

**2017 Fall**

# **“Phase Equilibria *in* Materials”**

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**Eun Soo Park**

**Office: 33-313**

**Telephone: 880-7221**

**Email: [espark@snu.ac.kr](mailto:espark@snu.ac.kr)**

**Office hours: by an appointment**

# Chapter 11. Ternary phase Diagrams

## Intermediate Phases

Intermediate phases may **melt congruently** or **incongruently**.  
They may occur as **either binary or ternary phases**.

# 11.1 Congruently melting intermediate phases

## 11.1. Binary intermediate phases

### 1) Two ternary eutectic reactions

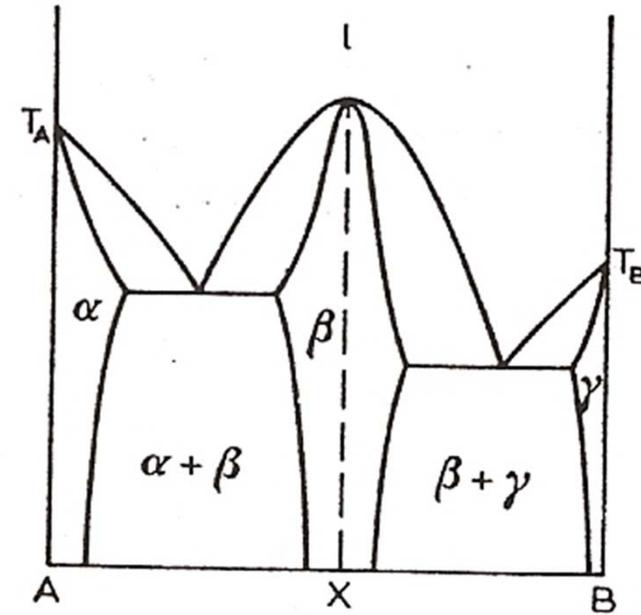
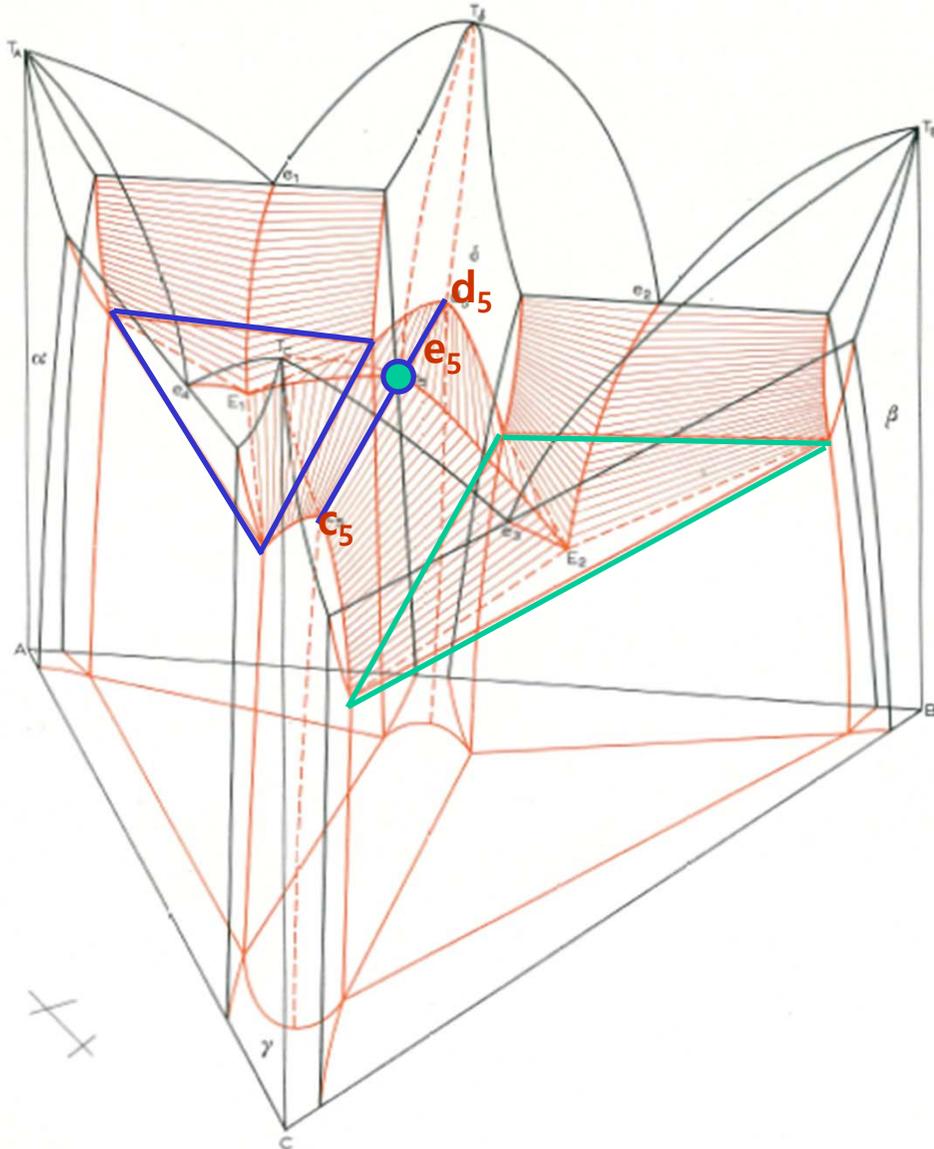


Fig. 78. Phase diagram with a congruent intermediate phase.

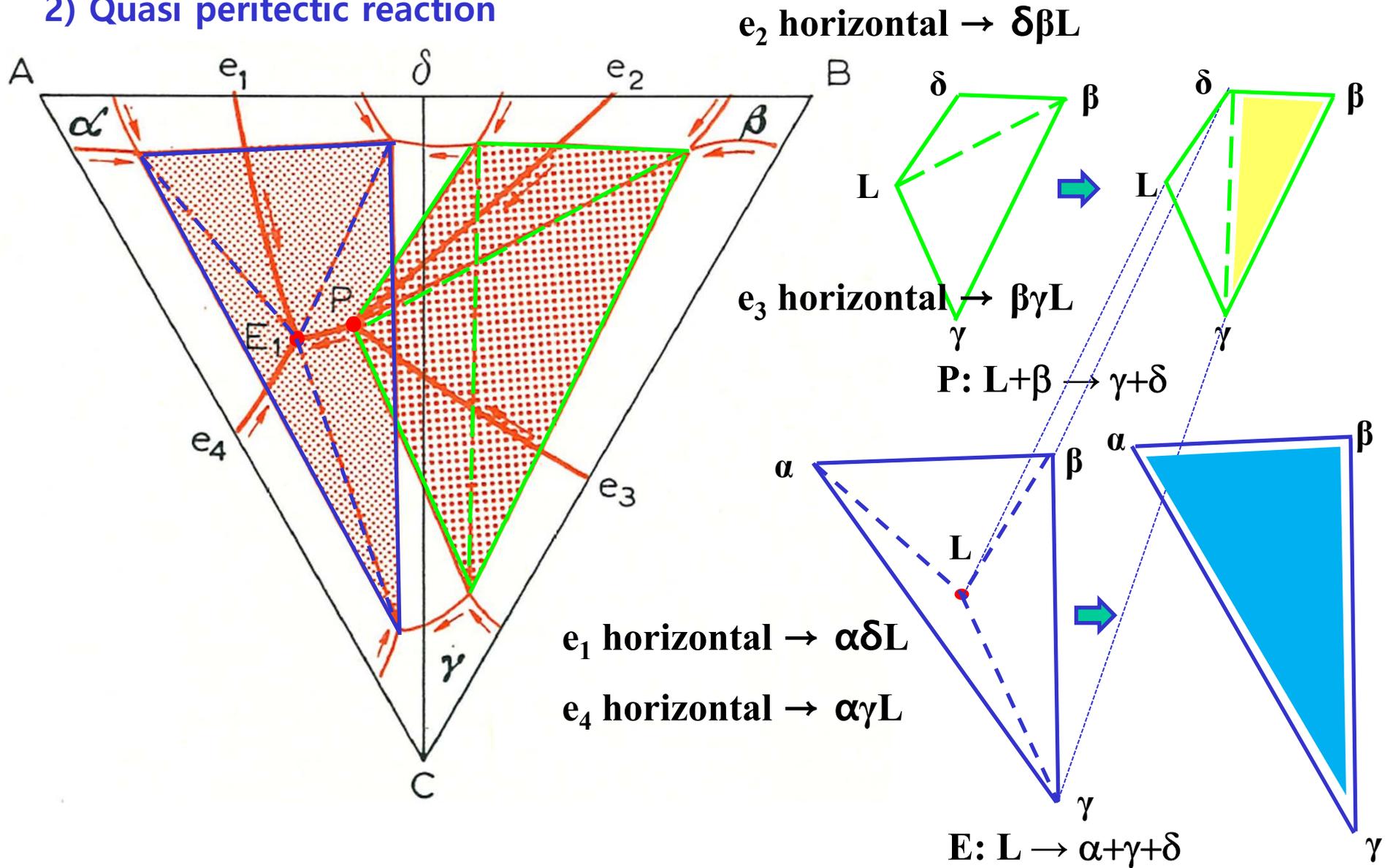
the eutectic point  $e_5$  on the quasi-binary section  $\delta C$  is saddle point.

the straight line is the quasi-binary eutectic horizontal  $c_5e_5d_5$ .

# 11.1 Congruently-melting intermediate phases

- Binary intermediate phases

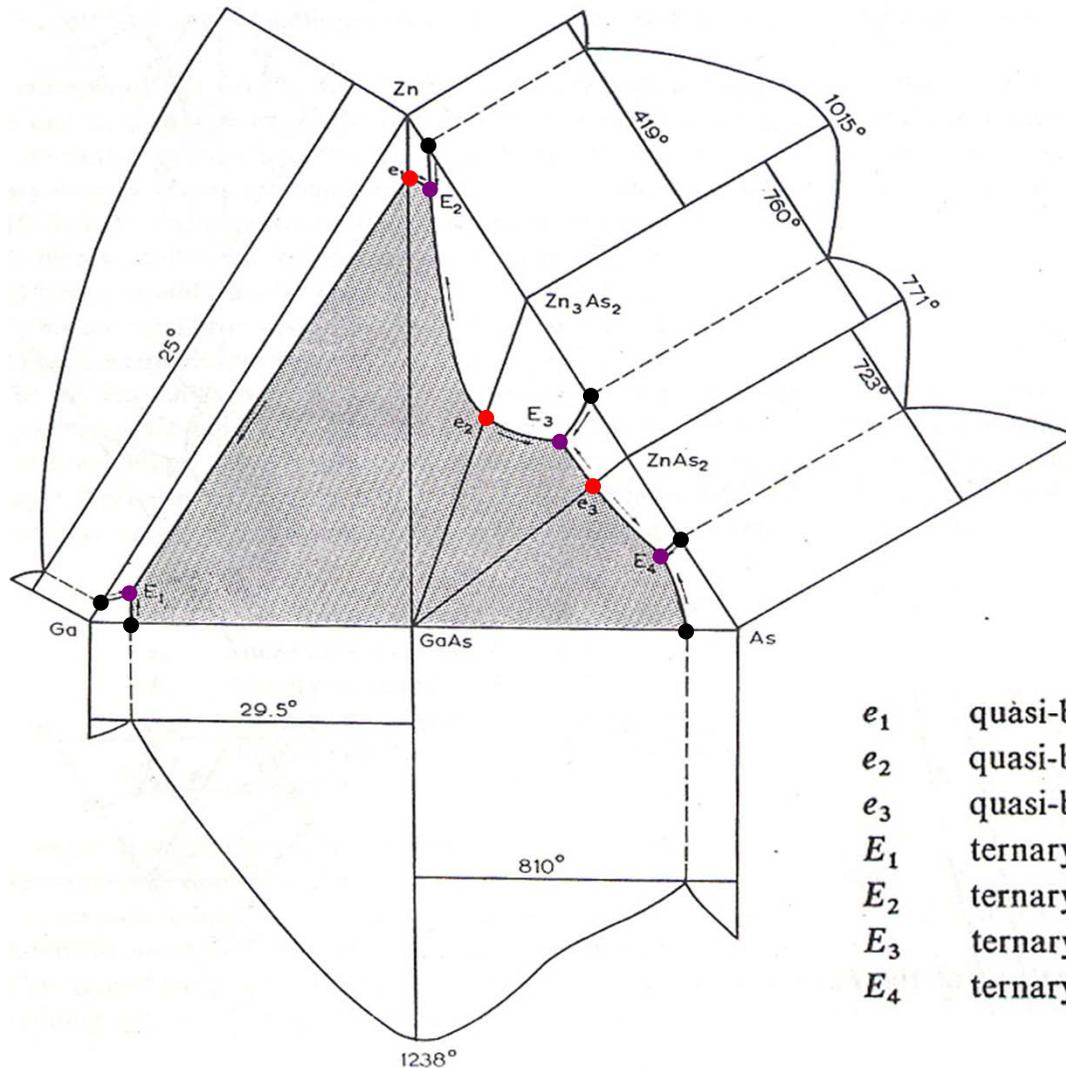
## 2) Quasi peritectic reaction



# 11.1 Congruently-melting intermediate phases

## a) Binary intermediate phases

: Quasi binary eutectic rxn. between Ga, As and Zn



- The region in which GaAs is the primary phase to crystallize from the liquid is lightly shaded.
- It illustrates the dominating behavior of the **high melting phase GaAs** in this system
- For clarity, **no solid solubility** between any of the phases has been indicated.

