Electron Transport

SNU/NUKE/EHK

Ion-Atom Scattering

- ✓ Elastic scattering by electrons of the target atom
- ✓ Inelastic scattering by electrons of the target atom
- ✓ Elastic scattering by the nucleus of the target atom
- ✓ Inelastic scattering by the nucleus of the target atom

Electron Interactions

➢ Catastrophic events

- hard collision (large energy–loss) Moller scattering ($e^-e^- \rightarrow e^-e^-$): δ–ray emission
- hard collision (large energy–loss) Bhabha scattering ($e^+e^- \rightarrow e^+e^-$): δ–ray emission
- hard bremsstrahlung emission ($e^{\pm}N \rightarrow e^{\pm}\gamma N$), and
- positron annihilation "in–flight" and at rest ($e^+e^- \rightarrow \gamma\gamma$).

Electron Interactions (cont.)

➢ Soft events

- low–energy Moller (Bhabha) scattering, (modeled as part of the collision stopping power),
- atomic excitation ($e^{\pm}A \rightarrow e^{\pm}A^{*}$) via soft collision (modeled as another part of the collision stopping power),
- soft bremsstrahlung (modeled as radiative stopping power), and
- elastic electron (positron) multiple scattering from atoms, ($e^{\pm}N \rightarrow e^{\pm}N$).

Hard Bremsstrahlung Production

- ✓ ($e^{\pm}N \rightarrow e^{\pm}\gamma N$),
- ✓ Bremsstrahlung production is the creation of photons by electrons (or positrons) in the field of an atom.
- ✓ There are two possibilities. The predominant mode is the interaction with the atomic nucleus. This effect dominates by a factor of about Z over the three-body case where an atomic electron recoils. (nucleus $B \propto Z^2$ vs. electron $B \propto Z$)
- ✓ The de-acceleration and acceleration of an electron scattering from nuclei can be quite violent, resulting in emission of energy of up to the total kinetic energy of the incoming charged particle.

Hard Bremsstrahlung Production (cont.)

- ✓ The two-body effect ($e^{\pm}N \rightarrow e^{\pm}\gamma N$) can be taken into account through the total cross section and angular distribution kinematics.
- ✓ The three-body case ($e^{\pm}N \rightarrow e^{\pm}e^{-}\gamma N$) is conventionally treated only by inclusion in the total cross section of the two body-process.
- ✓ The two-body process can be modeled using one of the Koch and Motz formulae (Reviews of Modern Physics Vol. 31(4) Oct. 1959).
- ✓ The bremsstrahlung cross section scales with $Z(Z+\xi(Z))$, where $\xi(Z)$ is the factor accounting for three-body case where the interaction is with an atomic electron. These factors can be taken from the work of Tsai.
- ✓ The total cross section depends approximately like $1/E_{\gamma}$.

Hard Bremsstrahlung Production (cont.)

✓ The cross section can be written as the sum of two terms

$$\boxed{\frac{d\sigma}{dk}} = \frac{d\sigma_n}{dk} + Z\frac{d\sigma_e}{dk}$$

where $d\sigma_n/dk$ represents the bremsstrahlung of energy k in m_0c^2 unit produced in the field of the screened atomic nucleus, and $Z(d\sigma_e/dk)$ represents the bremsstrahlung produced in the field of the Z atomic electrons.

✓ It can be rewritten as $\frac{d\sigma}{dk} = \left(1 + \frac{\eta}{Z}\right)\frac{d\sigma_n}{dk},$

where η is the cross section ratio $\eta = \frac{d\sigma_e}{dk} \Big/ \left(\frac{1}{Z^2}\frac{d\sigma_n}{dk}\right)$

Hard Bremsstrahlung Production (cont.)

✓ The radiation integral for electron of energy E_0 is

$$\phi_{rad} \equiv \frac{1}{E_0} \int_0^{k_{max}} k \, \frac{d\sigma_{Brem}}{dk} \, dk$$

✓ The Koch and Motz formulae are

$$\phi_{rad} \equiv \frac{1}{E_0} \int_0^{k_{max}} k \, \frac{d\sigma_{Brem}}{dk} \, dk$$

in completely screened nuclear field and

$$\phi_{rad,electron} = 4\alpha r_0^2 \, Z \, \ell n \left(530 \, Z^{-2/3}\right)$$

in completely screened electron field,
where r_0 = classical electron radius and α = the fine structure constant.

Moller (Bhabha) Scattering

- ✓ Moller Scattering ($e^-e^- \rightarrow e^-e^-$)
- ✓ Bhabha scattering ($e^+e^- \rightarrow e^+e^-$)

- ✓ Moller and Bhabha scattering are collisions of incident electrons or positrons with atomic electrons.

- ✓ It is conventional to assume that these atomic electrons are "free" ignoring their atomic binding energy.

- ✓ The electrons in the e^-e^+ pair can annihilate and be recreated, contributing an extra interaction channel to the cross section.

$$\frac{Q_{max}}{E_{in}} = \frac{E_{in} - E_{out,min}}{E_{in}} = \frac{4mM}{(m+M)^2}$$

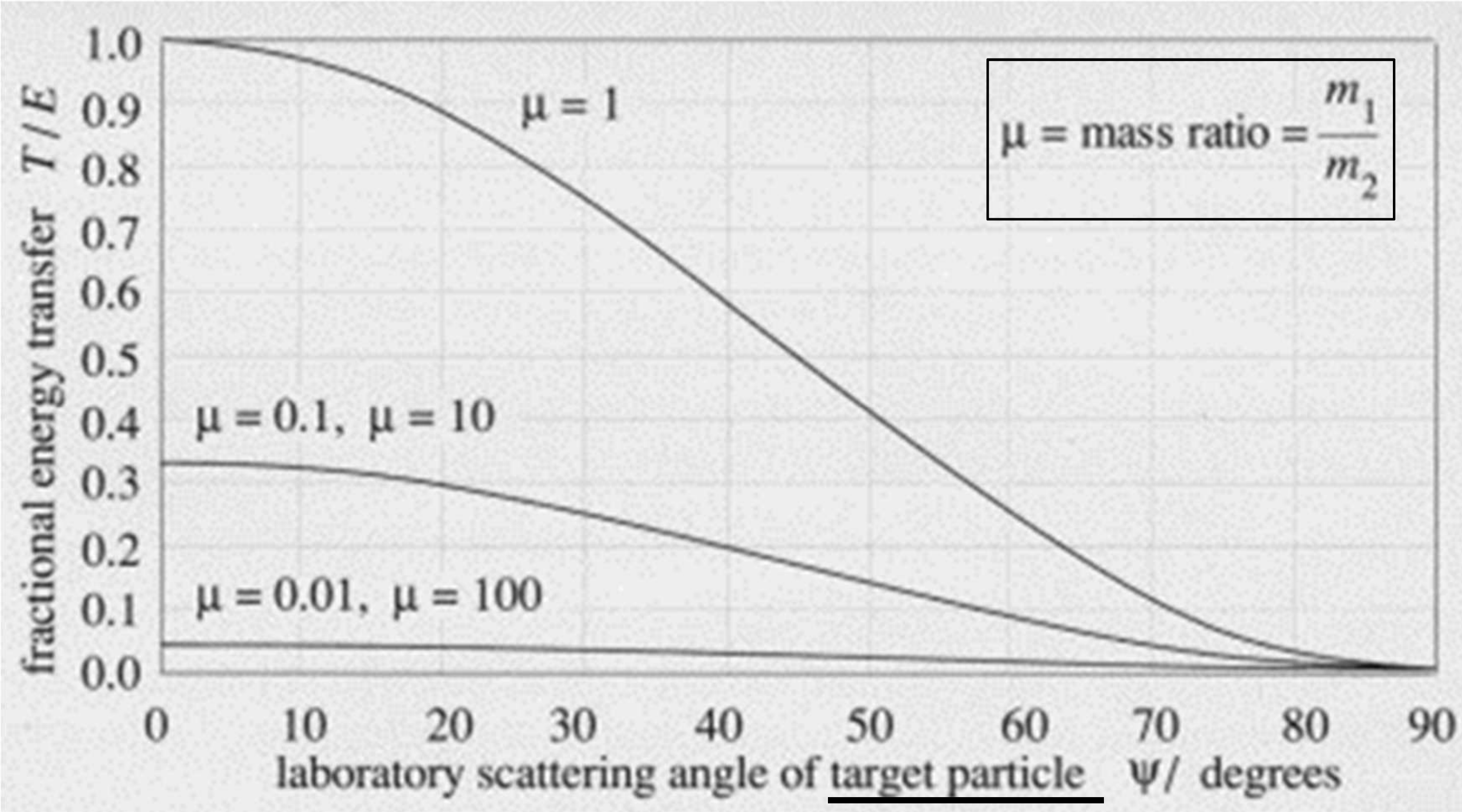
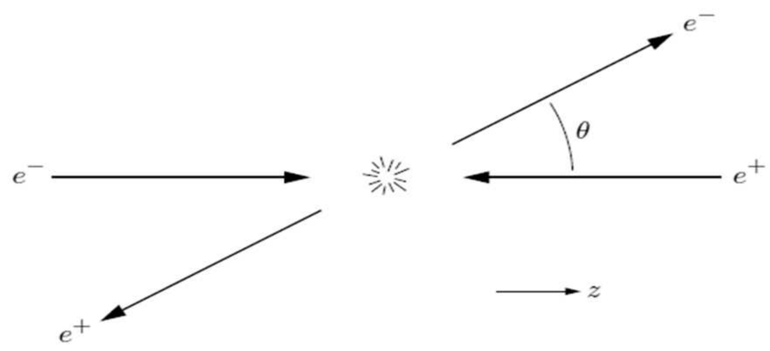


Figure 2-12 *Energy transfer in elastic collisions*. Fraction T / E of energy transferred from the projectile to the target in an elastic collision as function of the laboratory scattering angle Ψ of the target. The fraction of transferred energy is symmetric with respect to a mass exchange $m_1 \rightarrow m_2$, $m_2 \rightarrow m_1$, it has a maximum for equal masses, $m_1 = m_2$, and it decreases with increasing scattering angle Ψ.

Moller (Bhabha) Scattering (cont.)

- ✓ In the e–e– case, the "primary" electron can only give at most half its energy to the target electron if we adopt the convention that the higher energy electron is always denoted "the primary". This is because the two electrons are indistinguishable. In the e+e– case, the positron can give up all its energy to the atomic electron.
- ✓ Moller and Bhabha cross sections scale with Z for different media. The cross section scales approximately as $1/v^2$, where v is the velocity of the scattered electron.
- ✓ Many more low energy secondary particles are produced from the Moller interaction than from the bremsstrahlung interaction.

- Bhabha scattering ($e^+e^- \rightarrow e^+e^-$)
  - elastic positron–electron scattering where outgoing particles are distinguishable.



