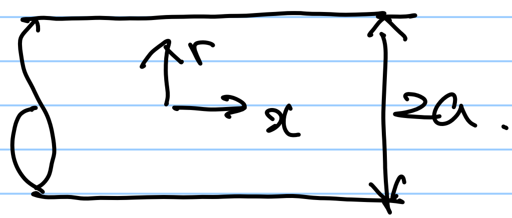
(high-freq. component decays faster)

3 Transient flow in Poiseuille-flow formation.

- circular pipe, fluid is at rest initially
- $\frac{dP}{dx} = -G$ is imposed suddenly



$$\frac{\partial u}{\partial t} = \frac{G}{\rho} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right)$$

BC : $u = 0$ @ $r = a$ (for all t's)

$u = 0$ @ $t = 0$ (for $0 \leq r < a$)

- Let's use the same method as for Couette-flow formation.

$$: \omega(r,t) = \underbrace{\frac{G}{4\mu}(a^2 - r^2)}_{\text{steady state.}} - \underbrace{u(r,t)}_{\text{instantaneous (unsteady) velocity.}}$$

$$\rightarrow \frac{\partial \omega}{\partial t} = \nu\left(\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r}\frac{\partial \omega}{\partial r}\right). \quad \omega(a,t) = 0, \quad \omega(r,0) = \frac{G}{4\mu}(a^2 - r^2).$$

↳ "ω" should be a form of $J_0\left(\lambda_n \cdot \frac{r}{a}\right) \cdot \exp\left(-\lambda_n^2 \frac{\nu t}{a^2}\right)$.

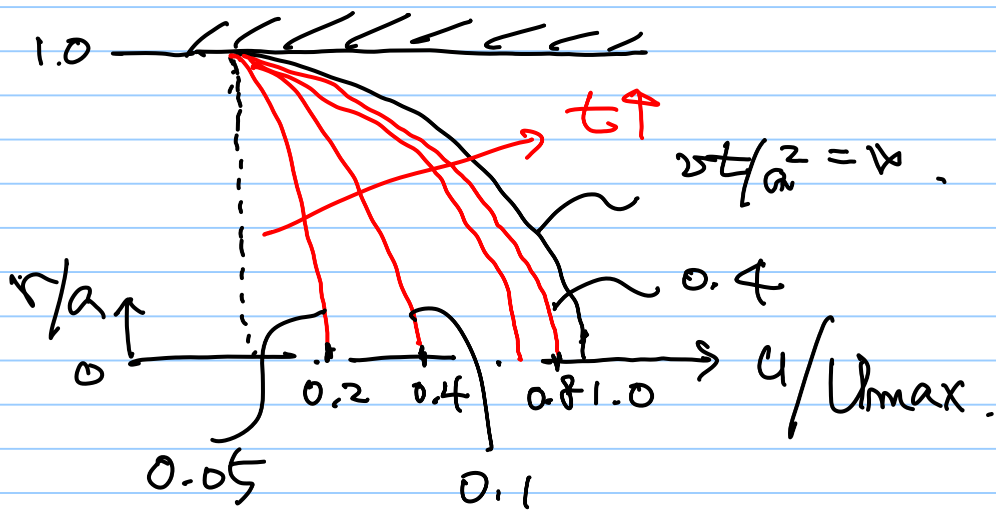
J_0: Bessel fn of 1^{st} kind, order 0

λ_n : root of $J_0(\lambda) = 0$.

$$\therefore\ \omega(r,t) = \frac{G}{4\mu}\sum_{n=1}^{\infty} A_n \cdot J_0\left(\lambda_n \frac{r}{a}\right)\cdot \exp\left(-\lambda_n^2 \frac{\nu t}{a^2}\right).$$

apply I.C. ; $A_n = \dfrac{8a^2}{\lambda_n^3 \cdot J_1(\lambda_n)}$

$$\therefore\ u(r,t) = \frac{G}{4\mu}(a^2 - r^2) - \frac{2Ga^2}{\mu}\sum_{n=1}^{\infty} \frac{J_0(\lambda_n \cdot r/a)}{\lambda_n^3 \cdot J_1(\lambda_n)}\cdot \exp\left(-\lambda_n^2 \frac{\nu t}{a^2}\right).$$