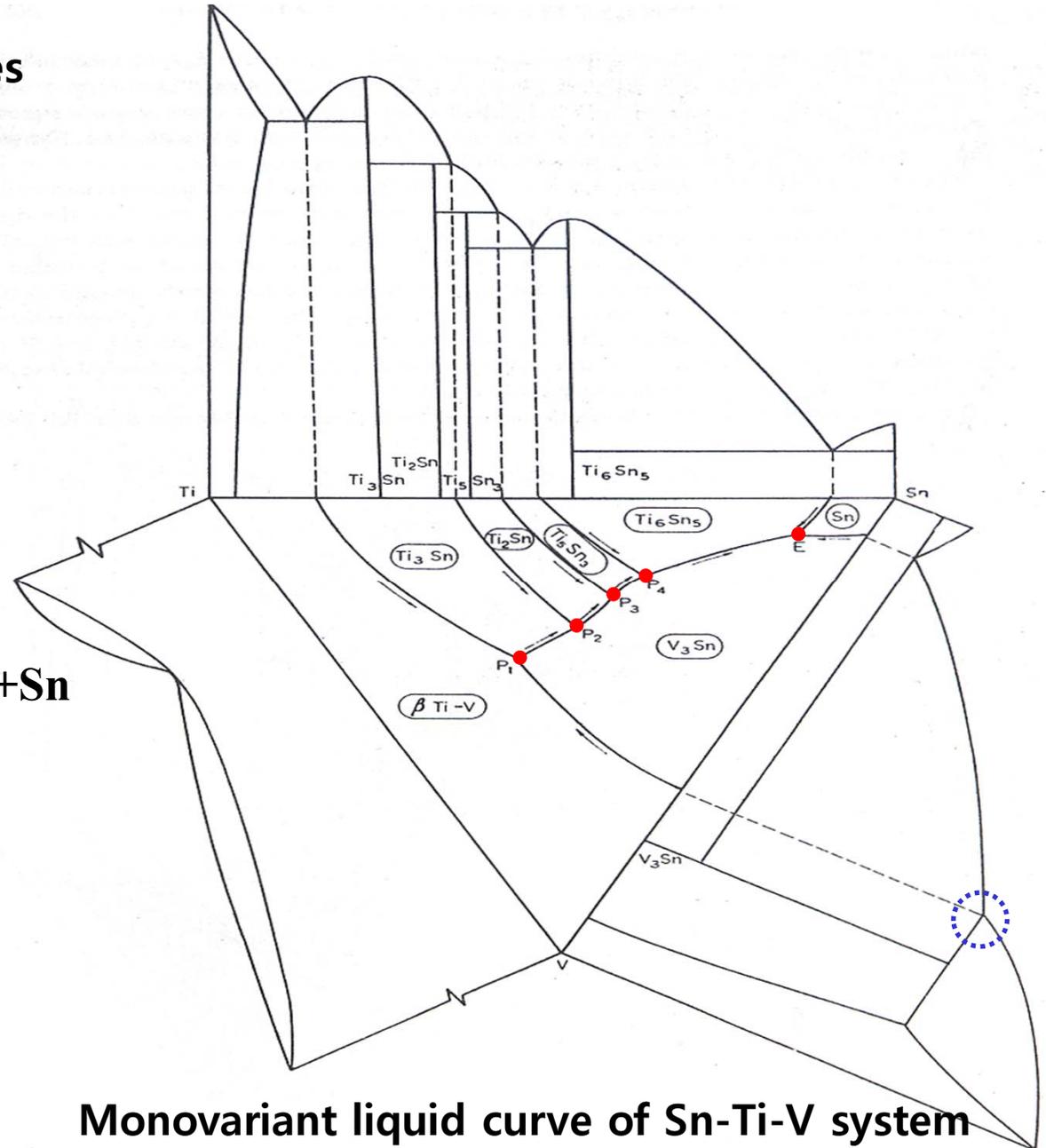
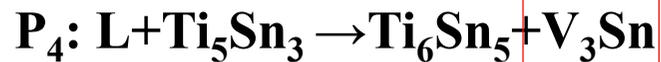
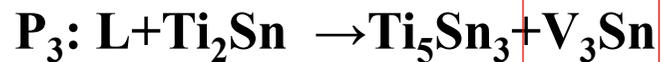
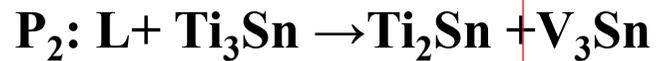
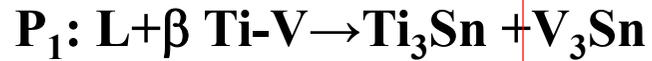
$e_1$	quasi-binary eutectic $l \rightleftharpoons \text{GaAs} + \text{Zn}$	at	414 °C,
$e_2$	quasi-binary eutectic $l \rightleftharpoons \text{GaAs} + \text{Zn}_3\text{As}_2$	at	972 °C,
$e_3$	quasi-binary eutectic $l \rightleftharpoons \text{GaAs} + \text{ZnAs}_2$	at	754 °C,
$E_1$	ternary eutectic $l \rightleftharpoons \text{GaAs} + \text{Zn} + \text{Ga}$	at	~ 20 °C,
$E_2$	ternary eutectic $l \rightleftharpoons \text{GaAs} + \text{Zn} + \text{Zn}_3\text{As}_2$	at	~410 °C,
$E_3$	ternary eutectic $l \rightleftharpoons \text{GaAs} + \text{Zn}_3\text{As}_2 + \text{ZnAs}_2$	at	~750 °C,
$E_4$	ternary eutectic $l \rightleftharpoons \text{GaAs} + \text{ZnAs}_2 + \text{As}$	at	~720 °C.

As-Ga-Zn system

# 11.1 Congruently-melting intermediate phases

- Binary intermediate phases

## Quasi peritectic reaction



Monovariant liquid curve of Sn-Ti-V system

# 11.1 Congruently-melting intermediate phases

- Binary intermediate phases

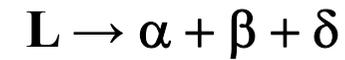
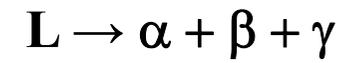
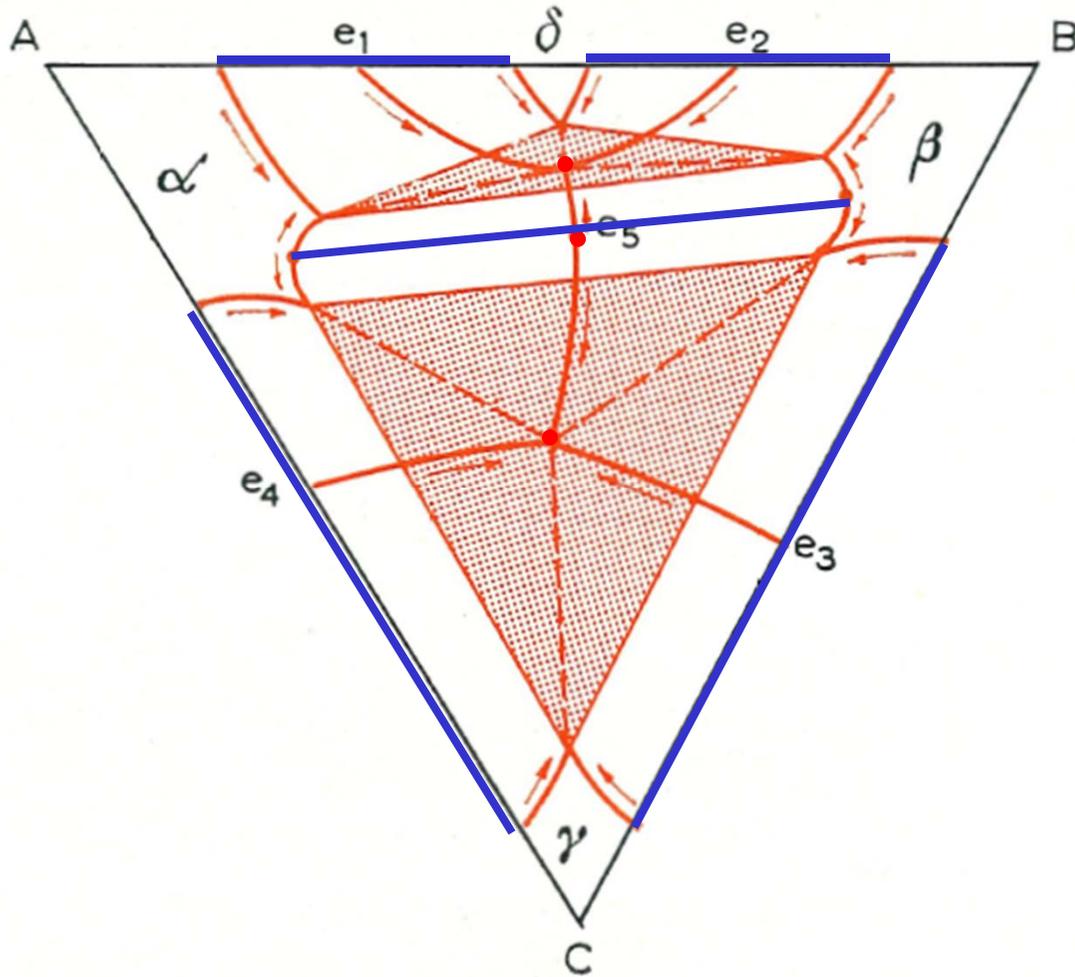
3) No quasi binary eutectic : **two ternary eutectic**

$$l \leftrightarrow \alpha + \beta + \gamma$$

$$l \leftrightarrow \alpha + \beta + \delta$$

$$l \leftrightarrow \alpha + \gamma + \delta$$

$$l \leftrightarrow \beta + \gamma + \delta$$



$e_5$ : saddle point

## 11.1 Congruently-melting intermediate phases

- Binary intermediate phases: **Kurnakov rule**

### 1) Case1: with only binary congruent intermediate phases

$$K = E = c_2 + 1 = q + 1 = m + 1$$

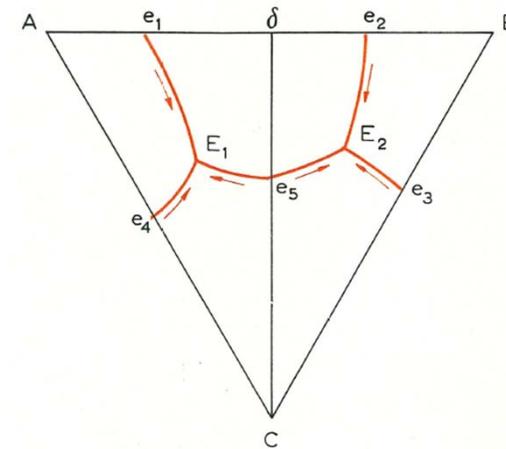
$K$  = # of secondary triangles

$E$  = # of ternary eutectic points

$c_2$  = binary congruent intermediate phases

$q$  = quasi binary reaction

$m$  = saddle point



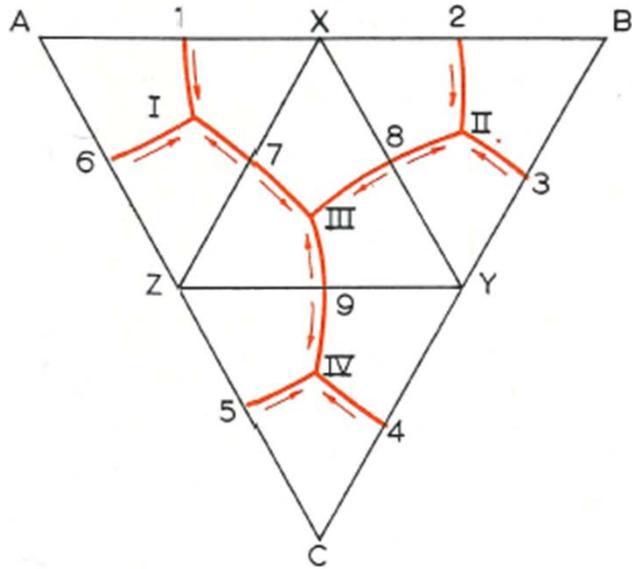
### 2) Case2: with only ternary congruent intermediate phases

