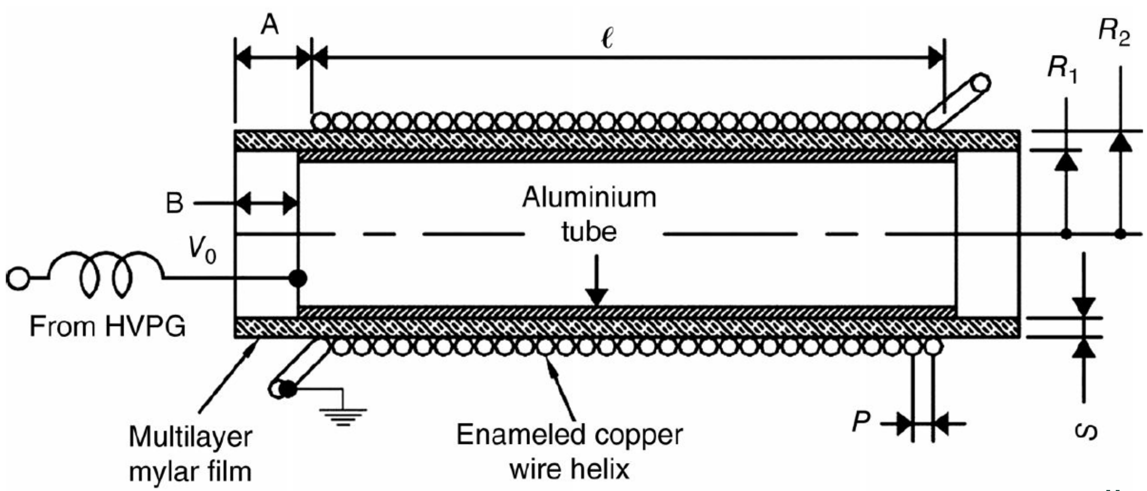

- If points A and B and C and D are connected, then the load can be placed either across points E to H with points F and G connected or across points F to G with points E and H connected. → This change simply reverse the polarity of the pulse generated in the load.
- If points E and F and G and H are connected, then the load can be placed either across points A to D with points B and C connected or across points B to C with points A and D connected.

# Stacked Blumlein



# [Optional] Helical lines

- For rectangular pulses with pulse durations on the order of a few microseconds, the physical lengths of the transmission lines become prohibitive – on the order of 1 km for a 10 μs pulse.
- The helical line storage element can be realized by helical winding of a conductor of circular or rectangular cross section over a metallic cylinder.




