Pulsed Voltage and Current Measurements

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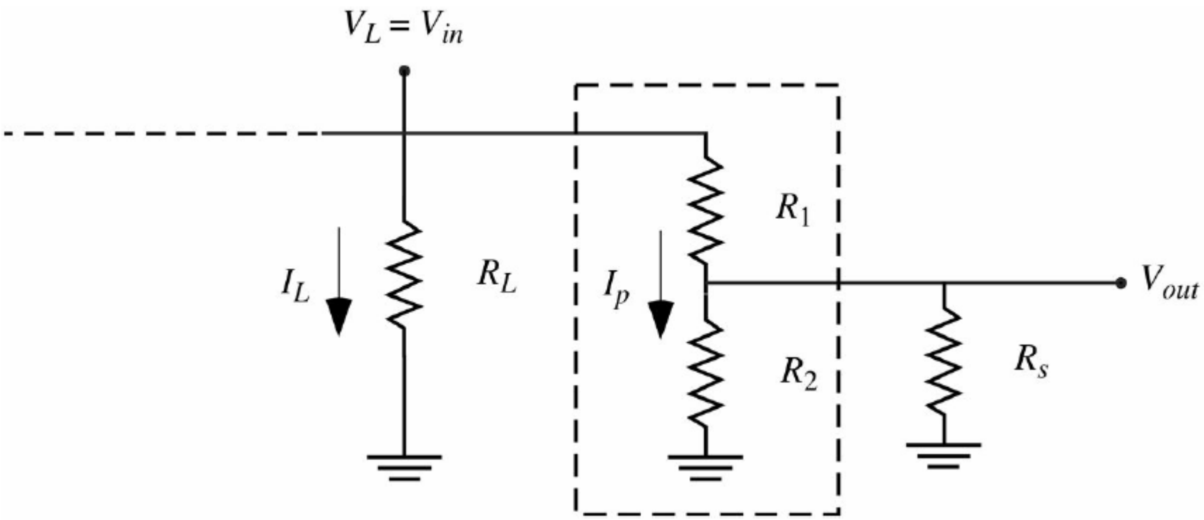
Resistive voltage divider

- A resistive divider is a conventional method for measuring the potential difference V_L across a load R_L.



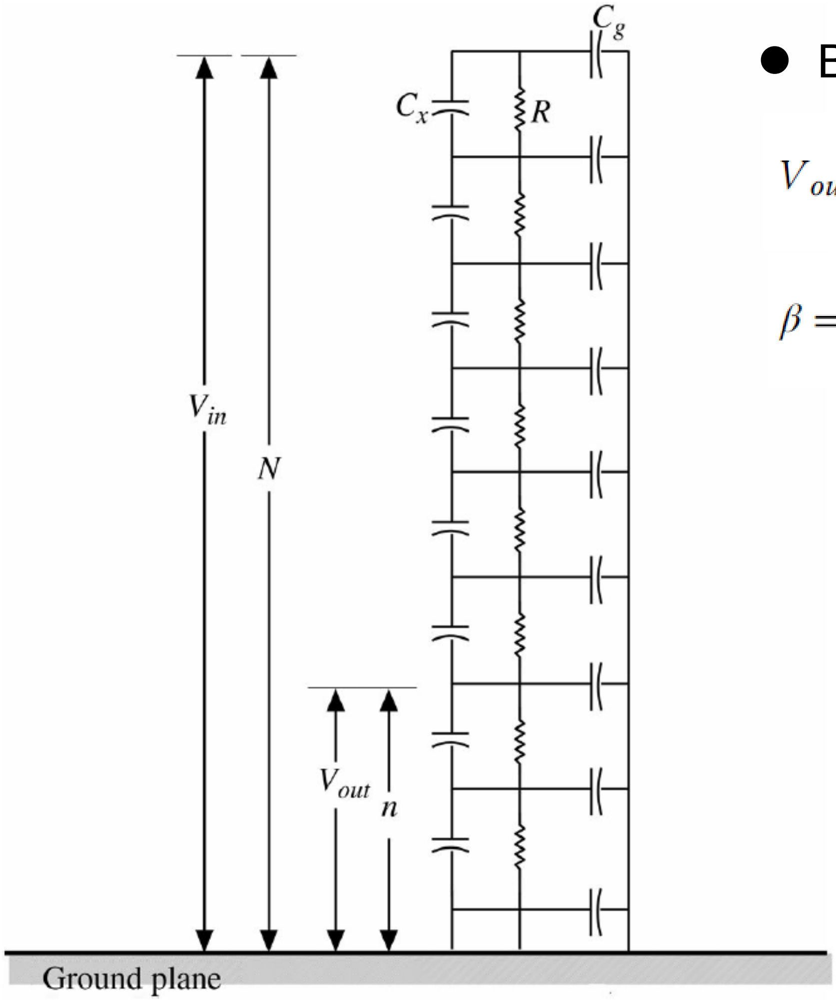
- Ideal resistive divider:

$$I_L \gg I_p \qquad R_S \gg R_2$$

$$\frac{V_{out}}{V_{in}} = \frac{R_S \parallel R_2}{R_S \parallel R_2 + R_1} \approx \frac{R_2}{R_2 + R_1}$$

Effect of stray capacitance

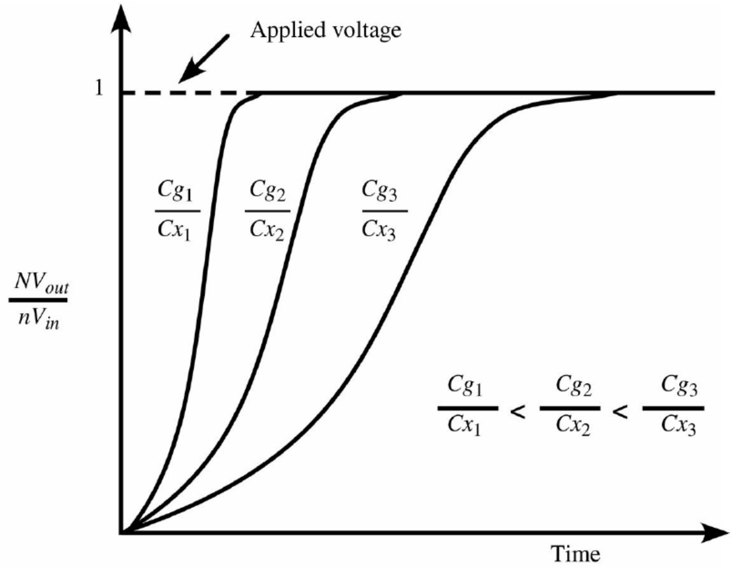
- Whereas in theory the resistive divider has an ideal frequency response, in practice, the frequency response is limited by stray reactance.