$$K = E = 2c_3 + 1 = 2/3q + 1 = 2/3m + 1$$

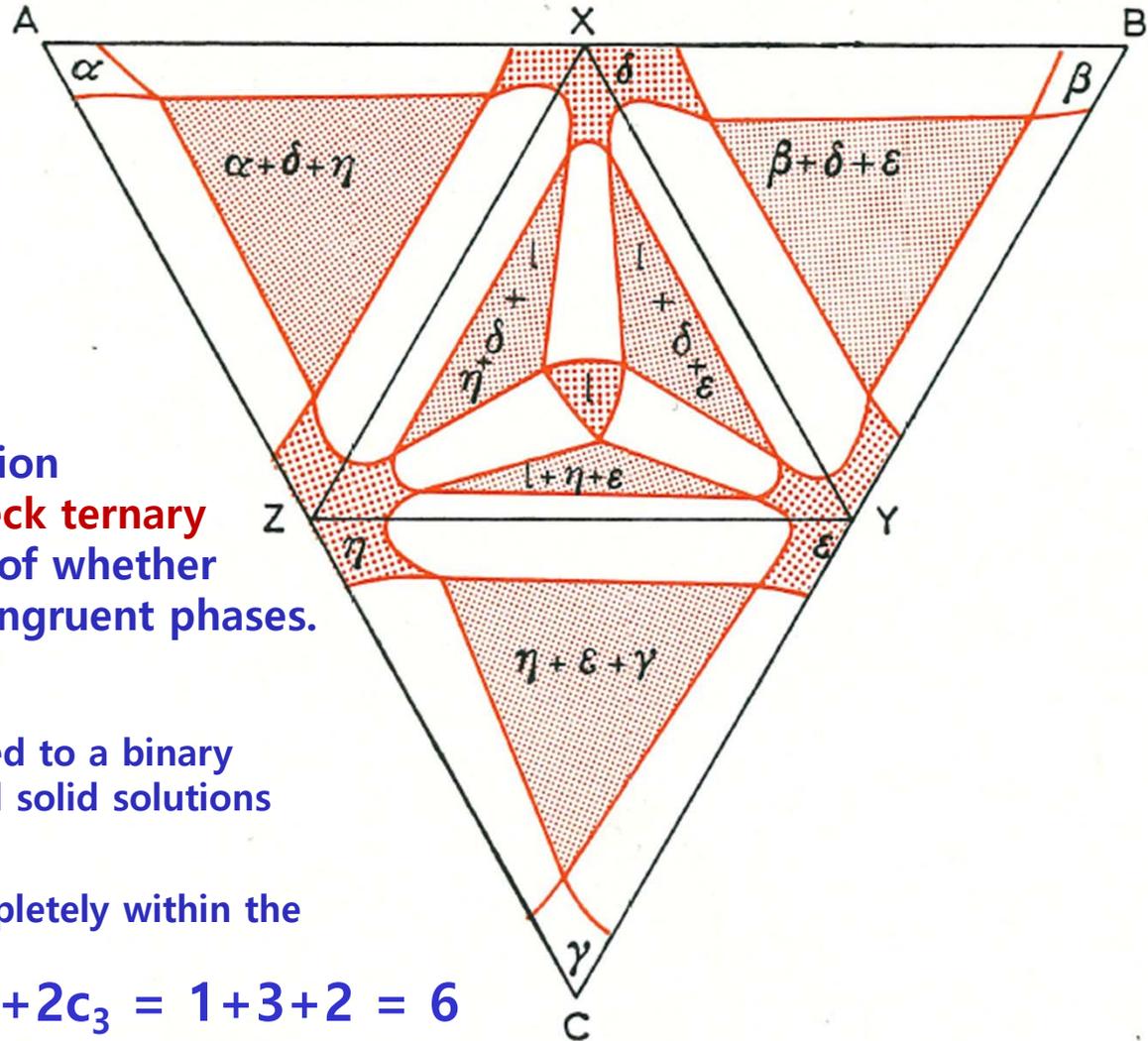
$c_3$  = ternary congruent intermediate phases

### 3) Case3: with both binary and ternary congruent intermediate phases

$$K = E = 1 + c_2 + 2c_3 = q + 1 - c_3 = m + 1 - c_3$$



- Isothermal section at a temperature just above the lowest melting ternary eutectic (III)



- Rhines has noted that the relation  $k=1+c_2+2c_3$  can be used to check ternary isothermal section irrespective of whether they contain congruent or incongruent phases.

-  $K$ : # of 3 phase tie triangles,

- $C_2$ : # of single phase regions joined to a binary edge (excluding the  $\alpha$ ,  $\beta$ ,  $\gamma$  terminal solid solutions based on components A, B and C),

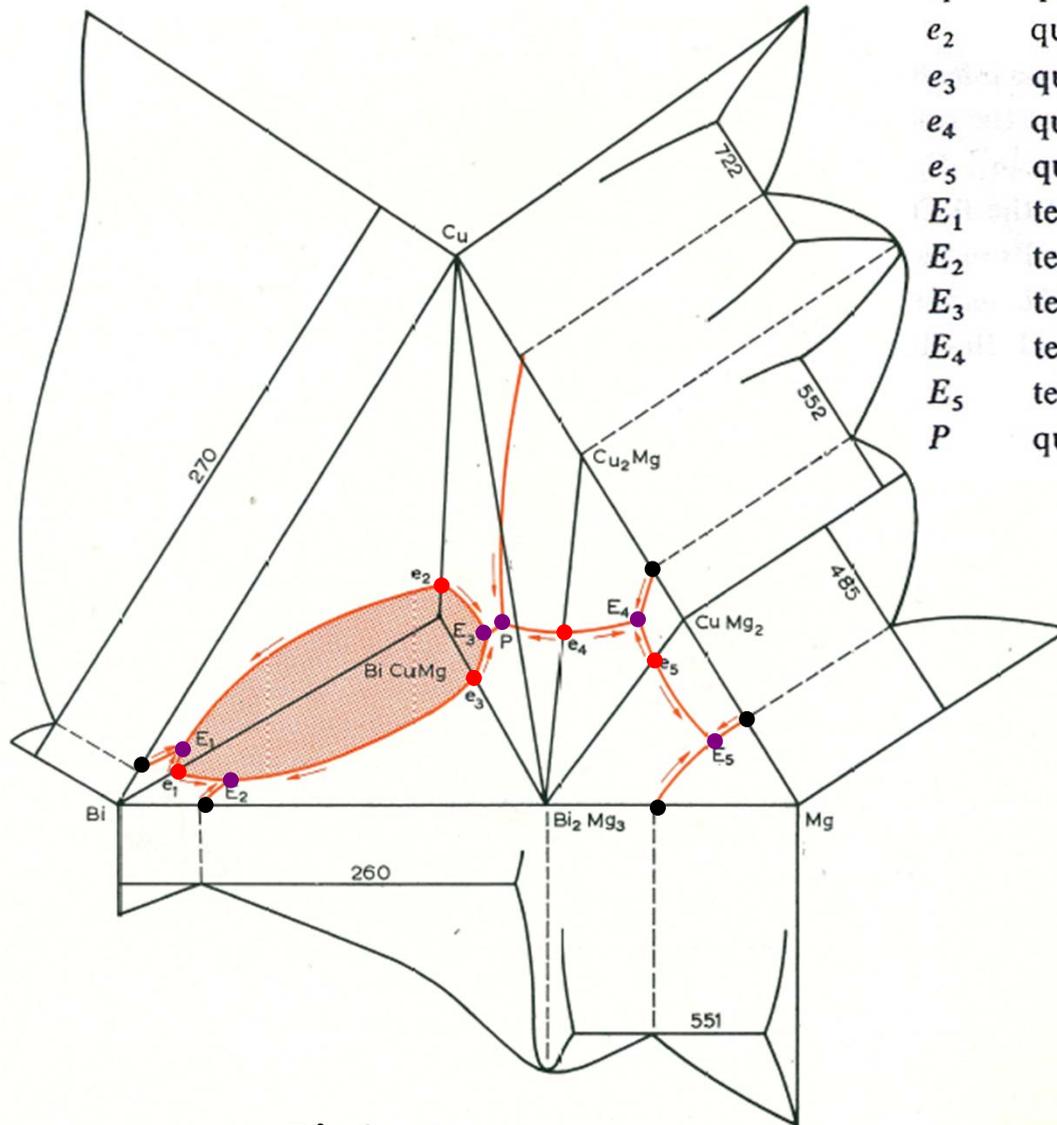
- $C_3$ : # of single phase regions completely within the ternary system.

$$K = E = 1 + c_2 + 2c_3 = 1 + 3 + 2 = 6$$

- The Kurnakove and Rhines' rules are useful in checking the construction of ternary systems and their isothermal sections when intermediate phases are involved.

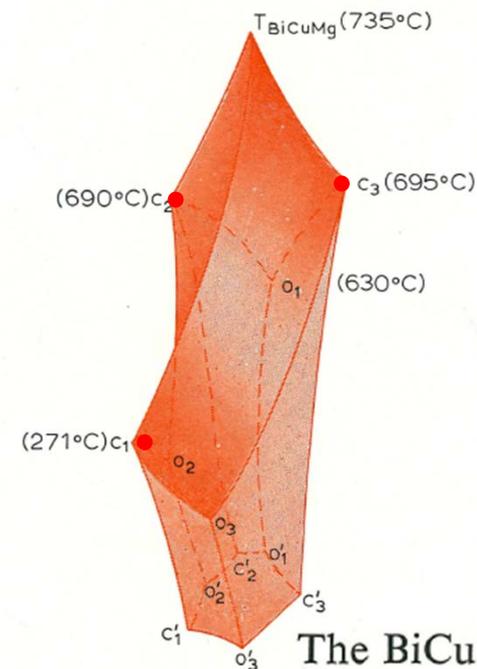
# 11.1 Congruently-melting intermediate phases

**b) Ternary intermediate phase: behaves as a pure metal in that it freezes isothermally and its appearance is associated with a maximum on the liquidus/solidus surfaces**



Bi-Cu-Mg system

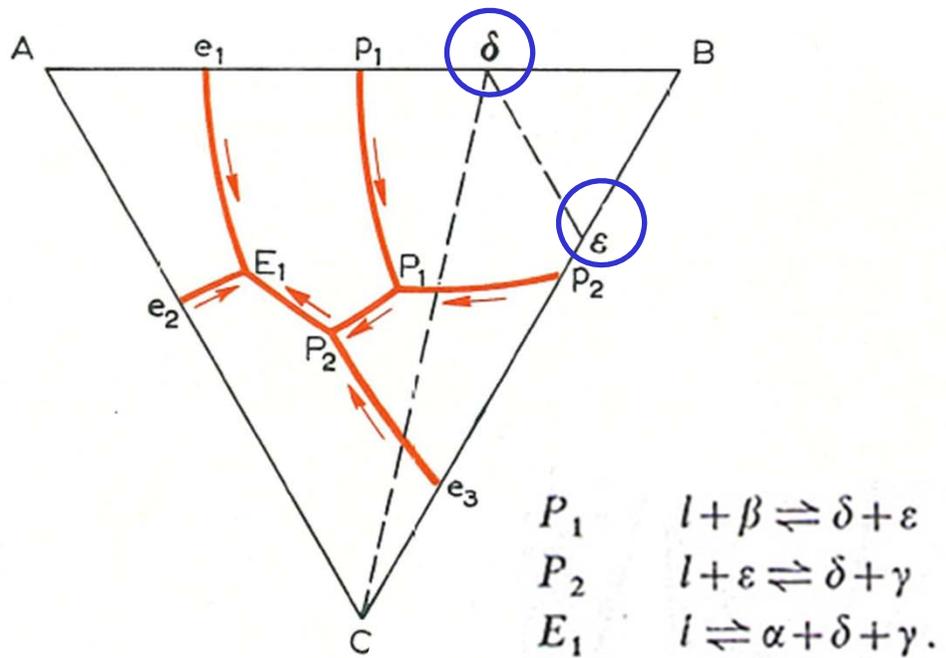
- e<sub>1</sub> · quasi-binary eutectic  $l \rightleftharpoons \text{Bi} + \text{BiCuMg}$  at  $\sim 271^\circ\text{C}$
- e<sub>2</sub> quasi-binary eutectic  $l \rightleftharpoons \text{Cu} + \text{BiCuMg}$  at  $690^\circ\text{C}$
- e<sub>3</sub> quasi-binary eutectic  $l \rightleftharpoons \text{Bi}_2\text{Mg}_3 + \text{BiCuMg}$  at  $695^\circ\text{C}$
- e<sub>4</sub> quasi-binary eutectic  $l \rightleftharpoons \text{Bi}_2\text{Mg}_3 + \text{Cu}_2\text{Mg}$  at  $655^\circ\text{C}$
- e<sub>5</sub> quasi-binary eutectic  $l \rightleftharpoons \text{Bi}_2\text{Mg}_3 + \text{CuMg}_2$  at  $557^\circ\text{C}$
- E<sub>1</sub> ternary eutectic  $l \rightleftharpoons \text{Bi} + \text{Cu} + \text{BiCuMg}$  at  $265^\circ\text{C}$
- E<sub>2</sub> ternary eutectic  $l \rightleftharpoons \text{Bi} + \text{Bi}_2\text{Mg}_3 + \text{BiCuMg}$  at  $255^\circ\text{C}$
- E<sub>3</sub> ternary eutectic  $l \rightleftharpoons \text{Cu} + \text{Bi}_2\text{Mg}_3 + \text{BiCuMg}$  at  $630^\circ\text{C}$
- E<sub>4</sub> ternary eutectic  $l \rightleftharpoons \text{Bi}_2\text{Mg}_3 + \text{Cu}_2\text{Mg} + \text{CuMg}_2$  at  $546^\circ\text{C}$
- E<sub>5</sub> ternary eutectic  $l \rightleftharpoons \text{Mg} + \text{Bi}_2\text{Mg}_3 + \text{CuMg}_2$  at  $470^\circ\text{C}$
- P quasi-peritectic  $l + \text{Cu}_2\text{Mg} \rightleftharpoons \text{Cu} + \text{Bi}_2\text{Mg}_3$  at  $660^\circ\text{C}$



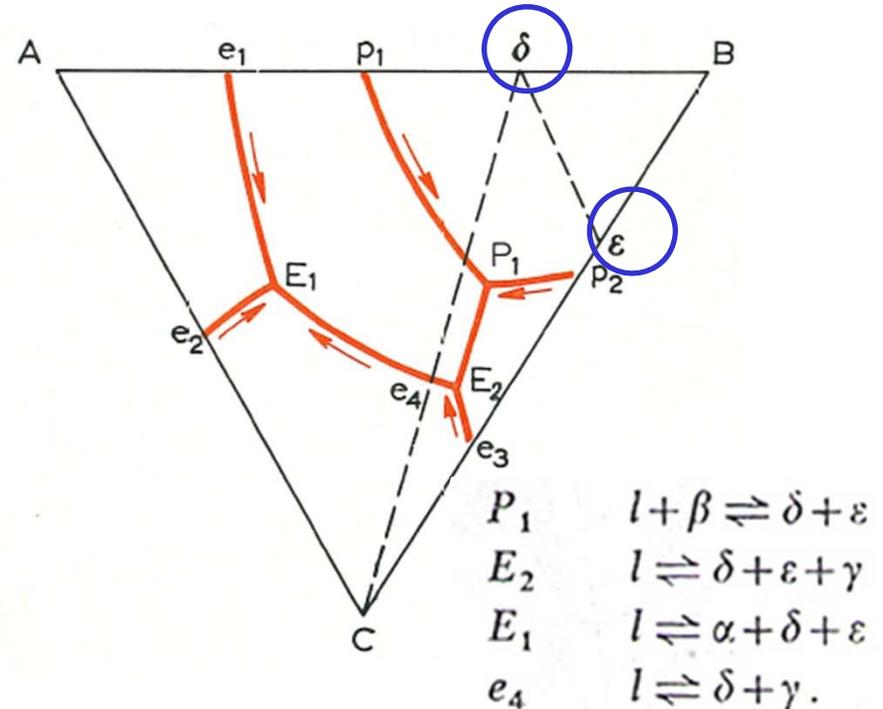
The BiCuMg phase region.

