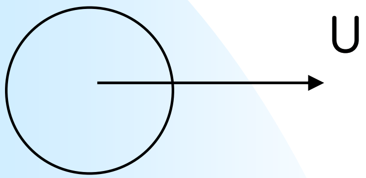
• Particle Motion and Transport

what kinds of forces exerted on particle should be considered to determine particle motion?



- drag force
- random force due to collision of surrounding gas molecules: diffusional force
- gravitational force
- electrical Coulomb force
- thermal force
- Inertial force

Is the particle really following the streamline of gas or is it deviating from the streamline?

We need to understand how the above mentioned force influences the motion of particles. Since the particle can be very small, therefore, we can not always assume no-slip condition. When do we need to consider "slip"?

- Drag force

  Drag force = fu

  f: friction coefficient

Drag on a single sphere : Stokes law

To begin to understand the dynamical behavior of aerosol particles we consider the drag force exerted on an aerosol particle as it moves in a fluid. To calculate the drag force exerted by a fluid on a particle moving in that fluid we must solve the equations of fluid motion to determine the velocity and pressure fields around the particle.

The velocity $\mathbf{u} = [u_1, u_2, u_3]$ and pressure p in an incompressible Newtonian fluid are governed by the equation of continuity,

$$\frac{\partial u_k}{\partial x_k} = 0 \tag{8.14}$$

and the Navier–Stokes equations (Bird et al., 1960),

$$\rho\left[\frac{\partial u_j}{\partial t} + u_k \frac{\partial u_j}{\partial x_k}\right] = \frac{\partial p}{\partial x_j} + \mu \frac{\partial^2 u_j}{\partial x_k \partial x_k} + \rho g_j \qquad j = 1, 2, 3 \tag{8.15}$$

Navier-Stokes Equation

By introducing a characteristic velocity u_0 and length l, the continuity and Navier-Stokes equations can be made dimensionless,

$$\frac{\partial u_k^*}{\partial x_k^*} = 0 \tag{8.16}$$

$$\frac{\partial u_j^*}{\partial t} + u_k^* \frac{\partial u_j^*}{\partial x_k^*} = -\frac{\partial p^*}{\partial x_j^*} + \frac{1}{\mathrm{Re}} \frac{\partial^2 u_j^*}{\partial x_k^* \, \partial x_k^*} \tag{8.17}$$

where the asterisk indicates a dimensionless variable. The *Reynolds number* $\mathrm{Re} = u_0 l \rho / \mu$, the ratio of inertial to viscous forces in the flow. For flow around a submerged body, l can be chosen as a characteristic dimension of the body, say its diameter, and u_0 can be chosen as the speed of the undisturbed fluid upstream of the body.† We will be interested in steady state situations.

When viscous forces dominate inertial forces, $\mathrm{Re} \ll 1$, the type of flow that results is called a *creeping flow* or low-Reynolds number flow, in which case the

Creeping flow for small Re numbers

continuity and Navier–Stokes equations become

$$\frac{\partial u_k^*}{\partial x_k^*} = 0 \tag{8.18}$$

$$\frac{1}{\mathrm{Re}} \frac{\partial^2 u_j^*}{\partial x_k^* \, \partial x_k^*} = \frac{\partial p^*}{\partial x_j^*} \tag{8.19}$$

The solution of these equations for the velocity and pressure distribution around a sphere in creeping flow was first obtained by Stokes. The assumptions invoked to obtain that solution are (1) an infinite medium, (2) a rigid sphere, and (3) no slip at the surface of the sphere. Using the spherical coordinate system defined in Figure 8.3, the two non-zero velocity components and the pressure are given by (Bird et al., 1960, p. 132),

$$u_r = u_\infty \left[1 - \tfrac{3}{2}(R_p/r) + \tfrac{1}{2}(R_p/r)^3\right]\cos\theta \tag{8.20}$$

$$u_\theta = -u_\infty \left[1 - \tfrac{3}{4}(R_p/r) - \tfrac{1}{4}(R_p/r)^3\right]\sin\theta \tag{8.21}$$

$$p = p_0 - \tfrac{3}{2}(\mu u_\infty/R_p)(R_p/r)^2 \cos\theta \tag{8.22}$$

where R_p is the sphere radius, p_0 is the pressure in the plane $z = 0$ far from

Pressure force on the sphere

To obtain the normal force on the sphere we integrate the component of the pressure acting perpendicularly to the surface. (See Figure 8.4.A.)