- Bellaschi (1933) showed that

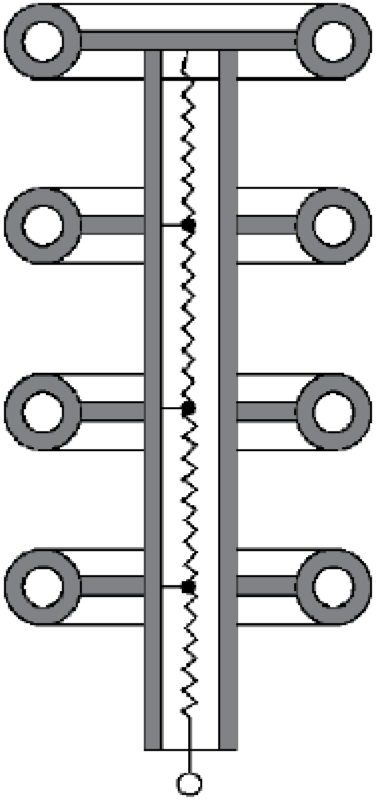
$$V_{out} = \frac{n}{N} V_{in} + \frac{V_i}{\pi} \sum_{k=1}^{\infty} \frac{2 \sin\left(k\pi(n/N)\right)}{k \cos\left(k\pi\right)} \times \frac{e^{-\beta \cdot t}}{1 + k^2\pi^2(C_C/C_G)}$$

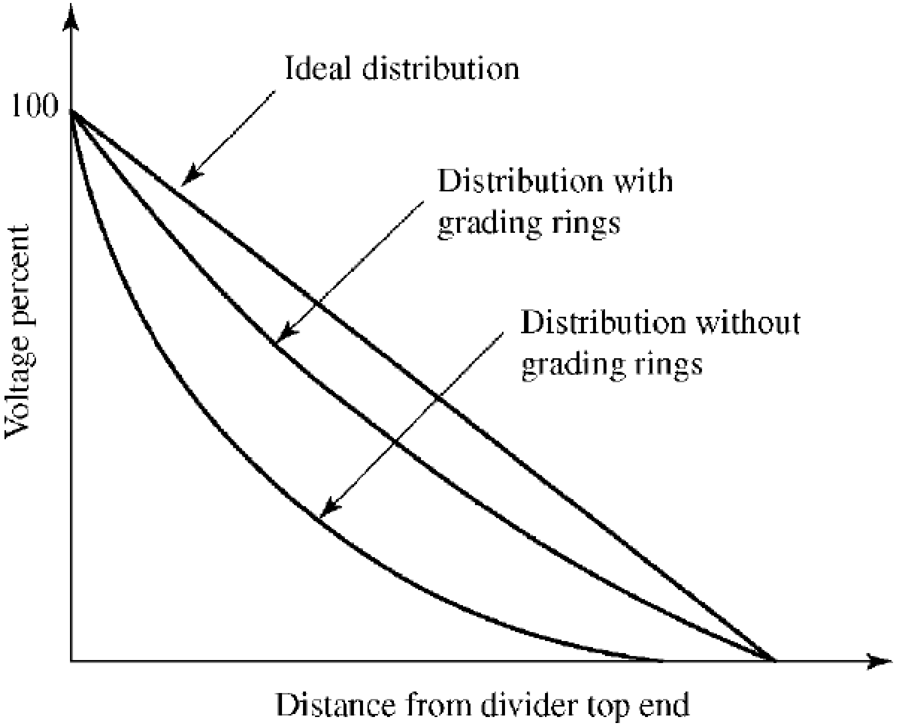
$$\beta = \frac{k^2\pi^2}{R_0\left(C_G + k^2\pi^2 C_C\right)}$$



Use of grading rings

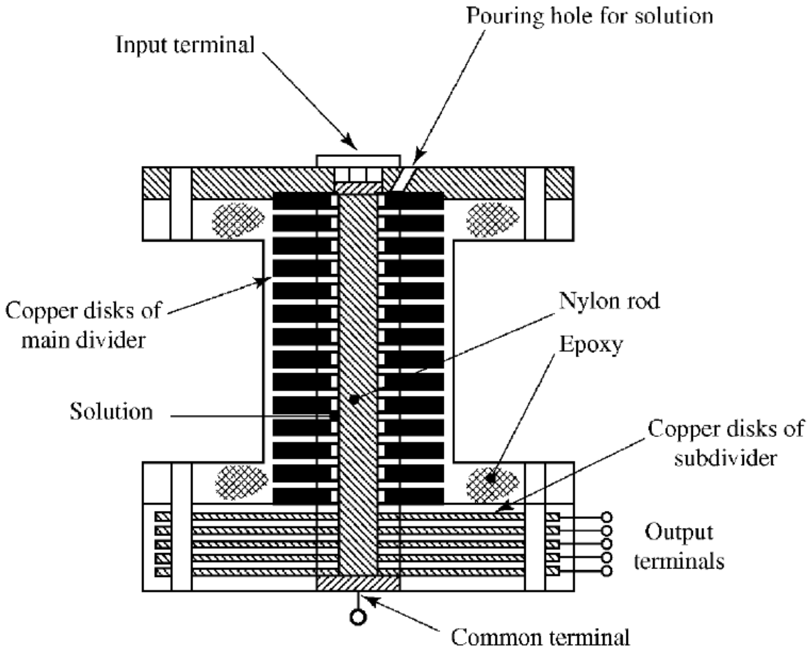
- The grading rings affect the voltage distribution along the divider length by making it more uniform, improving the frequency response and reducing the probability of electrical breakdown near the top end of the divider.