$$Z_0 = \frac{(R_2 - R_1)}{d_h}\sqrt{\frac{\mu}{\epsilon}}$$

$d_h$: Diameter of helical conductor

$$T_T = \frac{l'}{v_p} = \frac{(2\pi R_{avg})n_h}{v_p}$$

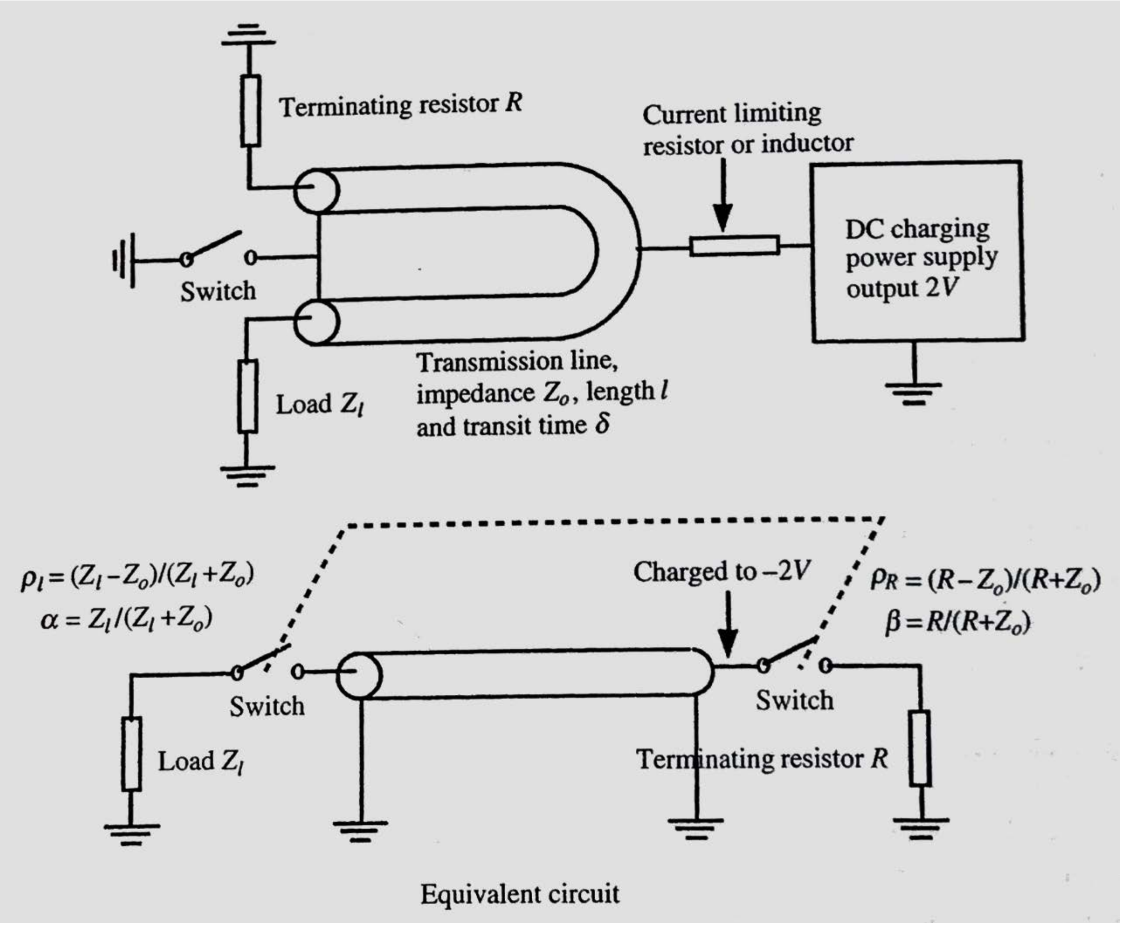
$l'$: Unfolded length of wire helix

$R_{avg}$: Average radius of helical winding

$n_h$: Total number of turns

# Self-matching PFL

- In this circuit, at one end of the line the inner conductor is connected to a terminating resistor $R$ and at the other it is connected to a load $Z_l$, where $Z_l \neq Z_0$.



# Self-matching PFL

- From the lattice diagram, the voltage on the load can be found to be

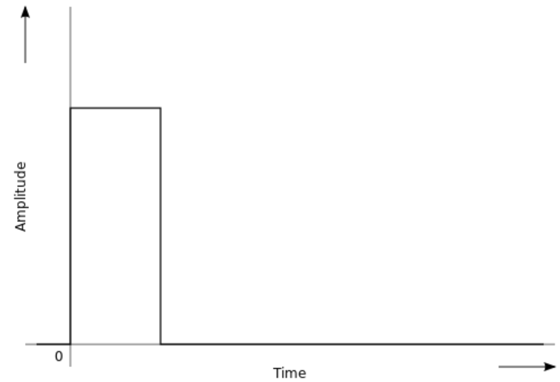
$$v_l(t) = -2V \begin{bmatrix} \alpha + (\beta - 1)(1 + \rho_l)u(t - \delta) \\ +\rho_R(\alpha - 1)(1 + \rho_l)u(t - 2\delta) \\ +\rho_R\rho_l(\beta - 1)(1 + \rho_l)u(t - 3\delta) \\ +\rho_R^2\rho_l(\alpha - 1)(1 + \rho_l)u(t - 4\delta) \\ + \cdots \end{bmatrix}$$

where $u(\Delta t) = 1$ for $\Delta t > 0$, $u(\Delta t) = 0$ for $\Delta t < 0$
and $\Delta t = (t - n\delta)$, $n = 1, 2, 3, \ldots$

- If the value of the terminating resistor is made equal to the impedance of the line, i.e. $R = Z_0$, then $\rho_R = 0$ and $\beta = 1/2$. Then, the load voltage becomes

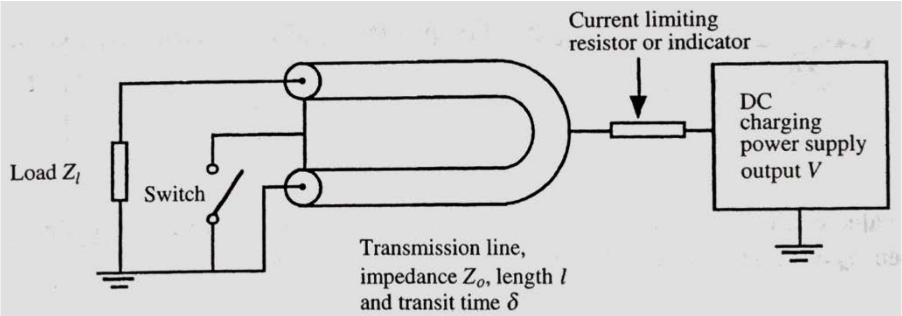
$$V_l(\boldsymbol{s}) = -\frac{2V\alpha}{\boldsymbol{s}}(1 - e^{-s\delta})$$

  - ➢ A single rectangular pulse with duration $\delta$ independently of the value of $Z_l$, thus the PFL is self-matching.