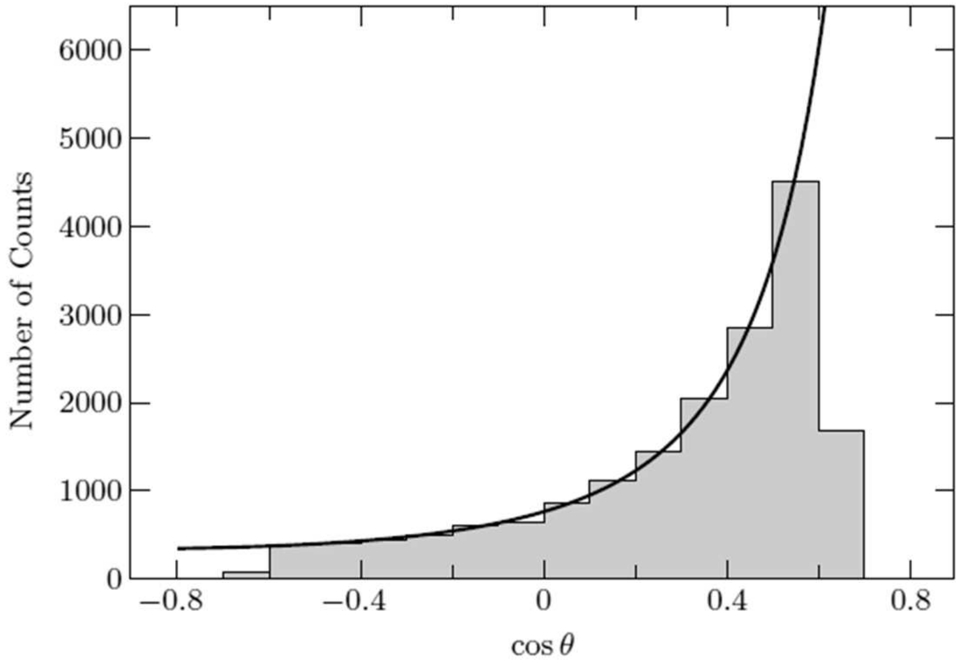
in COM frame

  - Bhabha's theoretical formula (1935)

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2 E_{\rm cm}^2}\left[\frac{1+\cos^4\frac{\theta}{2}}{\sin^4\frac{\theta}{2}} - \frac{2\cos^4\frac{\theta}{2}}{\sin^2\frac{\theta}{2}} + \frac{1+\cos^2\theta}{2}\right]$$

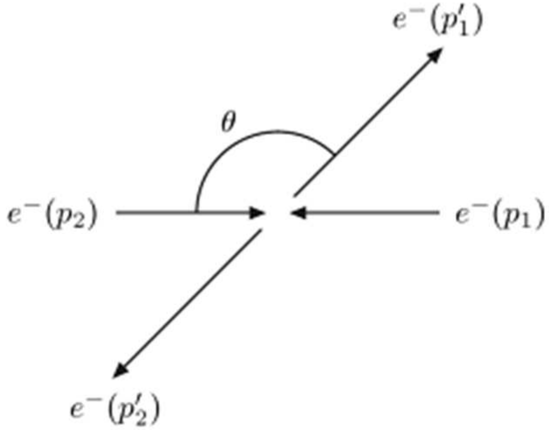
where θ is the scattering angle of <u>electron</u>

- Bhabha scattering ($e^+e^- \rightarrow e^+e^-$)
  - Cross section in COM frame depends only on E and θ at E (GeV to TeV) >> m_e (MeV).



measurement taken at Standford's SPEAR collider (early 1970s):
The number of events (and hence the differential cross section) increases as θ decreases, and goes to infinity in the limit $\theta \rightarrow 0$.

- Moller scattering ($e^-e^- \rightarrow e^-e^-$)
  - elastic electron-electron scattering



in COM frame

$$\boxed{\frac{d\sigma}{d\Omega}} = \frac{d\sigma}{d\Omega}(\theta) = \frac{d\sigma}{d\Omega}(-\theta) = \frac{d\sigma}{d\Omega}(\pi - \theta)$$

because the outgoing particles are indistinguishable.

Positron Annihilation

- ✓ Two-photon "in-flight" annihilation ($e^+e^- \rightarrow 2\gamma$) can be modeled using the cross section formulae of Heitler.

$$\boxed{\sigma(Z,E)} = \frac{Z\pi r_0^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right)-\frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

E = total energy of the incident positron

$\gamma = E/m_e c^2$

r_0 = classical electron radius

- ✓ It is conventional to consider the atomic electrons to be free, ignoring binding effects.
- ✓ Three and higher-photon annihilations ($e^+e^- \rightarrow n\gamma$ [$n>2$]) as well as one-photon annihilation, which is possible in the Coulomb field of a nucleus ($e^+e^-N \rightarrow \gamma N^*$), can be ignored as well.

Positron Annihilation (cont.)

- ✓ The higher-order processes are very much suppressed relative to the two-body process (by at least a factor of 1/137) while the one-body process competes with the two-photon process only at very high energies where the cross section becomes very small.

- ✓ If a positron survives until it reaches the transport cut-off energy, it can be converted into two photons (annihilation at rest).

Range in water: positron vs. electron

TABLE I: SIMULATED AND EXPERIMENTAL POSITRON RANGE IN WATER.

β_{max}(kev)		Mean range PeneloPET (mm)	Max. range PeneloPET (mm)	Mean range [17] (mm)	Max range [17] (mm)
635	^{18}F	0.61	2.3	0.64	2.3
960	^{11}C	1.04	3.9	1.03	3.9
1,190	^{13}N	1.31	5.1	1.32	5.1
1,720	^{15}O	2.00	7.9	2.01	8.0
1,899	^{68}Ga	2.21	8.9	2.24	8.9
3,400	^{82}Rb	4.24	16.7	4.29	16.5

source: Cal-Gonzalez et al., “Positron range effects in high resolution 3D PET imaging”

E(kev)	Electron range (mm)
1	68.3×10^{-6}
10	2.64×10^{-3}
100	0.143
500	1.76
1,000	4.43
2,000	9.85

Continuous Energy Loss

- ✓ One method to account for the energy loss to sub-threshold (soft bremsstrahlung and soft collisions) is to assume that the energy is lost continuously along its path.

- ✓ The formalism that may be used is the Bethe-Bloch theory of charged particle energy loss as expressed by Berger and Seltzer and in ICRU 37.

  [1] M. J. Berger, S. M. Seltzer, Stopping powers and ranges of electrons and positrons, (National Bureau of Standarts Report, NBSIR 82-2550 A, 1982).
  [2] ICRU, Report No. 37, 1984, Stopping powers for electrons and positrons. (International Commission on Radiation Units and Measurements, Bethesda, MD, 1984).

- ✓ This continuous energy loss scales with the Z of the medium for the collision contribution and Z^2 for the radiative part.

Bethe-Bloch formula for the *average* energy loss of heavy ($M>>m_e$) charged particle: (interaction dominated by collision with electrons)

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right] [\cdot \rho]$$

density

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307$ MeV g^{-1} cm^2

$T_{max} = 2 m_e c^2 \beta^2 \gamma^2 / (1 + 2\gamma m_e/M + (m_e/M)^2)$
[Max. energy transfer in single collision]

z : Charge of incident particle
M : Mass of incident particle

Z : Charge number of medium
A : Atomic mass of medium

I : Mean excitation energy of medium
δ : Density correction [transv. extension of electric field]

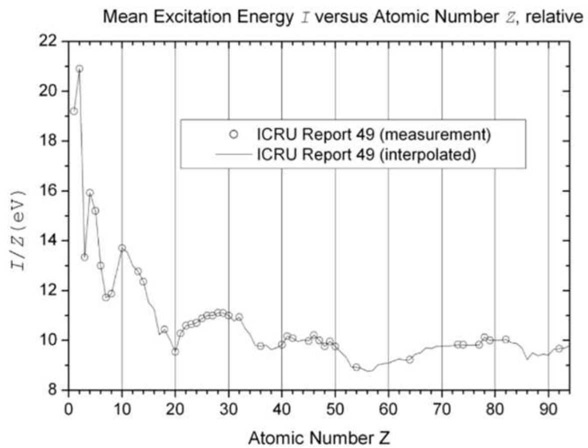
$N_A = 6.022 \cdot 10^{23}$
[Avogardo's number]

$r_e = e^2/4\pi\varepsilon_0 m_e c^2 = 2.8$ fm
[Classical electron radius]

$m_e = 511$ keV
[Electron mass]

$\beta = v/c$
[Velocity]

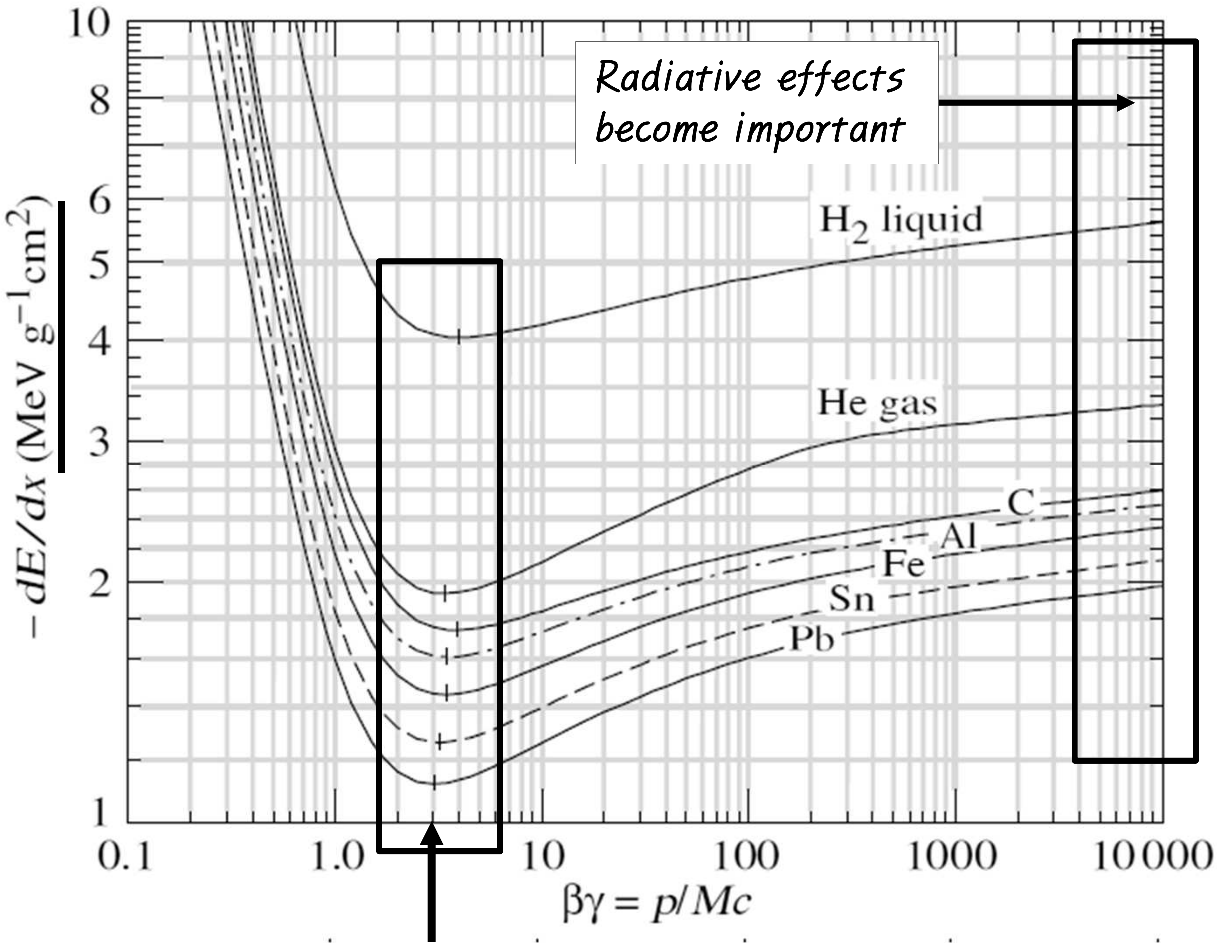
$\gamma = (1-\beta^2)^{-2}$
[Lorentz factor]