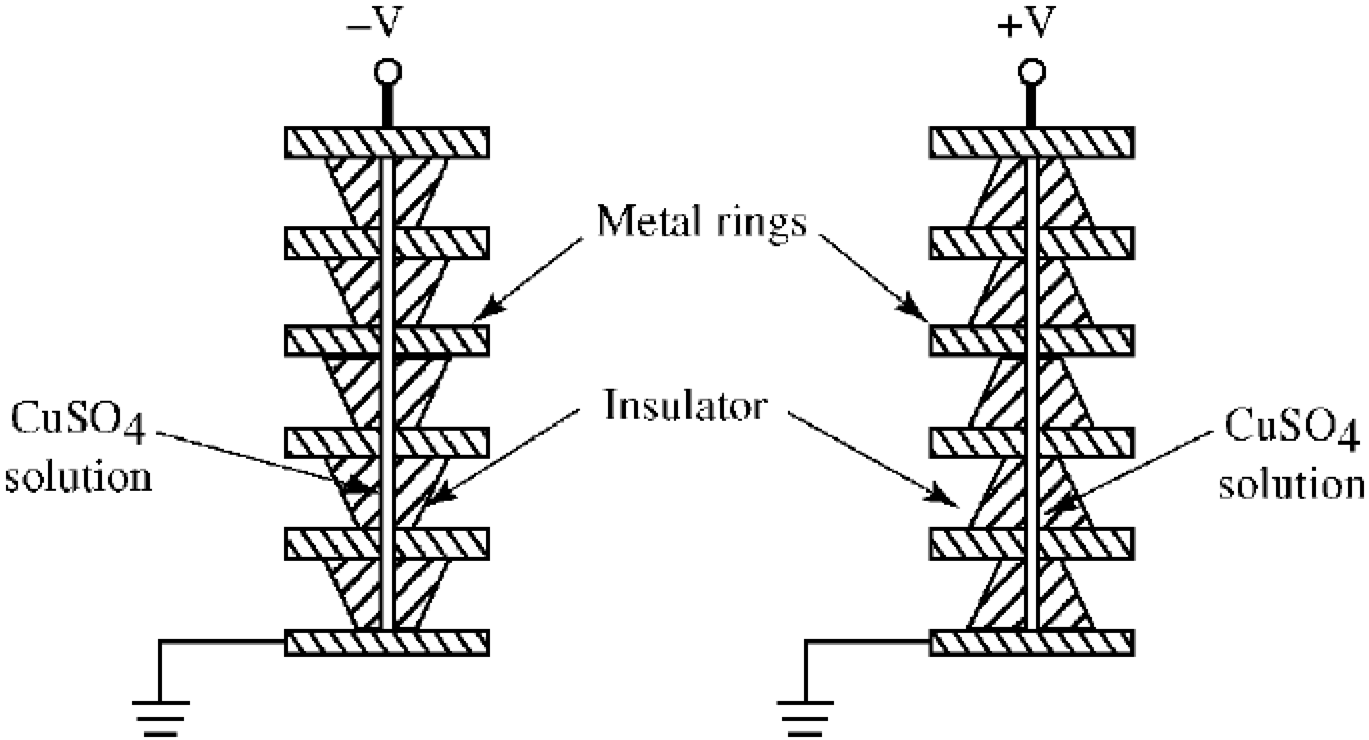
Dividers with liquid resistors

- Advantages of using liquid resistors
  - The overall inductance is lower because of the absence of connecting leads between adjacent resistor elements, which also minimizes corona effects.
  - Capability of handling very high power densities.
  - Self-healing capability in the event of accidental electrical breakdown.
  - The divider ratio is unaffected, in spite of changes in the resistivity caused by variations in temperature, field gradient, or frequency.
- Copper sulfate ($CuSO_4$) liquid dividers are very popular in nanosecond pulsed power technology for measuring pulse waveforms with amplitudes of millions of volts.



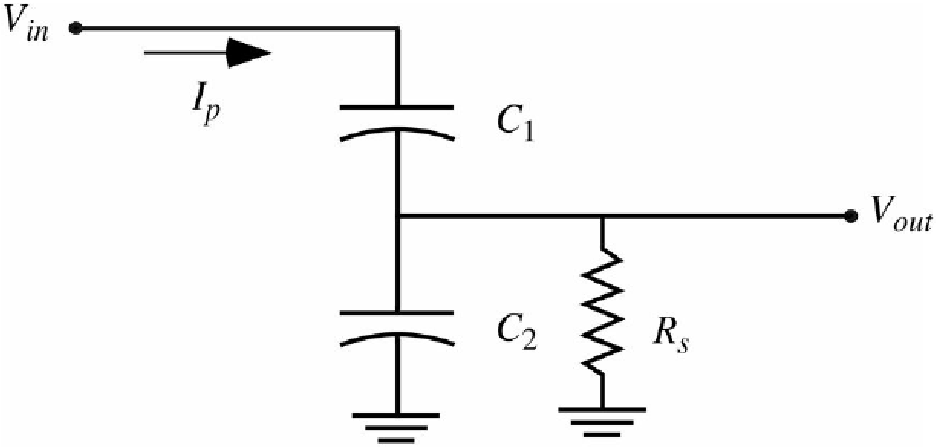
Dividers with liquid resistors

- When $CuSO_4$ dividers are used for measurement of pulsed voltages in vacuum, special care needs to be taken to prevent surface flashover across the insulator.
- In vacuum, the insulator angle used depends on the polarity of the measured voltage.



Capacitive dividers

- In a capacitor divider, $C_1 \ll C_2$, so that the majority of the voltage drop is across C_1 and the voltage output across C_2 is low enough to be taken to the oscilloscope for waveform display.
- Circuit equation



$$\frac{dV_{in}}{dt} = \left(\frac{C_1 + C_2}{C_1}\right)\frac{dV_{out}}{dt} + \frac{V_{out}}{R_S C_1}$$

⇨ $$\frac{V_{in}}{V_{out}} = \left(\frac{C_1 + C_2}{C_1}\right) + \frac{1}{sR_S C_1}$$

$$\omega_{3dB} = \frac{1}{R_S(C_1 + C_2)}$$

- Low-frequency response (ω ≪ ω_{3dB}, slowly-rising pulse)

$$V_{out} = \frac{C_1}{C_1 + C_2} V_{in}$$

Similar to resistive divider

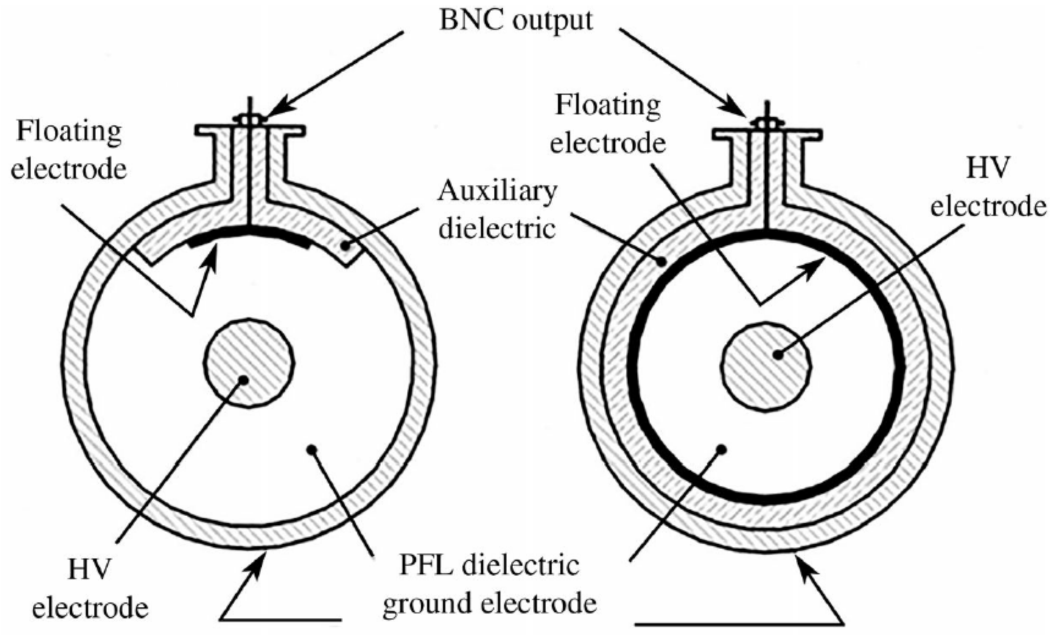
- High-frequency response (ω ≪ ω_{3dB}, slowly-rising pulse)

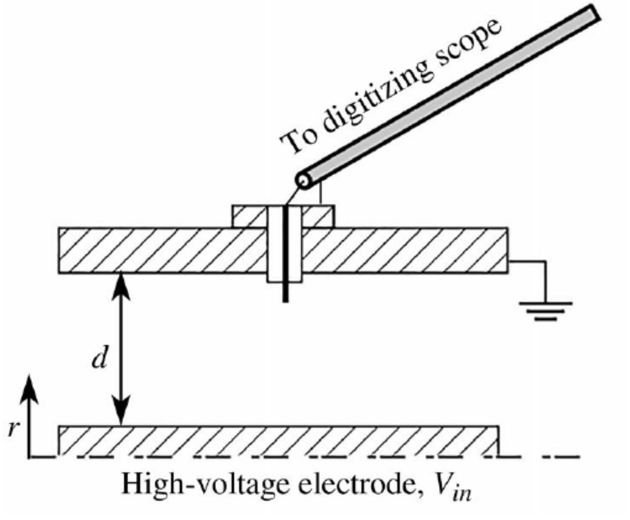
$$V_{out} = R_S C_1 \frac{dV_{in}}{dt}$$

