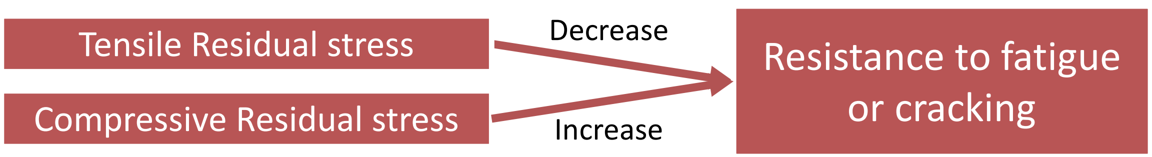
Chapter 12. Plastic Deformation Behavior and Models for Material

Seoul National University System Health & Risk Management

Contents

12.1 Introduction

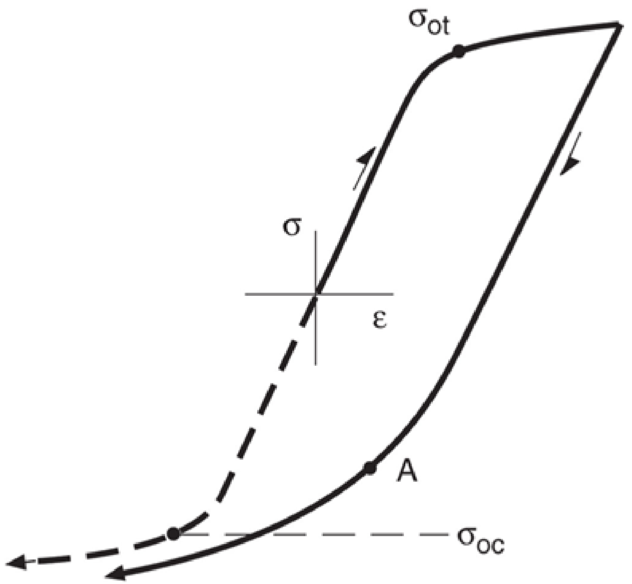
- **Plastic Deformation**
  - Deformation beyond the point of yielding that is not strongly time dependent (Assumption: Always-present time-dependent (creep) deformations are relatively small)
- **Significance of Plastic Deformation (12.1.1)**
  - Cause the “residual stress”



  - Fatigue analysis (S-N curve & mean stress effects)
    - ➢ Traditional method: Stress-based approach (Chapter 10)
      - Based on Elastic analysis
      - Hence, it involves rough empirical adjustments for plastic deformation
      - Needed adjustments are especially large at short lives and at high stresses

12.1 Introduction

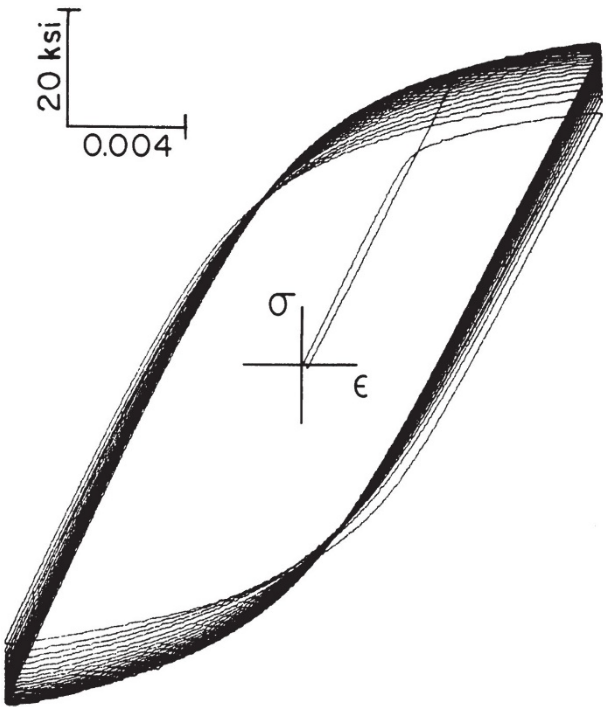
- **Unloading – Bauschinger effect (12.1.2)**
  - *Bauschinger effect*
    - Yielding on unloading generally occurs prior to the stress reaching the yield strength σ_{OC} for monotonic compression, as at point A
  - Significance of the unloading stress-strain paths
    - Need to be described for use in predicting behavior after unloading from a severe load, as in estimating residual stresses



<Fig. 12.1> Unloading stress-strain curve with Bauschinger effect

12.1 Introduction

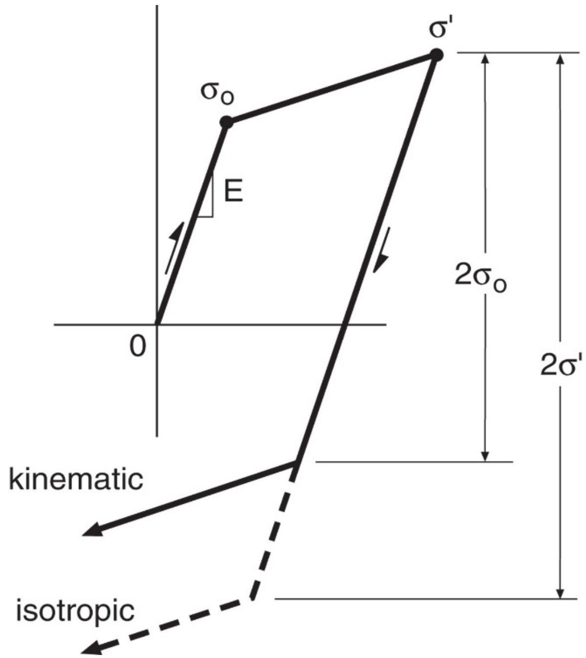
- **Cyclic loading (12.1.2)**
  - Complexity of cyclic loading
    - ➢ The behavior is observed to gradually change with the number of applied cycles
  - Demand for “Rheological models”
    - ➢ For the strain-based approach to fatigue (Chapter 14), at least approximate modeling of the cyclic loading behavior is needed
    - ➢ *Rheological models* (Chapter 5) are found to be useful for this purpose



<Fig. 12.2> Stress-strain response in 2024-T4 aluminum for 20 cycles of completely reversed strain at $\varepsilon_a = 0.01$

12.1 Introduction

- **Additional comments (12.1.3)**
  - Deformation theory
    - Also called the total strain theory of plasticity
    - We consider the *deformation theory* rather than the more advanced *incremental theory*
  - Kinematic hardening
    - The rheological models used are consistent with the behavior called *Kinematic hardening* which predicts a Bauschinger effect as observed in real materials
    - The alternative choice of *isotropic hardening* is not employed, as it is a poor model for real materials



<Fig. 12.3> Differing unloading behavior for kinematic and isotropic hardening

12.1 Introduction

- **Objectives**
  - Become familiar with basic forms of stress-strain relationships, including fitting data to these and representing them with spring and slider rheological models.

    (Chapter 12.2)

  - Employ deformation plasticity theory to explore the effects of multiaxial states of stress on stress-strain behavior

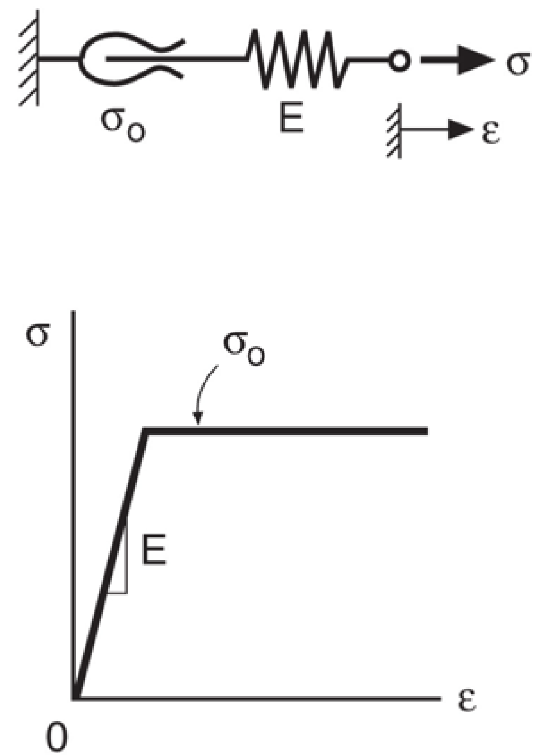
    (Chapter 12.3)

  - Analyze unloading and cyclic loading behavior for both rheological models and for real materials, including cyclic stress-strain curves, irregular variation of strain with time, and transient behavior such as mean stress relaxation

    (Chapter 12.4-12.5)

12.2 Stress-Strain Curves

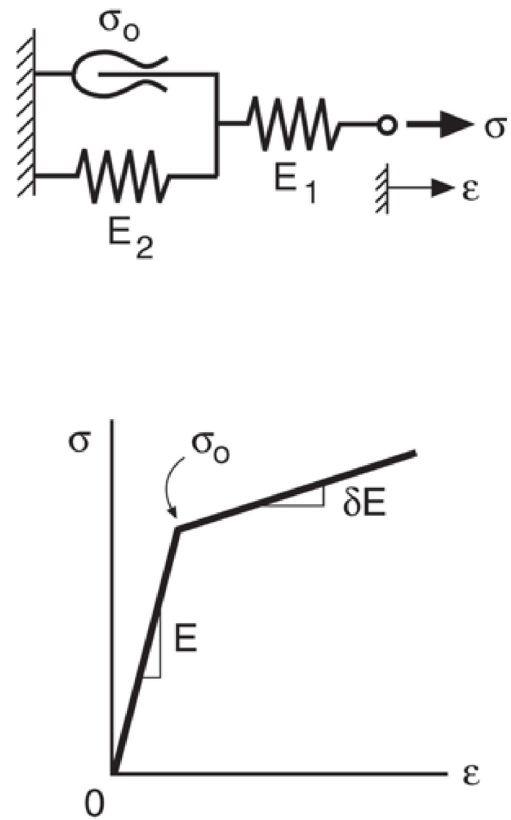
- **Elastic, Perfectly Plastic Relationship (12.2.1)**
  - Equations
    - $\sigma = E\varepsilon,\ (\sigma \le \sigma_0)$
    - $\sigma = \sigma_0,\ (\varepsilon \ge \sigma_0/E)$
    - $\varepsilon = \varepsilon_e + \varepsilon_p = \sigma/E + \varepsilon_p,\ (\varepsilon \ge \sigma_0/E)$
  - Application
    - For the initial yielding behavior or certain metals and other materials
    - Used as a simple idealization to make rough estimates, even where the stress-strain curve has a more complex shape