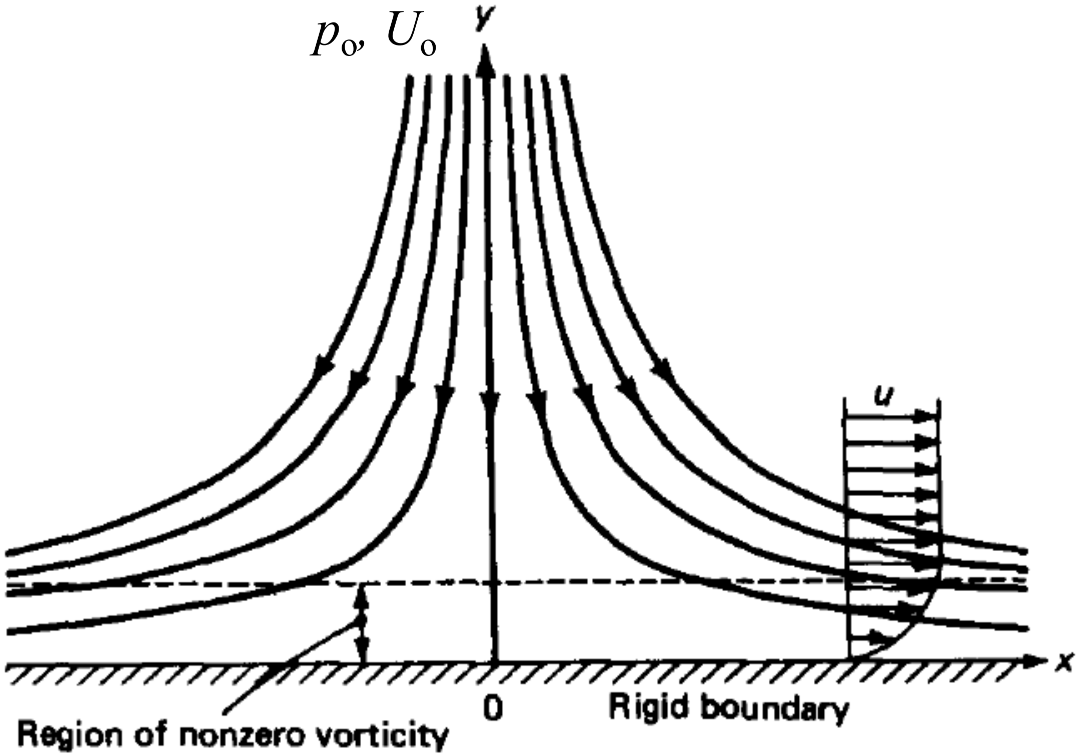


i) initially, the central core flow accelerates resulting in a boundary-layer like behavior.

ii) steady state, @ $t^* = \nu t / a^2 \simeq 0.75$

if $a = 5$ mm, air, $t \simeq 1.25$ sec.