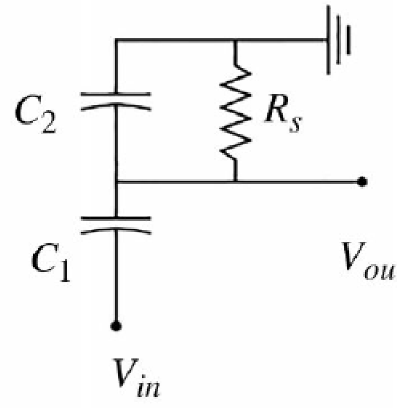
V-dot probe (or D-dot probe)

Configurations of capacitive dividers

- The most common design of a capacitive voltage divider is to use the natural capacitance existing between the electrodes and creating a low-voltage leg of the capacitive divider by installing a small capacitance created with a dielectric and a floating electrode near the ground plane.







Electro-optical techniques

- Electro-optical techniques for measuring voltage involve the electric field induced birefringence of some insulating materials. A monochromatic probing light beam is modulated by the unknown electric field to be measured and the voltage waveform can be recovered by unfolding the modulation information.
- Compared to other methods of pulsed voltage measurement, electro-optical techniques have the unique feature of freedom from electromagnetic interference.
- Refractive index of electro-optical transducer is given by

$$n = n_0 + aE + bE^2 + \cdots$$

- Kerr effect (J. Kerr, 1875)

$$\Delta n = \lambda \underset{\substack{\uparrow \\ \text{Kerr constant}}}{K} E^2$$

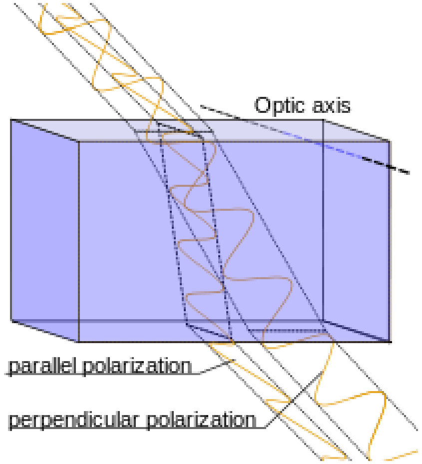
→ Kerr cell

- Pockels effect (F. Pockels, 1893)

$$\Delta n = n_0^3 \gamma E$$

→ Pockels cell

Birefringence



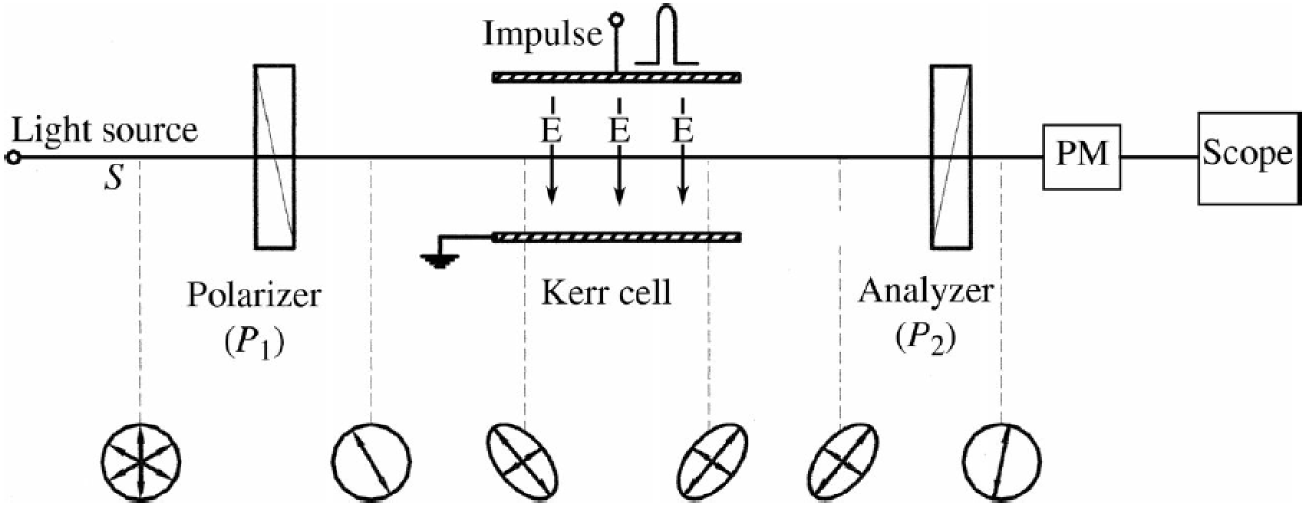
Kerr cell

- The applied electric field induces birefringence in the Kerr medium, resulting in the two component beams travelling at different propagation speeds as they pass through the Kerr medium, yielding a phase difference Δ upon exiting the cell, given by

$$\Delta = 2\pi \frac{l}{\lambda}(n_e - n_n) = 2\pi K l E^2$$

- The intensity of the transmitted light is given by

$$I = I_0 \sin^2\left(\frac{\Delta}{2}\right) = I_0 \sin^2(\pi K l E^2)$$



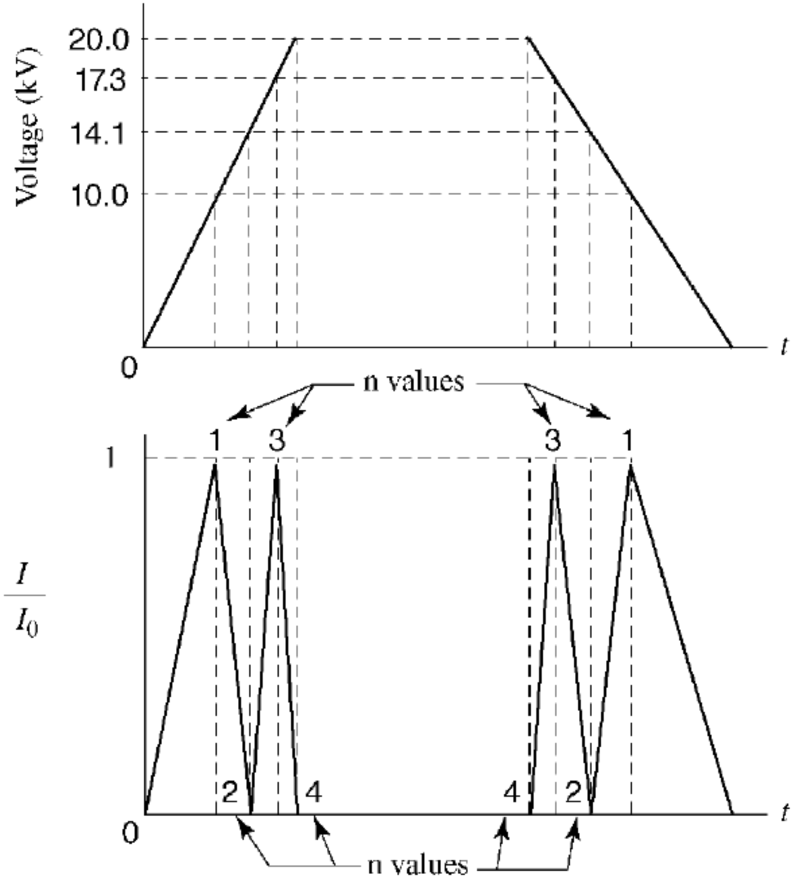
Kerr cell

- The first maximum occurs at the value corresponding to the half wave voltage

$$\frac{\Delta}{2} = \pi K l E_M^2 = \frac{\pi}{2} \quad \Rightarrow \quad E_M = \frac{1}{\sqrt{2Kl}}$$