<Fig. 12.4> (a) Stress-strain curves and rheological models for elastic, perfectly plastic behavior

12.2 Stress-Strain Curves

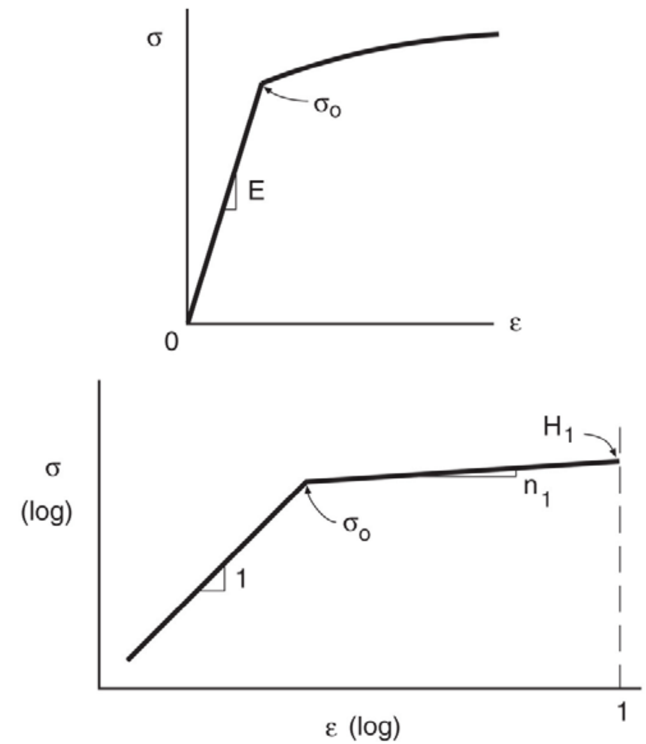
- **Elastic, Linear-Hardening Relationship (12.2.2)**
  - Equations
    - $\sigma = E\varepsilon, \qquad (\sigma \le \sigma_0)$
    - $\sigma = (1-\delta)\sigma_0 + \delta E\varepsilon, \qquad (\sigma \ge \sigma_0)$
    - $\varepsilon = \sigma_0/E + (\sigma - \sigma_0)/(\delta E), \quad (\sigma \ge \sigma_0)$
    - $\delta E = (\sigma - \sigma_0)/(\varepsilon - \varepsilon_0)$
      where δ is *reduction factor*
  - Rheological model
    - $\varepsilon = \sigma/E_1 + (\sigma - \sigma_0)/E_2, \qquad (\sigma \ge \sigma_0)$
    - Hence, $E = E_1, \quad \delta E = E_1E_2/(E_1+E_2)$



<Fig. 12.4> (b) Stress-strain curves and rheological models for elastic, linear-hardening behavior

12.2 Stress-Strain Curves

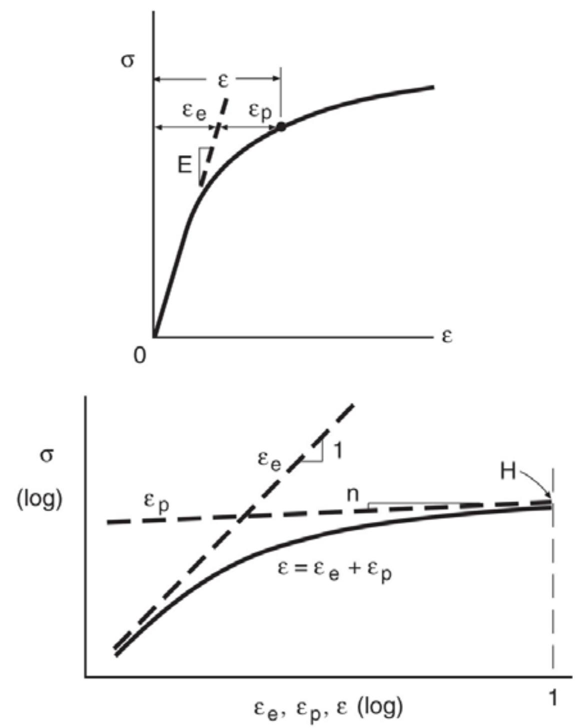
- **Elastic, Power-Hardening Relationship (12.2.3)**
  - Equations
    - $\sigma = E\varepsilon, \qquad (\sigma \le \sigma_0)$
    - $\sigma = H_1 \varepsilon^{n_1}, \qquad (\sigma \ge \sigma_0)$
    - $\varepsilon = (\sigma / H_1)^{1/n_1}, \qquad (\sigma \ge \sigma_0)$
  - Characteristics
    - *Strain hardening exponent* n_1
      - Typically in the range 0.05 to 0.4 for metals
    - Additional constant H_1
      - The value of σ at $\varepsilon = 1$
    - σ_0, H_1, n_1 are dependent each other
      ➔ $\sigma_0 = E(H_1/E)^{1/(1-n_1)}$



<Fig. 12.5> (a) Stress-strain curves on linear and logarithmic coordinates for an elastic, power-hardening relationship

12.2 Stress-Strain Curves

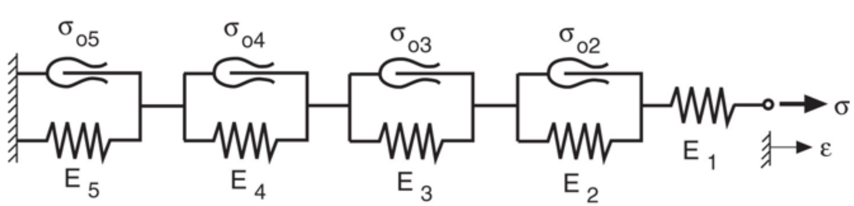
- **Ramberg-Osgood Relationship (12.2.4)**
  - Equations
    - $\sigma = E\varepsilon, \qquad (\sigma \leq \sigma_0)$
    - $\sigma = H\varepsilon_p^n, \qquad (\sigma \geq \sigma_0)$
    - $\varepsilon = \varepsilon_e + \varepsilon_p = \sigma/E + (\sigma/H)^{1/n}, \qquad (\sigma \geq \sigma_0)$
  - Characteristics
    - *<u>Strain hardening exponent</u>* n
      - Defined differently than the previous n_1
    - Additional constant H
      - The value of σ at $\varepsilon_p = 1$
    - Smooth curve for all values of σ
      - No distinct yield point
      - Thus, a yield strength is defined as the stress corresponding to a given plastic strain offset, such as $\varepsilon_{p0} = 0.002$
        ➔ $\sigma_0 = H(0.002)^n$
    - Ramberg-Osgood form is often applied to true stresses and strains for tension tests

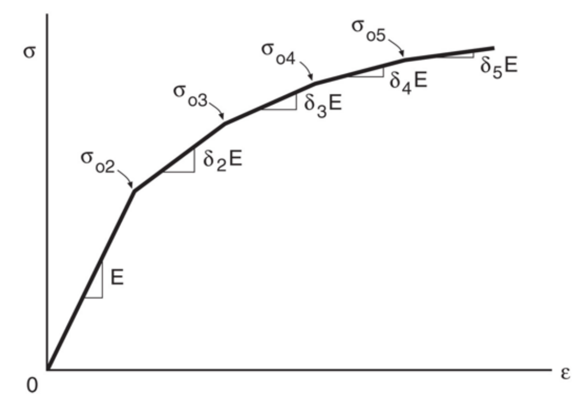


<Fig. 12.5> (b) Stress-strain curves on linear and logarithmic coordinates for the Ramberg-Osgood relationship

12.2 Stress-Strain Curves

- **Rheological Modeling of Nonlinear Hardening (12.2.5)**
  - Idea
    - ➢ Series of straight line segments ➔ Approximate the nonlinear hardening
    - ➢ The first segments ends
      - at the yield strength for the elastic, power-hardening case
      - at a low stress where the ε_p is small for the Ramberg-Osgood case
    - ➢ Equations
      - $\sigma = \sigma_{0i} + E_i \varepsilon_i, \quad (\sigma > \sigma_{0i})$
      - $\varepsilon = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_j$
      - $\varepsilon = \sigma / E_1 + (\sigma - \sigma_{02})/E_2 + \cdots \left(\sigma - \sigma_{0j}\right)/E_j$
      - $d\sigma / d\varepsilon = \delta_j E = \frac{1}{1/E_1 + 1/E_2 + \cdots + 1/E_j}$





<Fig. 12.6> Multistage spring and slider model for nonlinear-hardening stress–strain curves

