convective mass transfer

Parameters in convective mass transfer

$$\frac{\text{momentum diffusivity}}{\text{mass diffusivity}} = \mathrm{Sc} \equiv \frac{\nu}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

$$\frac{\text{thermal diffusivity}}{\text{mass diffusivity}} = \mathrm{Le} \equiv \frac{k}{\rho c_p D_{AB}}$$

$$\frac{\text{molecular mass transport resistance}}{\text{convective mass transfer resistance}} = Sh = \frac{k_c L}{D_{AB}} = \frac{-d\left(c_A - c_{A,s}\right)/dy\Big|_{y=0}}{\left(c_{A,s} - c_{A,\infty}\right)/L}$$

Dimensional analysis

transfer of mass from the walls of a circular conduit to a fluid

variables: tube diameter, fluid density, fluid viscosity, fluid velocity, fluid duffusivity, mass-transfer coefficient (6)

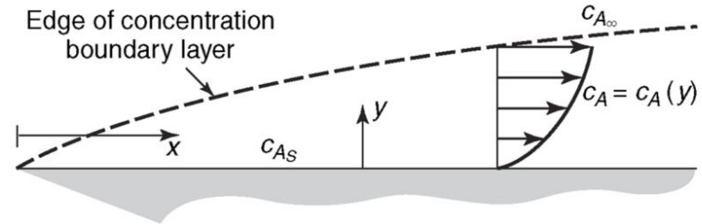
$$\mathrm{Sh} = f\left(\mathrm{Re}, \mathrm{Sc}\right)$$

transfer of mass from vertical wall to adjacent fluid with natural convection

variables: characteristic length, fluid diffusivity, fluid density, fluid viscosity, buoyant force, mass-transfer coefficient (6)

$$\mathrm{Sh} = f\left(\mathrm{Gr}_{AB}, \mathrm{Sc}\right) \qquad \mathrm{Gr}_{AB} = \frac{L^3 \rho g \Delta\rho_A}{\mu^2}$$

Laminar concentration boundary layer



$$v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} = \nu \frac{\partial^2 v_x}{\partial y^2}$$

$$momentum: \quad \frac{v_x}{v_\infty} = 0 \text{ at } y = 0 \quad \text{and} \quad \frac{v_x}{v_\infty} = 1 \text{ at } y = \infty$$

$$v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}$$

$$thermal: \quad \frac{T - T_s}{T_\infty - T_s} = 0 \text{ at } y = 0 \quad \text{and} \quad \frac{T - T_s}{T_\infty - T_s} = 1 \text{ at } y = \infty$$

$$v_x \frac{\partial c_A}{\partial x} + v_y \frac{\partial c_A}{\partial y} = D_{AB} \frac{\partial^2 c_A}{\partial y^2}$$

$$concentration: \quad \frac{c_A - c_{A,s}}{c_{A,\infty} - c_{A,s}} = 0 \text{ at } y = 0 \quad \text{and} \quad \frac{c_A - c_{A,s}}{c_{A,\infty} - c_{A,s}} = 1 \text{ at } y = \infty$$

The rate at which mass enters or leaves the boundary layer at the surface is so small that it does not alter the velocity profile.

heat | mass

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = \left(T_\infty - T_s\right)\left[\frac{0.332}{x}\mathrm{Re}_x^{1/2}\right]$$

$$\left.\frac{dc_A}{dy}\right|_{y=0} = \left(c_{A,\infty} - c_{A,s}\right)\left[\frac{0.332}{x}\mathrm{Re}_x^{1/2}\right]$$

$$\frac{q_y}{A} = h_x\left(T_s - T_\infty\right) = -k\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$N_{A,y} = k_c(C_{A,s} - C_{A,\infty}) = -D_{AB}\left.\frac{\partial c_A}{\partial y}\right|_{y=0}$$

Pr=1,
Sc=1

$$h_x = -\frac{k}{T_s - T_\infty}\left.\frac{\partial T}{\partial y}\right|_{y=0} = \frac{0.332k}{x}\mathrm{Re}_x^{1/2}$$

$$k_c = \frac{D_{AB}}{x}\left[0.332\,\mathrm{Re}_x^{1/2}\right]$$

$$\frac{h_x x}{k} = \mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}$$

$$\frac{k_c x}{D_{AB}} = \mathrm{Sh}_x = 0.332\,\mathrm{Re}_x^{1/2}$$

Pr≠1,
Sc≠1

$$\frac{\delta}{\delta_t} = \mathrm{Pr}^{1/3} \qquad \frac{h_x x}{k} = \mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\frac{\delta}{\delta_c} = \mathrm{Sc}^{1/3} \qquad \frac{k_c x}{D_{AB}} = \mathrm{Sh}_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Sc}^{1/3}$$

$$\frac{hL}{k} = \mathrm{Nu}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Pr}^{1/3}$$

$$\frac{\bar{k}_c L}{D_{AB}} = \mathrm{Sh}_L = 0.664\,\mathrm{Re}_L^{1/2}\,\mathrm{Sc}^{1/3}$$

