Radiation Interaction with Matter

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Kyoung-Jae Chung

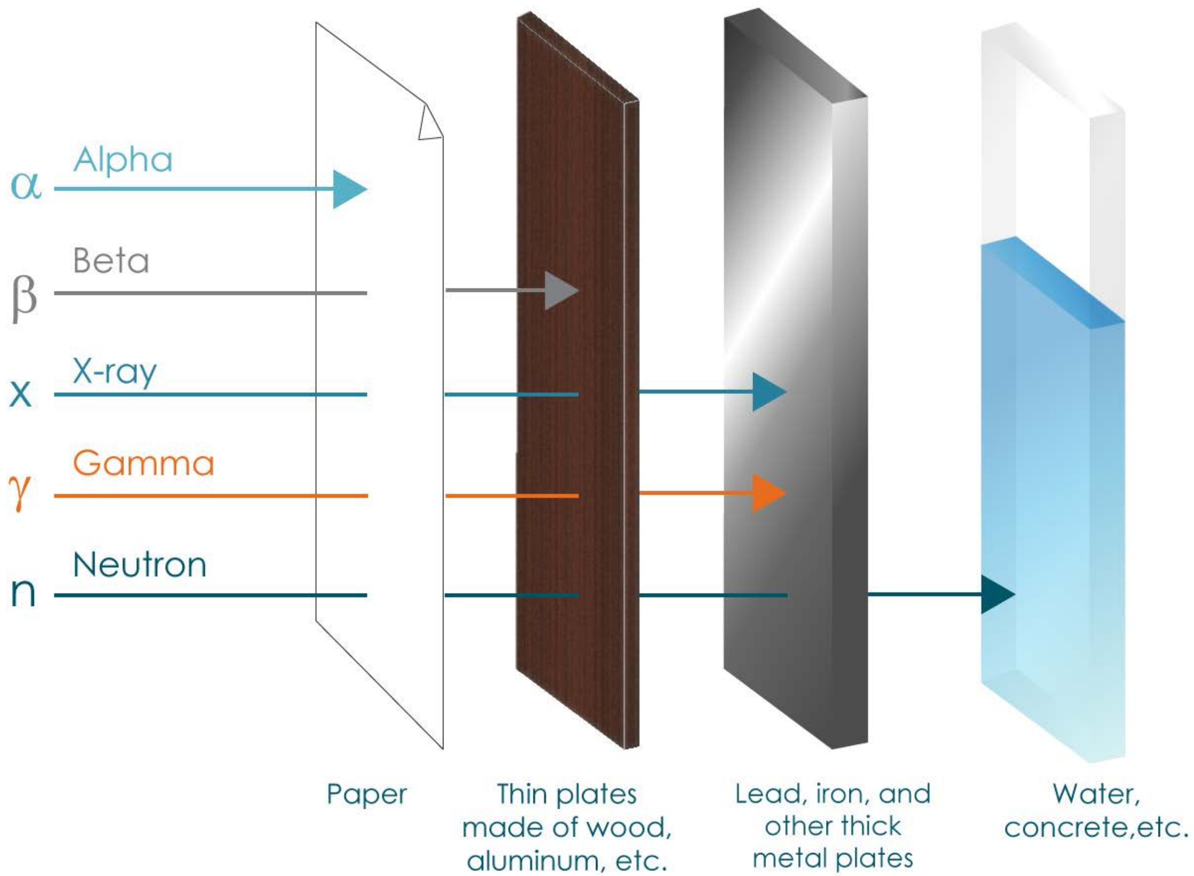
Department of Nuclear Engineering

Seoul National University

Introduction

- Particulate or wave
- Charged or neutral

❖ Energy loss in air
  - ~0.25 keV/mm for β
  - ~100 keV/mm for α



Introduction

- Radiation emitted by radioactive nuclides, both inside and outside our bodies, interacts with our tissues.
- Photons (EM waves) are far more abundant than matter in our universe; for every nucleon there are about 10^9 photons.
- Cosmic rays and the subatomic debris they create during interactions in the atmosphere also impinge on us (e.g. ~10^9 neutrinos/$cm^2 \cdot s$).
- For radiation to produce biological damage, it must first interact with the tissue and ionize cellular atoms, which, in turn, alter molecular bonds and change the chemistry of the cells. Likewise, for radiation to produce damage in structural and electrical materials, it must cause interactions that disrupt crystalline and molecular bonds.
- Such radiation must be capable of creating ion-electron pairs and is termed ionizing radiation (directly ionizing or indirectly ionizing).
- In this lecture, we study how the ionizing radiations interact with matter. Particular emphasis is given to how the radiations are attenuated as they pass through a medium, and to quantify the rate at which they interact and transfer energy to the medium.

Attenuation of neutral particle beams

- The interaction of a photon or neutron with constituents of matter is dominated by short-range forces. Consequently, unlike charged particles, neutral particles move in straight lines through a medium, punctuated by occasional “point” interactions, in which the neutral particle may be absorbed or scattered or cause some other type of reaction.
- The interaction may be a scattering of the incident radiation accompanied by a change in its energy. A scattering interaction may be elastic or inelastic.
- It is important to note that, for both elastic and inelastic scattering, unique relationships between energy exchanges and angles of scattering arise from conservation of energy and linear momentum.