- The measured intensity ratio

$$\frac{I(t)}{I_0} = \sin^2\left(\frac{\pi}{2}\frac{E^2(t)}{E_M^2}\right)$$

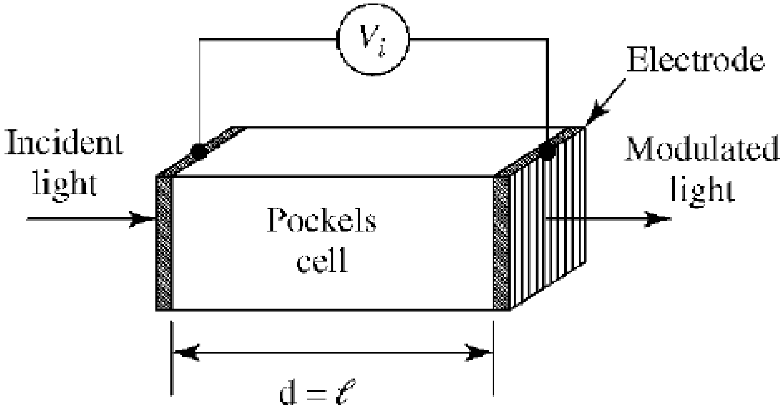


Pockels cell

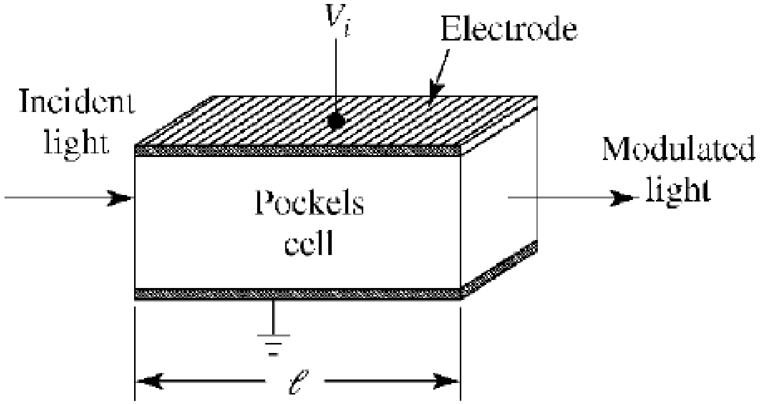
- The phase difference Δ upon exiting the cell is given by

$$\Delta = 2\pi\frac{l}{\lambda}(n_e - n_n) = 2\pi\frac{l}{\lambda}n_0^3\gamma\frac{V_i}{d}$$

- The Pockels cell can be configured for longitudinal modulation, where the direction of the incident light beam is parallel to the electric field, or transverse modulation, where the direction of the incident beam is perpendicular to the direction of the electric field.
- For longitudinal modulation ($l = d$): $$\Delta = \frac{2\pi}{\lambda}n_0^3\gamma V_i$$



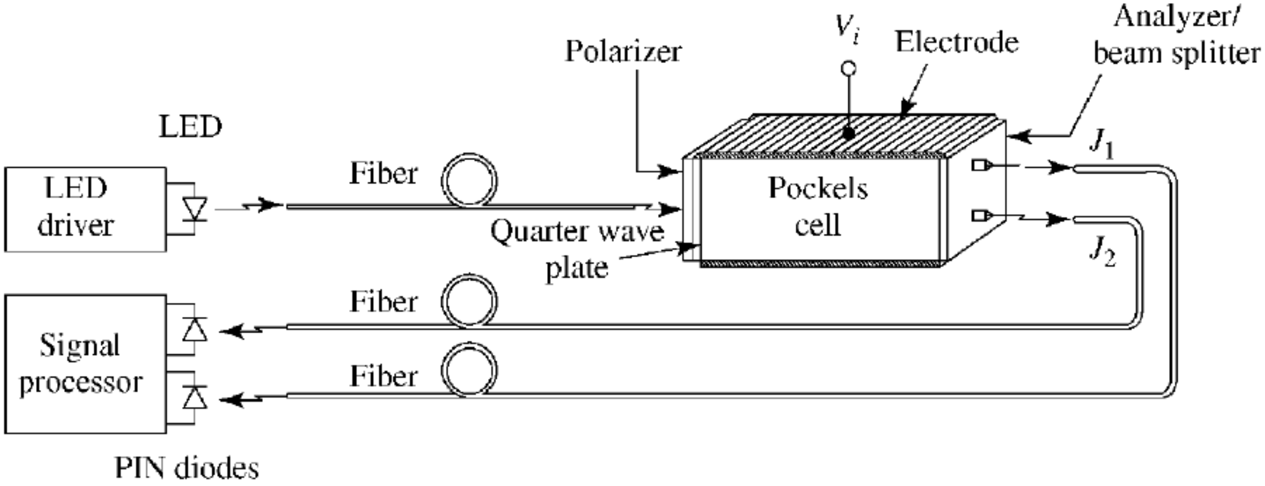
Longitudinal modulation



Transverse modulation

Pockels cell: transverse modulation

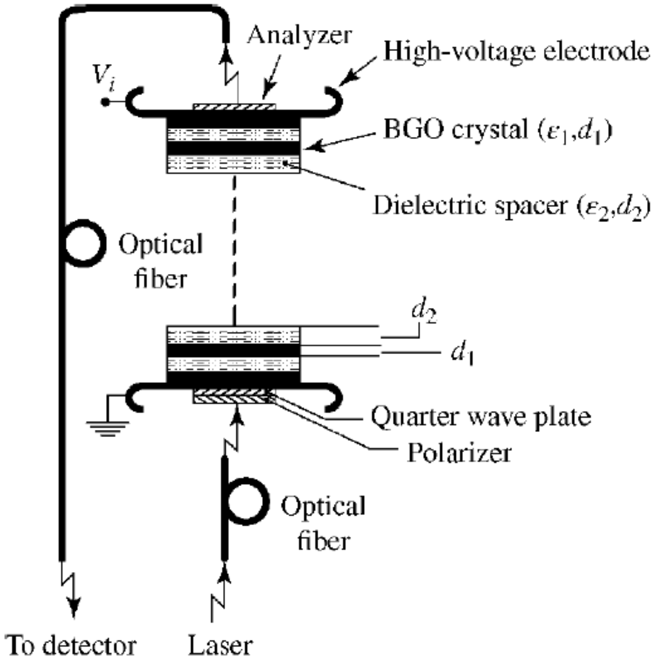
- The light beam passes through the components of the transducer comprised of polarizer, quarter wave plate, and the Pockels cell.
- At the entrance to the Pockels cell, the beam is circularly polarized, but is converted to an elliptically polarized beam after being modulated by the applied voltage on the Pockels cell.
- The analyzer splits the incident beam, of intensity J_0, into two mutually perpendicular beams of intensities J_1 and J_2. These light beams are then transmitted through optical fibers and converted to electrical signals by PIN diodes.
- For voltage values higher than 3 kV, it is necessary to attenuate the input voltage by means of a voltage divider.