The z-component of the pressure at a point on the surface at the azimuthal angle θ is $-p\cos\theta$, where the negative sign is needed since the pressure acts in the $-z$ direction if $-90° < \theta < 90°$. Then the normal force over the entire surface is

$$F_n = \int_0^{2\pi}\int_0^{\pi}\left(-p|_{r=R_p}\cos\theta\right)R_p^2\sin\theta\, d\theta\, d\phi \qquad (8.23)$$

Since $p|_{r=R_p} = p_0 - \frac{3}{2}(\mu u_\infty/R_p)\cos\theta$,

$$F_n = 2\pi\mu R_p u_\infty \qquad (8.24)$$

Friction force on the sphere

The z-component of the tangential force is $(-\tau_{r\theta})\sin\theta$, as shown in Figure 8.4.B. Thus, the tangential force over the entire surface is

$$F_t = \int_0^{2\pi}\int_0^{\pi}\left(\tau_{r\theta}|_{r=R_p}\sin\theta\right)R_p^2\sin\theta\,d\theta\,d\phi$$

$$= 4\pi\mu R_p u_\infty \tag{8.28}$$

The total drag force exerted by the fluid on the sphere is

$$F_{\text{drag}} = F_n + F_t = 6\pi\mu R_p u_\infty \tag{8.29}$$

Correction for higher Re number flow (larger particles)

At Re = 1, the drag force predicted by Stokes law is 13 percent low due to the neglect of the inertial terms in the equation of motion. The correction to account for higher Reynolds numbers is (Re $\leq$ 2)*

$$F_{\text{drag}} = 6\pi\mu R_p u_\infty \left[1 + \tfrac{3}{16}\,\text{Re} + \tfrac{9}{160}\,\text{Re}^2 \ln 2\,\text{Re}\right] \qquad (8.31)$$

*To gain a feeling for the order of magnitude of Re for typical aerosol particles at $T = 20°\text{C}$, $p = 1$ atm in air falling at their terminal velocities:

D_p, μm	20	60	100	300
Re	0.02	0.4	2	20

Entire range of particles

To account for the drag force over entire range of Reynolds number, we can express the drag force in terms of an empirical *drag coefficient* C_D as

$$F_{\mathrm{drag}} = C_D A_p \rho \left(u_\infty^2/2\right) \tag{8.32}$$

where A_p is the projected area of the body normal to the flow. Thus, for a spherical particle of diameter D_p,

$$F_{\mathrm{drag}} = \tfrac{1}{8}\pi C_D \rho D_p^2 u_\infty^2 \tag{8.33}$$

where

$$C_D = \begin{cases} 24/\mathrm{Re} & \mathrm{Re} < 0.1 \text{ (Stokes law)} \\ (24/\mathrm{Re})\left[1 + \frac{3}{16}\mathrm{Re} + \frac{9}{160}\mathrm{Re}^2 \ln(2\,\mathrm{Re})\right] & 0.1 < \mathrm{Re} < 2 \\ (24/\mathrm{Re})\left[1 + 0.15\,\mathrm{Re}^{0.687}\right] & 2 < \mathrm{Re} < 500 \\ 0.44 & 500 < \mathrm{Re} < 2 \times 10^5 \end{cases} \tag{8.34}$$

• Non-Continuum effect

: Aerosol particles are small. The particle diameter is often comparable to the distance that gas molecules travel between collisions with other gas molecules.

The key dimensionless group that defines the interaction of the surrounding fluid relative to the particle is Knudsen number $Kn = \frac{2\lambda}{d_p}$

where λ is the mean free path of gas.

So, Knudsen number is the ratio of two length scales.

$$\lambda = \frac{\mu}{0.499P\left(\frac{8M}{\pi RT}\right)^{1/2}}$$

$\lambda[m]$ P [pascal]

M: Molecular weight

μ: gas viscosity (kg/m·s)

The mean free path of air at T=298 K and P=1 atm is $6.51\times10^{-8}m = 0.0651\mu m$

If $d_p >> \lambda$, $Kn \longrightarrow$ 0 continuum regime

$f \sim d_p$ (Stokes flow)