- Linear interaction coefficient: the probability, per unit differential path length of travel, that a particle undergoes an ith type of interaction.

$$\mu_i \equiv \lim_{\Delta x \to 0} \frac{P_i(\Delta x)}{\Delta x}$$

m^{-1}

$$\boxed{\mu_i = \frac{1}{\lambda_i}}$$

Mean free path for ith interaction [unit: m]

$$\mu_t(E) = \sum_i \mu_i(E)$$

Scattering, absorption, etc

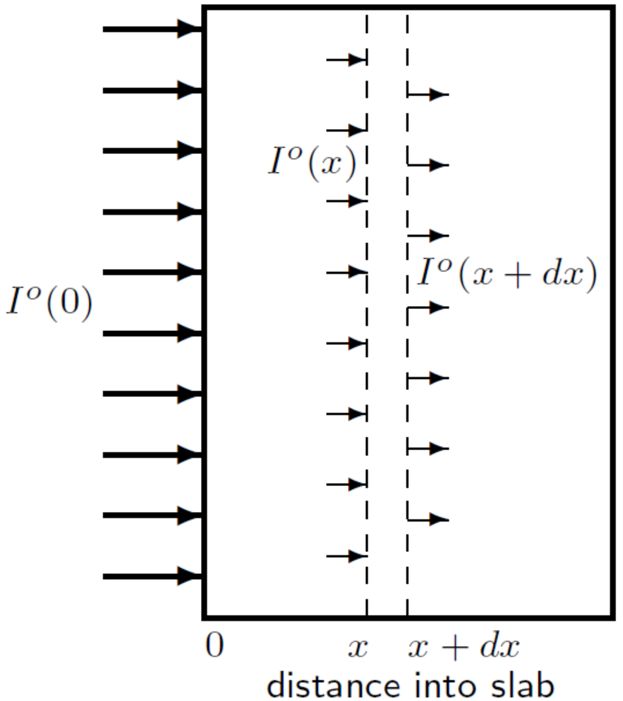
Attenuation of uncollided radiation

- The probability an uncollided particle interacts as it crosses Δx is

$$P(\Delta x) = \frac{I^o(x) - I^o(x+\Delta x)}{I^o(x)}$$

$$\mu_t \equiv \lim_{\Delta x \to 0} \frac{P(\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{I^o(x) - I^o(x+\Delta x)}{\Delta x} \frac{1}{I^o(x)} \equiv -\frac{dI^o(x)}{dx} \frac{1}{I^o(x)}$$

$$\Rightarrow \boxed{\frac{dI^o(x)}{dx} = -\mu_t I^o(x)}$$



- The solution for the uncollided intensity is

$$\boxed{I^o(x) = I^o(0) e^{-\mu_t x}}$$

- $P(x)$ that a particle interacts somewhere along a path of length x is

$$P(x) = 1 - \frac{I^o(x)}{I^o(0)} = 1 - e^{-\mu_t x}$$

Average travel distance before an interaction

- Let $p(x)dx$ be the probability that a particle interacts for the first time between x and $x + dx$. Then

$$p(x)dx = \{\text{prob. particle travels a distance } x \text{ without interaction}\} \times \{\text{prob. it interacts in the next } dx\}$$

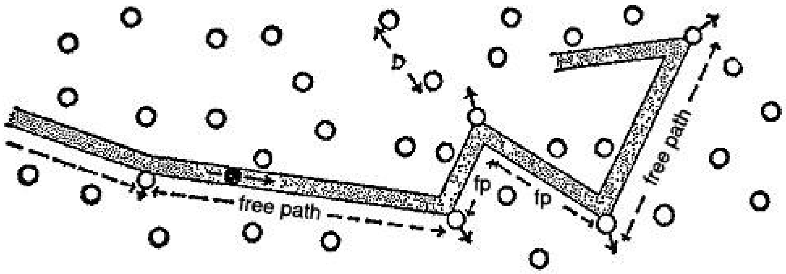
$$= \{\overline{P}(x)\}\{P(dx)\} = \{e^{-\mu_t x}\}\{\mu_t dx\} = \mu_t e^{-\mu_t x} dx.$$

- Mean-free-path length: the average distance x traveled by a neutral particle to the site of its first interaction, namely, the average distance such a particle travels before it interacts.

$$\bar{x} = \int_0^\infty x p(x) dx = \mu_t \int_0^\infty x e^{-\mu_t x} dx = \frac{1}{\mu_t} = \lambda_t \qquad \text{m}$$

- Half thickness: the thickness of a medium required for half of the incident radiation to undergo an interaction.

$$x_{1/2} = \frac{\ln 2}{\mu_t} = 0.693\,\bar{x}$$



Mean free path of a gas molecule

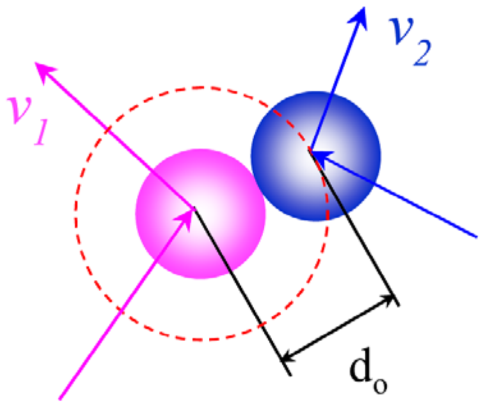
(Microscopic) cross section

- The linear coefficient $\mu_i(E)$ depends on the type and energy E of the incident particle, the type of interaction i, and the composition and density of the interacting medium.

$$\mu_i = \sigma_i N = \sigma_i \frac{\rho N_a}{A} \qquad \lambda_i = \frac{1}{\sigma_i N}$$

Target number density [m^{-3}]

Microscopic cross section for reaction i [m^2 or cm^2] : depends on the incident particle energy and target material.



- 1 barn = 1x10^{-28} m^2 or 1x10^{-24} cm^2

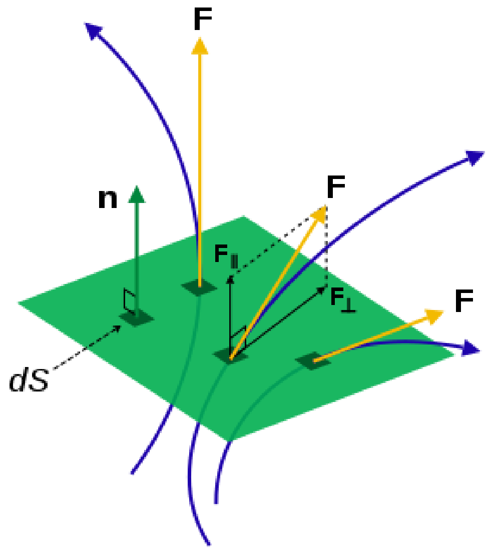
- Data on cross sections and linear interaction coefficients, especially for photons, are frequently expressed as the ratio of μ_i to the density ρ, called the mass interaction coefficient for reaction i.

$$\frac{\mu_i}{\rho} = \frac{\sigma_i N}{\rho} = \sigma_i \frac{N_a}{A}$$

Independent of target density [m^2/kg, cm^2/g]

Flux: flow rate per unit area

- The intensity of a beam is the flow (number per unit time) of radiation particles that cross a unit area perpendicular to the beam direction, and has dimensions, for example, of ($cm^{-2}\,s^{-1}$).