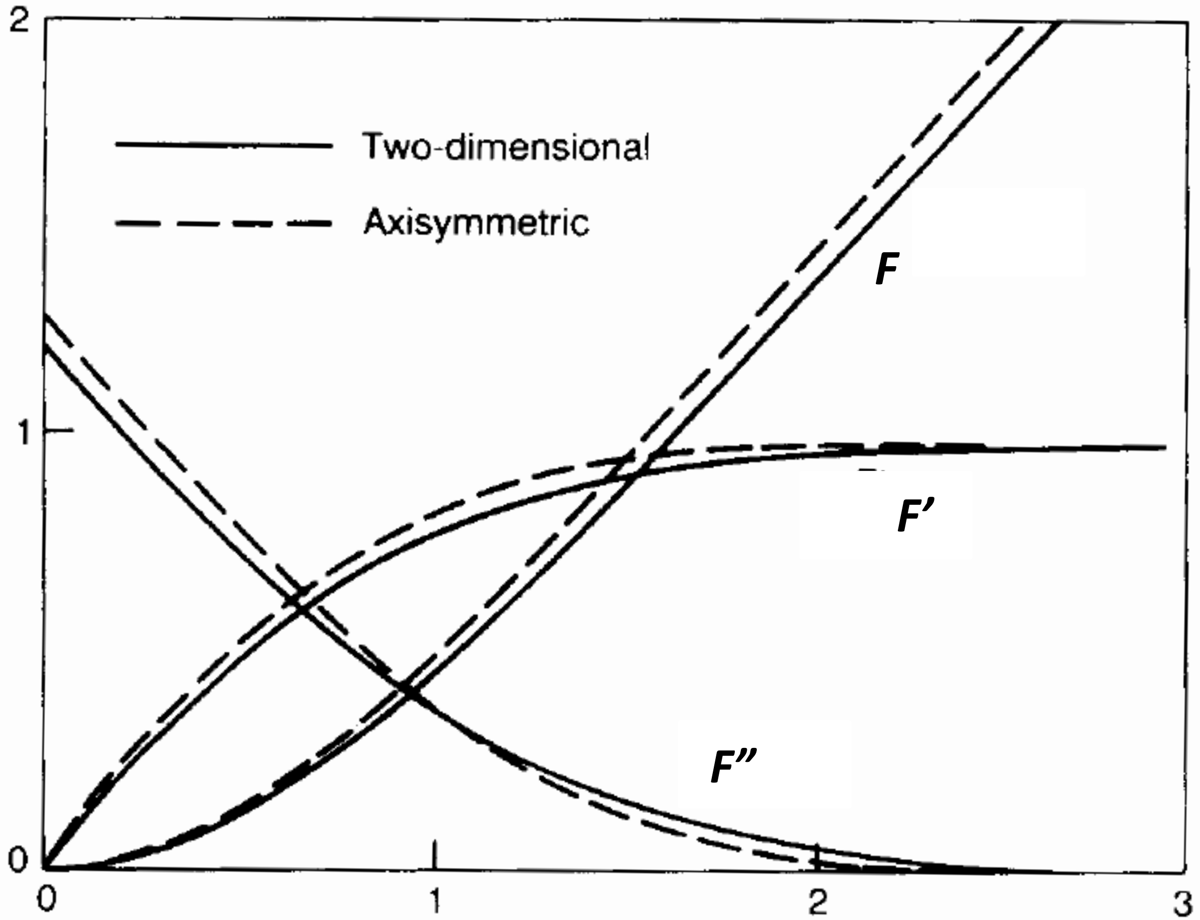
Stagnation (Point) Flow



From White "Viscous Fluid Flow"

- Cooling application like an impinging jet
- 2D plane, axisymmetric
- No-slip condition at the wall, u(x,0) = 0 → viscous region develops near the wall which has a constant thickness and has the effect of displacing the outer inviscid flow away from the wall.

Stagnation (Point) Flow



$$\eta = y\sqrt{\frac{B}{\nu}} = y\sqrt{\frac{2U_o}{\nu}}$$

$$\delta \simeq 2.4\sqrt{\frac{\nu}{2U_o}}$$

Numerical solutions for stagnation flow

	$F' = u/U$	
η	Plane $F''(0) = 1.23259$ $\eta^* = 0.6479$	Axisymmetric $F''(0) = 1.31194$ $\eta^* = 0.5689$
0.1	0.11826	0.12619
0.2	0.22661	0.24239
0.3	0.32524	0.34863
0.4	0.41446	0.44499
0.5	0.49465	0.53160
0.6	0.56628	0.60871
0.7	0.62986	0.67663
0.8	0.68594	0.73577
0.9	0.73508	0.78666
1.0	0.77787	0.82987
1.1	0.81487	0.86608
1.2	0.84667	0.89598
1.3	0.87381	0.92032
1.4	0.89681	0.93983
1.5	0.91617	0.95522
1.6	0.93235	0.96718
1.7	0.94578	0.97631
1.8	0.95684	0.98316
1.9	0.96588	0.98822
2.0	0.97322	0.99190
2.2	0.98386	0.99635
2.4	0.99055	0.99847
2.6	0.99464	0.99940
2.8	0.99705	0.99979
3.0	0.99843	0.99993