- BB formula is valid only if $v_{incident\ CP} >> v_{orbital\ electron} \sim 0.01c$.

energy loss of heavy charged particles:

dependence on mass A and charge Z of target nucleus



source: Particle Data Group, Review of Particle Physics, Physics Letters B 592 (2004)

Bethe–Bloch formula for energy loss of electron
in collision with <u>undistinguishable</u> particle, electron:

$$-\left\langle \frac{dE}{dx} \right\rangle_{\text{el.}} = K \frac{Z}{A} \frac{1}{\beta^2} \left[\ln \frac{m_e \beta^2 c^2 \gamma^2 T}{2I^2} + F(\gamma) \right]$$

T = kinetic energy of incoming electron (T_{max} = T/2)

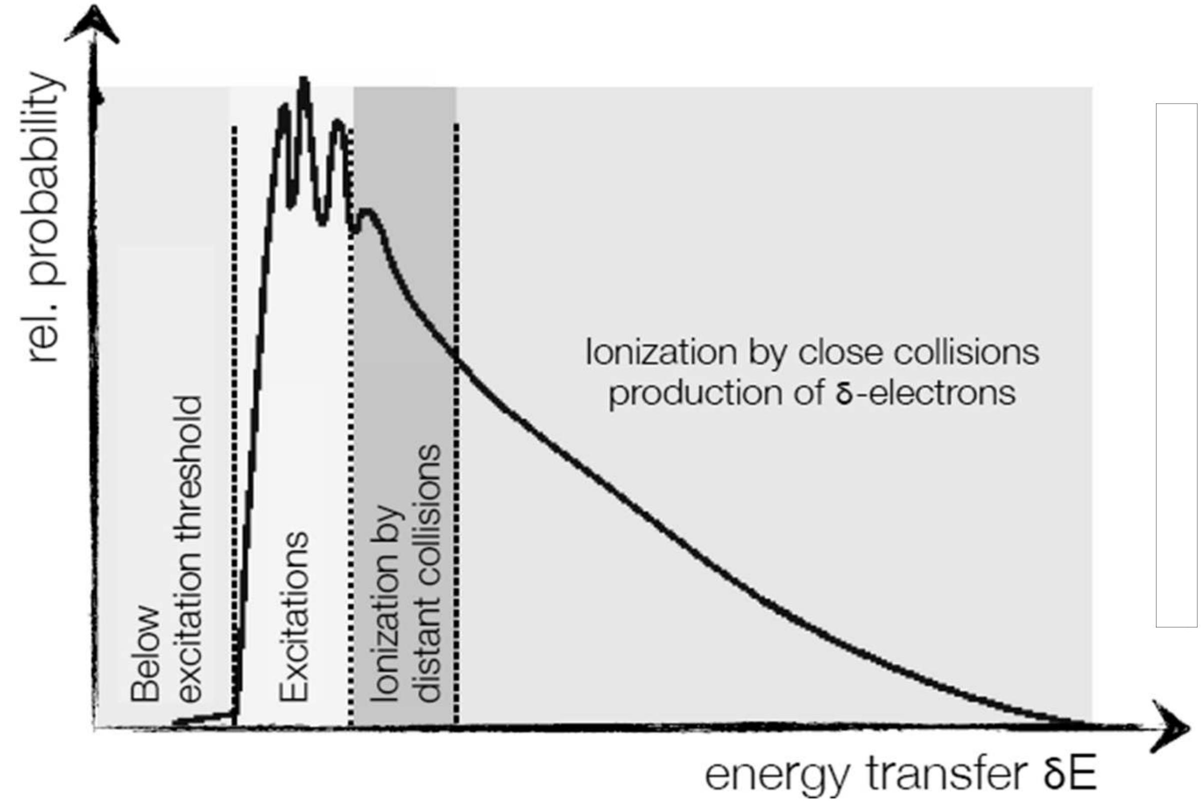
Note: different energy loss formula for electrons and positrons
at low energy as positrons are not identical with electrons.

$$-\left\langle \frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2 m_e c^2 \beta^2 \gamma^2 T_{\max}}{I^2} - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

Statistics of energy loss

- ✓ The energy loss is a statistical process. Number of collisions and energy loss varies from particle to particle.
- ✓ The distribution is usually asymmetric. Collisions with a small energy transfer are more probable than those with a large energy transfer.
- ✓ The tail at very high energy loss values are caused by rare collisions with small impact parameters. In these collisions e^- with high energies (keV), are produced, so-called δ-electrons.
- ✓ A result of the asymmetric is that the mean energy loss is larger than the most probable energy loss.
- ✓ For thin absorber, the energy loss can be described by the Landau distribution.
- ✓ For thick absorbers, the Landau distributions goes slowly into a Gaussian distribution.

energy loss "straggling"



$$\Delta E = \sum_{n=1}^{N} \delta E_n$$: *mean energy loss from Bethe-Bloch formula*

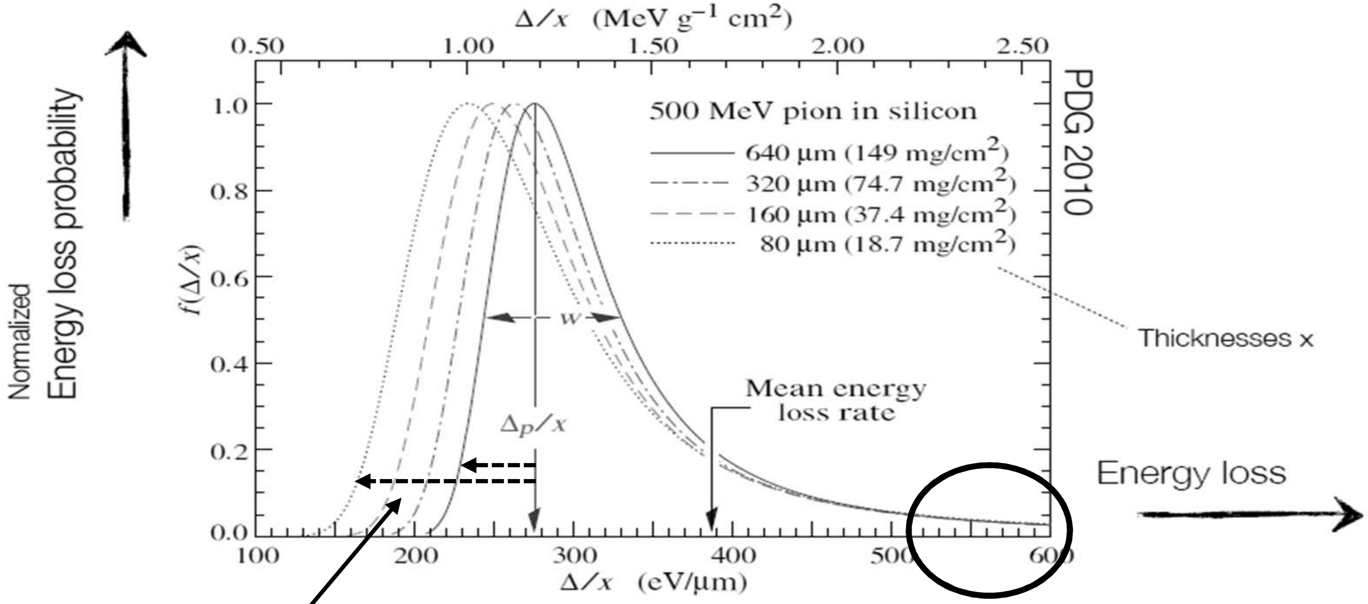
Landau distribution for thin absorber

✓ The fluctuations of energy loss by ionization of a charged particle in a thin layer of matter was theoretically described by Landau in 1944

Approximation:

$$f(\Delta/x) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\Delta/x - a(\Delta/x)_{\mathrm{mip}}}{\xi}\right) + e^{-\left(\frac{\Delta/x - a(\Delta/x)_{\mathrm{mip}}}{\xi}\right)}\right]$$

ξ: material constant



- broader with a thinner absorber due to a greater statistical uncertainty

Limitation of Landau's theory (infinite amount of energy loss? No!)

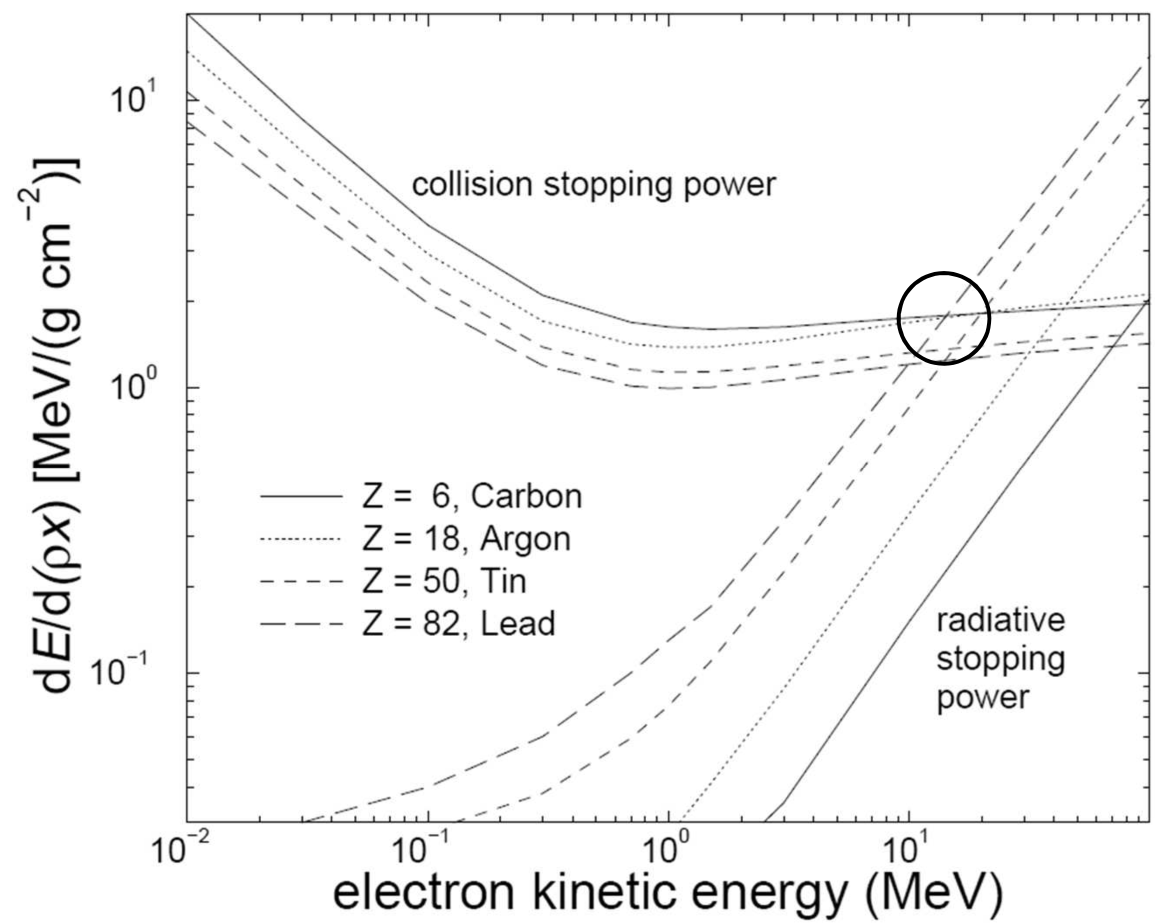
Continuous Energy Loss (cont.)