Reynolds analogy (Pr=1, Sc=1)

heat	mass
$f' = 2\frac{v_x}{v_\infty} = 2\frac{T - T_s}{T_\infty - T_s}$	
$\frac{d}{dy}\frac{v_x}{v_\infty}\Big\vert_{y=0} = \frac{d}{dy}\left(\frac{T - T_s}{T_\infty - T_s}\right)\Big\vert_{y=0}$	$\frac{\partial}{\partial y}\left(\frac{c_A - c_{A,s}}{c_{A,\infty} - c_{A,s}}\right)\Big\vert_{y=0} = \frac{\partial}{\partial y}\left(\frac{v_x}{v_\infty}\right)\Big\vert_{y=0}$
$h\left(T_s - T_\infty\right) == -k\frac{\partial}{\partial y}\left(T - T_s\right)\Big\vert_{y=0}$	$N_{A,y} = -D_{AB}\frac{\partial}{\partial y}\left(c_A - c_{A,s}\right)\Big\vert_{y=0} = k_c\left(c_{A,s} - c_{A,\infty}\right)$
$h = \frac{\mu c_p}{v_\infty}\frac{dv_x}{dy}\Big\vert_{y=0}$	$k_c = \frac{\mu}{\rho v_\infty}\frac{\partial v_x}{\partial y}\Big\vert_{y=0}$
$C_f \cong \frac{\tau_0}{\rho v_\infty^2 / 2} = \frac{2\mu}{\rho v_\infty^2}\frac{dv_x}{dy}\Big\vert_{y=0}$	$C_f \cong \frac{\tau_0}{\rho v_\infty^2 / 2} = \frac{2\mu}{\rho v_\infty^2}\frac{dv_x}{dy}\Big\vert_{y=0}$
$\frac{h}{\rho v_\infty c_p} = \frac{C_f}{2}$	$\frac{k_c}{v_\infty} = \frac{C_f}{2}$

Turbulent flow

heat

mass

$$\tau_{yx} = \mu \frac{d\bar{v}_x}{dy} - \overline{\rho v'_x v'_y}$$

$$\frac{q_y}{A} = -\rho c_p \left(\alpha + \varepsilon_H\right) \frac{d\bar{T}}{dy}$$

$$N_{A,y} = -\left(D_{AB} + \varepsilon_D\right) \frac{d\bar{c}_A}{dy}$$

$$C_f \equiv \frac{\tau_s}{\rho\left(v_\infty^2 / 2\right)}$$

Prandtl analogy

$$\mathrm{Nu} = \frac{(C_f / 2)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 5\sqrt{C_f / 2}\left(\mathrm{Pr} - 1\right)}$$

$$\mathrm{Sh} = \frac{\left(C_f / 2\right)\mathrm{ReSc}}{1 + 5\sqrt{C_f / 2}\left(\mathrm{Sc} - 1\right)}$$

von Karman analogy

$$\mathrm{Sh} = \frac{\left(C_f / 2\right)\mathrm{ReSc}}{1 + 5\sqrt{C_f / 2}\left\{\mathrm{Sc} - 1 + \ln\left[\left(1 + 5\mathrm{Sc}\right)/6\right]\right\}}$$

$$\mathrm{Nu} = \frac{\left(C_f / 2\right)\mathrm{Re}\,\mathrm{Pr}}{1 + 5\sqrt{C_f / 2}\left\{\mathrm{Pr} - 1 + \ln\left[\left(1 + 5\,\mathrm{Pr}\right)/6\right]\right\}}$$

Chilton-Colburn analogy

$$j_H = j_D$$

$$\frac{h}{\rho v_\infty c_p}(\mathrm{Pr})^{2/3} = \frac{k_c}{v_\infty}(\mathrm{Sc})^{2/3}$$

for turbulent pipe flow

$$\frac{hD}{k} = \mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3} \qquad \frac{\bar{k}_c D}{D_{AB}} = \mathrm{Sh} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Sc}^{1/3}$$

Table 28.1 Models for convective mass-transfer coefficients (dilute systems)

Model	Basic form	$f(D_{AB})$	Notes
Film theory	$k_c = \frac{D_{AB}}{\delta}$	$k_c \propto D_{AB}$	δ unknown, may be found when solute has high Sc
Penetration theory	$k_c = \sqrt{\frac{D_{AB} v_\infty}{\pi \delta}}$	$k_c \propto D_{AB}^{1/2}$	δ unknown, good model when homogeneous reaction within boundary layer or when solute has low Sc
Boundary-layer theory	$k_c = 0.664 \frac{D_{AB}}{L}(\mathrm{Re})^{1/2}(\mathrm{Sc})^{1/3}$	$k_c \propto D_{AB}^{2/3}$	Best way to scale k_c from one solute to another exposed to same hydrodynamic flow