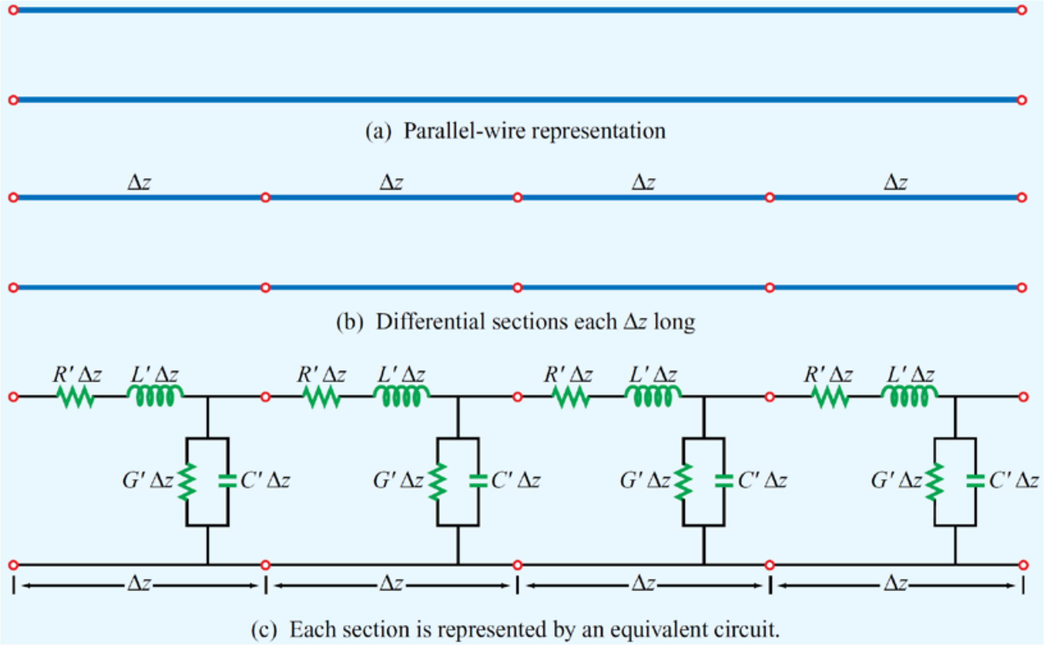
# Bi-directional or zero-integral PFL

- Some applications require the generation of a bi-directional pulse in which the polarity of the pulse changes sign in the middle of the pulse, causing the net integral of the pulse to be zero.

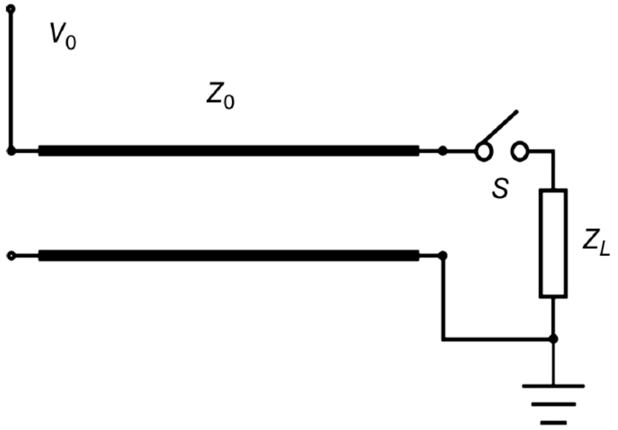


# Pulse forming network (PFN)

- A main disadvantage of the PFL is the speed of propagation of EM waves along transmission lines.
- The material used in transmission lines is some type of polymer plastic such as polypropylene, and the dielectric constant tens to be quite low ($\epsilon_r = 2{\sim}3$). Thus, it is impractical for making a long pulse over $1\ \mu s$.
- An alternative approach is to build a simulated line using a ladder network of inductors and capacitors.