From White “Viscous Fluid Flow”

Stagnation (Point) Flow

- Wall-shear stress

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)_{y=0} = \mu B x F_o'' \sqrt{\frac{B}{\nu}} = U_o F_o'' \sqrt{\mu \rho B}$$

$$C_f = \frac{\tau_w}{0.5\rho U_o^2} = \frac{2F_o''}{\sqrt{U_o x/\nu}} = \frac{2F_o''}{\sqrt{\mathrm{Re}_x}}$$

- Inverse variation of friction coefficient with square root of local Reynolds number is very common in boundary-layer flows.

MIDTERM. 10/31 (THU). 15:30 - 17:30

8 Similarity Solutions

- PDE → ODE by reducing the ~~#~~ of variables.
- limited application.

8-1) stagnation-point flow.

. potential flow analysis (refer to Inviscid Flow)

(2D) $z = x + iy$ $\Longrightarrow$ complex potential

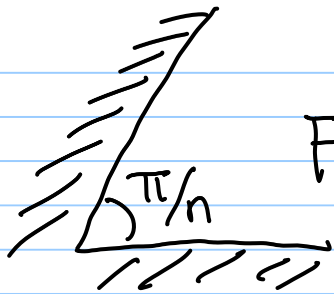
$F(z) = \phi + i\psi$

stream fcn.

vel. potential

complex velocity

$$W(z) \equiv \frac{dF}{dz} = \frac{\partial \phi}{\partial x} + i\frac{\partial \psi}{\partial x} = u - iv.$$



$F(z) = U_0 \cdot z^n$: in inviscid flow, the flow near a stagnation point of a body is described as

$(z = x + iy)$

$$\downarrow$$

$$\psi = 2U_0 xy \rightarrow u = \frac{\partial \psi}{\partial y} = 2U_0 x$$

$$v = -\frac{\partial \psi}{\partial x} = -2U_0 y$$

↳ slip @ $y = 0$ (wall)

However, viscous solution has a no-slip condition @ $y = 0$.

$$\rightarrow \psi_{viscous} = 2U_0 x \cdot f(y),$$ Hiemenz (1911).

$$\boxed{u = 2U_0 x f'(y)}, \quad \boxed{v = -2U_0 f(y)}$$

$$\lim_{y\to\infty} f(y) = y \quad \text{(approaching the inviscid sol.)}$$

① ψ_{viscous} satisfies the continuity by its definition.

② substitute u, v into the mtm eq.

x-mtm : $4U_0^2 x (f')^2 - 4U_0^2 x f f'' = -\frac{1}{\rho}\frac{\partial p}{\partial x} + 2U_0 \nu x f'''$

y-mtm : $4U_0^2 f f' = -\frac{1}{\rho}\frac{\partial p}{\partial y} - 2U_0 \nu f''$.

$\to p(x,y) = -2\rho U_0^2 f^2 - 2\rho U_0 \nu f' + g(x)$.

since, $\lim_{y\to\infty} p(x,y) = P_0$ (inviscid solution)

$(f \to y)$

$$\therefore\ -2\rho U_0^2 y^2 - 2\rho U_0 \nu + g(x) = P_0$$

$$g(x) = P_0 + 2\rho U_0^2 y^2 + 2\rho U_0 \nu .$$

Then, x-mntm eqn becomes

$$\frac{\nu}{2U_0} f''' + f f'' - (f')^2 + 1 = 0 .$$ ← 'x' disappears. (similarity achieved)

<u>BC</u>: $u(x,0) = 0 \Rightarrow f'(0) = 0$

$v(x,0) = 0 \rightarrow f(0) = 0 .$

Let, $\eta \equiv y\sqrt{\frac{B}{\nu}}$. $(B = 2U_0)$ as a similarity variable,

$\hookrightarrow \psi = x \cdot F(\eta)\sqrt{B\cdot\nu}$.

$$F''' + FF'' + 1 - (F')^2 = 0 .$$ $\left(\begin{array}{l} F(0) = F'(0) = 0 \\ F'(\infty) = 1 . \end{array} \right.$

→ if you choose η when $F' (= u/U_0) \simeq 1.0$

↳ 2.4

$$\therefore \delta = 2.4 \sqrt{\frac{\nu}{2U_0}}.$$

↳ thinning due to stream acceleration is balanced by thickening due to viscous diffusion.