12.3 Three-dimensional Stress-Strain Relationships (optional)

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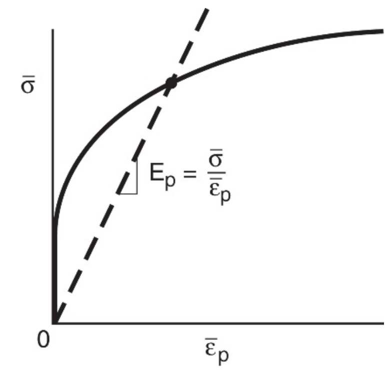
- **Need**
  - Stress components in more than one direction affects
    - Both a material's elastic stiffness and its yield strength
  - The state of stress continues to affect the behavior during the plastic deform.
- **Generalized Hooke's law**
  - Only the elastic portion of the strain
  - $\varepsilon_{ex} = \left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]/E$
  - $\varepsilon_{ey} = \left[\sigma_y - \nu(\sigma_x + \sigma_z)\right]/E$
  - $\varepsilon_{ez} = \left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]/E$
  - $\gamma_{exy} = \tau_{xy}/G, \quad \gamma_{eyz} = \tau_{yz}/G, \quad \gamma_{ezx} = \tau_{zx}/G,$
  - cf. $\varepsilon_x = \varepsilon_{ex} + \varepsilon_{px}, \quad \gamma_{xy} = \gamma_{exy} + \gamma_{pxy}$
- **Effective stress and strain (12.3.1)**
  - Effective stress $\bar{\sigma} = 1/\sqrt{2} \times \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$
  - Effective plastic strain $\bar{\varepsilon}_p = \sqrt{2}/3 \times \sqrt{\left(\varepsilon_{p1} - \varepsilon_{p2}\right)^2 + \left(\varepsilon_{p2} - \varepsilon_{p3}\right)^2 + \left(\varepsilon_{p3} - \varepsilon_{p1}\right)^2}$
  - Effective total strain $\varepsilon = \bar{\sigma}/E + \bar{\varepsilon}_p$

12.3 Three-dimensional Stress-Strain Relationships (optional)

- **Equations for plastic strains (12.3.1)**
  - Plastic modulus $E_p = \bar{\sigma}/\bar{\varepsilon}_p$
  - $\varepsilon_{px} = [\sigma_x - 0.5(\sigma_y + \sigma_z)]/E_p$
  - $\varepsilon_{py} = [\sigma_y - 0.5(\sigma_x + \sigma_z)]/E_p$
  - $\varepsilon_{pz} = [\sigma_z - 0.5(\sigma_x + \sigma_y)]/E_p$
  - $\gamma_{pxy} = 3\tau_{xy}/E_p, \quad \gamma_{pyz} = 3\tau_{yz}/E_p, \quad \gamma_{pzx} = 3\tau_{zx}/E_p$



<Fig. 12.8> Definition of the plastic modulus as the secant modulus to a point on the effective stress versus effective plastic strain curve

- **Equations for total strains (12.3.2)**
  - $\varepsilon_x = \varepsilon_{ex} + \varepsilon_{px} = [\sigma_x - \nu(\sigma_y + \sigma_z)]/E + [\sigma_x - 0.5(\sigma_y + \sigma_z)]/E_p$
    $= [\sigma_x - \bar{\nu}(\sigma_y + \sigma_z)]/E_t$
  - $\varepsilon_y = [\sigma_y - \bar{\nu}(\sigma_x + \sigma_z)]/E_t$
  - $\varepsilon_z = [\sigma_z - \bar{\nu}(\sigma_x + \sigma_y)]/E_t$
  - where $E_t = \bar{\sigma}/\bar{\varepsilon}, \quad \bar{\nu} = [\nu \times (\bar{\sigma}/E) + 0.5 \times \bar{\varepsilon}_p]/\bar{\varepsilon}$
  - Generalized Poisson's ratio $\bar{\nu}$
    - ➢ At small strain, $\bar{\nu} \to \nu$
    - ➢ At large strain, $\bar{\nu} \to 0.5$

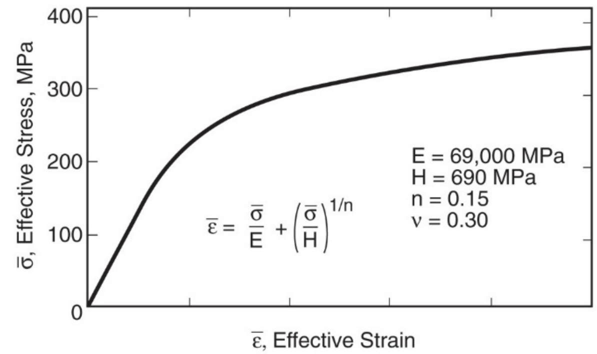
12.3 Three-dimensional Stress-Strain Relationships

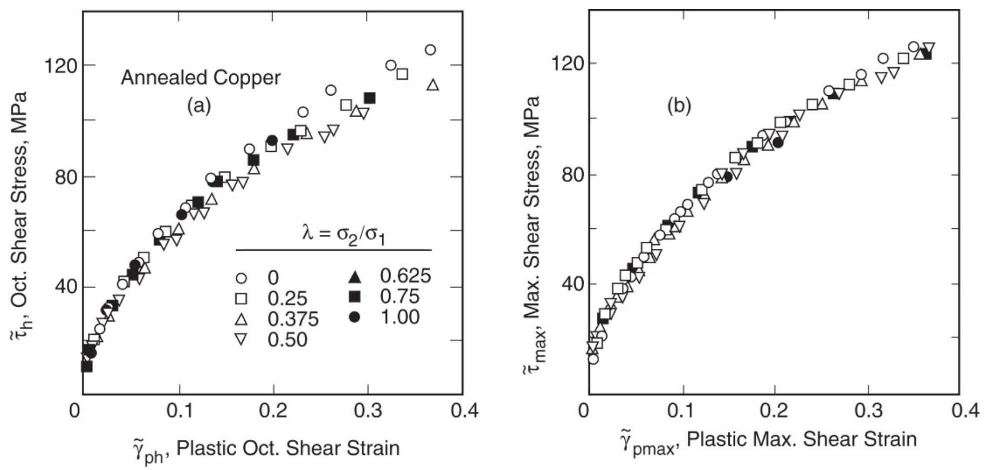
- **The Effective Stress-Strain Curve (12.3.3)**
  - For deformation plasticity theory, the curve relating effective stress and effective strain is the same as the uniaxial one
  - $\varepsilon_1 = f(\sigma_1) \quad (\sigma_2 = \sigma_3 = 0)$
  - $\bar{\varepsilon} = f(\bar{\sigma})$
- **Application to Plane Stress (12.3.4)**
  - Plane stress
    - $\sigma_x = \sigma_1, \quad \sigma_y = \sigma_2 = \lambda\sigma_1, \quad \sigma_z = \sigma_3 = 0$
  - Effective stress $\bar{\sigma} = \sigma_1\sqrt{1 - \lambda + \lambda^2}$
  - Total strain $\varepsilon_1 = \varepsilon_{e1} + \varepsilon_{p1}$
  - Elastic strain $\varepsilon_{e1} = (\sigma_1/E)(1 - \nu\lambda)$
  - Plastic strain $\varepsilon_{p1} = \left(\sigma_1/E_p\right)(1 - 0.5\lambda) = \left(\sigma_1\bar{\varepsilon}_p/\bar{\sigma}\right)(1 - 0.5\lambda)$
  - $\bar{\varepsilon}_p = \bar{\varepsilon} - \bar{\sigma}/E = f(\bar{\sigma}) - \bar{\sigma}/E$
  - Hence

$$\varepsilon_1 = \frac{1 - \nu\lambda}{E}\sigma_1 + \frac{(1 - 0.5\lambda)}{\bar{\sigma}}\sigma_1\left[f(\bar{\sigma}) - \frac{\bar{\sigma}}{E}\right] = f(\sigma_1)$$

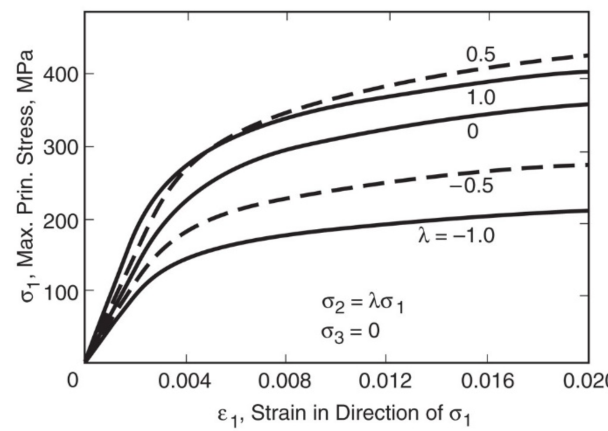
12.3 Three-dimensional Stress-Strain Relationships (optional)

- **Application to Plane Stress (12.3.4) – continued**
  - Key feature of deformation theory
    - ➢ It predicts a single curve relating $\bar{\sigma}$ and $\bar{\varepsilon}$ for all state of stress





<Fig. 12.7> Correlation of true stresses and true plastic strains from combined axial and pressure loading of thin-walled copper tubes in terms of (a) octahedral shear stress and strain, and (b) maximum shear stress and strain. (Adapted from [Davis 43]; used with permission of ASME.)



<Fig. 12.9> Estimated effect of biaxial stress on a Ramberg–Osgood stress–strain curve. (The constants correspond to a fictitious aluminum alloy.)