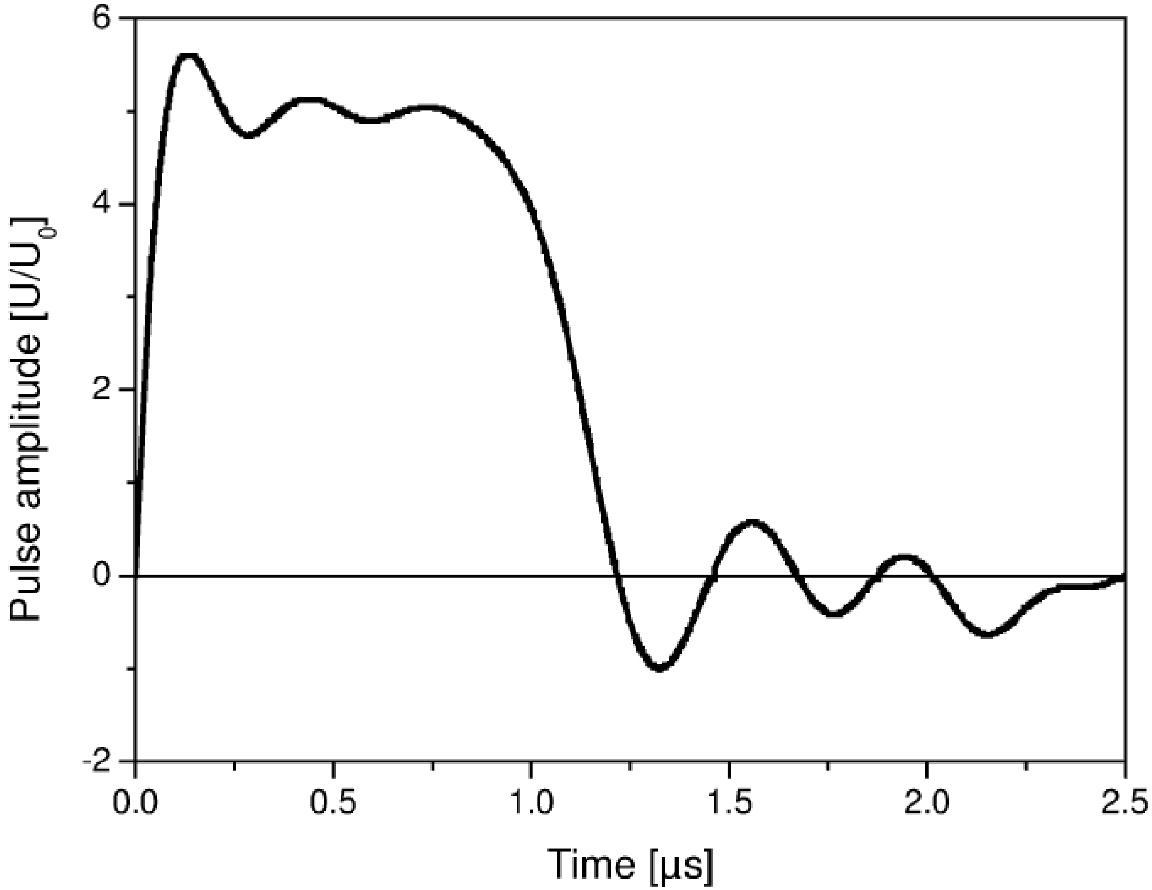
# Basic LC PFN




- Approximately, for $n > 10$

$$Z_N = \sqrt{\frac{L_N}{C_N}} = \sqrt{\frac{L}{C}} \qquad t_p = 2\delta = 2\sqrt{L_N C_N} = 2n\sqrt{LC}$$