- ✓ Charged particles can also polarize the medium in which they travel. This "density effect" is important at high energies and for dense media.
- ✓ Default density effect parameters are available from a 1982 compilation by Sternheimer, Seltzer and Berger and state-of-the-art compilations (as defined by the stopping-power guru Berger who distributes a PC-based stopping power program)

Continuous Energy Loss (cont.)

- ✓ Again, atomic binding effects are treated rather crudely by the Bethe-Bloch formalism. It assumes that each electron can be treated as if it were bound by an average binding potential.
- ✓ The use of more refined theories does not seem advantageous unless one wants to study electron transport below the K-shell binding energy of the highest atomic number element in the problem.

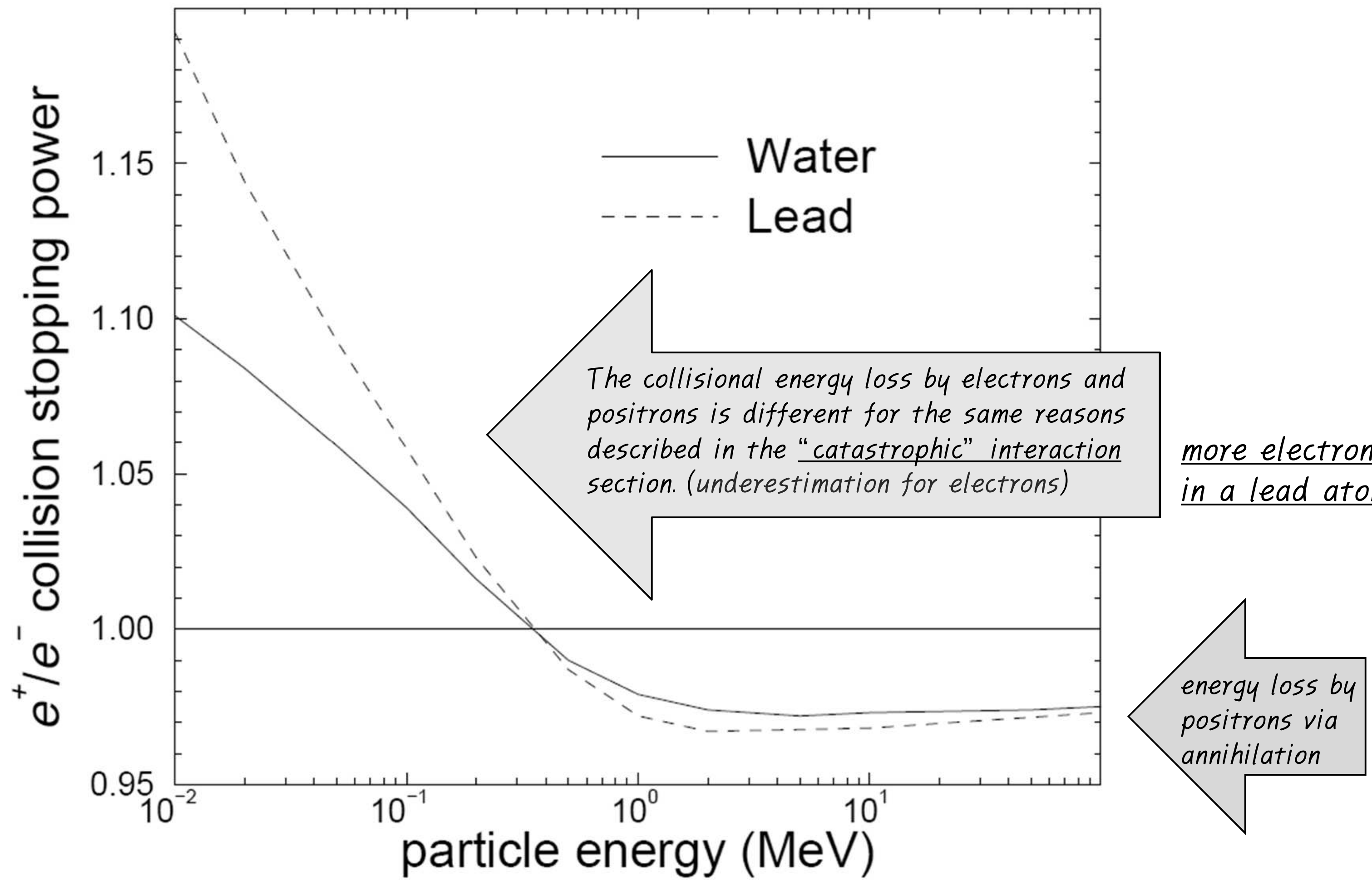
stopping power versus electron energy



Continuous Energy Loss (cont.)

- ✓ From the stopping power versus energy for different materials, the difference in the collision part is due mostly to the difference in ionisation potentials of the various atoms and partly to a Z/A difference, because the vertical scale is plotted in MeV/(g/cm^2), a normalisation by atomic weight rather than electron density.
- ✓ Note that at high energy the argon line rises above the carbon line. Argon, being a gas, is reduced less by the density effect at this energy.
- ✓ The radiative contribution reflects mostly the relative Z^2 dependence of bremsstrahlung production.

positron/electron collisional stopping power ratio

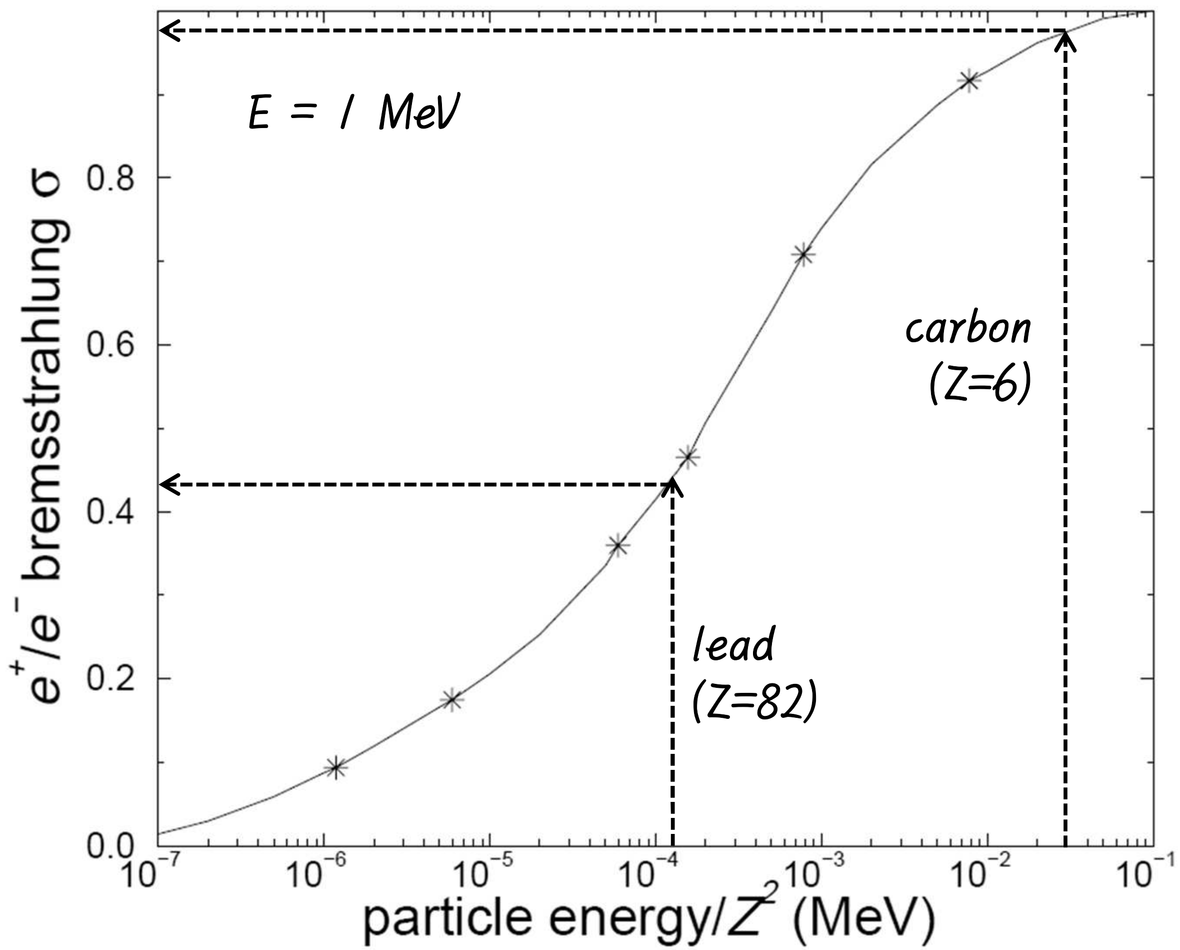


more electrons in a lead atom

Continuous Energy Loss (cont.)

- ✓ The collisional energy loss by electrons and positrons is different for the same reasons described in the "catastrophic" interaction section.
- ✓ Annihilation is generally not treated as part of the positron slowing down process and is treated discretely as a "catastrophic" event.
- ✓ The positron radiative stopping power is reduced with respect to the electron radiative stopping power. At 1 MeV, this difference is a few percent in carbon and 60% in lead.

positron/electron bremsstrahlung cross section ratio



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bremsstrahlung energy loss

$$\frac{dE}{dx} = 4\alpha N_A \,\frac{z^2 Z^2}{A} \left(\frac{1}{4\pi\epsilon_0} \frac{e^2}{mc^2} \right)^2 E \,\ln \frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. proportional to $1/m^2$ → main relevance for electrons
(α ~1/137, fine structure constant)

bremsstrahlung energy loss of electron (by Rossi): for E >~10 MeV, below which energy loss by ionization is dominant.

$$\frac{dE}{dx} = 4\alpha N_A \,\frac{Z^2}{A} r_e^2 \cdot E \,\ln \frac{183}{Z^{\frac{1}{3}}}$$

$$-\frac{dE}{dx} = \frac{E}{X_0} \qquad \text{with } \rho X_0 = \frac{\rho\, A}{4\alpha N_A \, Z^2 r_e^2 \,\ln \frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm^2]

vs. (collisional) mean free path

➡ $$E = E_0 e^{-x/X_0}$$

Rossi

After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy

[i.e. 63%]

Material	X_0 (g/cm^2) $= \rho X_o$	X_0 (cm)
H_2O	36.1	36.1
Air (NTP)	36.2	30050
H_2	63	$7 \cdot 10^5$
C	43	18.8
Polystyrol	43.8	42.9
Fe	13.8	1.76
Pb	6.4	0.56

– C. Grupen, *Teilchendetektoren*, BI-Wissenschaftsverlag, 1993
– W.R. Leo, *Techniques for Nuclear and Particle Physics Experiments*, Springer, 1987
– K. Kleinknecht, *Detektoren für Teilchenstrahlung*, B.G. Teubner, 1992
– D.H. Perkins, *Introduction to High Energy Physics*, Addison-Wesley, 1987

positron/electron total (collisional+radiative) stopping power

$$\left(\frac{dE}{dx}\right)_{\text{Tot}} = \left(\frac{dE}{dx}\right)_{\text{Ion}} + \left(\frac{dE}{dx}\right)_{\text{Brems}}$$

critical energy:

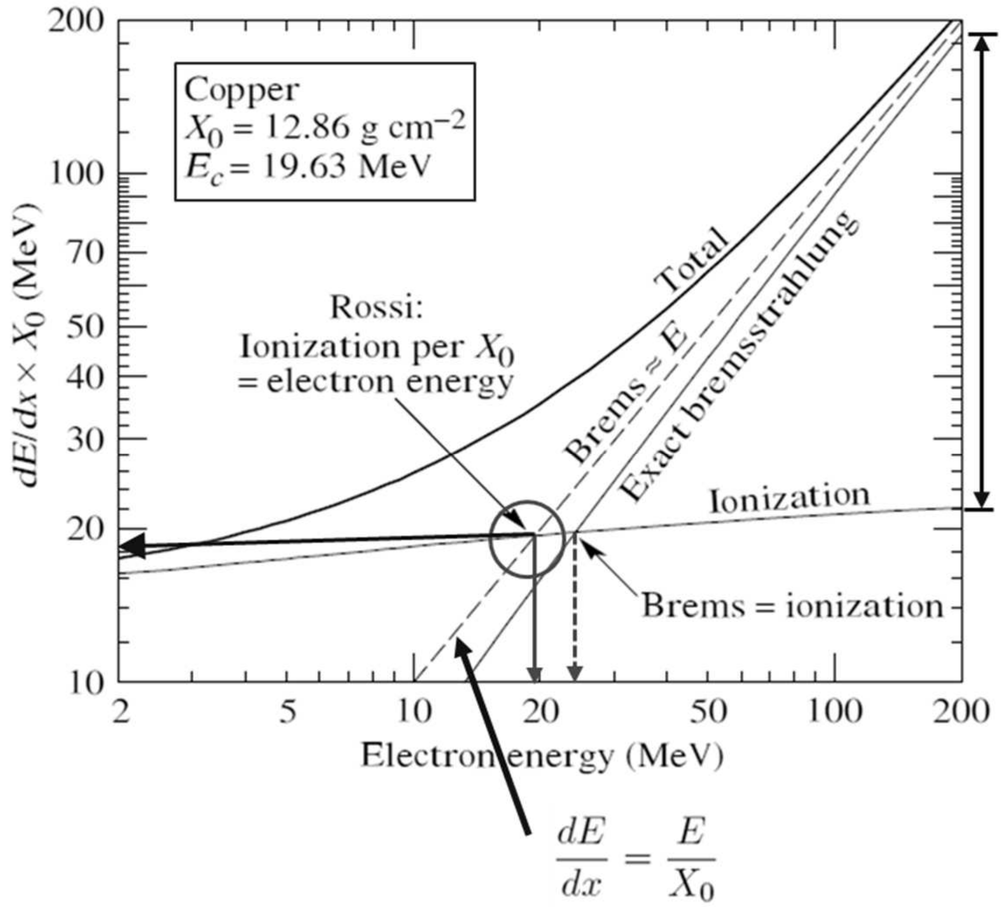
$$\left.\frac{dE}{dx}(E_c)\right|_{\text{Brems}} = \left.\frac{dE}{dx}(E_c)\right|_{\text{Ion}}$$

approximately,

$$E_c^{\text{Gas}} = \frac{710 \text{ MeV}}{Z + 0.92}$$

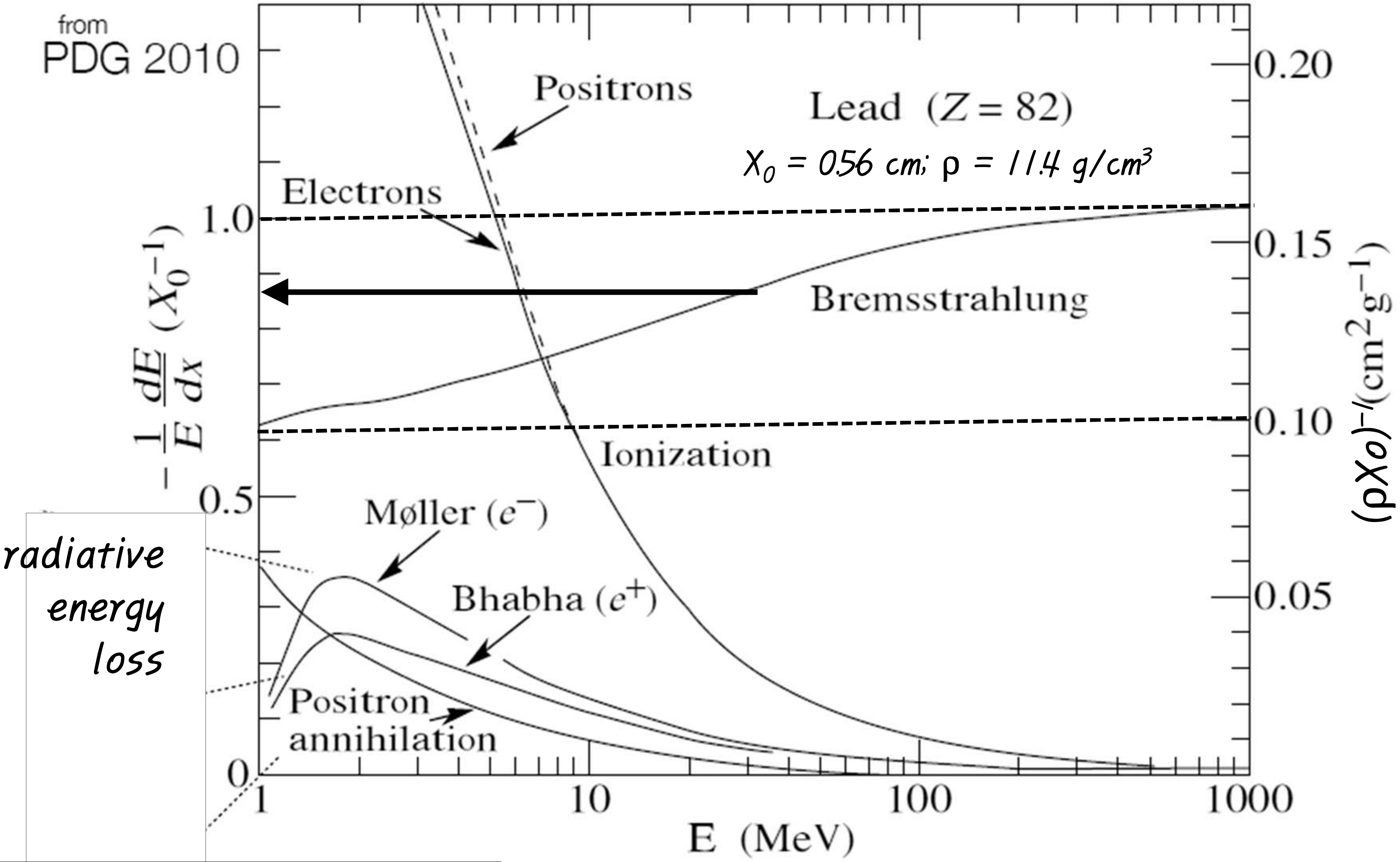
$$E_c^{\text{Sol/Liq}} = \frac{610 \text{ MeV}}{Z + 1.24}$$

$$-\frac{dE}{dx} = \frac{E}{X_0} \;\Rightarrow\; -\frac{1}{E}\frac{dE}{dx} = \frac{1}{X_0}$$



total energy loss of electrons

fractional energy loss per radiation length in lead as a function of electron or positron energy



$$-\frac{dE}{dx} = \frac{E}{X_0} \rightarrow -\frac{1}{E}\frac{dE}{dx} = \frac{1}{X_0}$$

Particle Data Group, *Review of Particle Physics*, Physics Letters B **592** (2004)

Multiple Scattering

- ✓ Elastic scattering of electrons and positrons from nuclei is predominantly small angle with the occasional large-angle scattering event.
- ✓ If it were not for screening by the atomic electrons, the cross section would be infinite. The cross sections are, nonetheless, very large.
- ✓ It is impractical to model all individual interactions discretely. There are several statistical theories that deal with multiple scattering. Some of these theories describe these "weak" interactions by accounting for them in a cumulative sense. These are the so-called "statistically grouped" interactions.

Multiple Scattering (cont.)

- ✓ The most popular such theory is the Fermi-Eyges theory, a small angle theory. This theory neglects large angle scattering and is unsuitable for accurate electron transport unless large angle scattering is somehow included.
- ✓ The most accurate theory is that of Goudsmit and Saunderson.
- ✓ A fixed step-size scheme permits an efficient implementation of Goudsmit-Saunderson theory and this has been done in ETRAN, ITS (E&P) and MCNP.

Multiple Scattering (cont.)

- ✓ The Moli`ere theory, although originally designed as a small angle theory, has been shown with small modifications to predict large angle scattering quite successfully.
- ✓ Owing to analytic approximations made by Molie`re theory, this theory requires a minimum step-size. as
- ✓ The Moli`ere theory ignores differences in the scattering of electrons and positrons, and uses the screened Rutherford cross sections instead of the more accurate Mott cross sections. However, the differences are known to be small.
- ✓ EGS4 uses the Moliere theory which produces results as good as Goudsmit-Saunderson for many applications and is much easier to implement in EGS4's transport scheme

Mott cross section

- ✓ The Mott cross section formula is the mathematical description of the elastic scattering of a high energy electron beam from an atomic nucleus-sized positively charged point in space.

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right)\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}$$

– Assumptions

1. The energy of the electron must be such that $\beta \approx 1$ so that β^4 can safely be set to unity. (a good approximation for E> 10 MeV)
2. The target is light nuclei as defined by $\frac{Z}{137} = \frac{Ze^2}{\hbar c} \ll 1$. (reasonable up to Calcium)

Mott cross section (cont.)

- The Mott cross section formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right)\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}$$

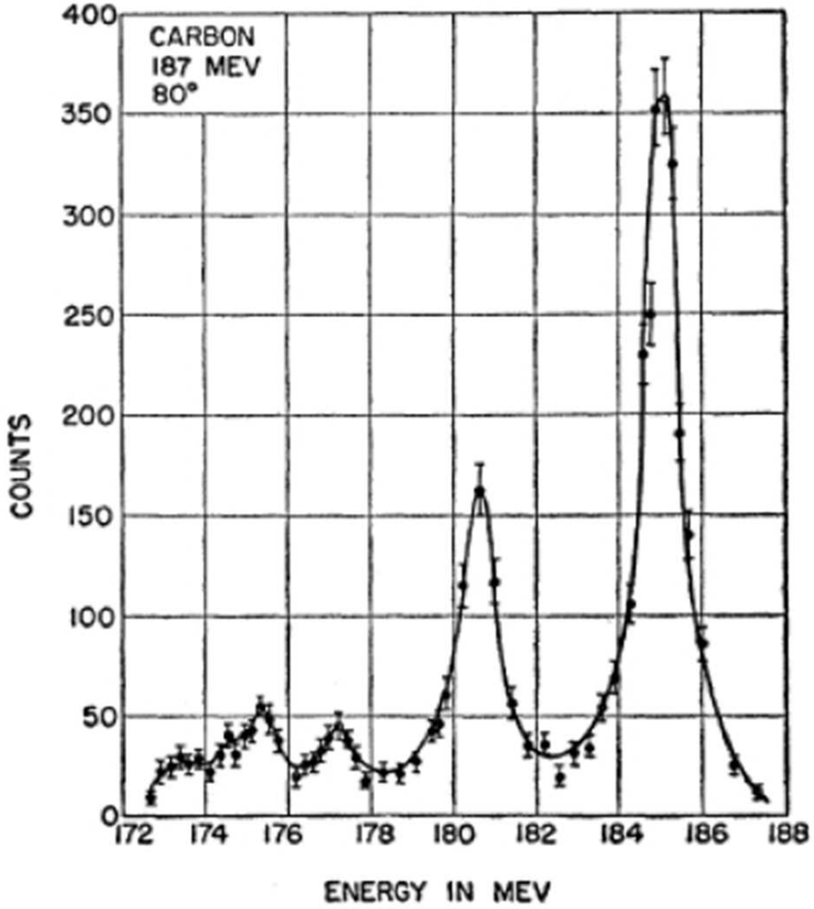
resembles the Rutherford scattering cross section.