## 11.2 incongruently-melting intermediate phases

a) ternary system formed when two of the Binaries contain incongruent intermediate phases

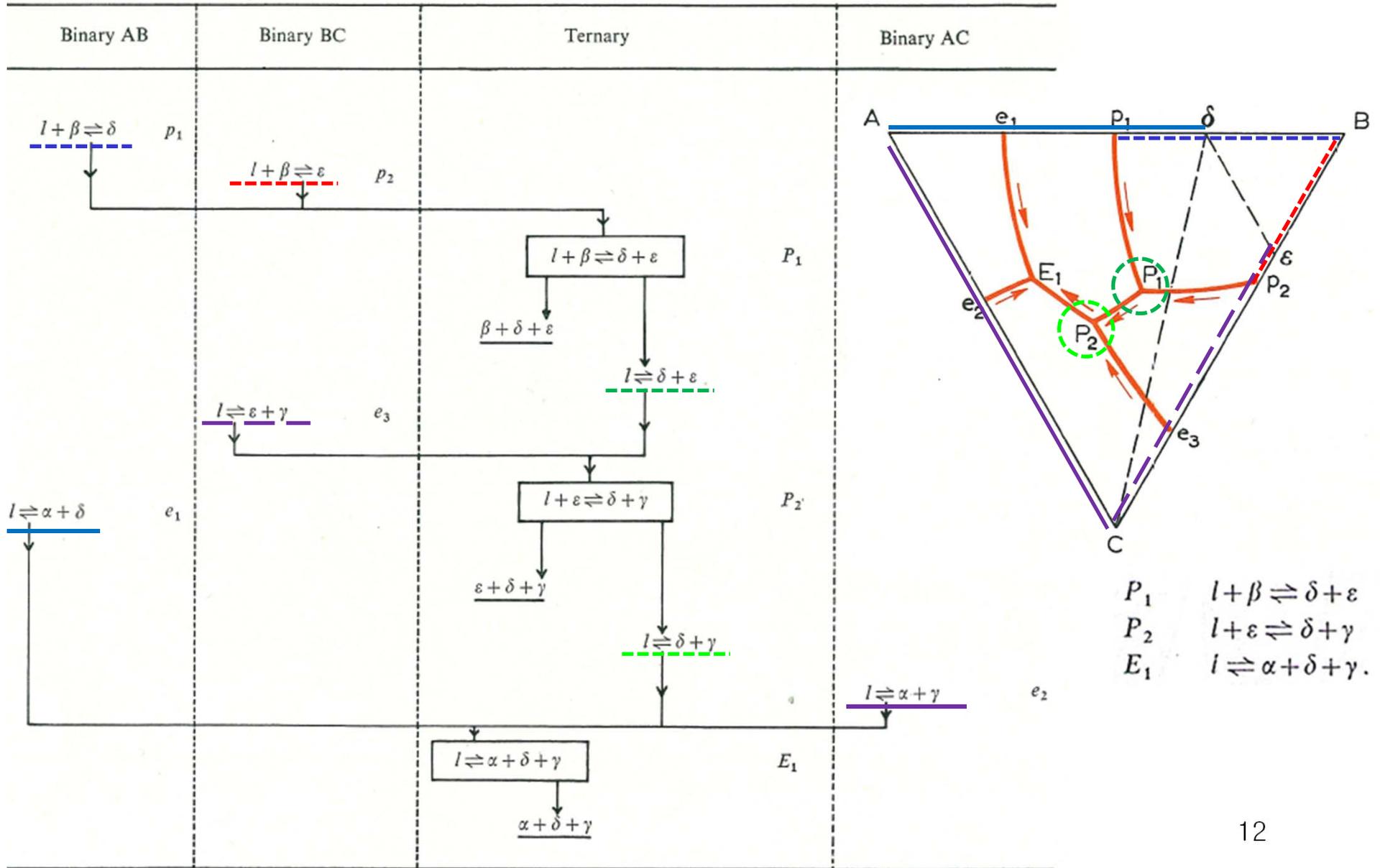


(a) Equilibria when the quasi-peritectic point P is located in the partial system AδC



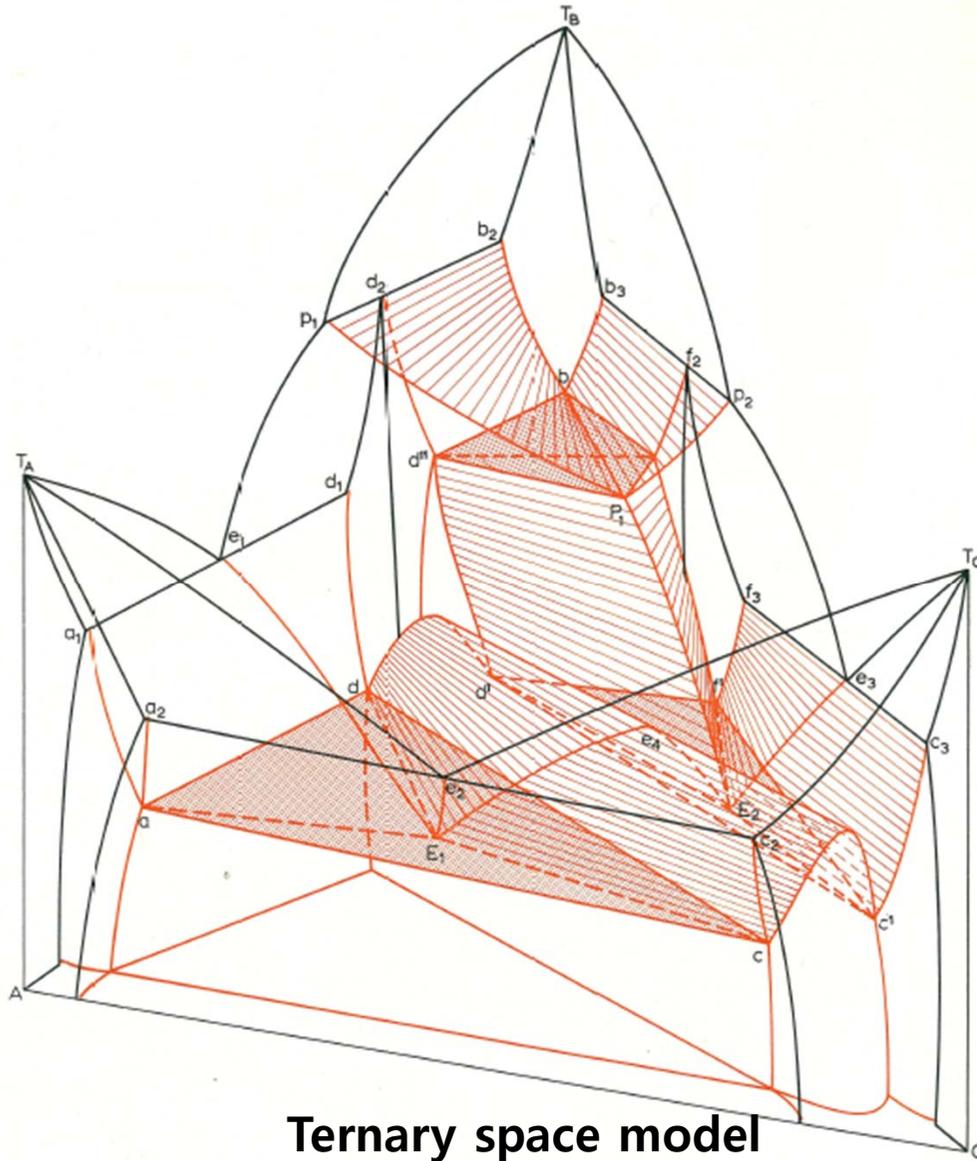
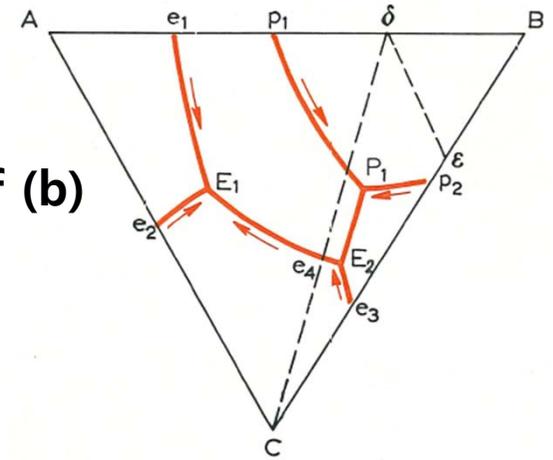
(b) Equilibria when the quasi-peritectic point P is located in the partial system Cδε

# Tabular representation of the ternary space model (a) :

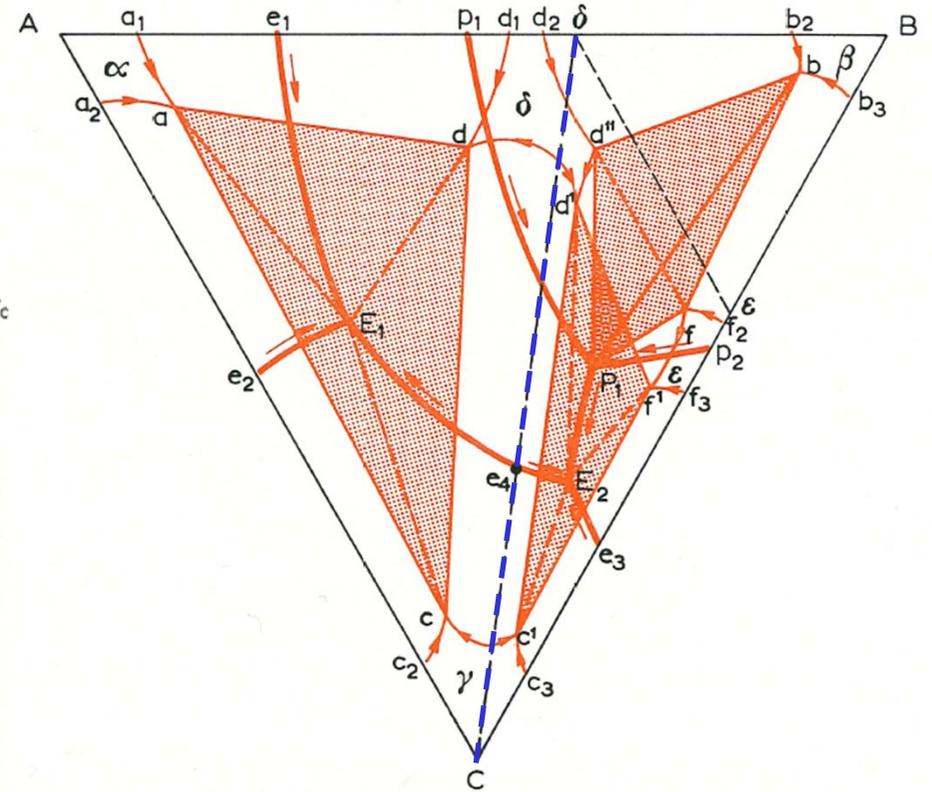


# 11.2 incongruently-melting intermediate phases

- Binary intermediate phases : ternary space model of (b)



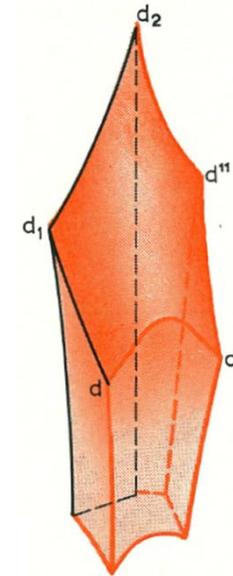
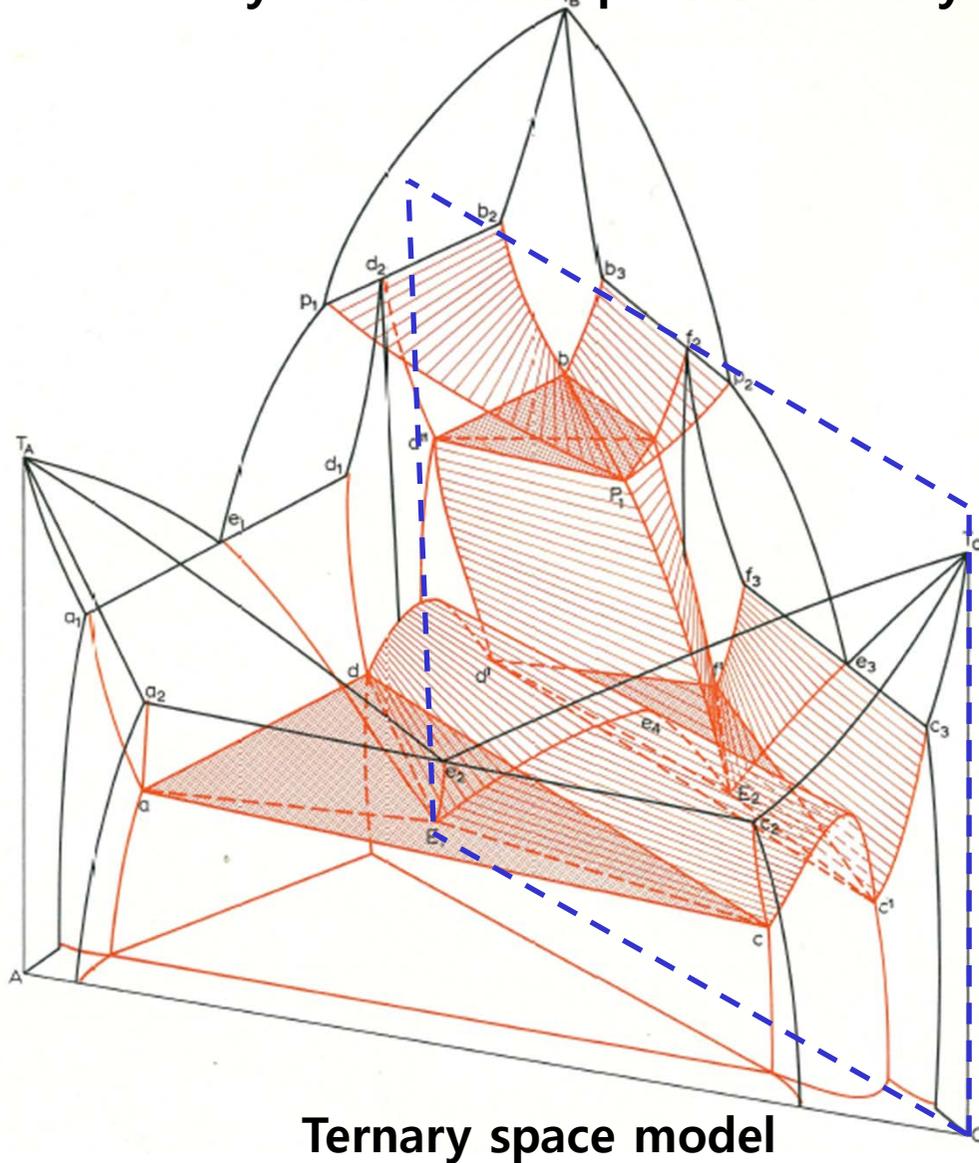
Ternary space model