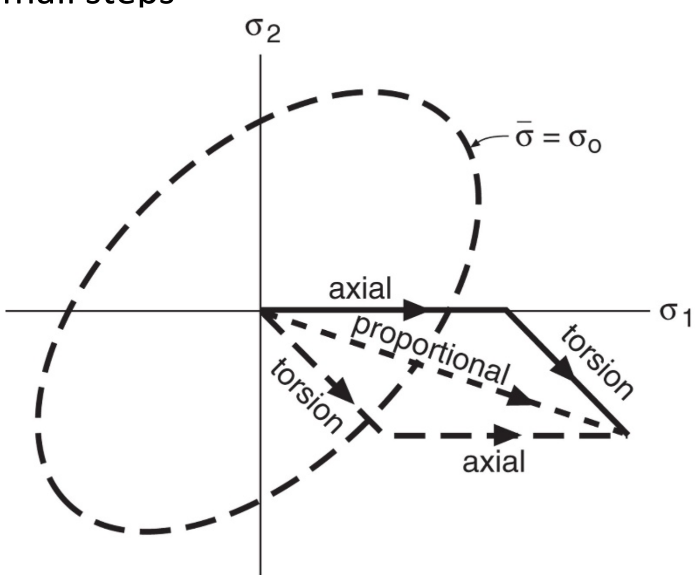
12.3 Three-dimensional Stress-Strain Relationships

- **Application to Plane Stress (12.3.4) – continued**
  - For an elastic, perfectly plastic material,
    - $\bar{\varepsilon} = f(\bar{\sigma})$ can be defined beyond yielding
    - $\sigma_1 = E\varepsilon_1/(1-\nu\lambda), \quad (\bar{\sigma} \le \sigma_0)$
    - $\sigma_1 = \sigma_0/\sqrt{1-\lambda+\lambda^2}, \quad (\bar{\varepsilon} \ge \sigma_0/E)$
  - Example: Ramberg-Osgood
    - $\bar{\varepsilon} = f(\bar{\sigma}) = \bar{\sigma}/E + (\bar{\sigma}/H)^{1/n}$
    - Using $\varepsilon_1 = \frac{1-\nu\lambda}{E}\sigma_1 + \frac{(1-0.5\lambda)}{\bar{\sigma}}\sigma_1\left[f(\bar{\sigma}) - \frac{\bar{\sigma}}{E}\right]$,

$$\varepsilon_1 = (1-\nu\lambda)\frac{\sigma_1}{E} + (1-0.5\lambda)(1-\lambda+\lambda^2)^{(1-n)/(2n)}\left(\frac{\sigma_1}{H}\right)^{1/n}$$

12.3 Three-dimensional Stress-Strain Relationships (optional)

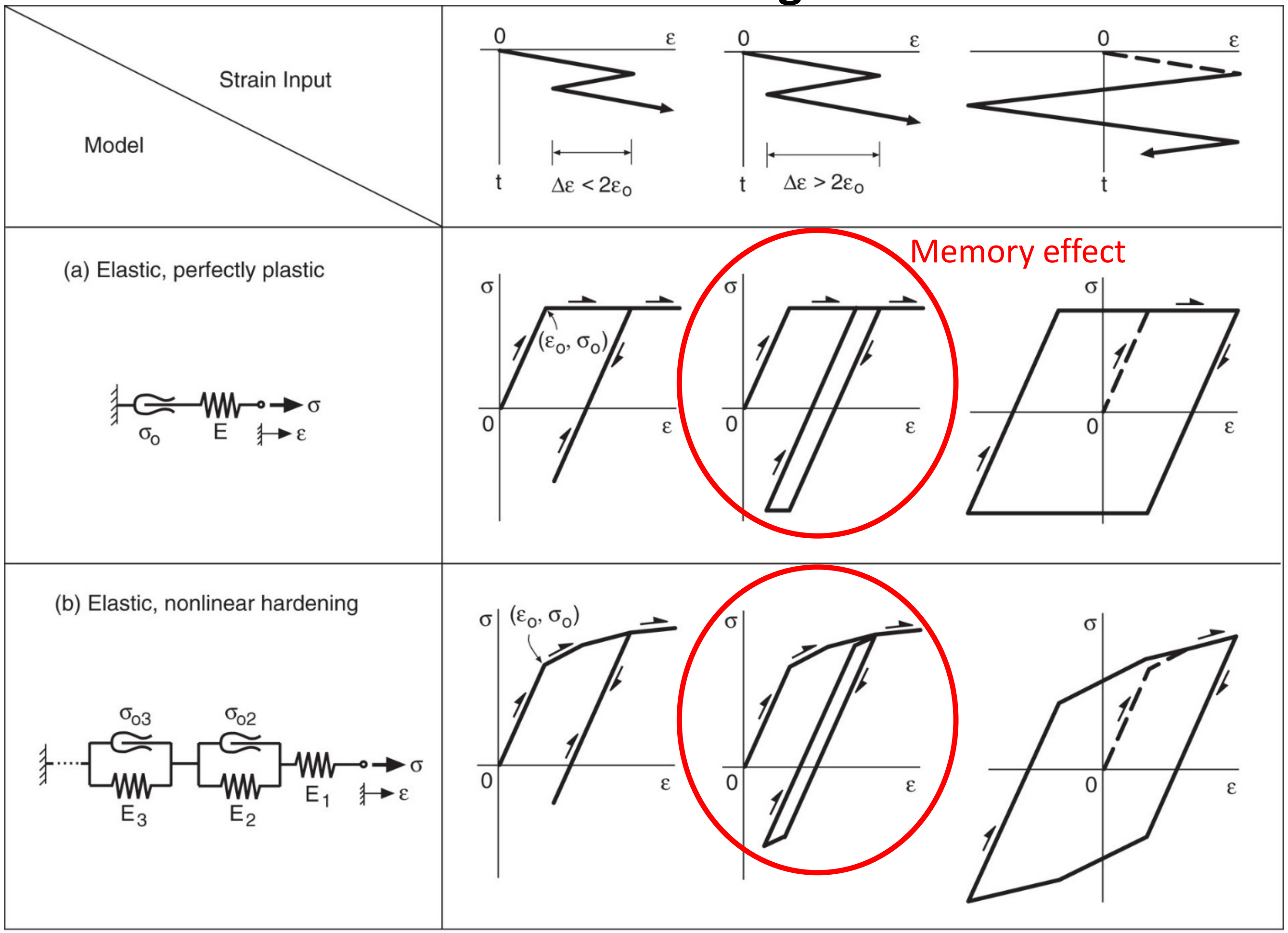
- **Deformation versus Incremental Plasticity Theories (12.3.5)**
  - Loading paths
    - ➢ The sequences of stressing correspond to variations in principal stresses
  - Loading path dependence
    - ➢ The situation of the plastic strains differing despite the final stresses being the same
    - ➢ To analyze such behavior, an *incremental plasticity theory* is needed, which is applied by following the loading path in small steps
  - Deformation versus Incremental
    - ➢ If all stresses are applied so that their magnitudes are proportional, and if no unloading occurs, then incremental plasticity theory gives the same result as deformation theory
    - ➢ If proportionality is preserved, but unloading does occur, as in cyclic loading, deformation theory can still be used



<Fig. 12.10> Three possible paths for combined axial and torsional loading of a thin-walled tube

12.4 Unloading and Cycling loading Behavior from Rheological Models

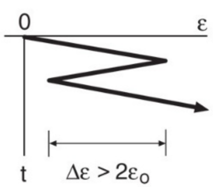
- **Some features of the behavior of rheological models**



<Fig. 12.11>Unloading and reloading behavior for two rheological models. The first strain history causes only elastic deformation during unloading, but the second one is sufficiently large to cause compressive yielding. The third history is completely reversed and causes a hysteresis loop that is symmetrical about the origin.

12.4 Unloading and Cycling loading Behavior from Rheological Models

- **Some features of the behavior of rheological models – continued**
  - Features
    - If the direction of loading is reversed, the behavior may be elastic until yielding again occurs on reloading
    - Reversed yielding occurs when the stress change since unloading reaches $\Delta\sigma = 2\sigma_0$
  - Memory effect
    - Reversed yielding occurs, causing a small loop to be formed, following which the stress-strain path rejoins the original path, then proceeding just as if the small loop had never occurred



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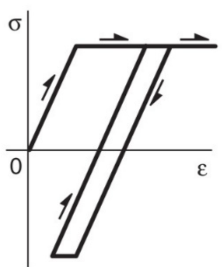
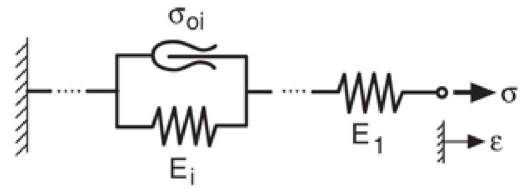


<σ-ε behavior>

12.4 Unloading and Cycling loading Behavior from Rheological Models

- **Unloading Behavior (12.4.1)**
  - The i^th stage of a multistage parallel spring-slider model
    - $\sigma' = +\sigma_{oi} + E_i \varepsilon_i'$
    - $\sigma'' = -\sigma_{oi} + E_i \varepsilon_i'$
      - ε_i' is *locked* until the resistance of the slider is overcome, that is, at the point of reversed yielding