- If an extended charge distribution (vs. a point target) is considered, it becomes

$$\frac{d\sigma}{d\Omega} = \left(\frac{Ze^2}{2E}\right)\frac{\cos^2(\frac{1}{2}\theta)}{\sin^4(\frac{1}{2}\theta)}|F(q)|^2$$

where F(q) is the form factor for the electron screening.

Electron Inelastic scattering and Nucleus Structure



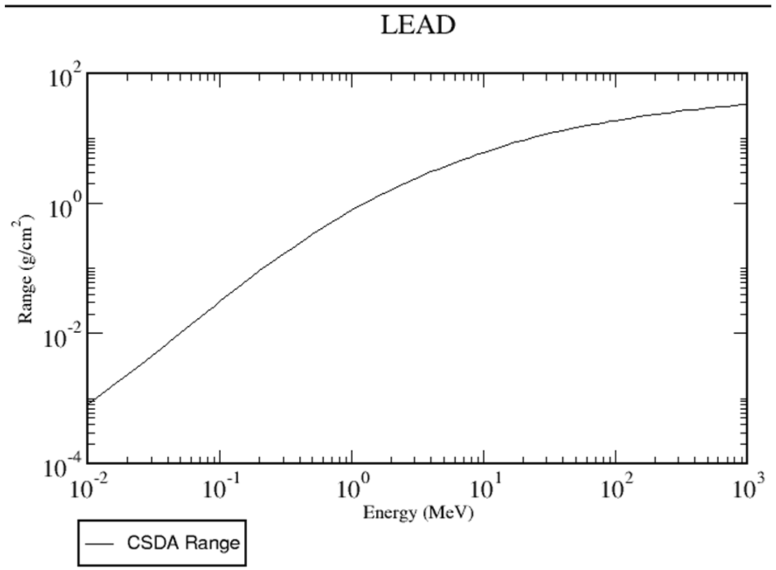
A plot of electron intensity vs. scattered electron energy of 187 MeV electrons scattered off a carbon-12 target. Note the distinct peaks with energy differences corresponding to the energies associated with excited states of the target.

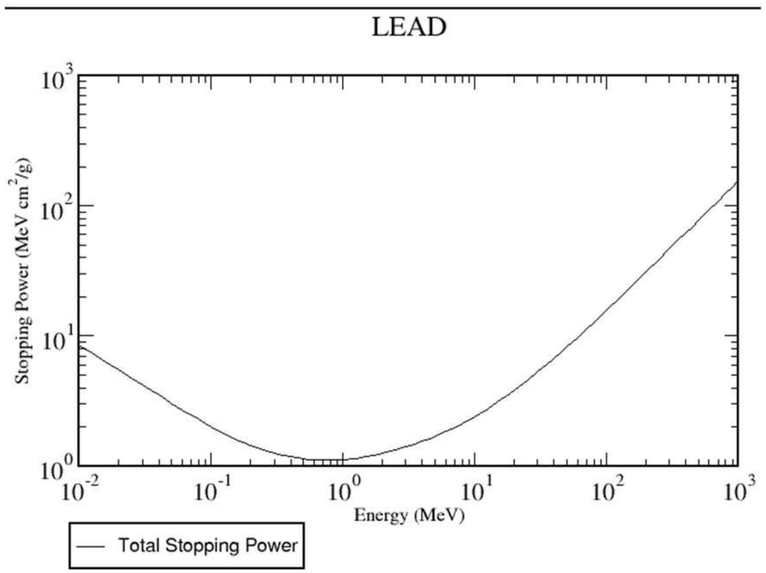
Electron Transport Mechanics:

CSDA (continuously slowing-down approximation)

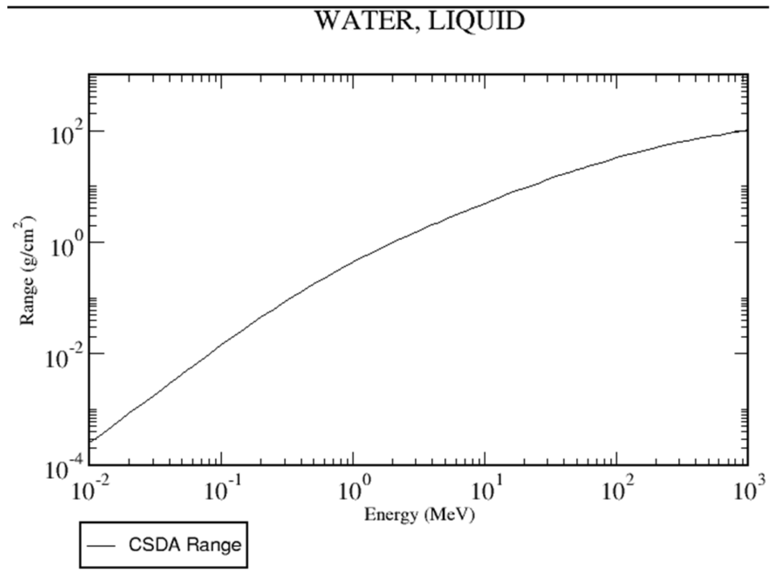
1. Energy is being lost "continuously" to sub-threshold knock-on electrons and bremsstrahlung (i.e., no secondaries are created). The rate of energy loss at every point along the track is assumed to be equal to the same as the total stopping power.
2. Energy-loss fluctuations are neglected (i.e., no energy-loss straggling).

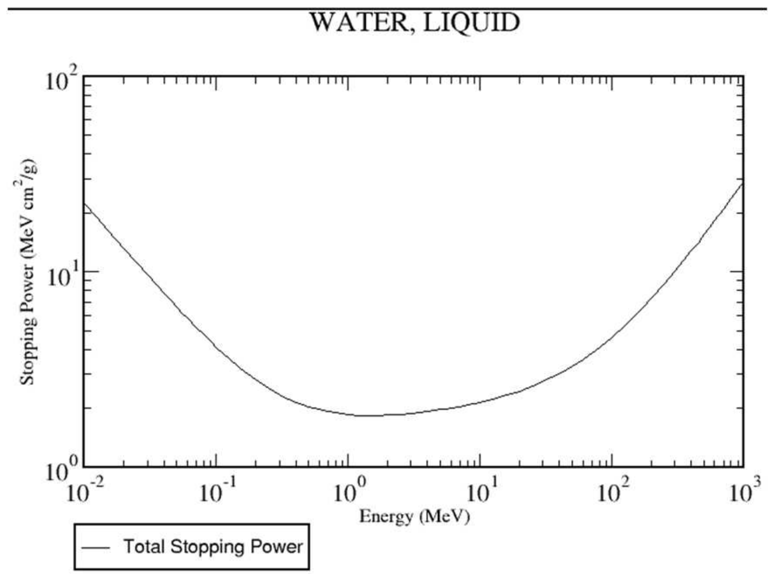
ρ_{Pb} = *11.34 g/cm³*





ρ_{H2O} = *1.0 g/cm³*





source: NIST Physical Measurement Laboratory

Electron Transport Mechanics:

stopping power for continuos energy loss

- ✓ In continuous slowing down approximation (CSDA), the projectile is assumed to lose energy continuously along its path and the slowing-down process is completely characterized by the (linear) stopping power S(E), which defined as the average energy loss per unit path length,

$$S(E) = -\frac{\mathrm{d}E}{\mathrm{d}s}$$

- ✓ CSDA completely neglects energy straggling, i.e., fluctuations in the energy loss due to the discreteness of the energy transfers in inelastic and radiative interactions and to the randomness of the number of these interactions.

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

- ✓ The CSDA range of an electron with kinetic energy E is given by

$$R(E) = \int_{E_{\mathrm{abs}}}^{E} \frac{\mathrm{d}E'}{S(E')} \quad R(E) = \int_{E}^{E_{abs}} ds(E') = \int_{E}^{E_{abs}} \frac{-dE'}{S(E')} = \int_{E_{abs}}^{E} \frac{dE'}{S(E')}$$

where E_{abs} is the 'absorption' energy, i.e., the energy at which the electron is assumed to be effectively absorbed in the medium

- ✓ If an electron starts its trajectory with kinetic energy E, the energy loss W after a path length s (would be randomly chosen) is determined by

$$s = \int_{E-W}^{E} \frac{\mathrm{d}E'}{S(E')} = R(E) - R(E-W)$$

- ✓ To calculate the energy loss as a function of the path length, W(s), we only need to know the CSDA range as a function of energy, R(E).

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

- ✓ Step 1. Set the energy E_1 of an electron from STOCK at start
- ✓ Step 2. Select s_1 from $s_1 = -[\ln \xi / \Sigma_t(E_1)]$
- ✓ Step 3. Calculate $R_2(E_1 - W_1) = R_1(E_1) - s_1$, read the energy $E_2 = E_1 - W_1$ corresponding to R_2 from Table and determine W_1.
- ✓ Step 4. If $W_1 <$ ECUT or PCUT, then $E_{dep} = W_1$ and go to Step 6. Or If $W_1 >$ AE or AP(=PCUT), then stock the secondary particle at AE or AP and go to Step 6. (Otherwise, go to Step 5 with ECUT$<W_1<$AE)
- ✓ Step 5. Operate the routine of (CSDA and multiple scattering) for subsets of $t_{1,i}$ ($i = 1, 2, \ldots, W_1$/ESTEPE)

  $t_{1,1} + t_{1,2} + \ldots + t_{1,(W1/ESTEPE)} = s_1$; $t_{1,i} = s_1/(W_1/ESTEPE)$

  *ESTEPE = the max. fractional electron energy loss per electron step

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

- ✓ Step 6. If $E_2 < ECUT$, then $E_{dep} = E_2$ and go to Step 1.
  Or If $E_2 > AE$, $E_1 = E_2$ and go to Step 2.
  Otherwise, $W_1 = E_2$. (Go to Step 7)
- ✓ Step 7. Operate the routine of (CSDA and multiple scattering) for subsets of $t_{1,i}$ ($i = 1, 2, . . . , W_1/ESTEPE$)

  $t_{1,1} + t_{1,2} + . . . + t_{1,(W1/ESTEPE)} = s_1$; $t_{1,i} = s_1/(W_1/ESTEPE)$

  *ESTEPE = the max. fractional electron energy loss per electron step
- ✓ Step 9. Go to Step 1.

Electron Transport Mechanics:

stopping power for continuous energy loss (cont.)

* The number of subsets t_i (l = 1, 2, ⋯, n) also can be determined by selecting the n according to the Poisson distribution: $t_i = s/n$.

  – The probability distribution for a number n (≥ 0) of collisions in a path length s is approximated to Poisson distribution:

$$P(n) = \exp(-s/\lambda)\frac{(s/\lambda)^n}{n!}$$ with the mean $\langle n \rangle = s/\lambda$.

where $\lambda(W) = 1/N\sigma_{elas}(W)$ is the mean free path b/w elastic collision and $\sigma_{elas}(W)$ is the elastic scattering cross section of the secondary electron of energy W.