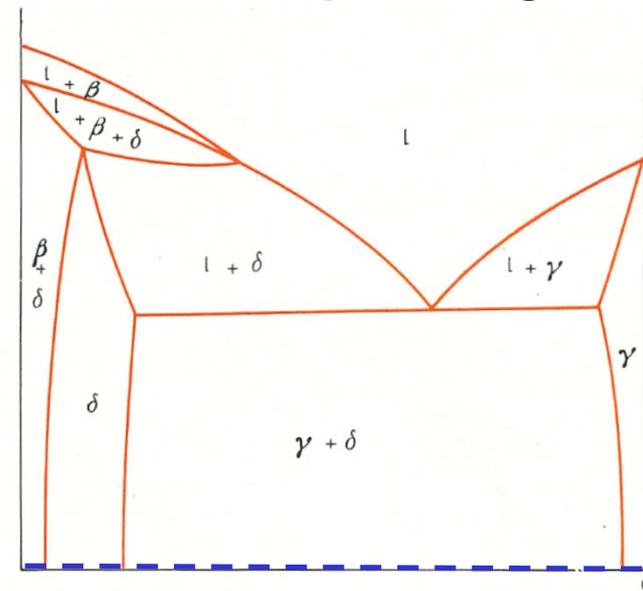
Projection of equilibria

# 11.2 incongruently-melting intermediate phases

- Binary intermediate phases : ternary space model of (b)



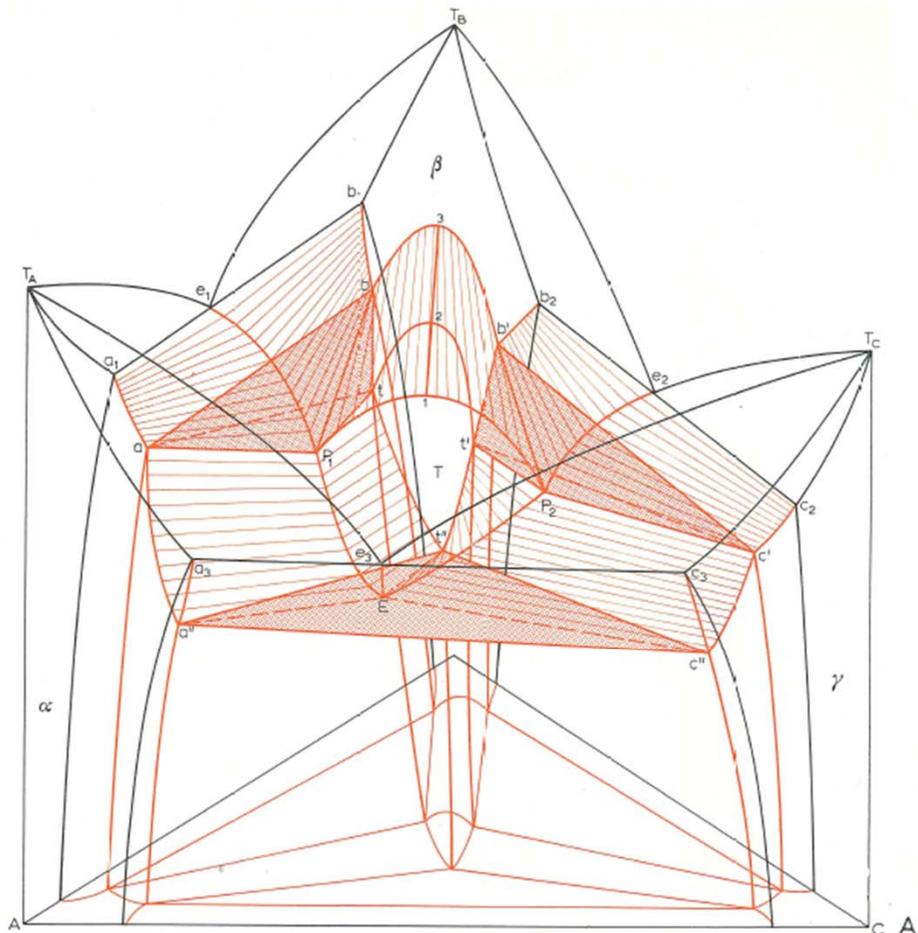
$\delta$  phase region



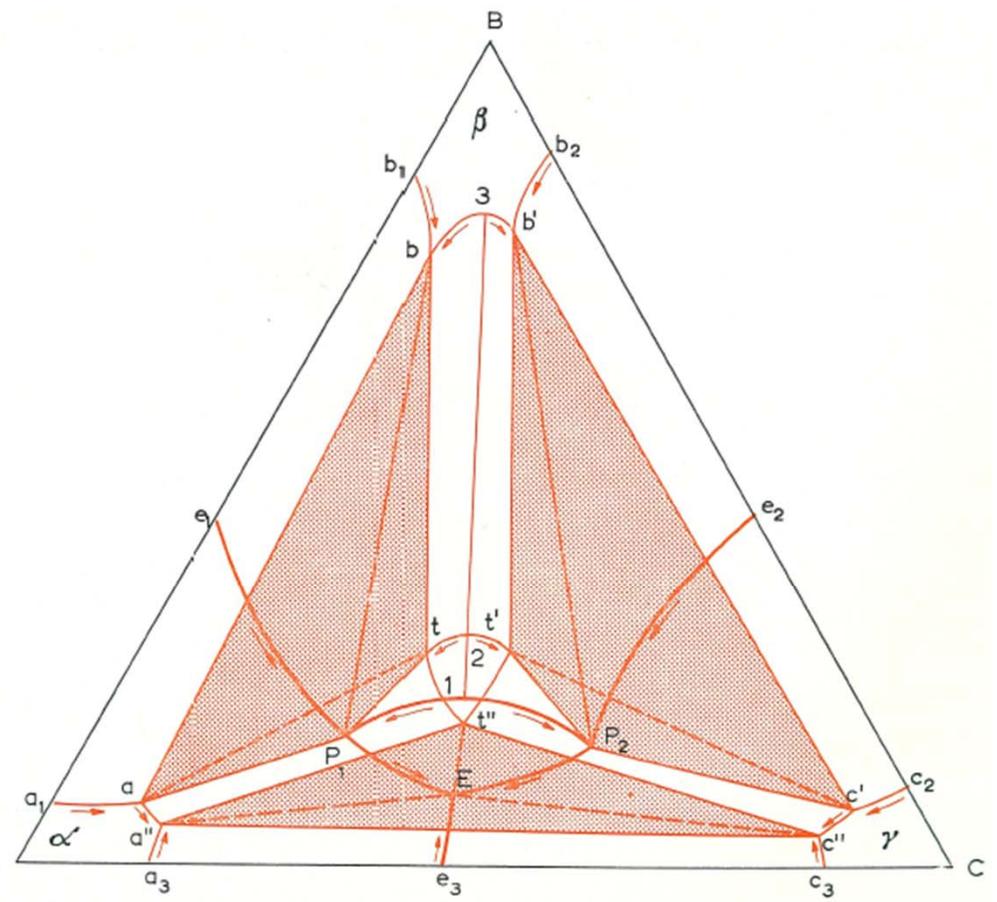
Vertical section from C to  $\delta$   
 : near quasi-binary nature  
 ~ not quasi-binary