    - The change in stress causing reversed yielding in the i^th stage

$$\Delta\sigma'' = \sigma' - \sigma'' = 2\sigma_{oi}$$

    - The contribution of the i^th stage to the change in strain

$$\Delta\varepsilon_i = (\Delta\sigma - 2\sigma_{oi})/E_i$$

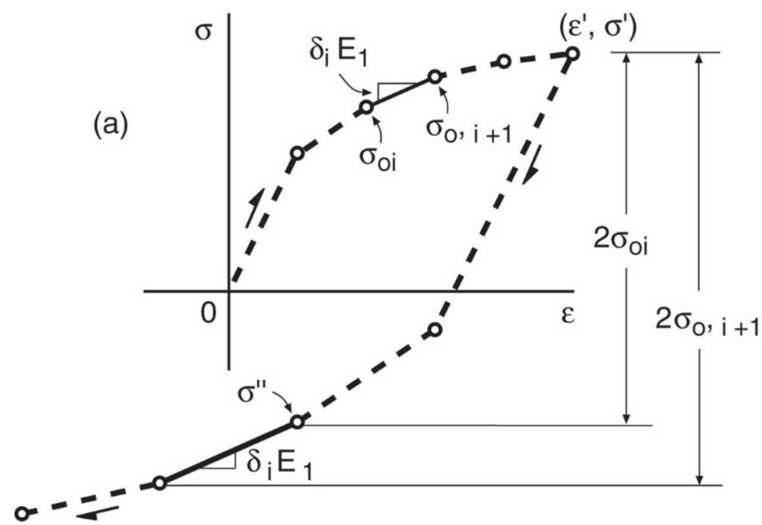
    - Hence, the total change in strain

$$\Delta\varepsilon = \frac{\Delta\sigma}{E_1} + \frac{\Delta\sigma - 2\sigma_{o2}}{E_2} + \cdots + \frac{\Delta\sigma - 2\sigma_{oj}}{E_j}$$

    - Then, with $\sigma = \sigma' - \Delta\sigma,\ \varepsilon = \varepsilon' - \Delta\varepsilon$,

$$\frac{d\sigma}{d\varepsilon} = \frac{1}{\frac{1}{E_1} + \frac{1}{E_2} + \cdots \frac{1}{E_j}}$$

      - Same as for the monotonic one



<Fig. 12.12> (a) Unloading behavior of spring and slider rheological models showing doubling of segment lengths with the slope unchanged

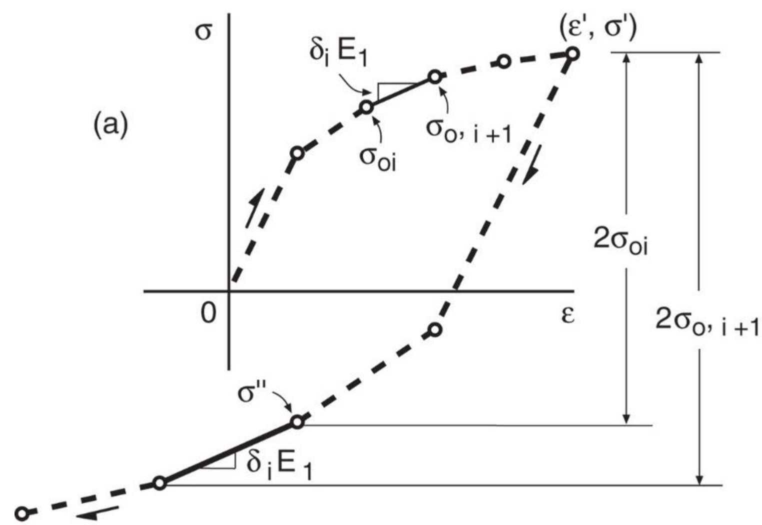
12.4 Unloading and Cycling loading Behavior from Rheological Models

- **Unloading Behavior (12.4.1) – continued**
  - The i^{th} stage of a multistage parallel spring-slider model – continued
    - Unloading curve has the same shape as the monotonic curve, differing in that it is expanded by a scale factor of two
      - If the monotonic reponse is $\varepsilon = f(\sigma)$

$$\frac{\Delta\varepsilon}{2} = f\left(\frac{\Delta\sigma}{2}\right)$$

    - The initial loading in compression

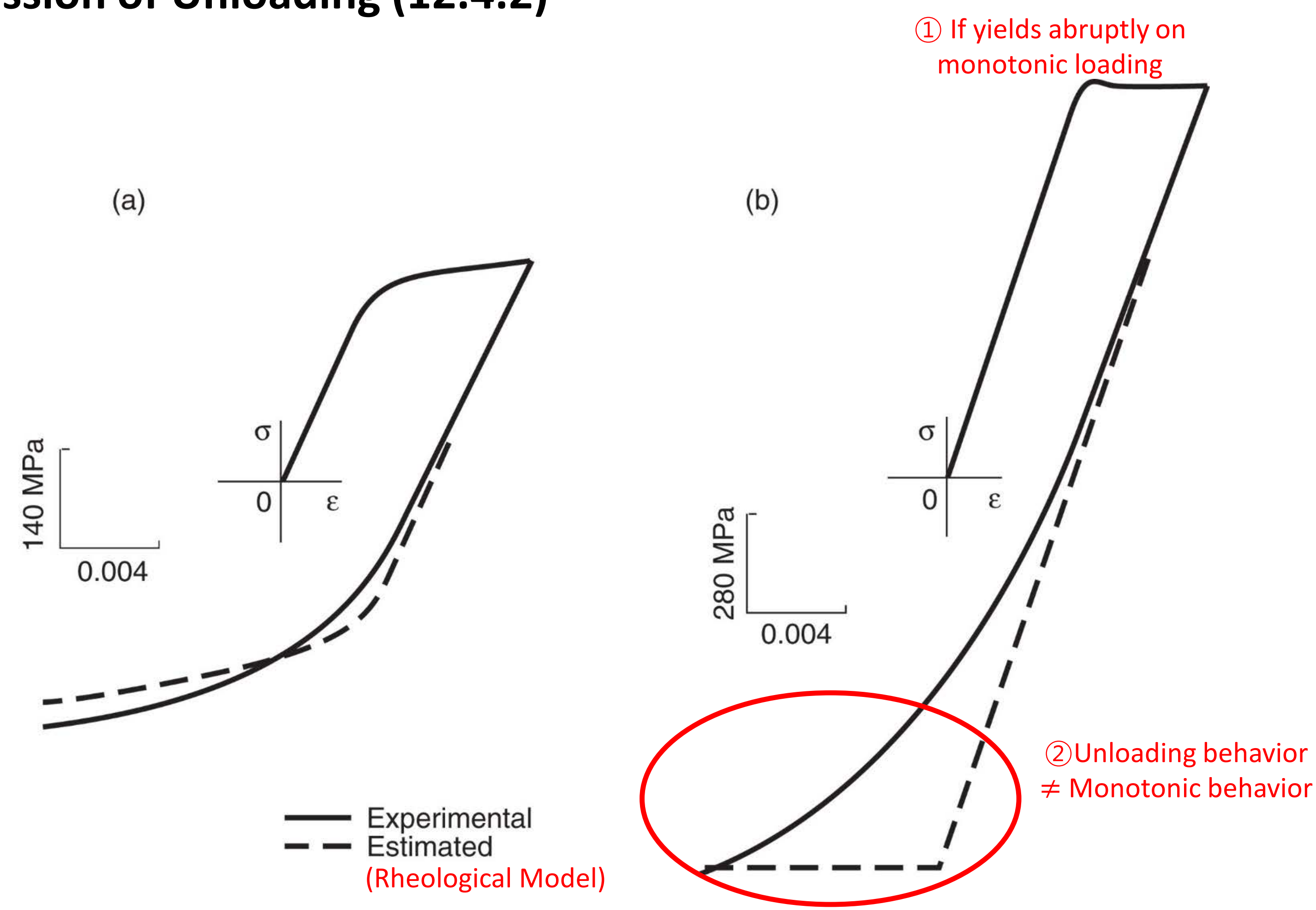
$$\varepsilon = -f(-\sigma)$$



<Fig. 12.12> (a) Unloading behavior of spring and slider rheological models showing doubling of segment lengths with the slope unchanged

12.4 Unloading and Cycling loading Behavior from Rheological Models (optional)

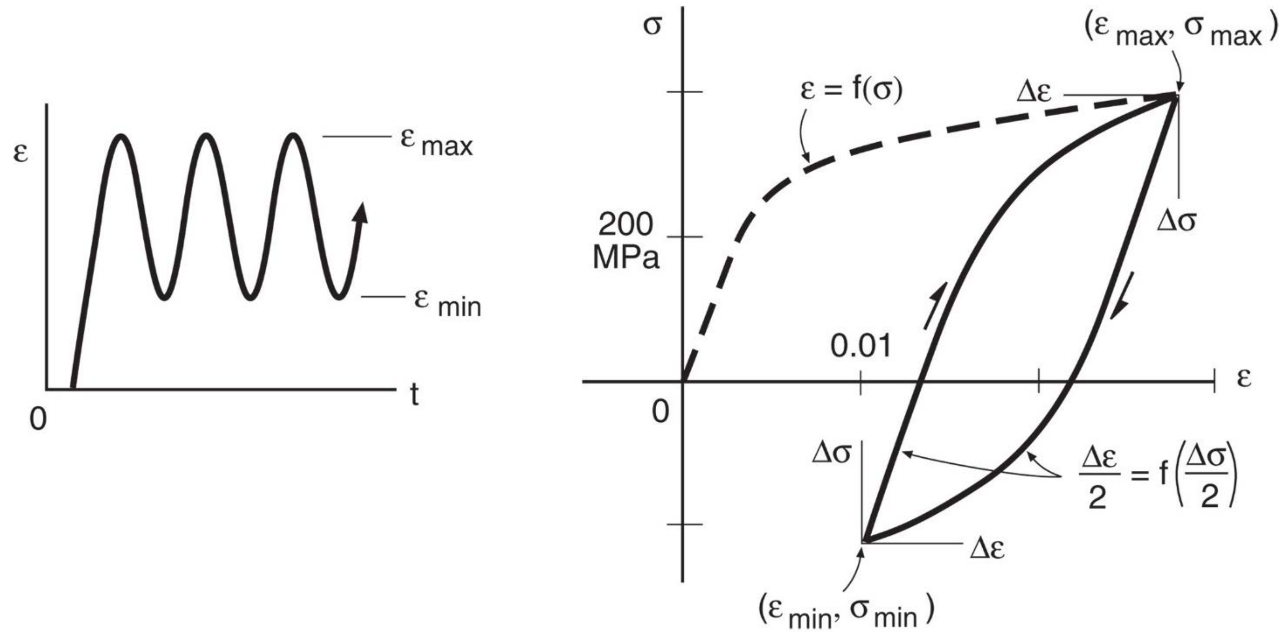
- **Discussion of Unloading (12.4.2)**



<Fig. 12.13> Monotonic tension followed by loading into compression for (a) aluminum alloy 2024-T4 and (b) quenched and tempered AISI 4340 steel. Unloading curves estimated from a factor-of-two expansion of the monotonic curve are also shown.