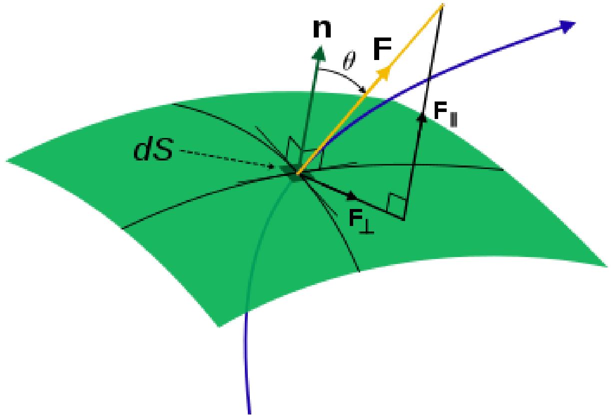


- Flux is given by

$$\phi = nv \qquad\qquad \Gamma = nv$$

- Fluence

$$\Phi = \int_{t_1}^{t_2} \phi dt$$



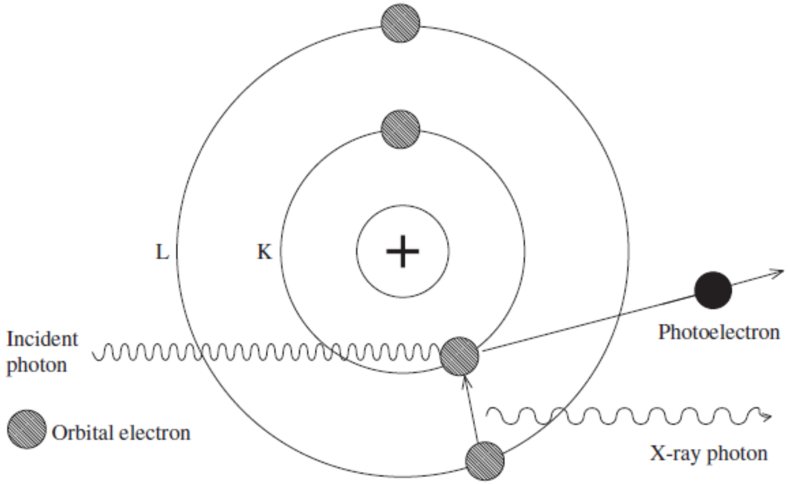
- Example of flux
  - Particle flux [#/m^2·s]
  - Mass flux [kg/m^2·s]
  - Momentum flux [N·s/m^2·s]
  - Heat or energy flux [J/m^2·s]

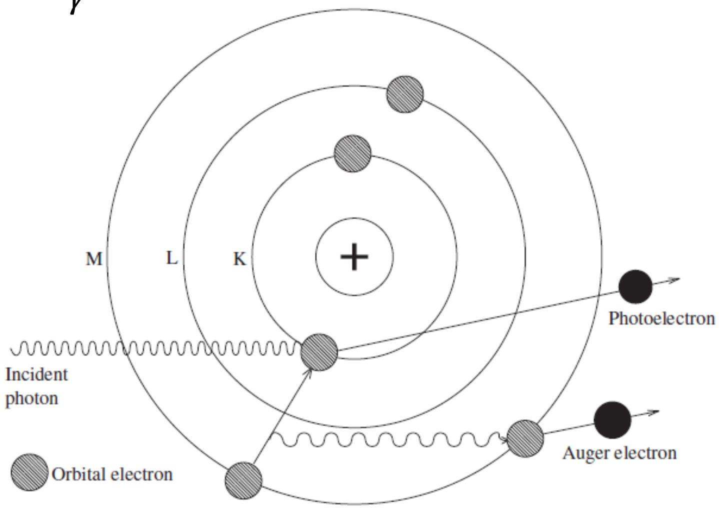
Photon interaction: (1) photoelectric effect

- During the photoelectric effect, energy from the incident photon is absorbed by an inner shell electron, which is then ejected out of the atom.
- The energy of the ejected electron can be determined based on the energy of the incoming photon and the binding energy that held the electron to the atom:

$$E_e = E_\gamma - E_b$$

- As consequence of the ejection of an inner shell electron, electrons from outer shells will cascade down from outer to inner shells. → characteristic X-ray or Auger electron emission.
- Rough approximation of cross section: $\sigma_{pe} \propto \frac{Z^{4\sim5}}{E_\gamma^{3.5}}$



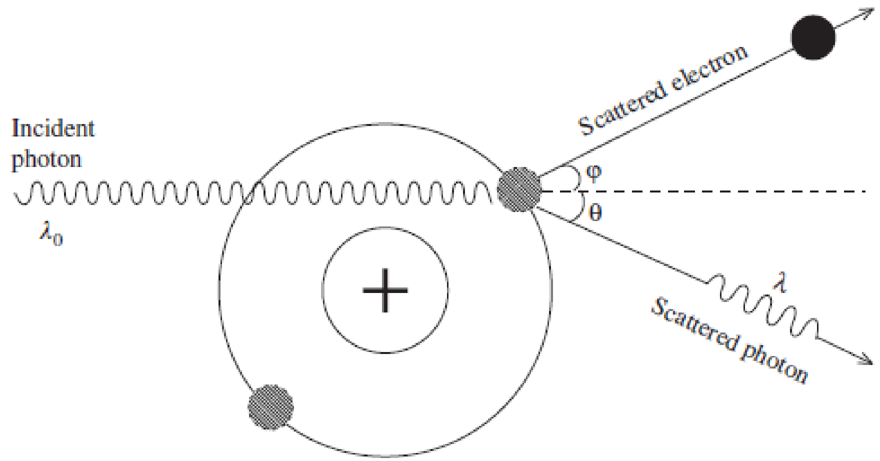


Photon interaction: (2) Compton scattering

- Compton scattering refers to the inelastic scattering of photons from free or loosely bound electrons which are at rest. Since the electron is almost free, it may also get scattered as a result of the collision.
- The energy or wavelength of the scattered photon: [H/W] Derive the following.