# 11.2 incongruently-melting intermediate phases

b) one ternary intermediate phase and all three binary eutectic

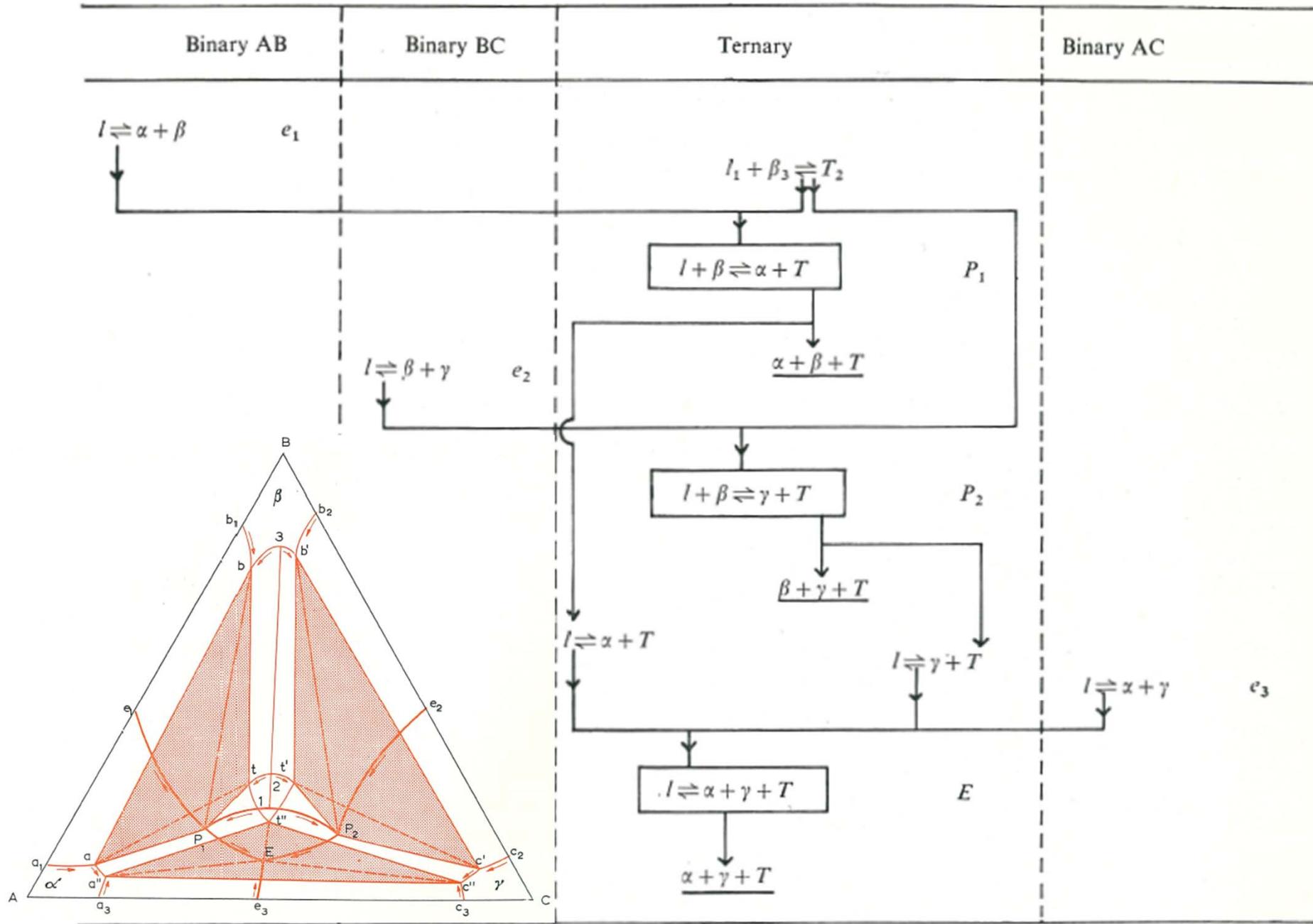


Ternary space model



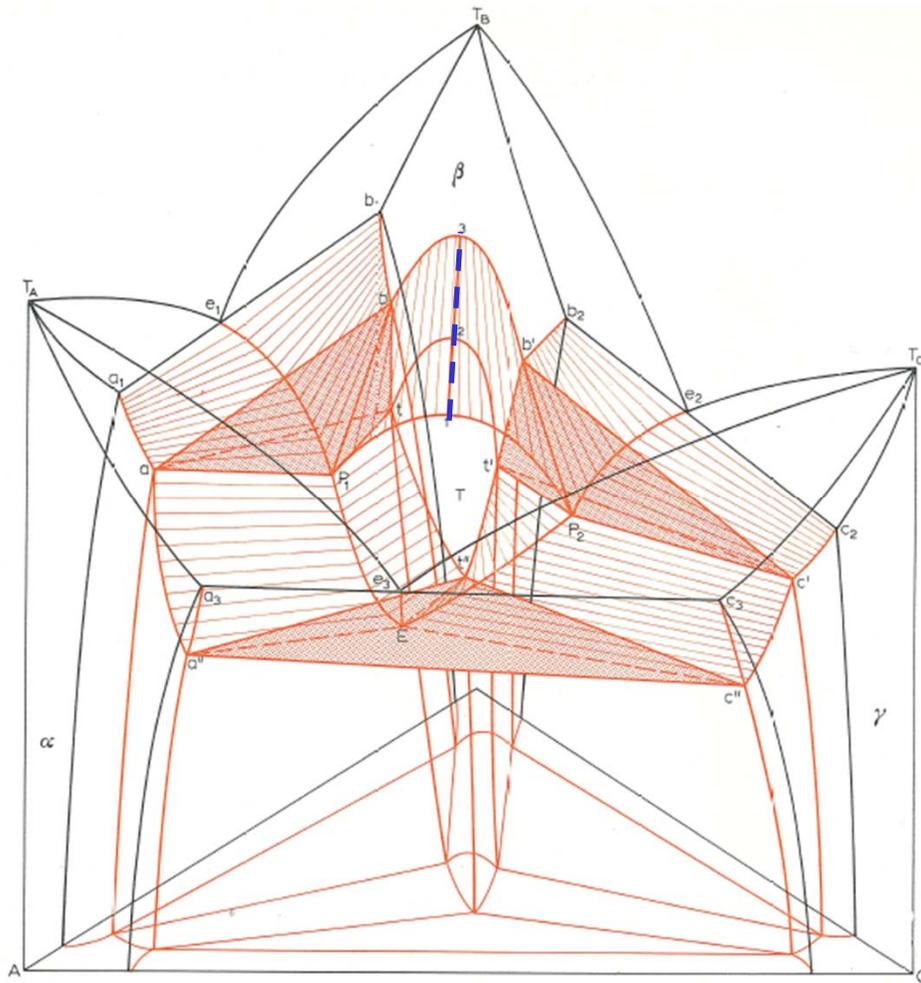
Projection on the concentration triangle

# Tabular representation of the ternary equilibria, e.g. Al-Mg-Zn system

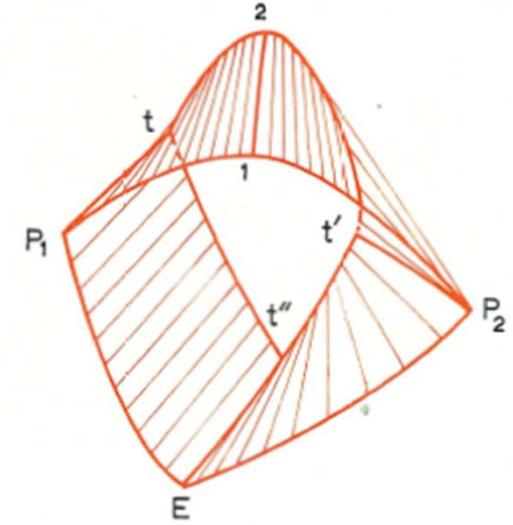


# 11.2 incongruently-melting intermediate phases

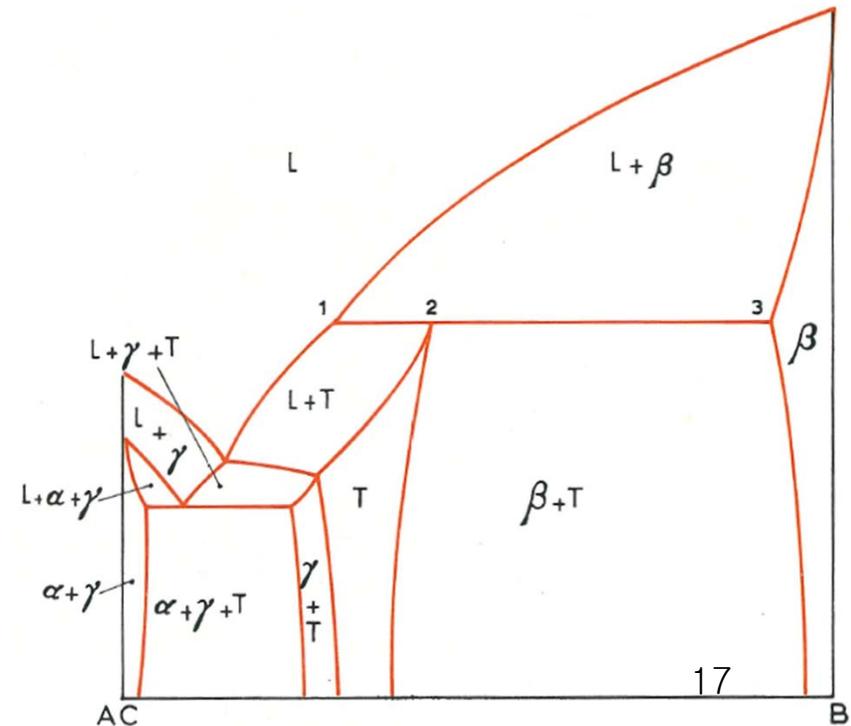
- Ternary intermediate phases



(a) T phase region



(b) L+T phase region



Vertical section along tie line 1-2-3



# Chapter 12. Ternary phase Diagrams

## Liquid Immiscibility

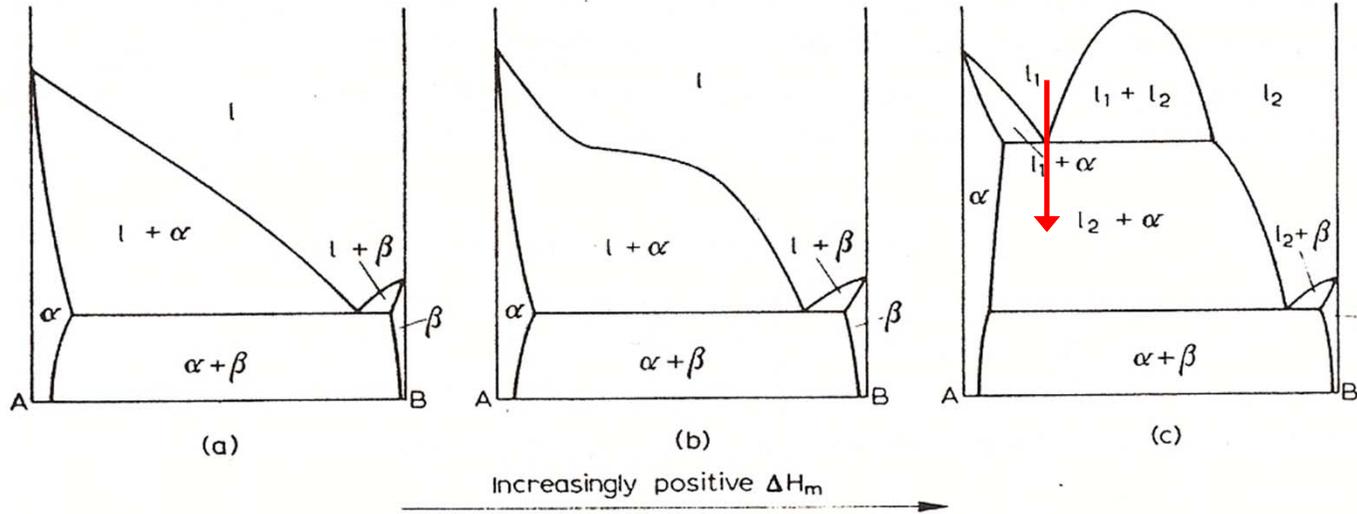
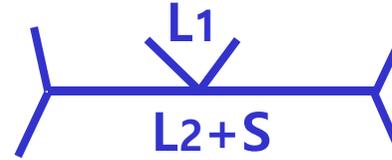
**Liquid immiscibility** in one or more of the binary systems can lead to either three-phase or four-phase equilibria in the ternary system.

**Immiscibility** can arise if either monotectic or syntectic reactions occur in the binary system; true ternary immiscibility is also possible.

# 1) Liquid immiscibility in binary system

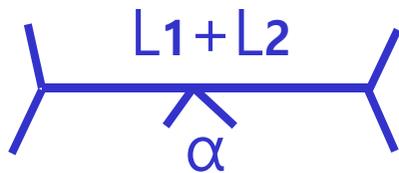
\* **Monotectic reaction:**

Liquid1  $\leftrightarrow$  Liquid2 + Solid

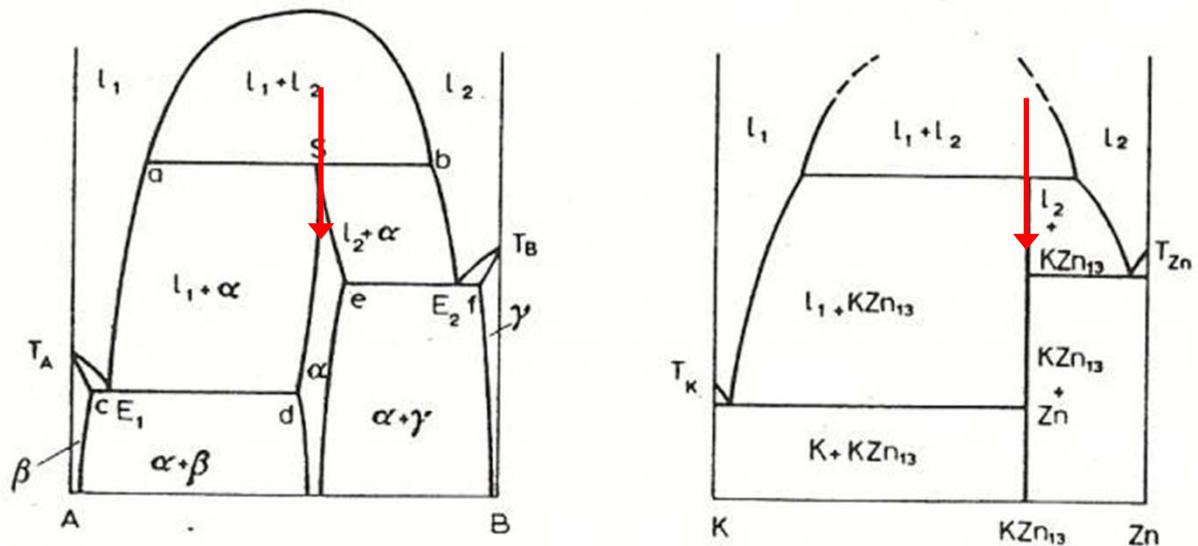


\* **Syntectic reaction:**

Liquid1 + Liquid2  $\leftrightarrow$   $\alpha$



K-Zn, Na-Zn,  
K-Pb, Pb-U, Ca-Cd



## 2) One binary liquid miscibility gap in ternary system

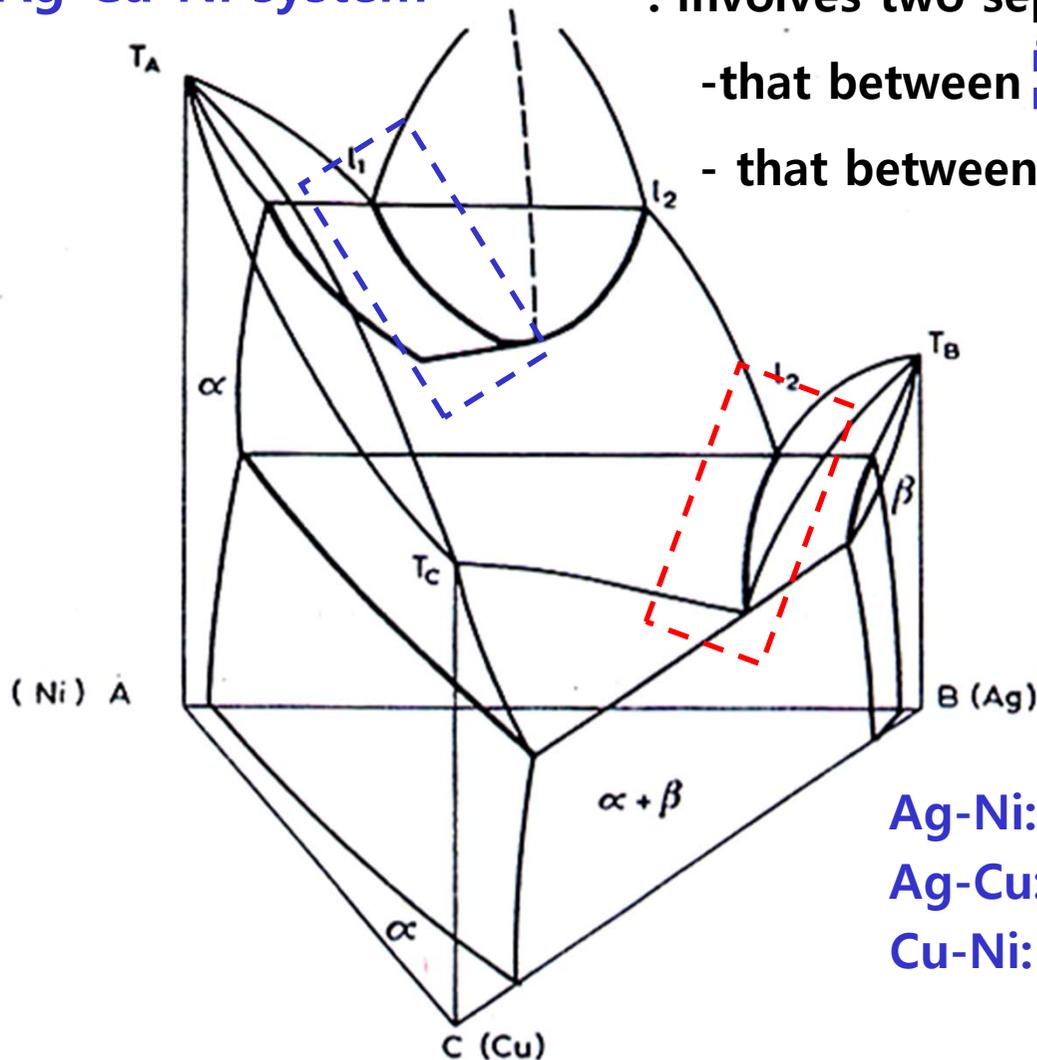
- Binary Monotectic, syntectic and metatectic reactions in combination with each other as well as with binary eutectic and peritectic reactions.