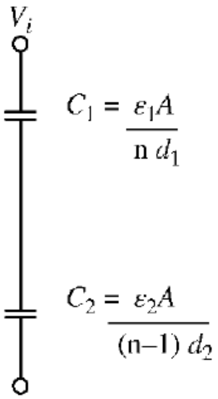


Pockels cell: longitudinal modulation

- The main advantage of this system is its capability to measure hundreds of kilovolts directly without the need for an external voltage divider.
- In the multi-segmented Pockels cell system (N crystals + (N-1) dielectrics), the electric field appearing across individual crystals can be written as

$$E = \frac{V_i}{Nd_1 + \left(\frac{\epsilon_1}{\epsilon_2}\right)(N-1)d_2}$$





Current viewing resistor (CVR)

- A current-viewing resistor (CVR) is a shunt with a highly accurate resistance R_{CVR} inserted in the current path. The voltage waveform developed across R_{CVR} is transmitted by a coaxial cable connected to an oscilloscope. In order for this signal to truly represent the current, the CVR should have negligible inductance and the resistive value should be low enough as not to disturb the original current flow.
- Energy capacity

$$E_{CVR} = R_{CVR} \int i^2(t)dt$$

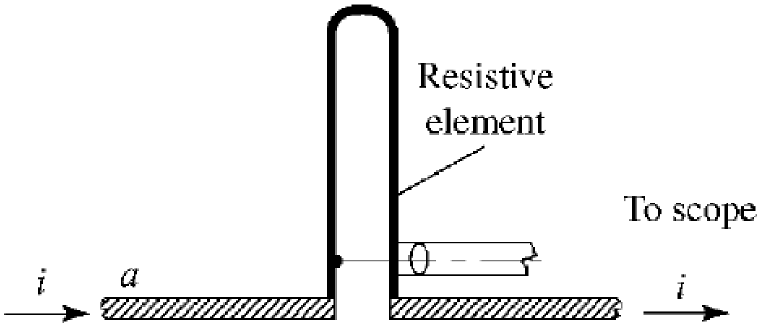
- Tolerance in resistance

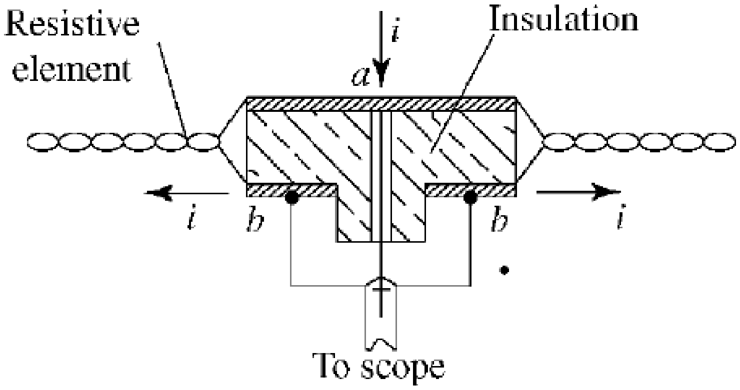
$$E_{CVR} = mc_pT = \rho V c_p T$$

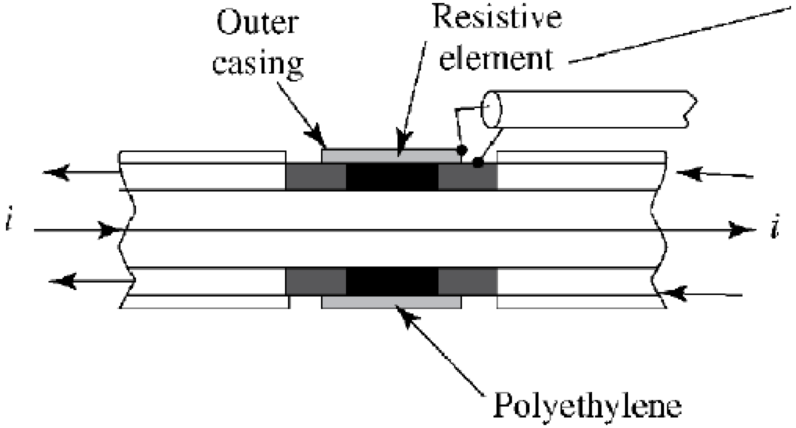
$$R' = R_{CVR}(1 + \alpha T)$$

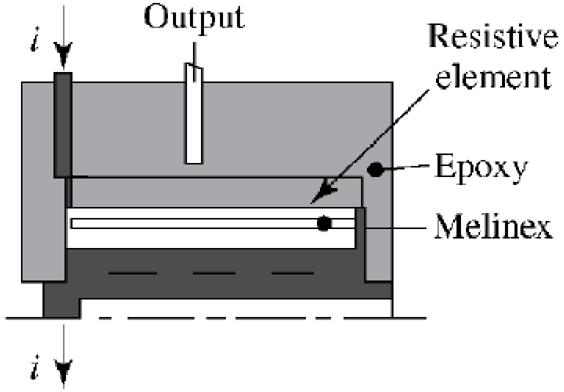
$$\Rightarrow \quad \frac{\Delta R_{CVR}}{R_{CVR}} = \frac{\alpha E}{\rho V c_p}$$