# Waveform of 5-element LC PFN



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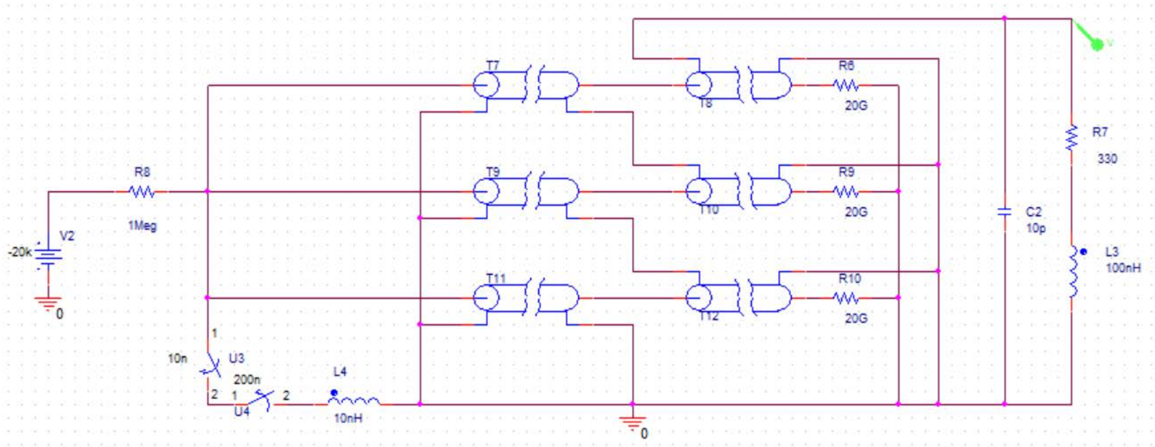
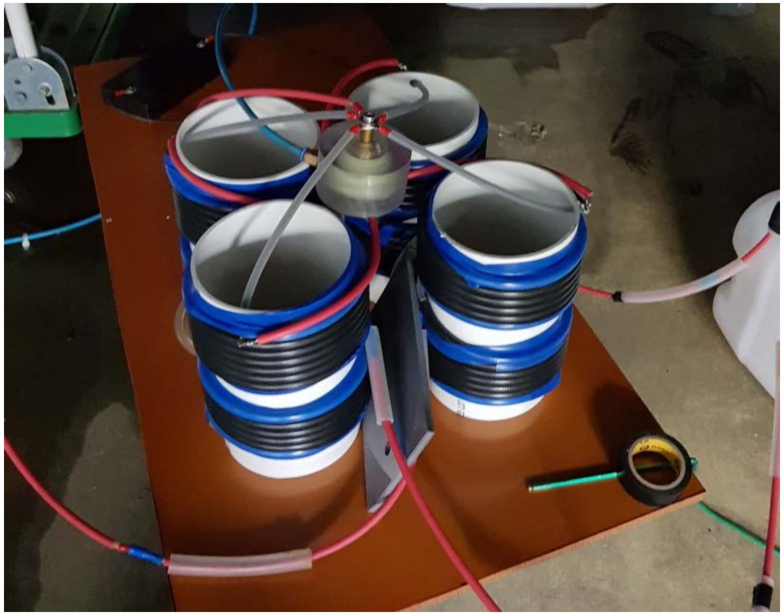
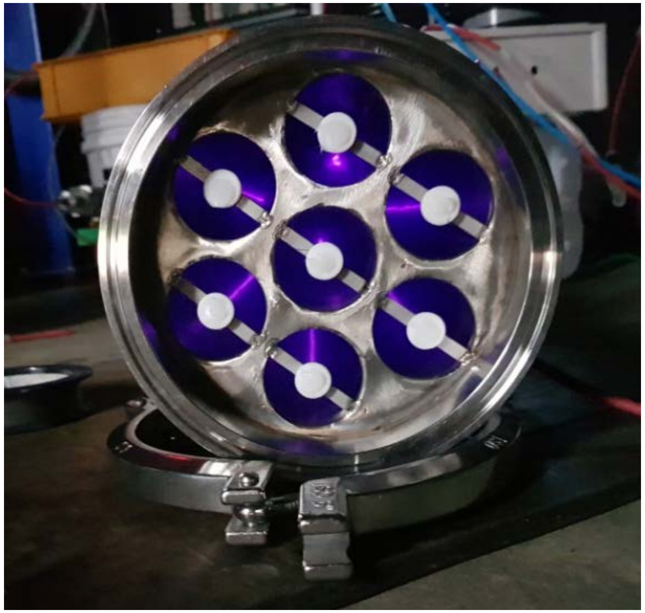
# Example of stacked Blumlein PFL




Cable spec (30 kV):
C’ = 108 pF/m
L’ = 328 nH/m

- $Z_0$ = 55 Ohm
- Matched load = 330 Ohm
- Transit time = 6 ns/m
- Pulse duration = 36 ns (3 m)