Electron Transport Mechanics:
typical electron tracks

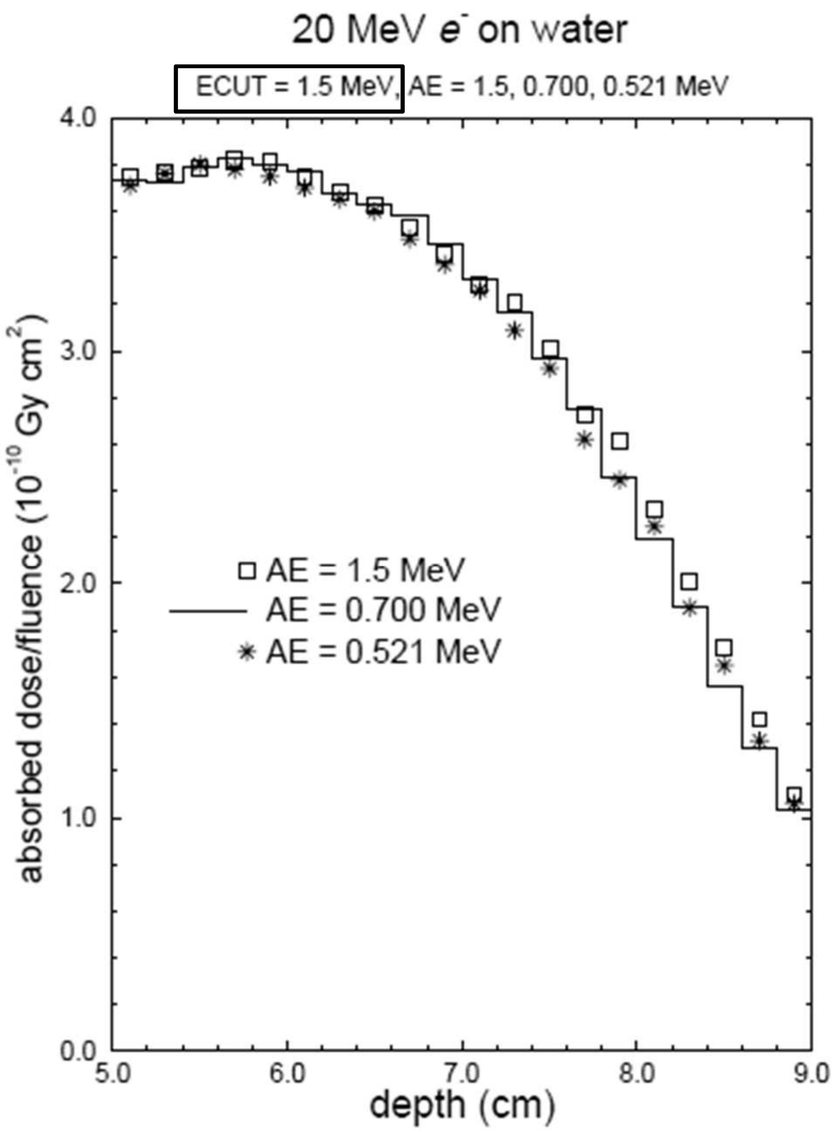
- An electron is being transported through a medium. Along the way energy is being lost "continuously" to sub-threshold knock-on electrons and bremsstrahlung.
- The track is broken up into small straight-line segments called multiple scattering substeps. In this case the length of these substeps was chosen so that the electron lost a selected fraction (ex. ESTEPE=0.01 in EGS4) of its subthreshold energy loss (W) during each substep.
- At the end of each of these substeps the multiple scattering angle is selected according to some theoretical distribution.
- Catastrophic events, here a single knock-on electron (or a hard bremsstrahlung), sets other particles in motion. These particles are followed separately in the same fashion. The original particle, if it does not fall below the transport threshold (AE or AP), is also transported.

Electron Transport Mechanics:

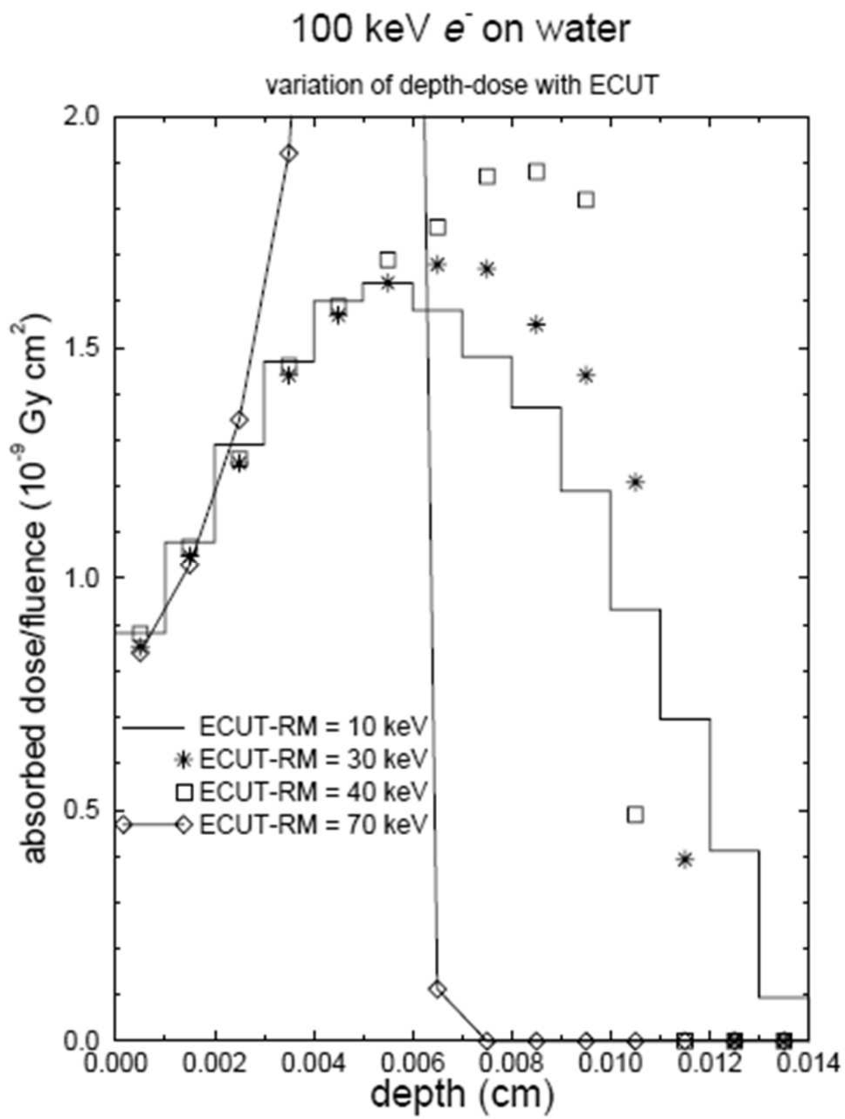
typical multiple scattering substeps

- ✓ A single electron <u>substep</u> is characterized by the length of total curved path-length to the end point of the substep, t. (This is a reasonable parameter to use because the number of atoms encountered along the way should be proportional to t.)
- ✓ At the end of all the substeps, the deflection from the initial direction, Θ, is sampled. Associated with the substeps is the average projected distance along the original direction of motion, s.
- ✓ The lateral deflection, ρ, the distance transported perpendicular to the original direction of motion, is often ignored by electron Monte Carlo codes. Such lateral deflections do occur as a result of multiple scattering.
- ✓ It is only the lateral deflection during the course of a subset which is ignored. One can guess that if the multiple scattering substeps are small enough, the electron track may be simulated more exactly.