12.4 Unloading and Cycling loading Behavior from Rheological Models

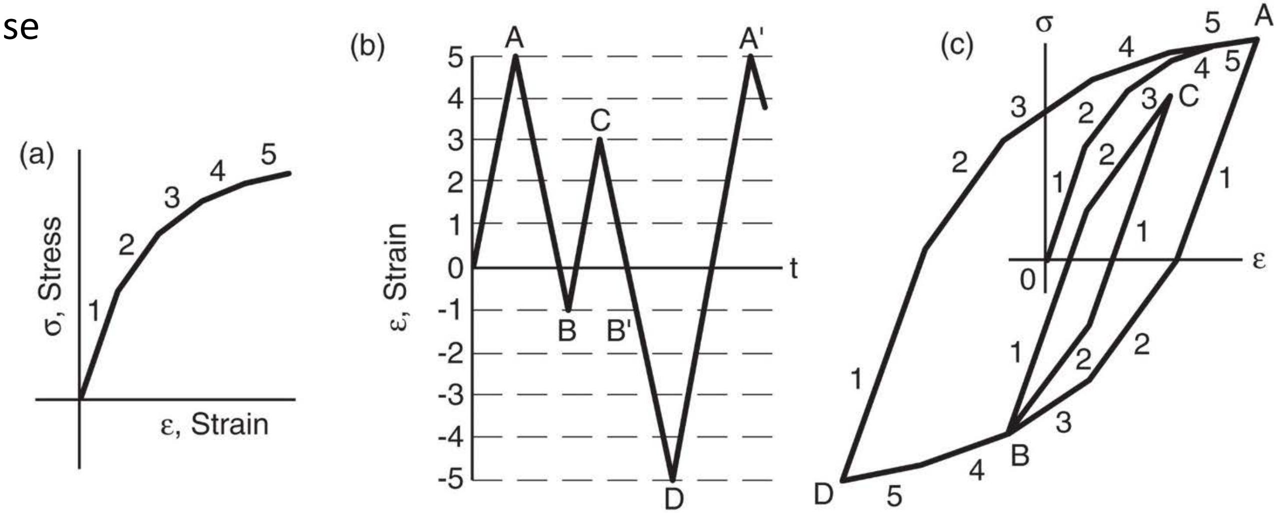
- **Cyclic Loading Behavior (12.4.3)**
  - *Hysteresis loop*
    - ➢ The stress-strain loop during constant amplitude cycling
    - ➢ Unloading branch: $\varepsilon = \varepsilon_{max} - 2f[(\sigma_{max} - \sigma)/2]$
    - ➢ Reloading branch: $\varepsilon = \varepsilon_{min} + 2f[(\sigma - \sigma_{min})/2]$



<Fig. 12.14> Stress–strain unloading and reloading behavior consistent with a spring and slider rheological model. The example curves plotted correspond to a Ramberg–Osgood stress–strain curve with constants as in Fig. 12.9.

12.4 Unloading and Cycling loading Behavior from Rheological Models

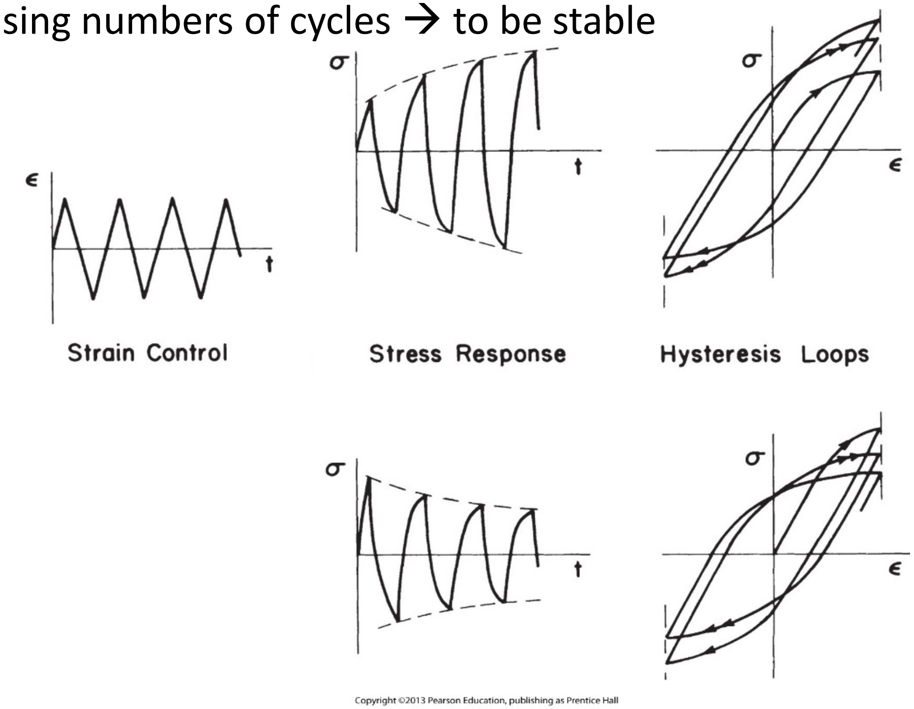
- **Application to Irregular Strain versus Time Histories (12.4.4)**
  - The rules for the behavior of a multistage spring and slider model
    - ➢ (1) Initially, and after each reversal of strain direction, segments are used in order, starting with the first
    - ➢ (2) Each segment may be used once in either direction with its original length. Thereafter, the length is twice the monotonic value and the segment retains the same slope
    - ➢ (3) An exception to rule (1) is that a segment (or portion thereof) must be skipped if its most recent use was not in the opposite direction of its impending use



<Fig. 12.15> Behavior of a multistage spring–slider rheological model for an irregular strain history. A model having the monotonic stress–strain curve (a) is subjected to strain history (b), resulting in stress–strain response (c). (Adapted from [Dowling 79b]; used with permission of Elsevier Science Publishers.)

12.5 Cyclic Stress-Strain Behavior of Real Materials

- **Cyclic Stress-Strain Tests and Behavior (12.5.1)**
  - *Cyclic-dependent hardening* and *Cyclic-dependent softening*
    - ➢ The stresses that are needed to enforce the strain limits usually change as the test progresses
    - ➢ Cyclic hardening or softening is usually rapid at first and decreases with increasing numbers of cycles → to be stable



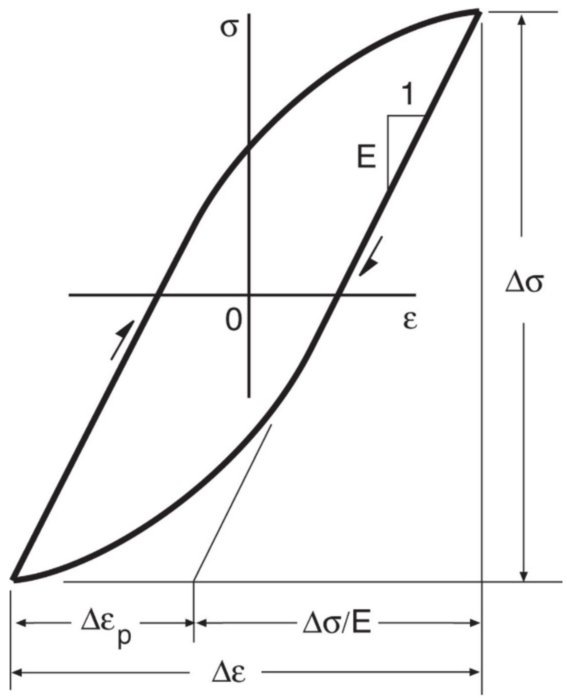
Cyclic-dependent hardening

Cyclic-dependent softening

<Fig. 12.16> Completely reversed controlled strain test and two possible stress responses, cycle-dependent hardening and softening. (From [Landgraf 70]; copyright © ASTM; reprinted with permission.)