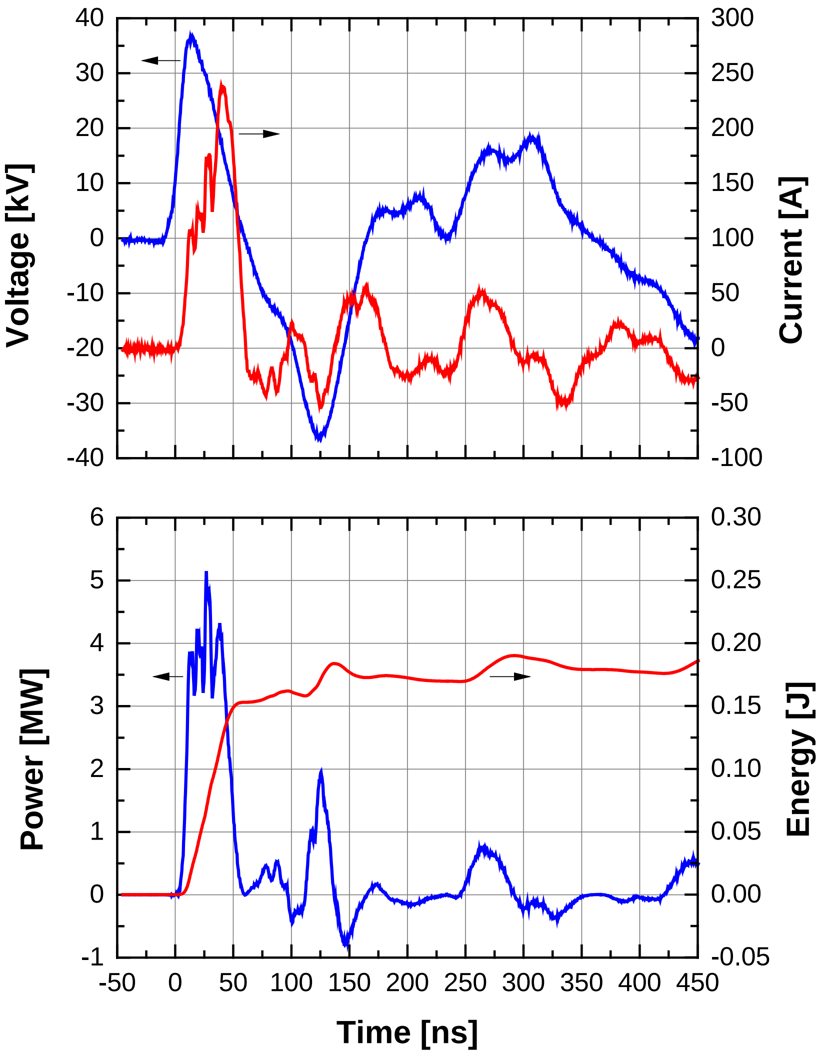




# Example of stacked Blumlein PFL

3-stage Blumlein pulse generator

- 20 kV charging, Load : 1m long corona reactor




## Summary

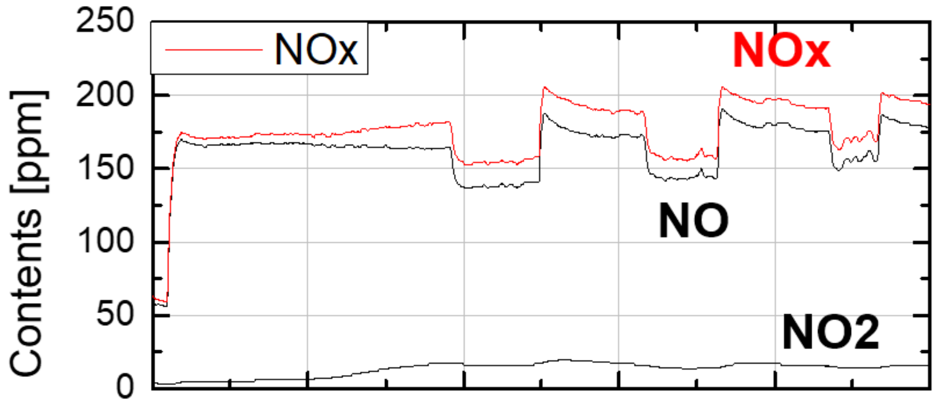
V_peak = 37 kV @ $V_0$ = -20 kV
I_peak = 230 A

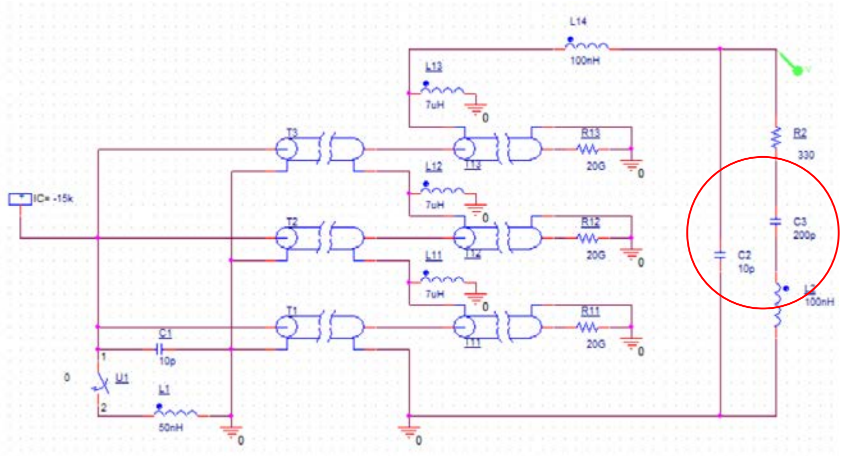
Peak power = ~5 MW
Energy = ~0.15 J/pulse
For 100 Hz operation, $P_{avg}$ ~ 15 W