Configurations of CVR









Configurations of CVR

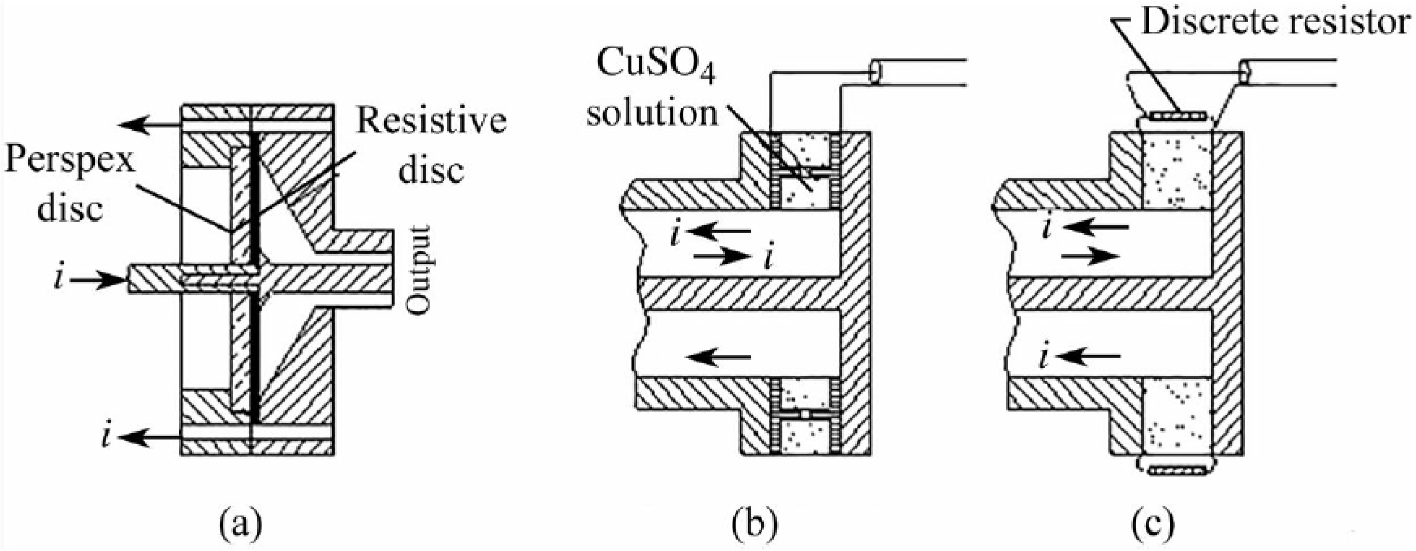


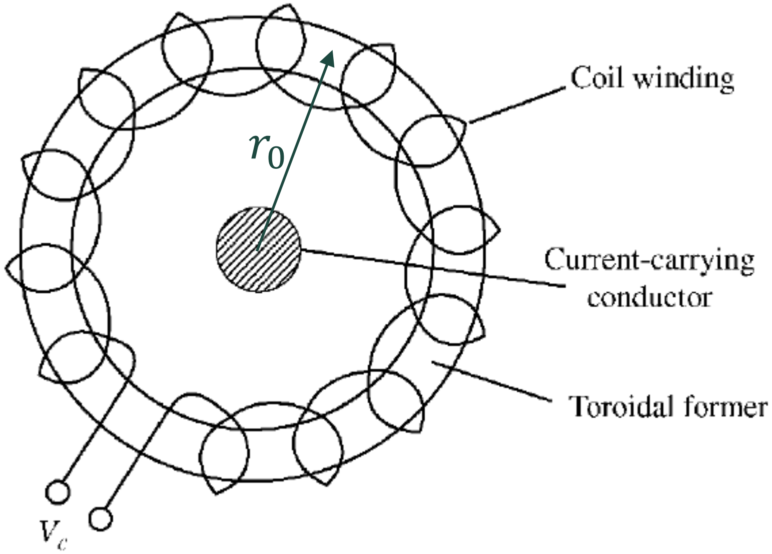
Figure 10.25 Disk current shunt configurations can use high-quality discrete resistors or those constituting liquids. (a) Radial disk current shunt. (b) Copper sulfate shunt. (c) Discrete resistor shunt.

Rogowski coil

- The rate of change of the magnetic flux density produced by the time-varying current induces a voltage in the conductor. This voltage can be quantitatively related to the current being measured. Rogowski coils can be constructed to either measure the current or its time derivative depending on the specific choices of components.
- Faraday's law:

$$V_c(t) = \frac{d\Phi}{dt} = \frac{d(NB(t)A)}{dt} = \frac{d}{dt}\left(NA\frac{\mu i(t)}{2\pi r_0}\right)$$

$$V_c(t) = \mu A \times \left(\frac{N}{l}\right) \times \frac{di(t)}{dt}$$



- The voltage induced in the coil is independent of the position of the current-carrying conductor within the window.
- Flux linkages are associated only with the minor turns, so it is valid for any shape and cross section of the toroid provided the coil completely encloses the current.

Compensated Rogowski coil

- The simple Rogowski is prone to errors caused by flux linkages Φ' within the window parallel to the major axis. This situation arises when the current-carrying conductor is not perpendicular to the toroidal plane, and an additional voltage $V_c' = d\Phi'/dt$ is added to V_c.
- In practical Rogowski coils, the voltage V_c' is cancelled by using bifilar winding or by introducing an additional turn.

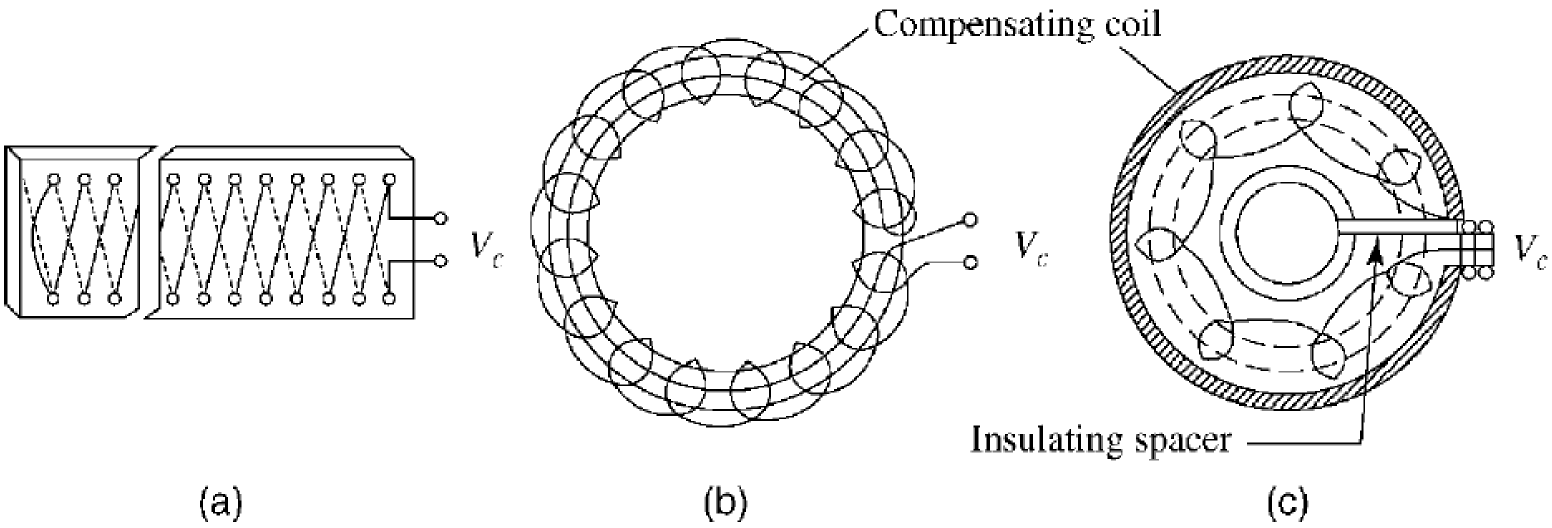


Figure 10.28 Compensated Rogowski coils may be constructed in a number of ways. (a) Bifiliar coil on a belt. (b) One-turn compensation within the coil. (c) One-turn compensation outside the coil.