- **Ag-Cu-Ni system**

: involves two separate three phase equilibria

- that between  $\alpha$ ,  $l_1$  and  $l_2$ , and

- that between  $\alpha$ ,  $\beta$  and  $l_2$



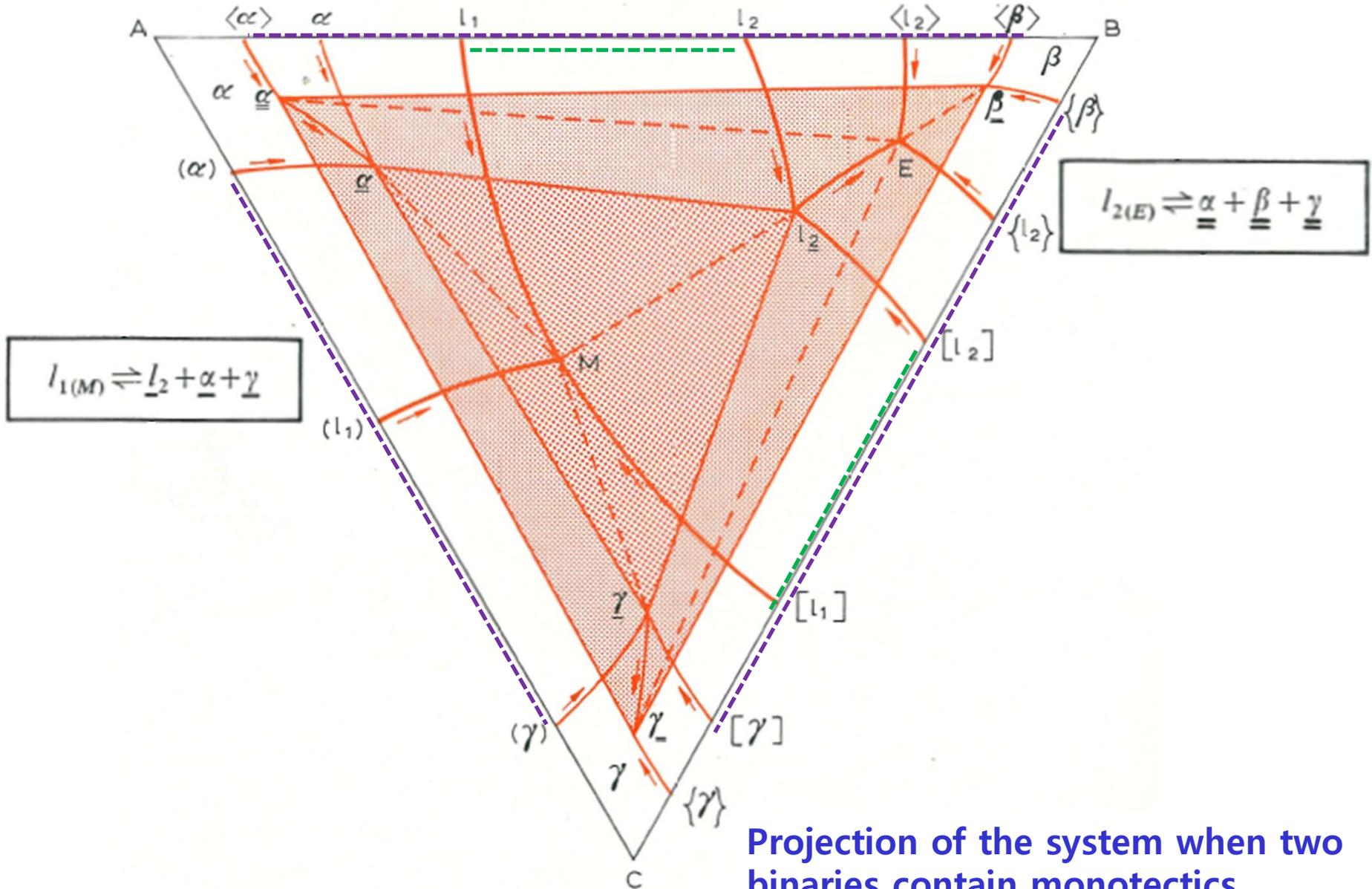
Ag-Ni: monotectic

Ag-Cu: eutectic

Cu-Ni: continuous series of solid soln

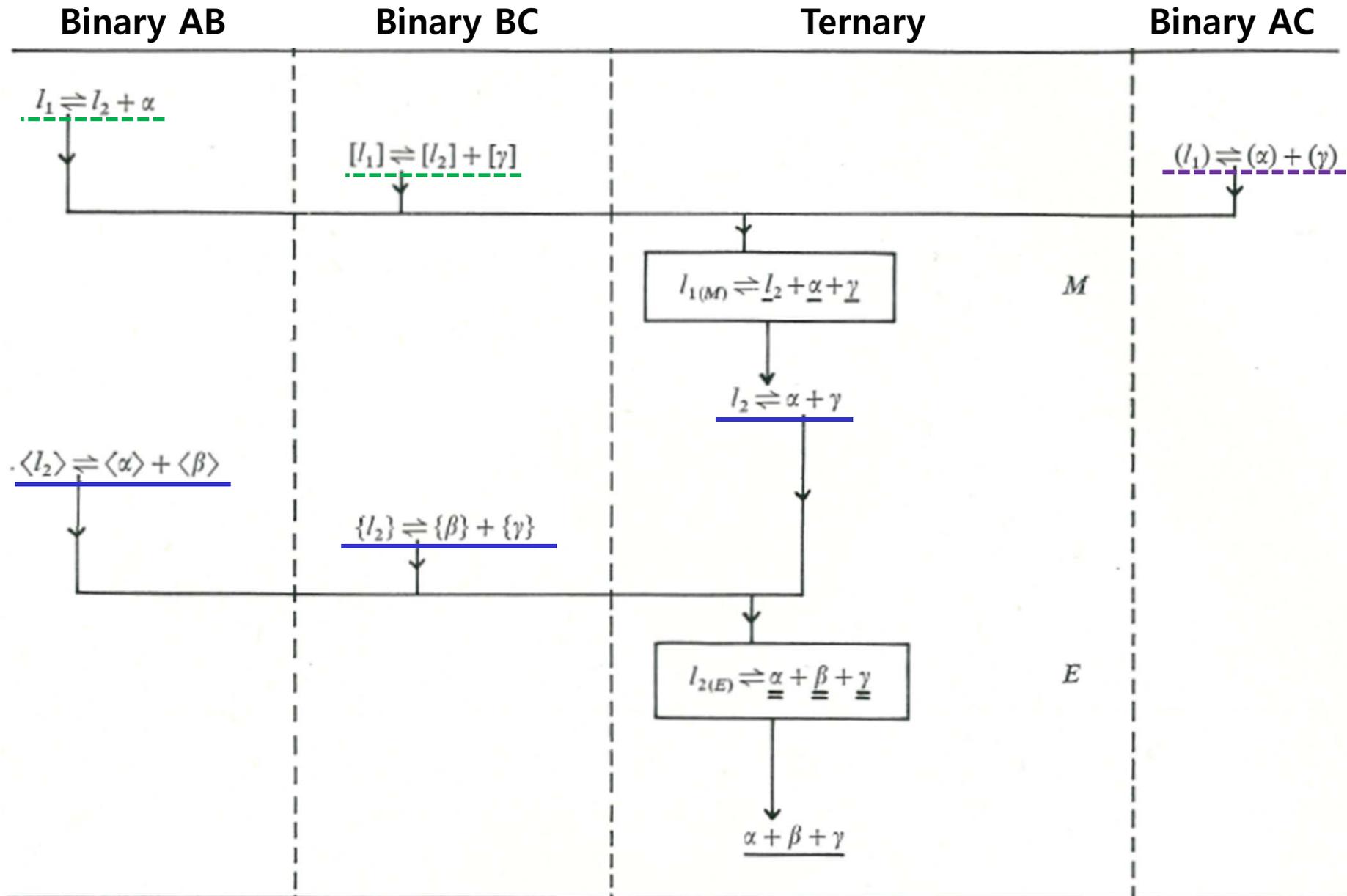
## 12.1. Two Binary Systems are Monotectic

- The AB and BC binaries are monotectics, the AC binary is eutectic.



Projection of the system when two binaries contain monotectics

\* Tabular foam of the system when two binaries contain monotectics

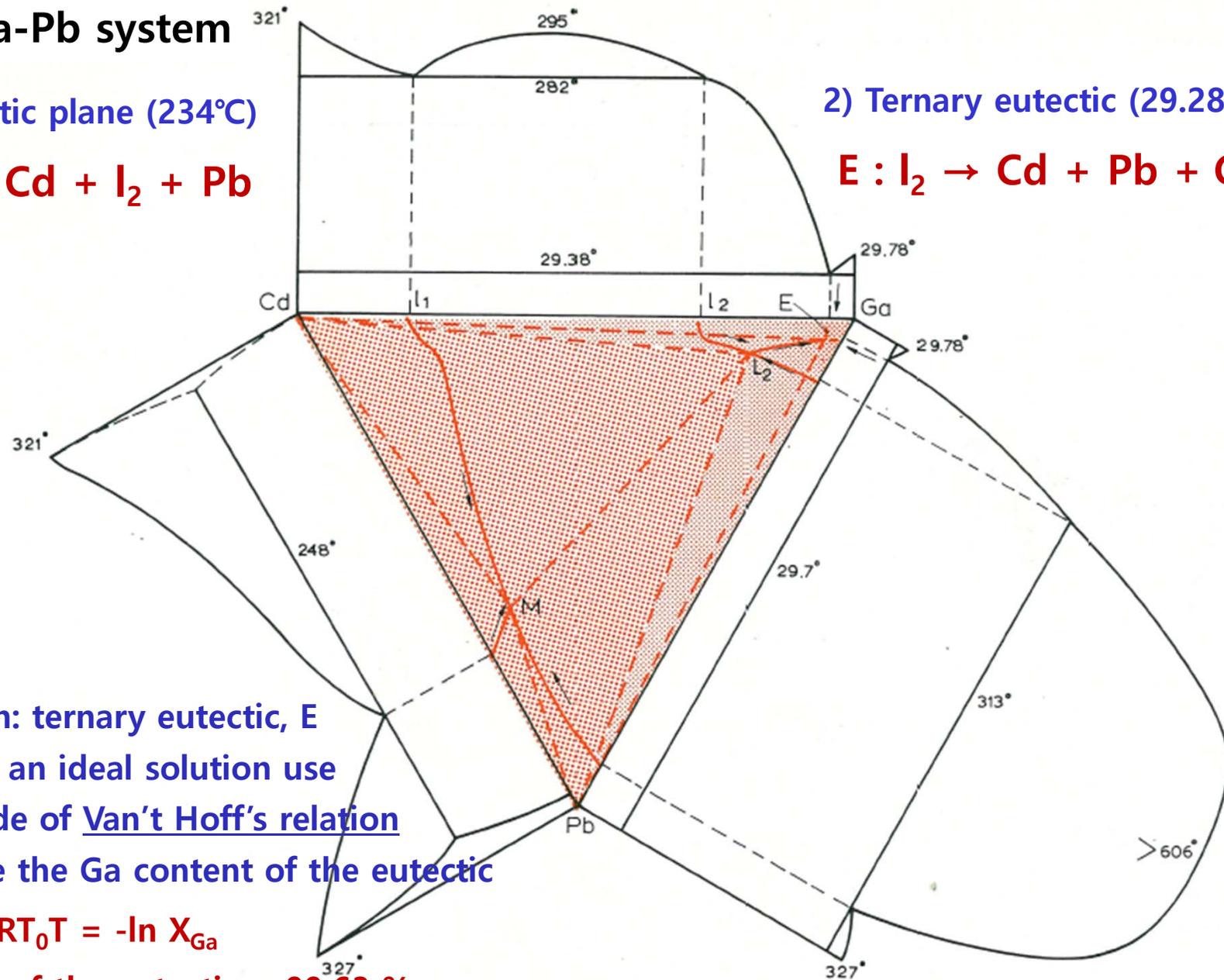


# The Cd-Ga-Pb system

1) Monotectic plane (234°C)



2) Ternary eutectic (29.28°C)



Assumption: ternary eutectic, E behaves as an ideal solution use can be made of Van't Hoff's relation to calculate the Ga content of the eutectic