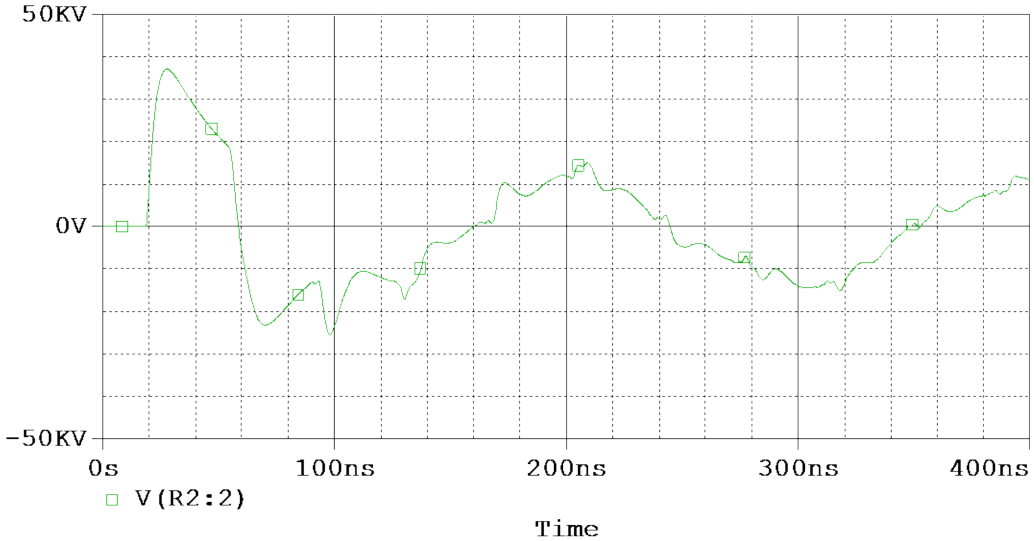
Huge instantaneous power
- → generation of high energy electrons
- → radical generation (O, N)
- → efficient removal of NOx

Low average power
- → low power consumption
- → high efficiency

# Example of stacked Blumlein PFL





# Implementation of transmission line theory

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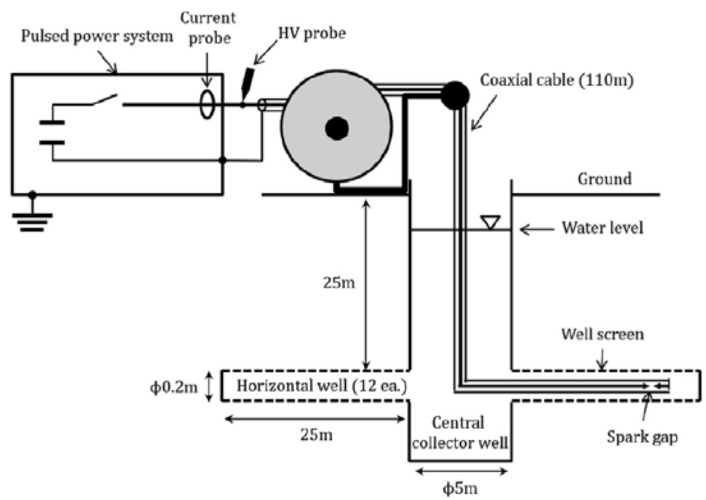
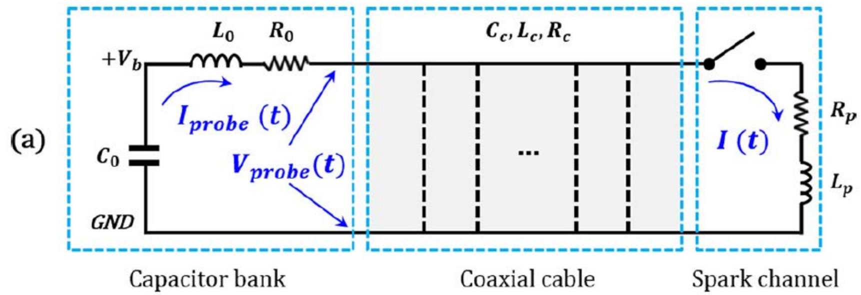
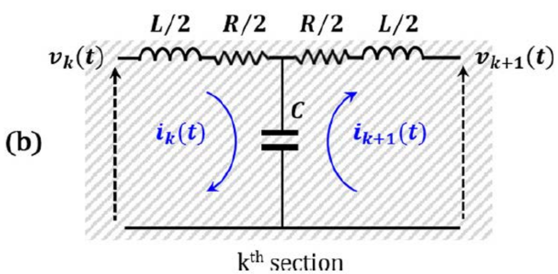
## Underwater spark discharge with long transmission line for cleaning horizontal wells