Rogowski coil parameters

- Sensitivity: [volts per amp, V/A]

$$K = \frac{V_0}{i} = \left(\frac{\mu_0 N A}{l}\right) \times \frac{1}{RC}$$

Integrator time constant

- High frequency response: The probe cannot record rise times faster than its inductive time constant, $L_c/(Z_0 + R_p)$, where R_p corresponds to the coil resistance. As L_c is proportional to $N^2 A$, a reduction in N improves the rise time but reduces the sensitivity. It is advisable to keep low N for good high-frequency response and then increase A for increased sensitivity.
- Low frequency response: The low-frequency response or the pulse width performance of a differentiating Rogowski depends on the integrator time constant (RC). To produce a minimum droop on the pulse width, RC should be chosen to be higher than 10 times the pulse width.

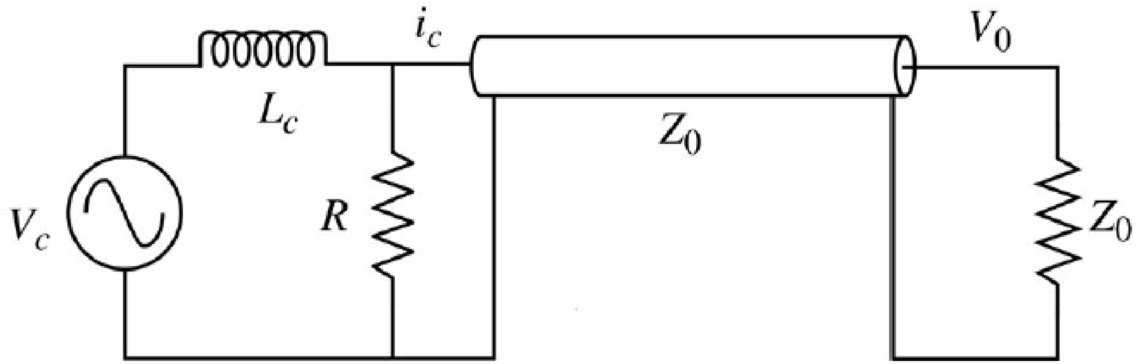
Self-integrating Rogowski coil

- A self-integrating Rogowski does not employ an external integrator. Instead, the coil inductance L_c in series with a low-value resistor R forms the L_cR integrating circuit.
- Under the conditions $\omega L_c \gg R$ and $L_c/R \gg \tau$, the entire coil voltage V_c is dropped across L_c.

$$V_c = \left(\frac{\mu_0 NA}{l}\right) \times \frac{di}{dt} \approx L_c \frac{di_c}{dt} \quad \Rightarrow \quad V_0 = i_c R = \left(\frac{\mu_0 NA \cdot R}{l \cdot L_c}\right) \cdot i$$

- Sensitivity: [volts per amp, V/A]

$$K = \frac{V_0}{i} = \frac{\mu_0 NA \cdot R}{l \cdot L_c} \propto \frac{1}{N}$$

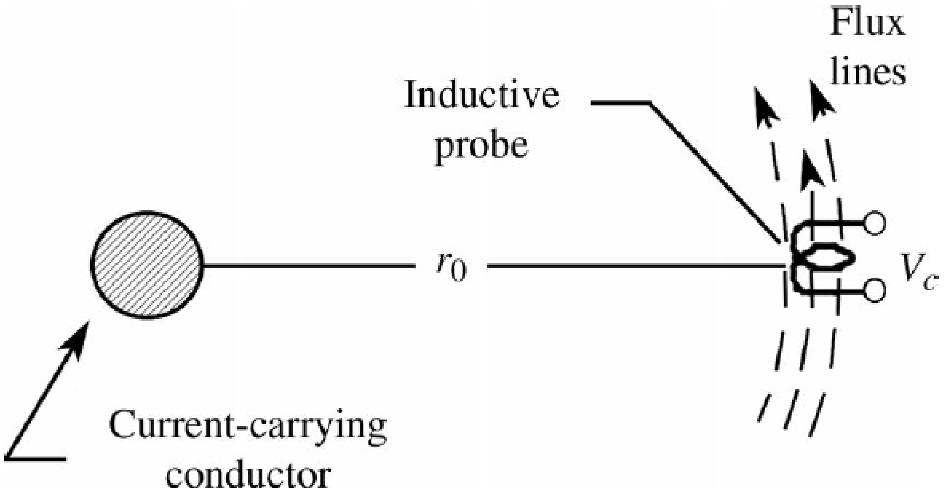


Inductive (B-dot) probe

- A B-dot or inductive probe is a particular case of a Rogowski coil, where the coil does not form a closed loop.

$$V_c = \left(\frac{\mu_0 N A}{2\pi r_0}\right) \times \frac{di}{dt} \propto \frac{dB}{dt} \equiv \dot{B}$$

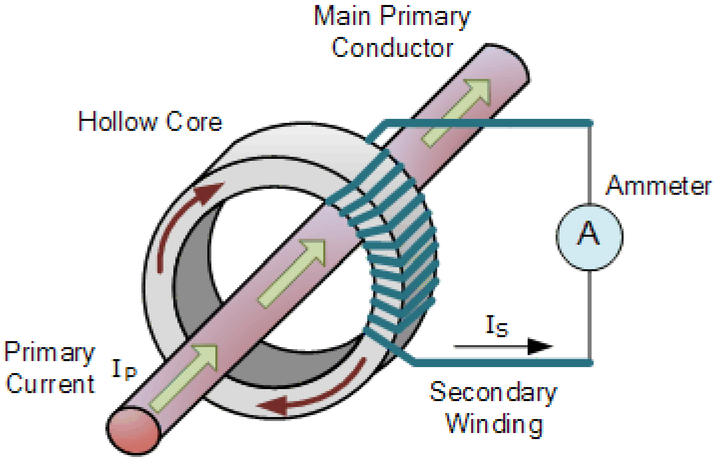
- The disadvantages of an inductive probe are that it is very sensitive to external flux linkage and mounting the probe such that the flux lines pass perpendicular to the coil cross section is difficult. Care must be taken since a B-dot probe needs fresh in situ calibration whenever the distance r_0 is charged.



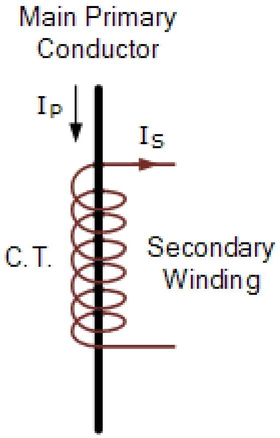
Current transformer (CT)

- A current transformer is a particular case of a Rogowski coil, employing iron core or ferrite core instead of air core, which results in a coupling coefficient of near unity.
- Under the conditions $\omega L_s \gg R_l$, where R_l is the resistance in the secondary circuit where the output voltage V_0 is developed.

$$V_0 = i_p(t)\frac{N_p}{N_s}R_l \quad \Rightarrow \quad V_0 = \frac{i_p(t)R_l}{N_s}$$



Construction



Circuit Symbol

Pearson coil

Magneto-optic current transformer

- A magneto-optic current transformer is based on Faraday effect, where the plane of polarization of a linearly polarized light, passing through a magneto-optic material subjected to a longitudinal magnetic field H is rotated by an angle ϕ. The Faraday rotation ϕ is given by

$$\phi = V\int_0^l H\cdot dl = VHl$$

Verdet constant