$$E_\gamma = \frac{E_{\gamma 0}}{1 + \frac{E_{\gamma 0}}{m_e c^2}(1 - \cos\theta)}$$

$$\lambda = \lambda_0 + \frac{h}{m_e c}(1 - \cos\theta)$$



- Extreme cases:

$\theta = 0$ $E_\gamma = E_\gamma^{max} = E_{\gamma 0}$

$\theta = \pi/2$ $E_\gamma = \frac{E_{\gamma 0}}{1 + \frac{E_{\gamma 0}}{m_e c^2}}$

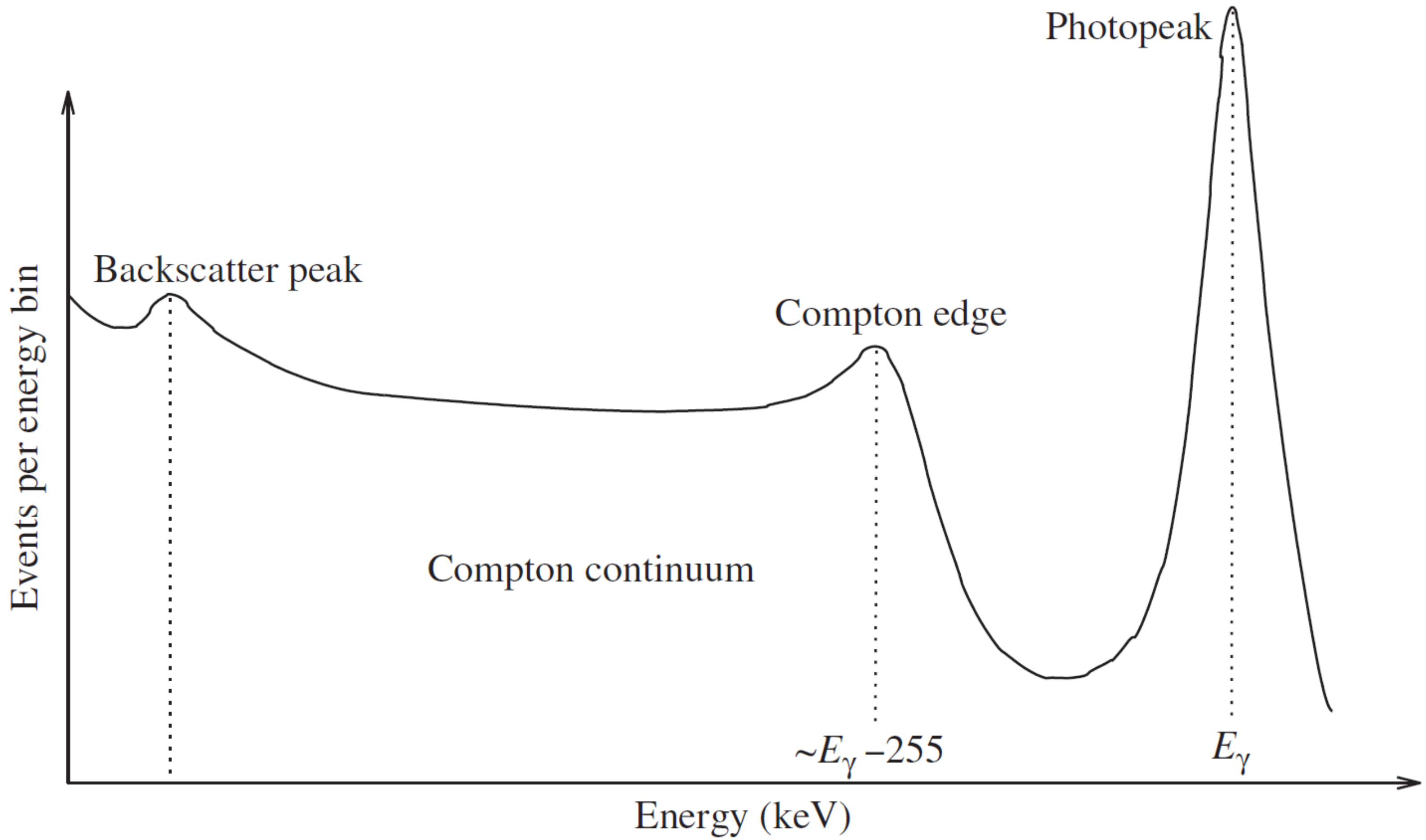
$\theta = \pi$ $E_\gamma = E_\gamma^{min} = \frac{E_{\gamma 0}}{1 + \frac{2E_{\gamma 0}}{m_e c^2}} \approx \frac{m_e c^2}{2} = 255\ \text{keV}$

[H/W] What is Thomson scattering?

[Optional] Typical γ-ray spectrum

- The maximum energy of the scattered electron can be calculated from

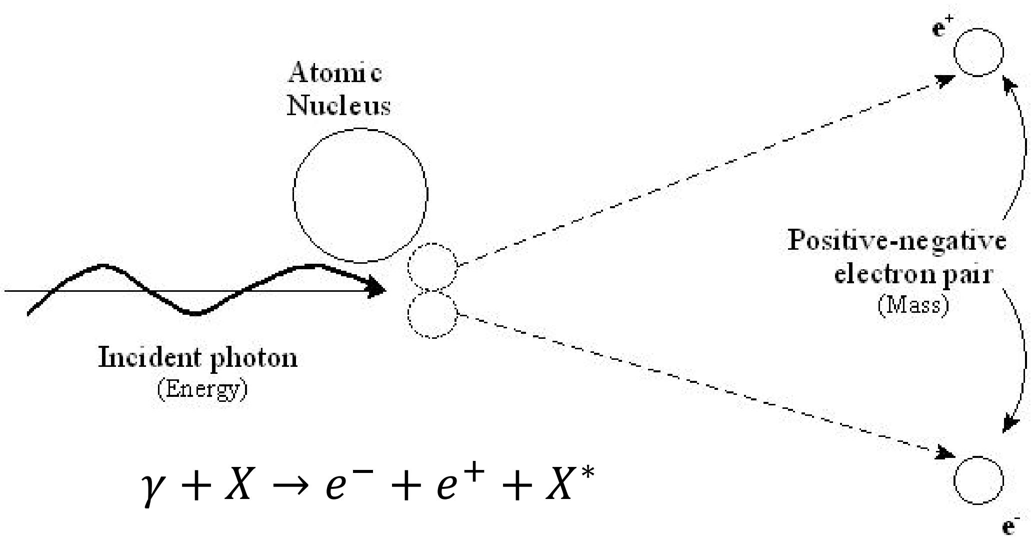
$$E_e^{max} = E_\gamma - 255\ \text{keV}$$