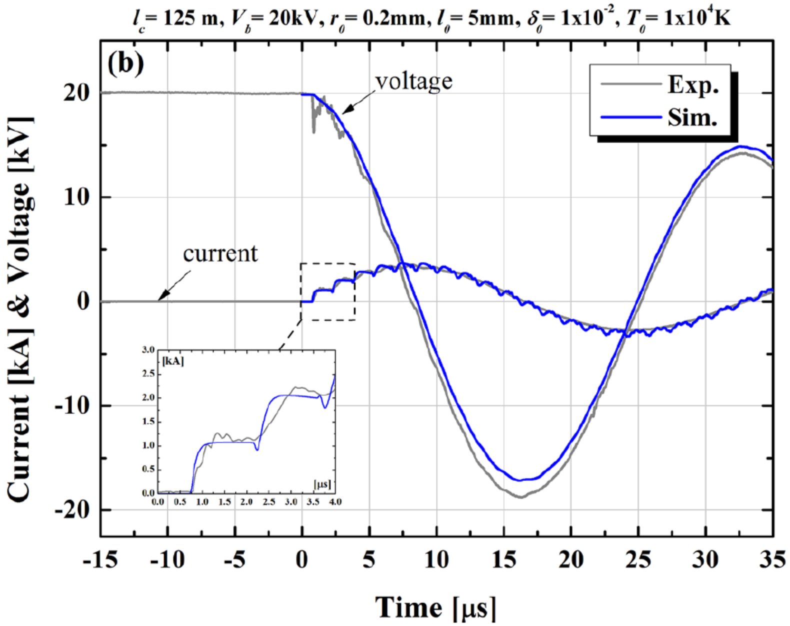
Kern Lee,[1] Kyoung-Jae Chung,[1,a)] Y. S. Hwang,[1] and C. Y. Kim[2]
[1]*Department of Nuclear Engineering, Seoul National University, Seoul 08826, South Korea*
[2]*Sun & Sea Co. Ltd., Seoul 06569, South Korea*




| Coaxial cable | |
|---|---|
| Length ($l_c$) | 15 – 125 m |
| Characteristic capacitance ($C'_c$) | 0.17 nF/m |
| Characteristic inductance ($L'_c$) | 0.20 $\mu$H/m |
| Characteristic resistance ($R'_c$) | 2.45 m$\Omega$/m |





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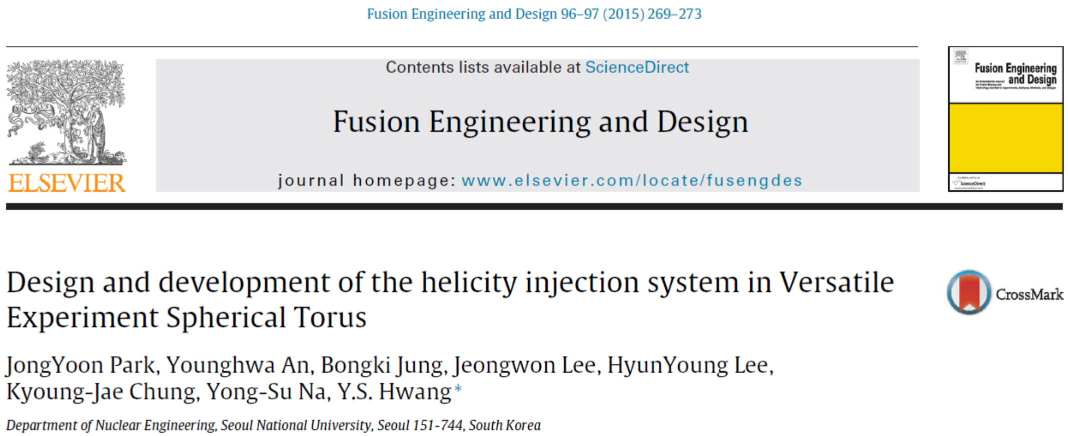
# Implementation of PFN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Design and development of the helicity injection system in Versatile Experiment Spherical Torus

JongYoon Park, Younghwa An, Bongki Jung, Jeongwon Lee, HyunYoung Lee, Kyoung-Jae Chung, Yong-Su Na, Y.S. Hwang*

*Department of Nuclear Engineering, Seoul National University, Seoul 151-744, South Korea*