Examples of Electron Transport: Parameter selection

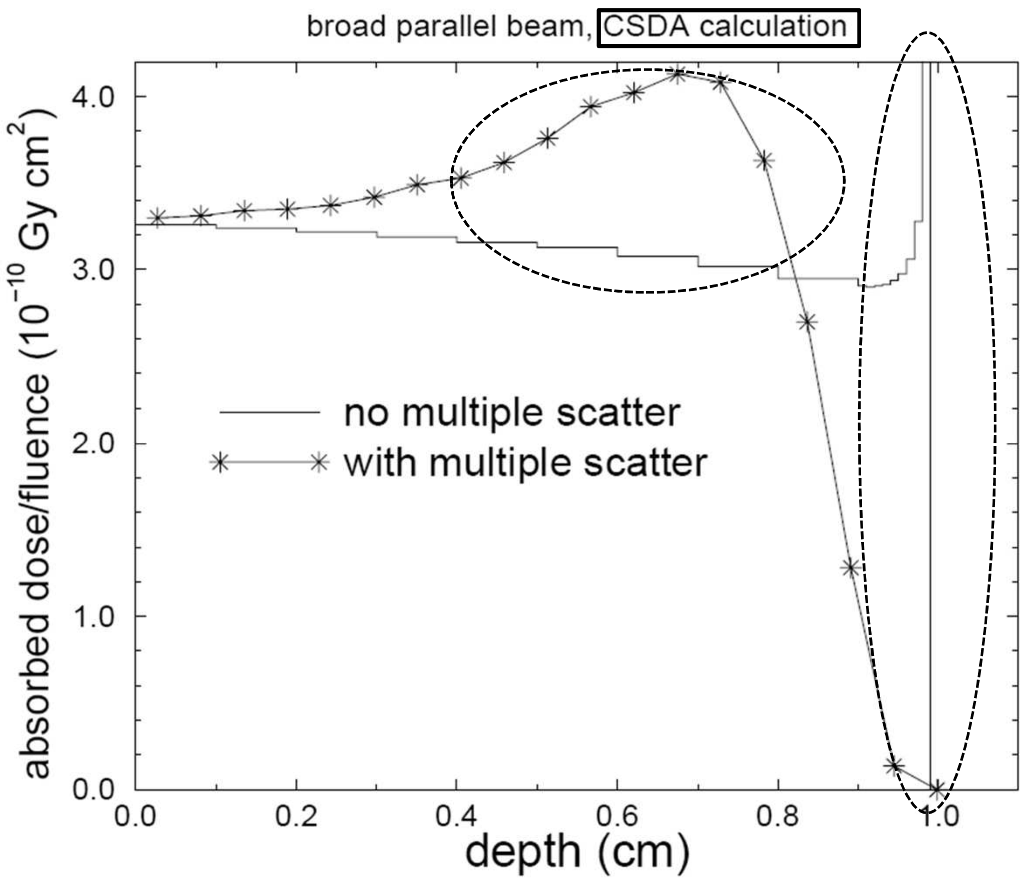


AE: secondary electron threshold

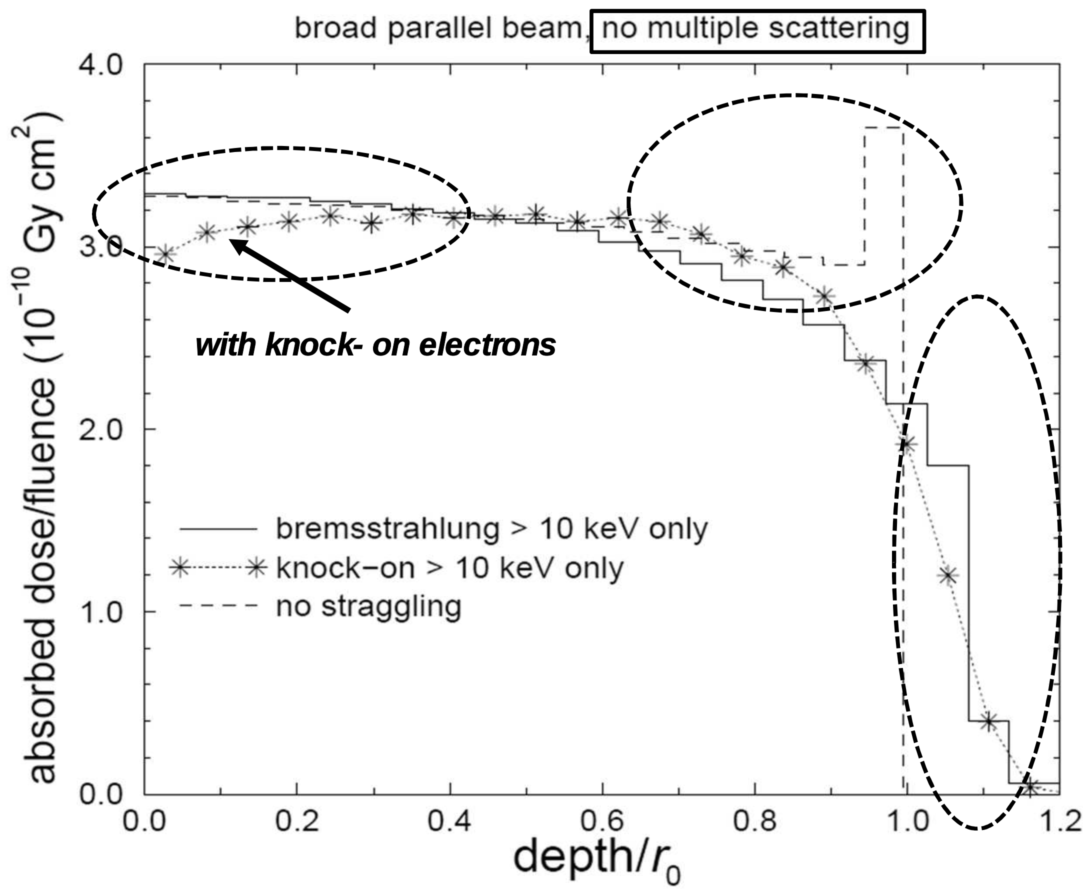


ECUT: electron transport cutoff

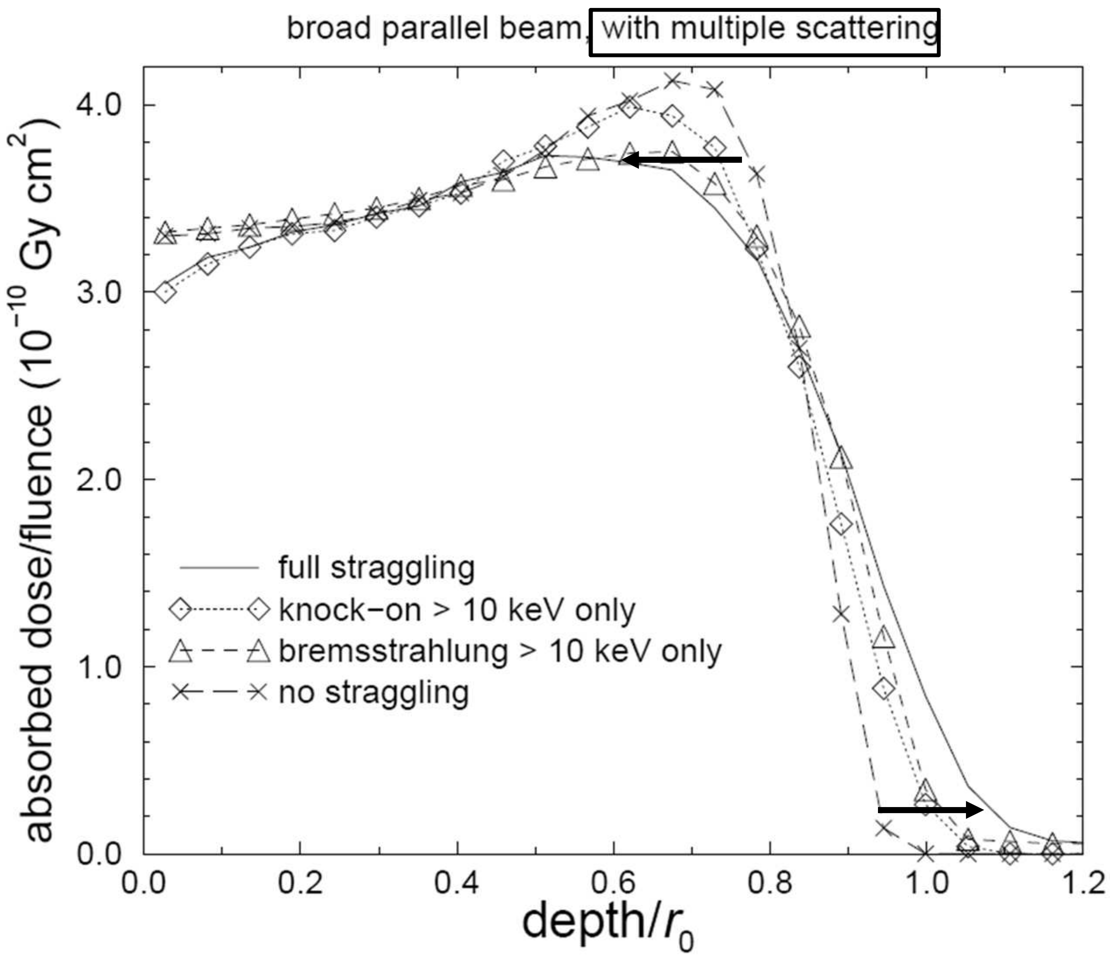
Examples of Electron Transport: w/ vs. w/o multiple scatterings

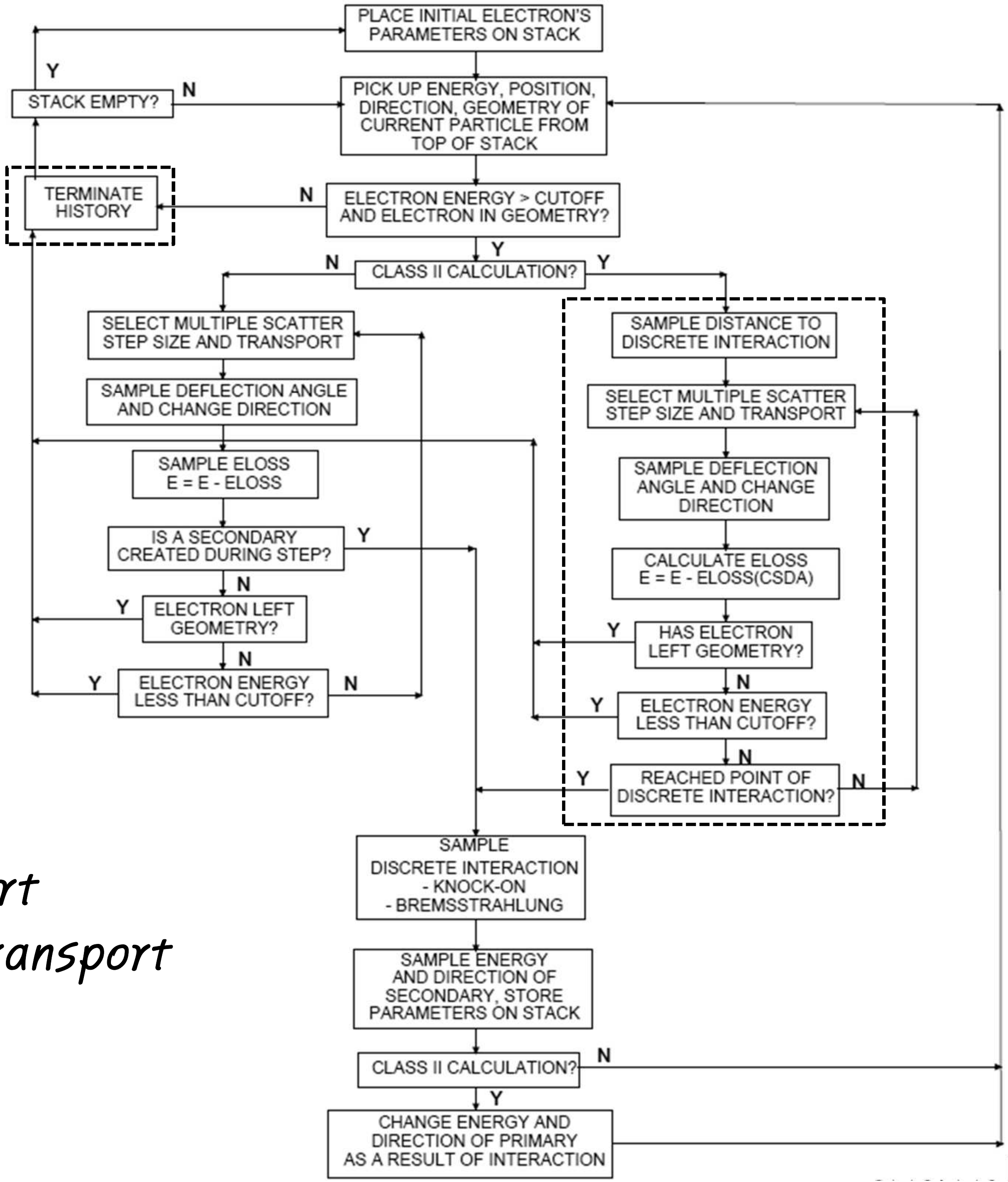


Examples of Electron Transport: w/o multiple scatterings



Examples of Electron Transport: w/ multiple scatterings





Flow Chart for Electron Transport (EGS4)

KE/EHK

Electron Transport Logic (cont.)

- ✓ Imagine that an electron's parameters (energy, direction, etc.) are on top of the particle stack. (STACK is an array containing the phase-space parameters of particles awaiting transport.)
- ✓ The electron transport routine picks up these parameters and first asks if the energy of this particle is greater than the transport cutoff energy, called ECUT. If it is not, the electron is discarded. ("Discard" means that the scoring routines are informed that an electron is about to be taken off the transport stack.)
- ✓ If there is no electron on the top of the stack, control is given to the photon transport routine. Otherwise, the next electron in the stack is picked up and transported.

Electron Transport Logic (cont.)

- ✓ If the original electron's energy was great enough to be transported, the distance to the next catastrophic interaction point is determined, exactly as in the photon case.
- ✓ The multiple scattering step-size t is then selected and the particle transported, taking into account the constraints of the geometry.
- ✓ After the transport, the multiple scattering angle is selected and the electron's direction adjusted. The continuous energy loss is then deducted.
- ✓ If the electron, as a result of its transport, has left the geometry defining the problem, it is discarded. Otherwise, its energy is tested to see if it has fallen below the cutoff as a result of its transport.

Electron Transport Logic (cont.)

- ✓ If the electron has not yet reached the point of interaction, a new multiple scattering step is effected. This innermost loop undergoes the heaviest use in most calculations because often many multiple scattering steps occur between points of interaction.

- ✓ If the distance to a discrete interaction has been reached, then the type of interaction is chosen. Secondary particles resulting from the interaction are placed on the stack as dictated by the differential cross sections, lower energies on top to prevent stack overflows.

- ✓ The energy and direction of the original electron are adjusted and the process starts all over again.