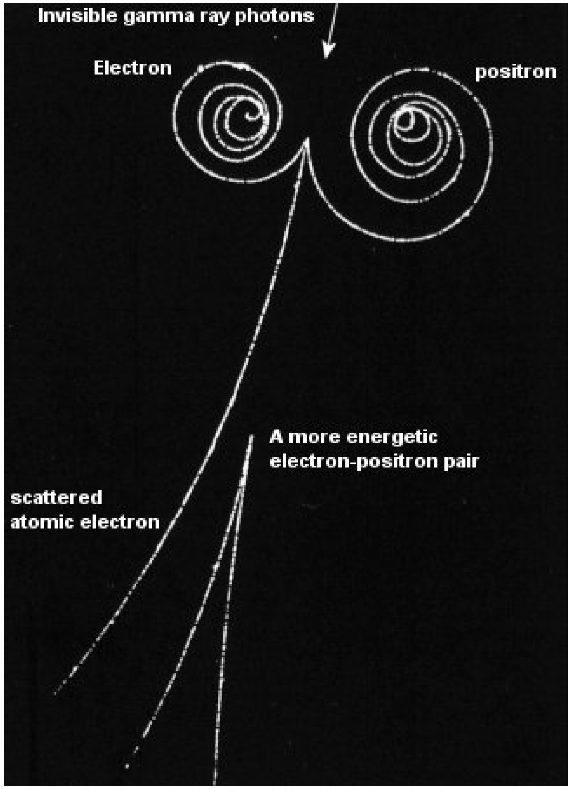
Photon interaction: (3) pair production

- Pair production is the process that results in the conversion of a photon into an electron-positron pair. Since photon has no rest mass, we can say that this process converts energy into mass according to $E = mc^2$.
- Pair production is the inverse process of the electron-positron annihilation.
- Pair production always takes place in a material, whereas electron-positron annihilation has no such requirement. Why?
- Threshold photon energy for pair production:

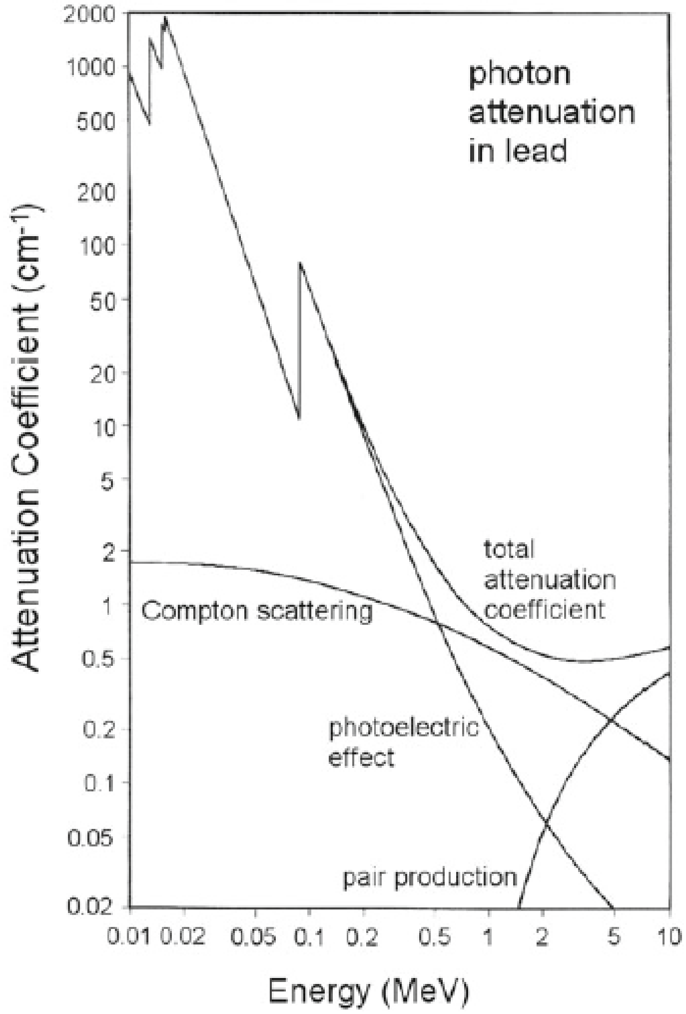
$$E_{\gamma,th} \geq 2m_e c^2 = 1.022 \text{ MeV}$$