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

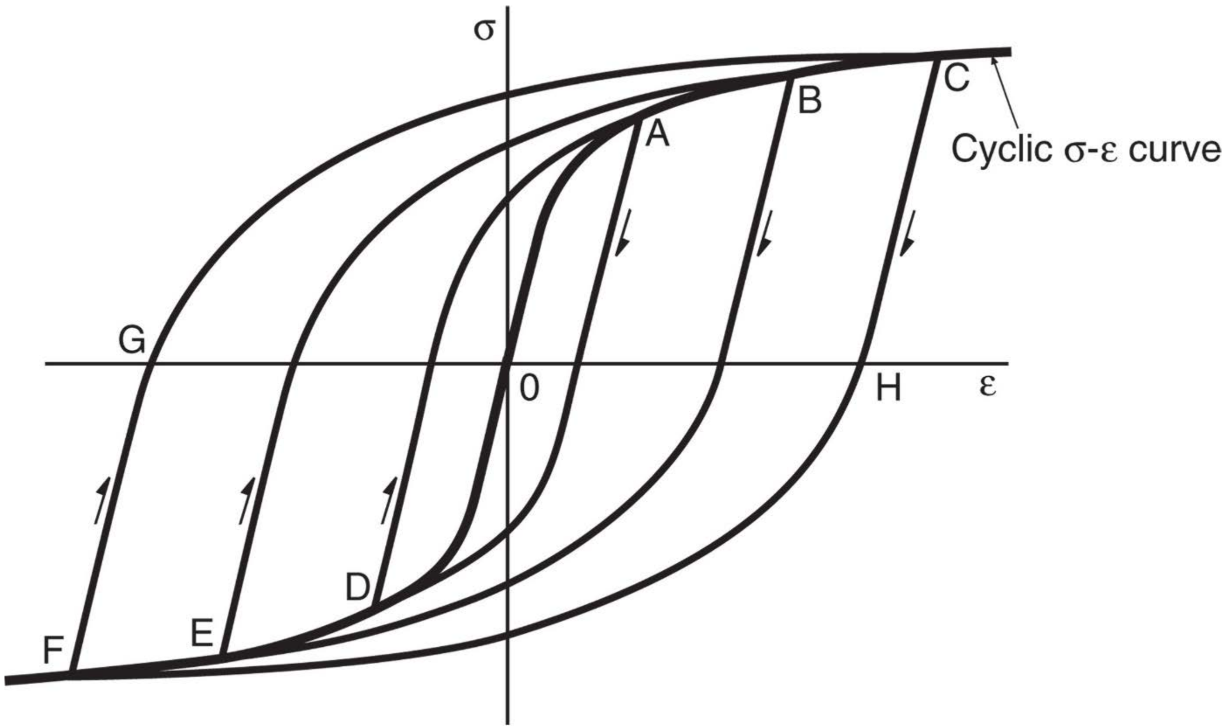
- **Cyclic Stress-Strain Tests and Behavior (12.5.1)**
  - Closed hysteresis loop (Stable behavior)
    - ➢ The stable hysteresis loops are nearly symmetrical with respect to tension and compression in most engineering metals
      - One exception is gray cast iron (asymmetric behavior)
    - ➢ Ductile polymers and their composites → asymmetric behavior



<Fig. 12.17> Stable stress–strain hysteresis loop

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

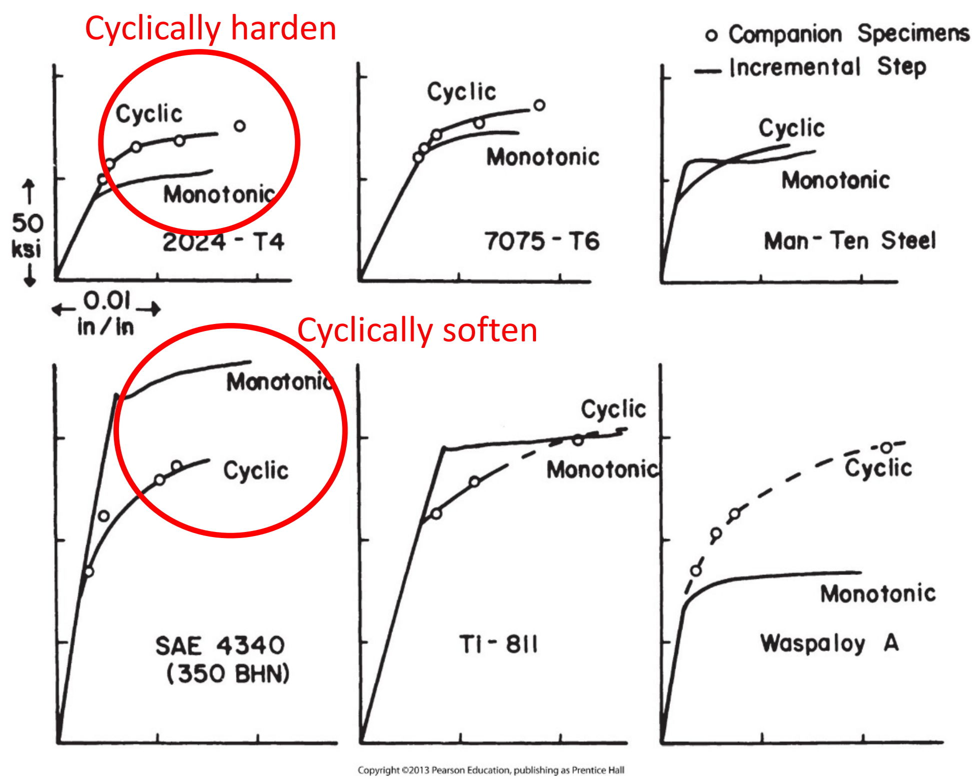
- **Cyclic Stress-Strain Curves and Trends (12.5.2)**
  - Stable behavior: The hysteresis loops from near the fatigue life(conventional)
  - *Cyclic stress-strain curves*
    - A line from the origin that passes through the tips of the loops (O-A-B-C below)
    - Always deviate smoothly from linearity → Ramberg-Osgood commonly used



<Fig. 12.19> Cyclic stress–strain curve defined as the locus of tips of hysteresis loops. Three loops are shown, A-D, B-E, and C-F. The tensile branch of the cyclic stress–strain curve is O-A-B-C, and the compressive branch is O-D-E-F.

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

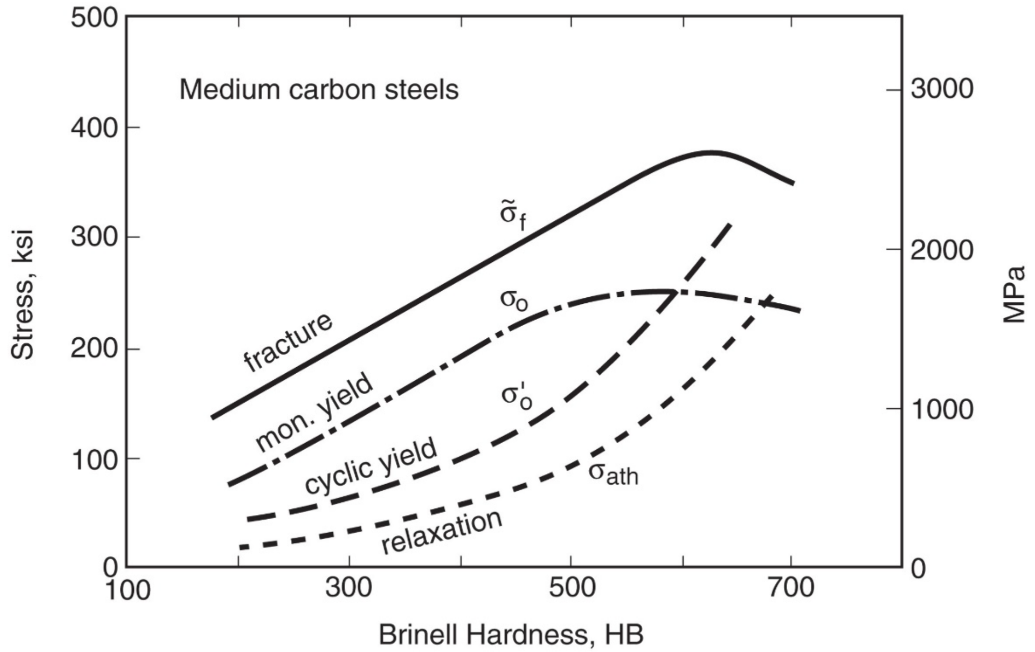
- **Cyclic Stress-Strain Curves and Trends (12.5.2) –continued**
  - *Cyclic stress-strain curves* –continued
    - Ramberg-Osgood form: $\varepsilon_a = \sigma_a/E + (\sigma_a/H')^{1/n'}$
      - For engineering metals, n' often ranges 0.1 to 0.2



<Fig. 12.20> Cyclic and monotonic stress–strain curves for several engineering metals. (From [Landgraf 69]; copyright © ASTM; reprinted with permission.)

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

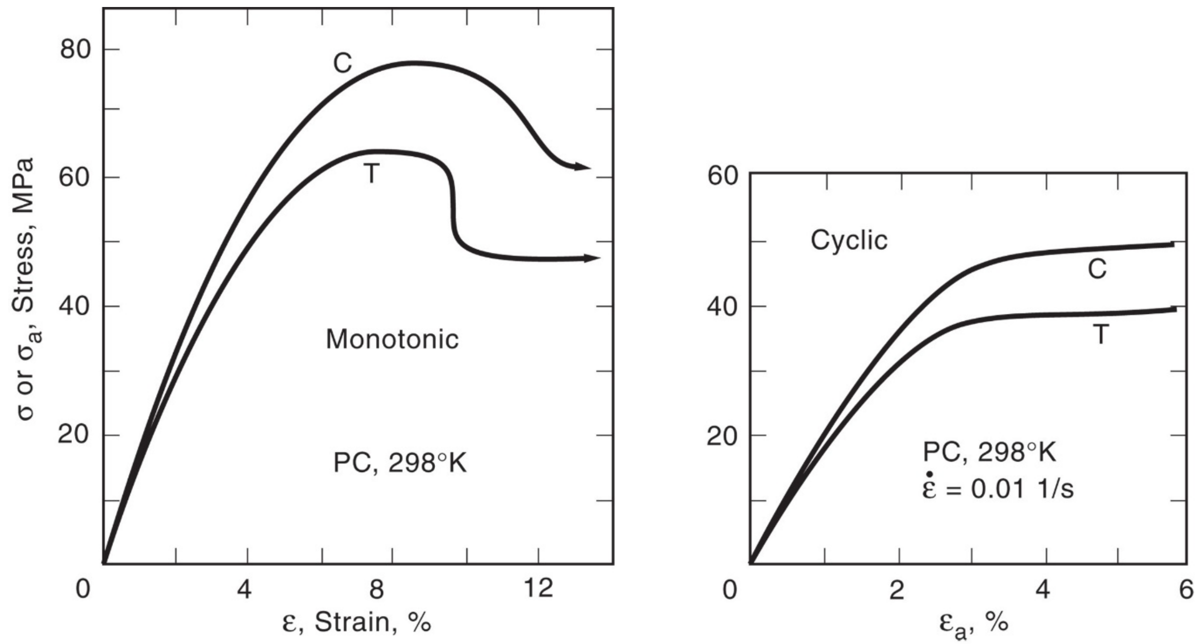
- **Cyclic Stress-Strain Curves and Trends (12.5.2) –continued**
  - The effects of alloy composition and processing of engineering metlas
    - ➢ The effects on the cyclic stress-strain behavior is different with that on the monotonic tension properties
    - ➢ In medium-carbon steels that are hardened by heat treatment, some of the effect usually is lost if cyclic loading occurs (shown below)



<Fig. 12.21> Average property trends for SAE 1045 steel and other medium-carbon steels as a function of hardness, including the true fracture strength, the monotonic and cyclic yield strengths, and the threshold stress amplitude for relaxation of mean stress. (Adapted from [Landgraf 88]; copyright © ASTM; reprinted with permission.)