* stagnation-point temperature distribution.

· known $u(x,y) \rightarrow T$.

· A similarity solution for T exists if T_w (wall temp.) and T_0 (upstream) are constant.

$$\rho C_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \Phi .$$

$$\text{let } \theta(\eta) = \frac{T - T_w}{T_0 - T_w}, \quad \eta = y\sqrt{\frac{B}{\nu}} = y\sqrt{\frac{2U_0}{\nu}} .$$

$$\Rightarrow \frac{d^2\theta}{d\eta^2} + P_r \cdot F(\eta) \cdot \frac{d\theta}{d\eta} = 0 . \quad \theta(0) = 0 . \quad \theta(\infty) = 1 .$$

same form as that for velocity field.

$$P_r \equiv \frac{\mu C_p}{k} \sim \frac{\text{diffusion of mtm}}{\text{diffusion of heat}} .$$

$$\frac{d\theta'}{d\eta} + P_r \cdot F(\eta) \cdot \theta' = 0 . \rightarrow \frac{d\theta'}{\theta'} + P_r \cdot F(\eta) \cdot d\eta = 0 .$$

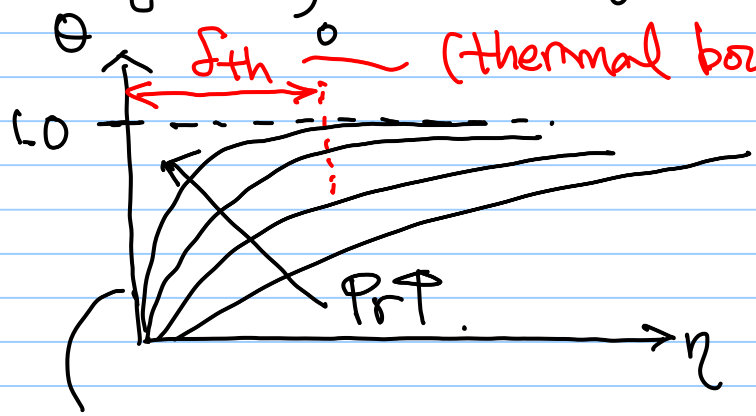
$$\therefore \theta' = C_1 \cdot \exp\left[-Pr\int_0^{\eta} F \cdot d\eta\right].$$

$$\theta = C_1 \int_0^{\eta} \left[\exp\left\{-Pr\int_0^{\eta} F\,d\eta\right]\right] d\eta + C_2.$$

By calculating C_1, C_2,

$$\theta(\eta) = \int_0^{\eta} \exp\left[-Pr\int_0^{\eta} F\,d\eta\right] d\eta \Big/ \boxed{\int_0^{\infty} \exp\left[-Pr\int_0^{\eta} F\,d\eta\right] d\eta} \qquad = \frac{1}{G(Pr)}$$

δ_{th} — (thermal boundary layer)



as $Pr \uparrow$, $\delta_{th} \downarrow$.

curve fit : $\dfrac{\delta}{\delta_{th}} \simeq P_r^{0.4}$

$$q_w = -k \frac{\partial T}{\partial y}\bigg|_{y=0} = -k\,(T_\infty - T_w)\cdot G(P_r)\sqrt{\frac{B}{\nu}}.$$

$\eta \left(= y\sqrt{\dfrac{B}{\nu}}\right).$

curve fit

$G(P_r) \simeq 0.57\,P_r^{0.4}$ (2D plane)

$\simeq 0.76\,P_r^{0.4}$ (axisymmetric)

∴ stagnation point flow

① thin viscous / thermal layers (boundary-layer like)

② favorable pressure grad. $(\partial p/\partial x < 0)$: accelerating flow

③ No separation.