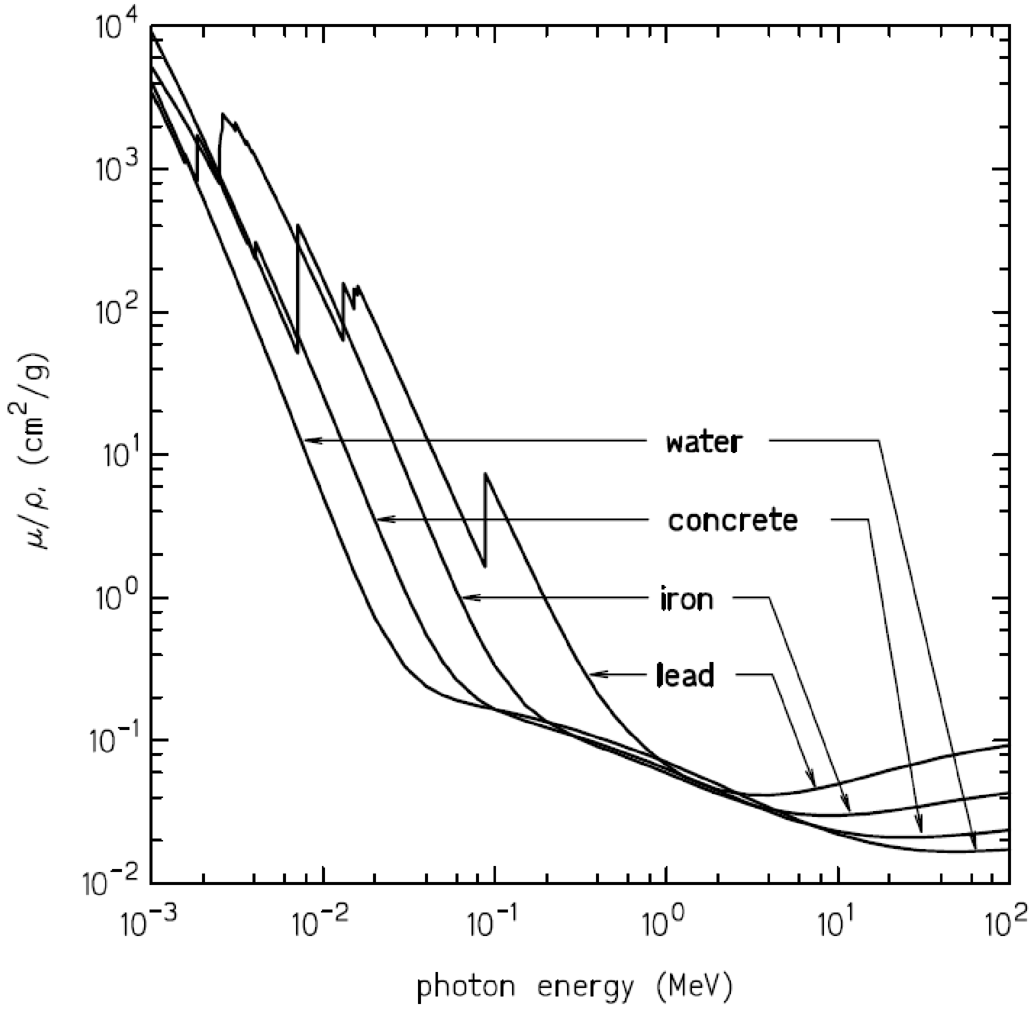


$$\gamma + X \rightarrow e^- + e^+ + X^*$$



Photon attenuation coefficient





Neutron interaction

- The interaction processes of neutrons with matter are fundamentally different from those for the interactions of photons. Whereas photons interact, more often, with the atomic electrons, neutrons interact essentially only with the atomic nucleus.
- The description of the interaction of a neutron with a nucleus involves complex interactions between all the nucleons in the nucleus and the incident neutron, and, consequently, fundamental theories which can be used to predict neutron cross-section variations in any accurate way are still lacking.

High-energy interactions (1 eV $< E <$ 20 MeV)

Elastic scattering cross sections
Angular distribution of elastically scattered neutrons
Inelastic scattering cross sections
Angular distribution of inelastically scattered neutrons
Gamma-photon yields from inelastic neutron scattering
Resonance absorption cross sections

Low-energy interactions ($<$ 1 eV)

Thermal-averaged absorption cross sections
Yield of neutron-capture gamma photons
Fission cross sections and associated gamma-photon and neutron yields

Attenuation of charged particle

- Charged particles such as beta particles, alpha particles, and fission fragments, are directly ionizing radiation and interact with the ambient medium primarily through the long-range electromagnetic (Coulomb) force.
- Charged particles traveling through matter lose energy in the following ways:
  - Coulomb interactions with electrons and nuclei
  - Emission of electromagnetic radiation (bremsstrahlung)
  - Nuclear interactions
  - Emission of Cerenkov radiation
- Since the radius of the nucleus is approximately 10^{-14} m and the radius of the atom is 10^{-10} m, one might expect that

$$\frac{\text{No. of interactions with electrons}}{\text{No. of interactions with nuclei}} = \frac{(10^{-10})^2}{(10^{-14})^2} \approx 10^8$$

⟹ Collisions with atomic electrons are more numerous than with nuclei

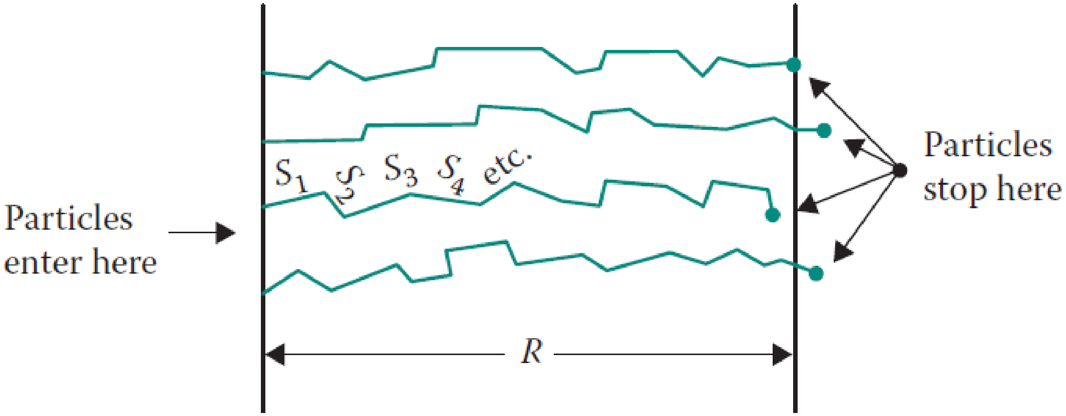
- Coulomb interaction: ionization (positive ions), excitation (line emissions)

Range of charged particles

- Energetic charged particles cause thousands of ionizations and excitations of the atoms along their path before they are slowed and become part of the ambient medium.
- The result of charged particle interaction with medium is a reduction in the energy of the particles as they pass through the medium.
- Stopping power (S) : the energy loss per unit length of the material a charged particle traverses. It is also called linear energy transfer (LET).
- Charged particles, unlike photons and neutrons, can travel only a certain maximum distance in matter, called the range (R), before they are stopped.