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

- **Cyclic Stress-Strain Curves and Trends (12.5.2) –continued**
  - Ductile polymers
    - ➢ It has monotonic yield strengths that are typically 20% to 30% higher in compression than in tension
    - ➢ This ratio is maintained in the cyclic stress-strain curves (shown below)

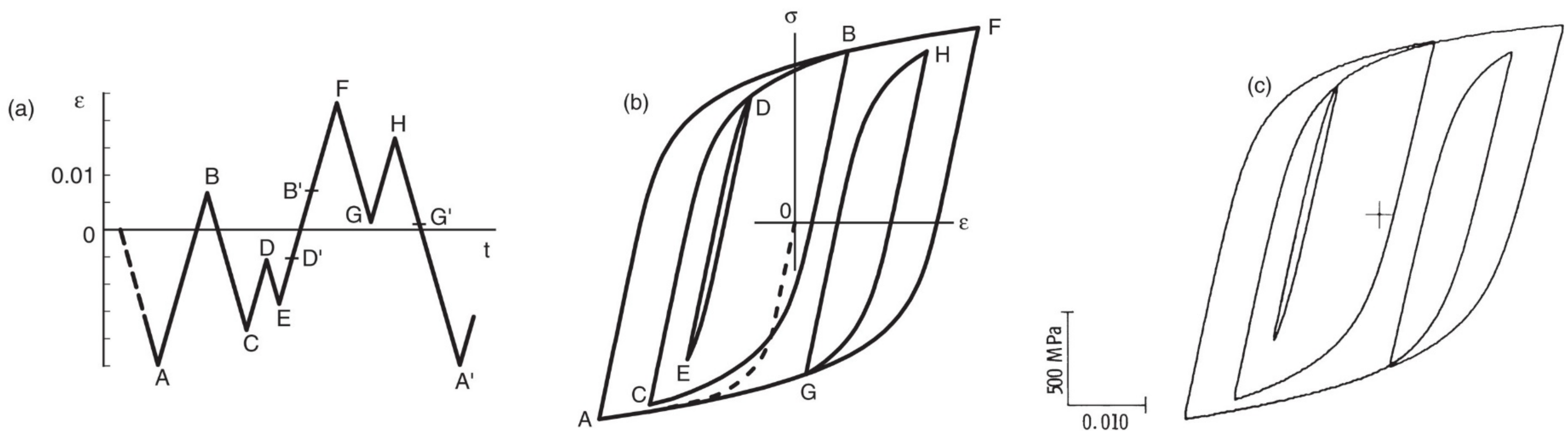


<Fig. 12.22> Monotonic (left) and cyclic (right) stress–strain curves for polycarbonate for both tension (T) and compression (C). (Adapted from [Beardmore 75]; used with permission.)

12.5 Cyclic Stress-Strain Behavior of Real Materials

- **Hysteresis Loop Curve Shapes (12.5.3)**
  - Hysteresis loop should have the same shape as the cyclic stress-strain curve, except for expansion by a scale factor of two
  - Ramberg-Osgood form using $\Delta\varepsilon/2 = f(\Delta\sigma/2)$

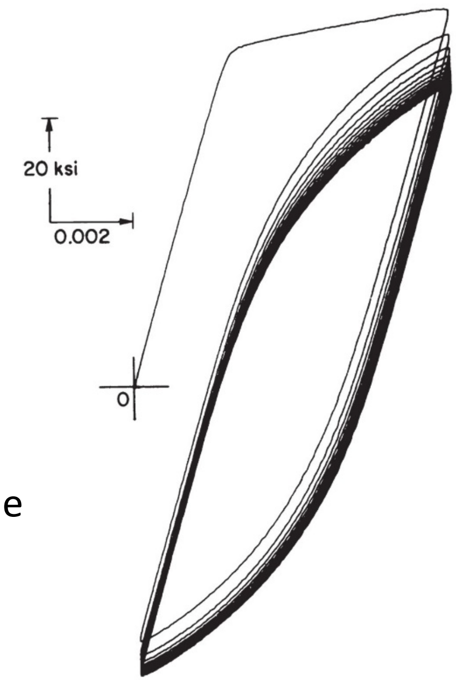
$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2H'}\right)^{1/n'}$$



<Fig. 12.24> Stable stress–strain response of AISI 4340 steel (σu = 1158 MPa) subjected to a repeatedly applied irregular strain history (a). The predicted response is shown in (b) and actual test data in (c). (Adapted from [Dowling 79b]; used with permission of Elsevier Science Publishers.)

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

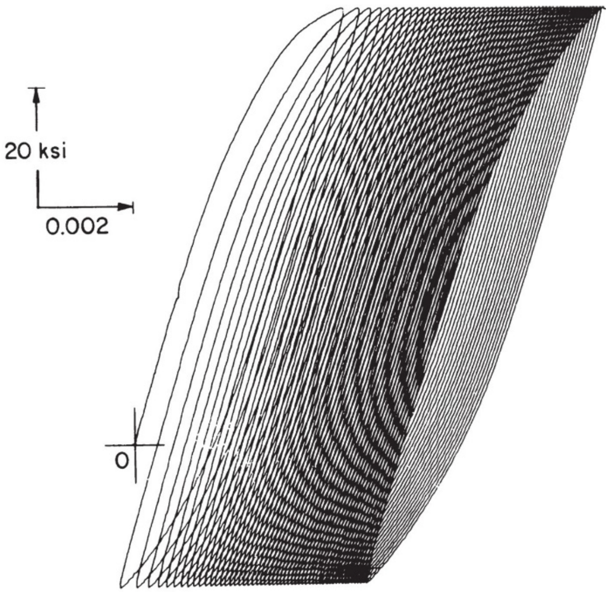
- **Transient Behavior; Mean stress Relaxation (12.5.4)**
  - Limitation of the Rheological model
    - Only stable behavior for cyclic loading
    - The transient behavior(cycle-dependent hardening or softening) is not predicted
    - Still exhibit 'stable' behavior even if the mean stress is not zero
  - *Cycle-dependent relaxation*
    - Real material behavior with the biased strain limits
    - With enough plasticity, the resulting mean stress gradually shift toward zero as increasing numbers of cycles
    - Sometimes a stable nonzero value is reached, or if the degree of plasticity is large, the mean stress may shift essentially to zero
    - Enhanced by cyclic softening occurring at the same time



<Fig. 12.25> Cycle-dependent relaxation of mean stress for an AISI 1045 steel. (From [Landgraf 70]; copyright © ASTM; reprinted with permission.)

12.5 Cyclic Stress-Strain Behavior of Real Materials (optional)

- **Transient Behavior; Mean stress Relaxation (12.5.4)**
  - Limitation of the Rheological model
    - ➢ Only stable behavior for cyclic loading
    - ➢ The transient behavior(cycle-dependent hardening or softening) is not predicted
    - ➢ Still exhibit 'stable' behavior even if the mean stress is not zero
  - *Cycle-dependent creep or ratchetting*
    - ➢ Real material with the biased stress limits
    - ➢ The mean strain increases with cycles
    - ➢ The mean strain shift may decrease its rate and stop, it may establish and constant rate, or it may accelerate and lead to a failure somewhat similar to that in a tension test



<Fig. 12.25> Cycle-dependent creep for an AISI 1045 steel. The specimen was previously yielded, so that the monotonic curve does not appear. (From [Landgraf 70]; copyright © ASTM; reprinted with permission.)

12.6 Summary

- **Stress-strain relationships**
  - ① Elastic, perfectly plastic, ② Elastic, linear-hardening, ③ Power-hardening, ④ Ramberg-Osgood
- **Three-dimensional states of stress**
  - Elastic portion: Hooke's law $\left(\varepsilon_{ex} = \left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]/E\right)$
  - Plastic portion: Deform. plasticity theory $\left(\varepsilon_{px} = \left[\sigma_x - 0.5\left(\sigma_y + \sigma_z\right)\right]/E_p\right)$
- **A key feature of deformation plasticity theory**
  - A single $\bar{\sigma}$ vs. $\bar{\varepsilon}_p$ curve which is independent from the state of stress.
- **Unloading Behavior**
  - Follow a path that is given by a factor-of-two expansion of the monotonic stress-strain curve

$$\Delta\varepsilon/2 = f(\Delta\sigma/2)$$

- **Cyclic loading Behavior**
  - Replace the monotonic curve with a special cyclic stress-strain curve, with the Ramberg-Osgood form often being used

$$\varepsilon_a = \sigma_a/E + (\sigma_a/H')^{1/n'}$$