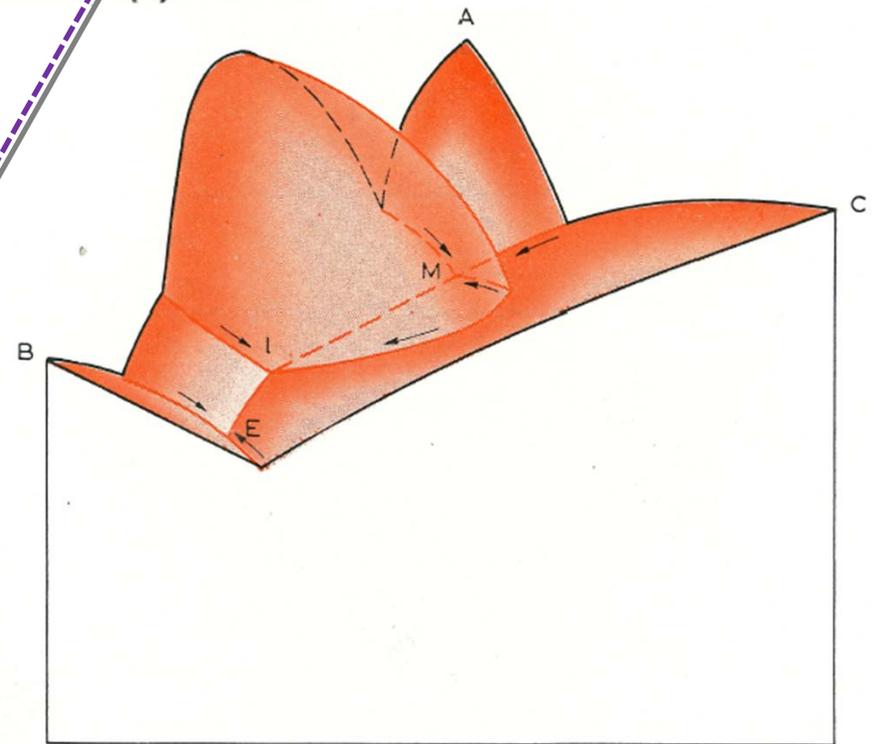
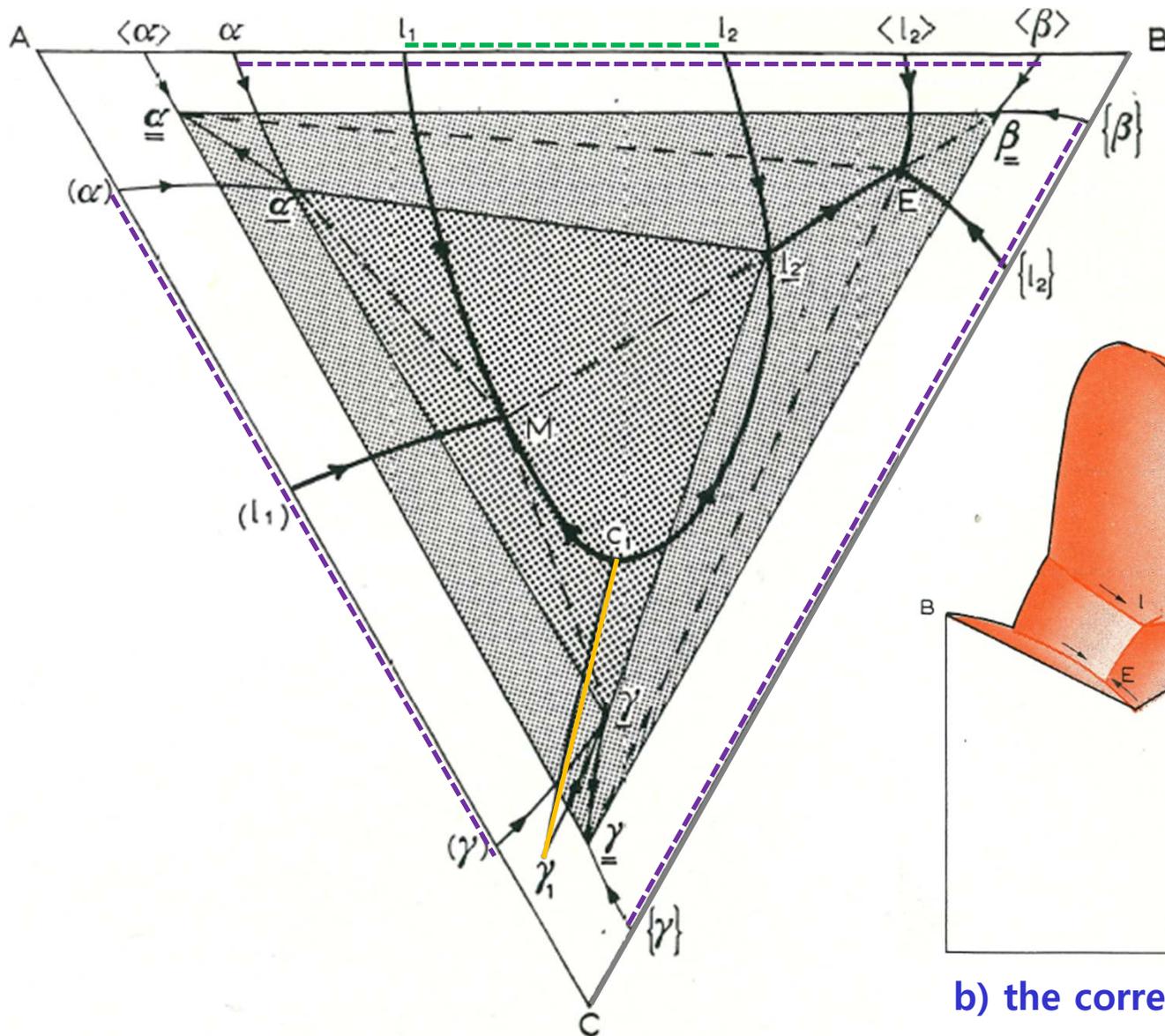
$$[L_{Ga}(T_0 - T)] / RT_0T = -\ln X_{Ga}$$

Ga content of the eutectic = 99.63 %

where  $L_{Ga}$  is the heat of fusion of Ga (1336 cal/g.-atom),  $T_0$  is the m.p. of Ga (302.93 °K),  $T$  is the ternary eutectic temperature,  $R$  the gas constant, and  $X_{Ga}$  the Ga content of the ternary eutectic  $E$ .

## 12.2. One Binary System is Monotectic      Liquid immiscibility in ternary system

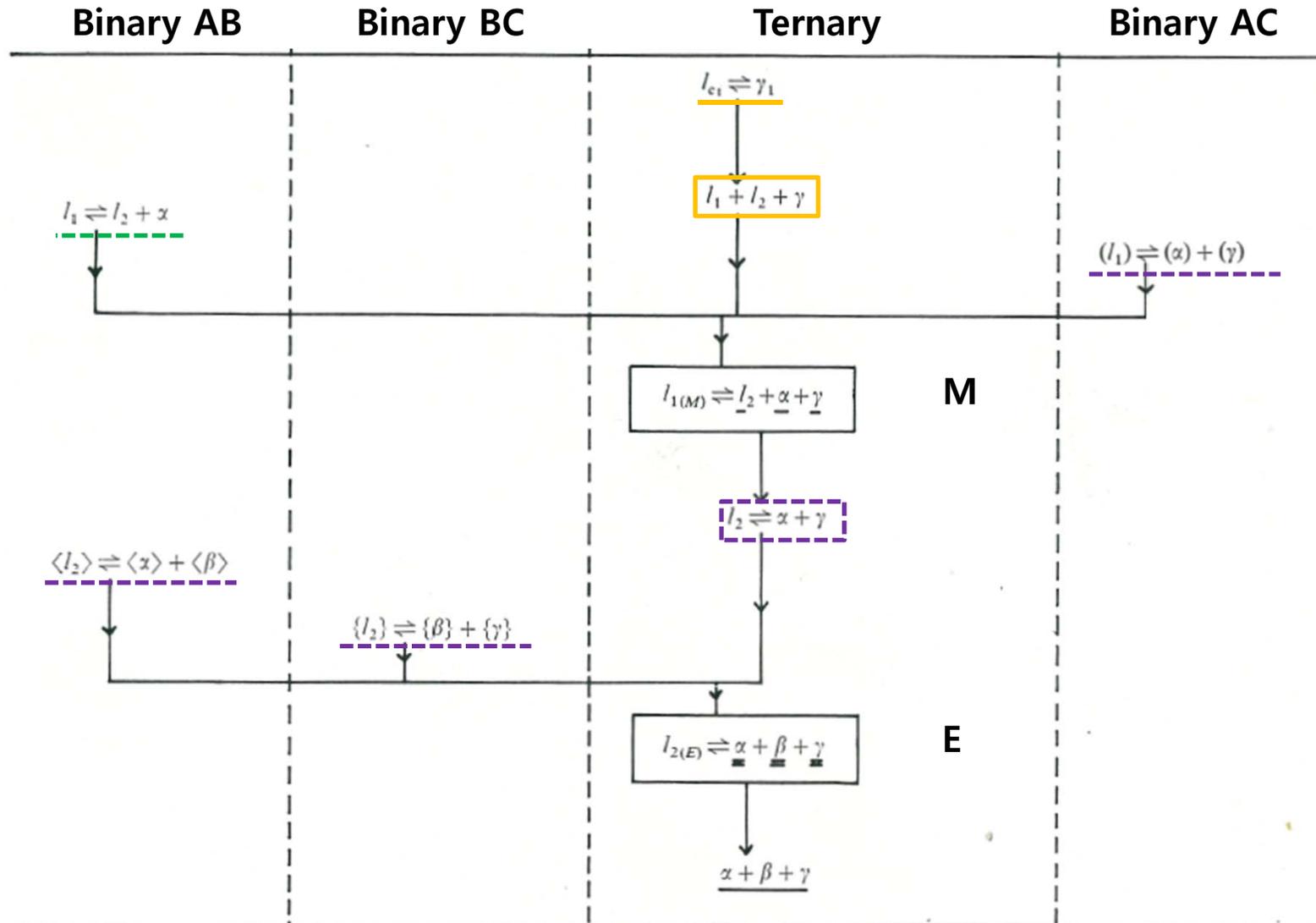
a) Projection of the system when only one binary is monotectic and two binaries are simple eutectic.



b) the corresponding liquidus surface

## 12.2. One Binary System is Monotectic

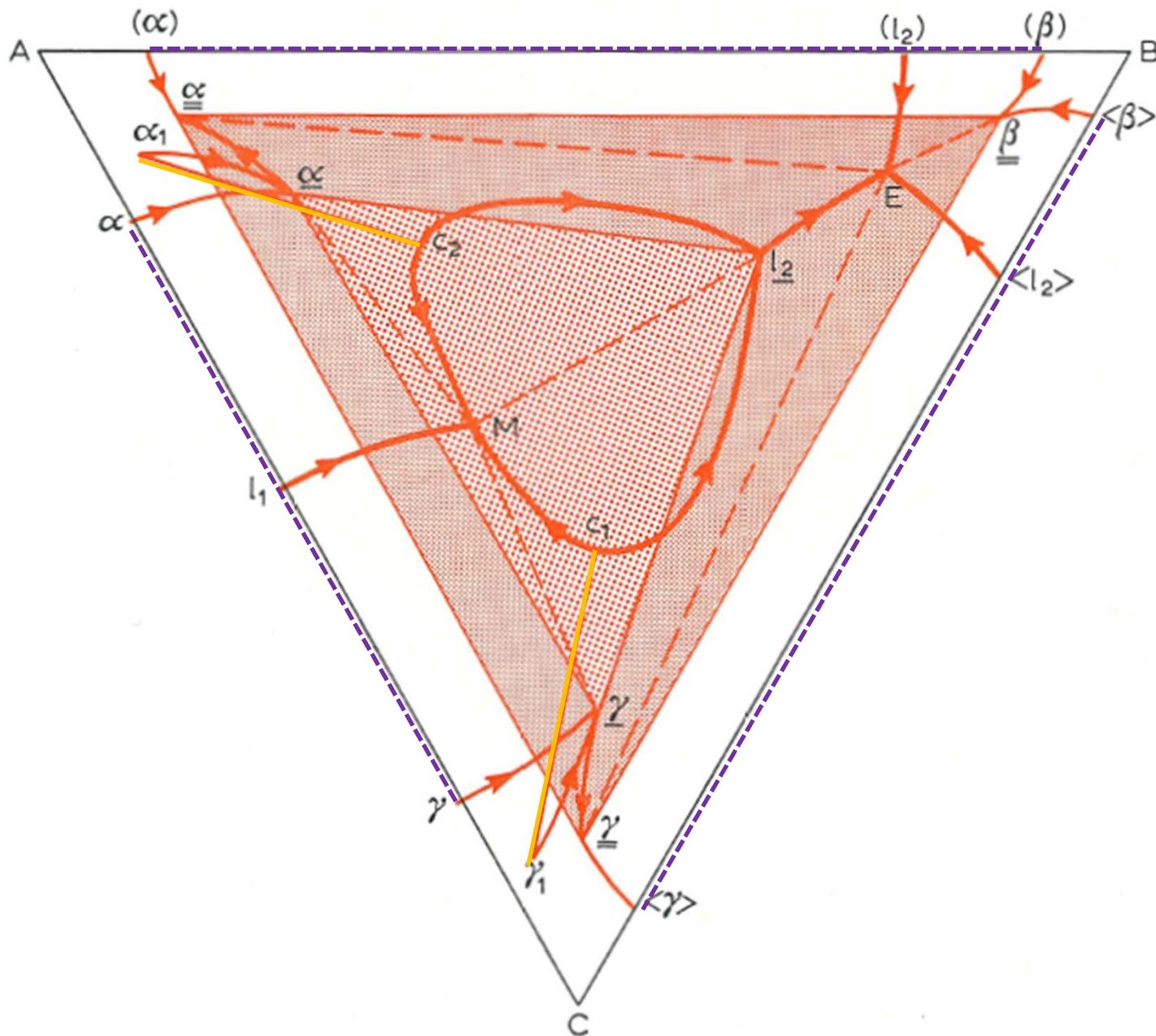
\* Tabular foam of the system when two binaries contain monotectics



\* ex)  $\text{Fe}_3\text{C}$ -FeS-Fe: partial system of C-Fe-S ternary

quasi-binary system Fe- $\text{Fe}_3\text{C}$ : monotectic/ Fe- $\text{Fe}_3\text{C}$  & Fe-FeS: simple eutectic

12.3. None of the Binaries contain liquid miscibility gaps  
but True Ternary Liquid Immiscibility Appears



### 12.3. True Ternary Liquid Immiscibility Appears

\* Tabular foam of the system when true ternary liquid immiscibility appears