$$S = -\frac{dE}{dx}$$

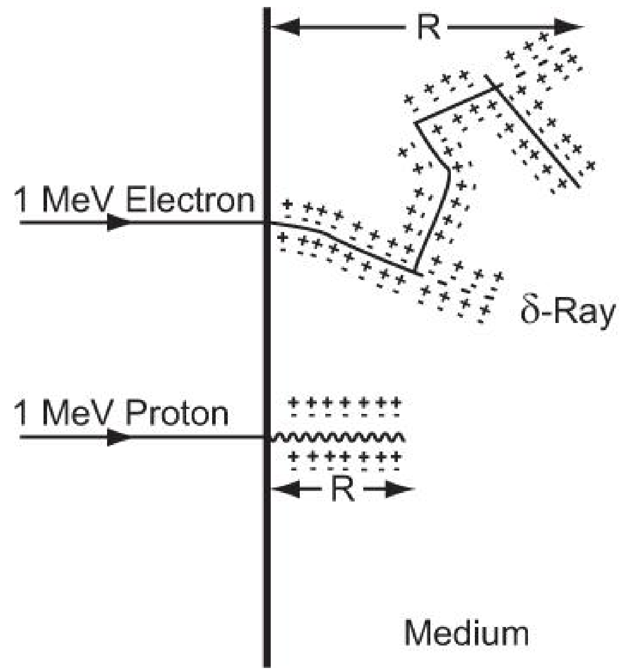
$$R = \int_E^0 \left(\frac{dE}{dx}\right)^{-1} dE$$



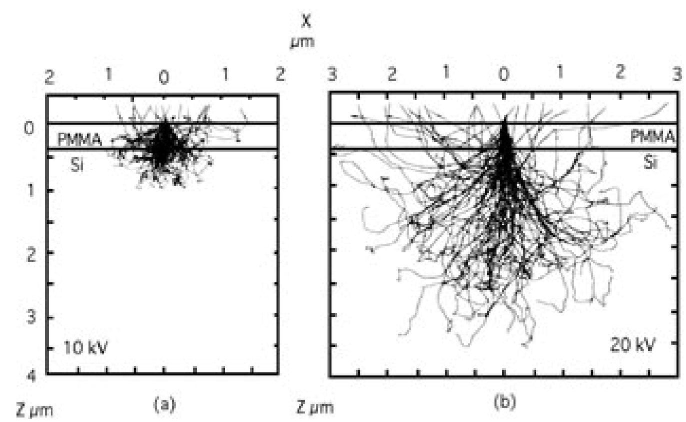
Q1. What is the difference between the range and the mean-free-path?

Range of charged particles

- Heavy charged particles

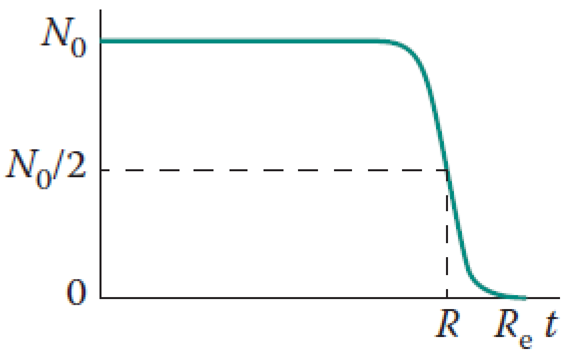


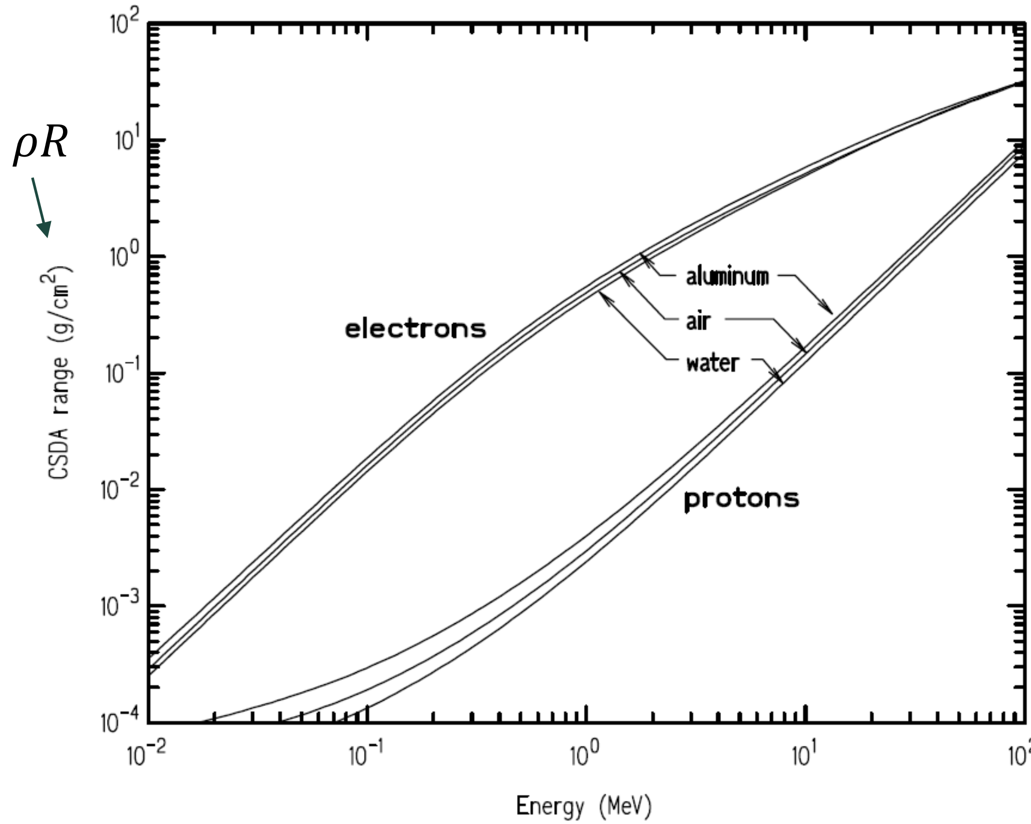
- Electrons (beta particles)



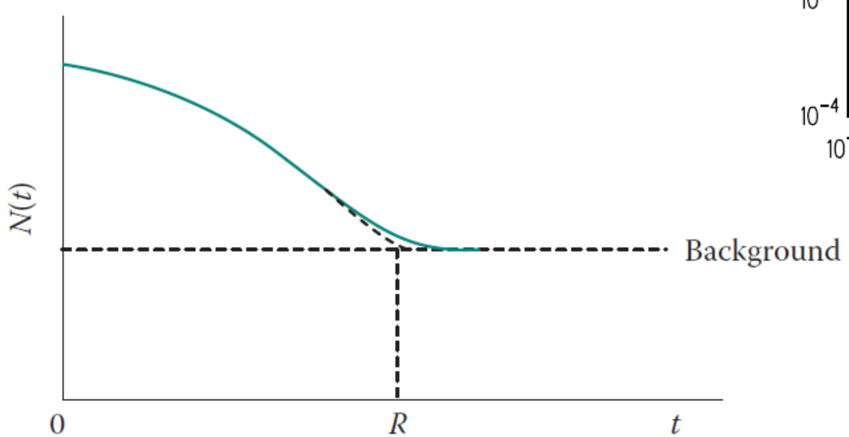
Range of charged particles

- Heavy charged particles

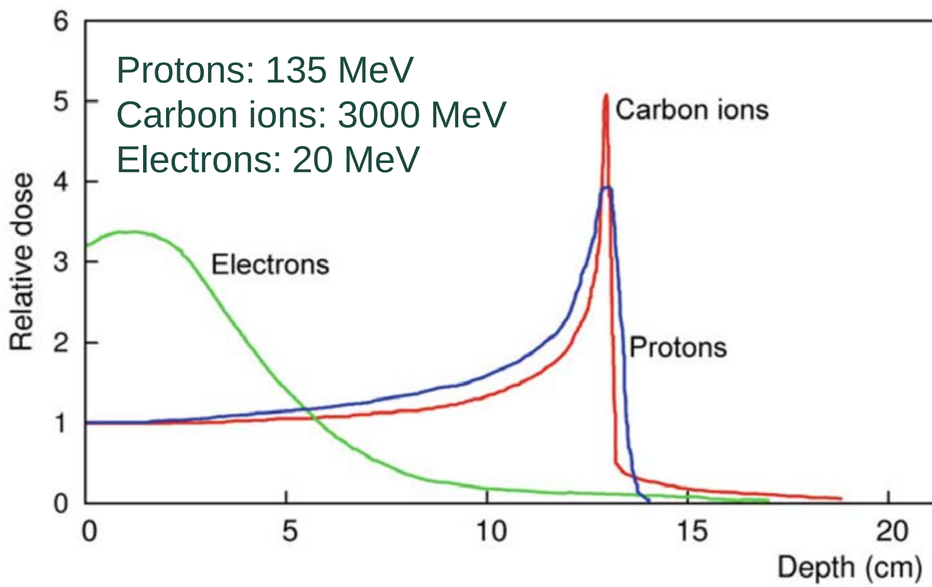




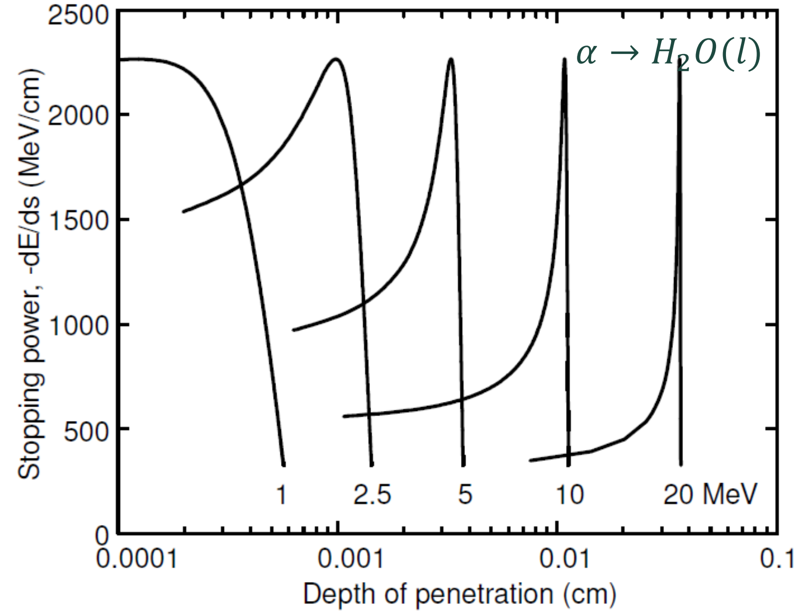
- Electrons (beta particles)



Bragg peak

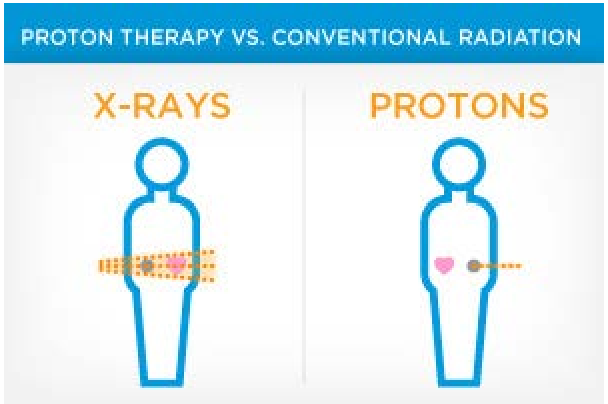


Protons: 135 MeV
Carbon ions: 3000 MeV
Electrons: 20 MeV
Carbon ions
Electrons
Protons
Relative dose
Depth (cm)
0
1
2
3
4
5
6
0
5
10
15
20



$\alpha \rightarrow H_2O(l)$
Stopping power, -dE/ds (MeV/cm)
Depth of penetration (cm)
1
2.5
5
10
20 MeV
0
500
1000
1500
2000
2500
0.0001
0.001
0.01
0.1

HEAVY ioN AND PARTICLE THERAPY
Public health patients are to be treated with lead ions while for patients with private health insurance we use gold ions!*
DOSE
197 Au
DEPTH
Claus Grupen 2014



PROTON THERAPY VS. CONVENTIONAL RADIATION
X-RAYS
PROTONS