$d_p << \lambda$ $Kn >> 1$ free molecular regime

$f \sim d_p^2$

Kinetic theory of gas

$$f = \frac{2}{3} d_p^2 \rho \left(\frac{2\pi kT}{M} \right)^{1/2} \left(1 + \frac{\pi\alpha}{8} \right)$$

gas gas molecular weight

α : accommodation coefficient ≈ 0.9

$d_p \sim \lambda$: transition regime

$$Drag = \frac{3\pi\mu d_p U}{C_c}$$

$$f = \frac{3\pi\mu d_p}{C_c}$$

C_c : Cunningham Slip connection factor

Non-continuum effect : too small particles

8.3. NON-CONTINUUM EFFECTS—CUNNINGHAM CORRECTION FACTOR

Stokes law is based on continuum fluid mechanics. When the particle diameter D_p approaches the same order as the mean free path λ of the suspending fluid (e.g., air), the resisting force offered by the fluid is smaller than that predicted by Stokes law. To account for non-continuum effects that become important as D_p becomes smaller and smaller, the Cunningham correction factor, C_c, is introduced into Stokes law, written now in terms of particle diameter D_p,

$$F_{\mathrm{drag}} = \frac{3\pi\mu u_\infty D_p}{C_c} \tag{8.35}$$

where

$$C_c = 1 + \frac{2\lambda}{D_p}\left[1.257 + 0.4\exp(-1.1D_p/2\lambda)\right] \tag{8.36}$$

Values of C_c as a function of D_p in air at 25°C ($\lambda = 0.065\ \mu$m) are given in Table 8.1. The limiting behavior of C_c for large and small particle diameter is

$$C_c = \begin{cases} 1 + (1.257)2\lambda/D_p & D_p \gg \lambda \\ 1 + (1.657)2\lambda/D_p & D_p \ll \lambda \end{cases} \tag{8.37}$$

TABLE 8.1. Cunningham Correction Factor C_c for Spherical Particles in Air at 20°C, 1 atm

D_p (μm)	C_c
0.001	216.
0.002	108.
0.005	43.6
0.01	22.2
0.02	11.4
0.05	4.95
0.1	2.85
0.2	1.865
0.5	1.326
1.0	1.164
2.0	1.082
5.0	1.032
10.0	1.016
20.0	1.008
50.0	1.003
100.0	1.0016

- Settling Velocity

$F_{gravity} = F_{drag}$

$$m_p g = f V_s$$

$$\frac{\pi}{6} d_p^3 (\rho_p - \rho) g = \frac{3\pi\mu d_p}{C_c} V_s$$

$$V_s = \frac{\rho_p g d_p^2}{18\mu} C_c (1 - \frac{\rho}{\rho_p})$$

define particle mobility

force

$$v_d = BF \qquad (= \frac{1}{f} m_p g)$$

$\therefore \frac{1}{f}$: particle mobility $\quad\therefore$ particle mobility becomes larger for small d_p

Settling velocity at standard condition, unit density

d_p(㎛)	Vs (cm/sec)
0.01	6.6×10^{-6}
0.1	8.62×10^{-5}
1.0	3.52×10^{-3}
10	3.07×10^{-1}

- Particle Acceleration

$$m_p \frac{dv_p(t)}{dt} = m_p g - \frac{3\pi\mu d_p}{C_c} v_p(t)$$

$$\frac{m_p}{f} \frac{dv_p(t)}{dt} = \frac{m_p g}{f} - v_p(t)$$

define $\dfrac{m_p}{f} \longrightarrow$ time scale

$= \tau = m_p B$: particle relaxation time

$= \dfrac{\rho_p d_p^2 C_c}{18\mu}$: time needed to adjust to surrounding fluid condition

It characterizes the time required for a particle to adjust or relax its velocity to a new condition of forces.

$$\boxed{\tau g = V_s}$$

$$\tau \frac{dv_p(t)}{dt} = \tau g - v_p(t)$$

$$t = 0$$

$$v_p(t) = 0$$

Sol.) $v_p(t) = V_s(1 - \exp(-t/\tau))$

$$t \to \infty$$

$$v_p(t) \to V_s$$

$t = \tau$ 63% of terminal settling velocity

ppt file for τ

small particles quickly adjust its velocity to the surrounding condition

Particle relaxation time

d_p	τ
0.1 ㎛	8.8×10^{-8} (sec)
1.0	3.6×10^{-6}
10	3.1×10^{-4}
100	3.1×10^{-2}

- Stopping distance

If there is no external force and particles have its initial velocity U_0, then the particles eventually stop.

Stopping distance is the distance that the particle travels before stopping.

$$m_p \frac{dv_p(t)}{dt} = -fv_p(t)$$

$$v_p(0) = U_0$$

$$v_p(t) = U_0 \exp(-t/\tau) = \frac{dx_p}{dt}$$

$$x_p(t) = U_0 \tau (1 - \exp(-t/\tau))$$

$$x_p(t \to \infty) = U_0 \tau = \frac{\rho_p d_p^2 U_0 C_c}{18\mu}$$: Stop distance

Stopping distance

d_p (㎛)	x_p (mm) (u=1m/s)
0.01	6.8×10^{-6}
0.1	8.8×10^{-5}
1.0	3.6×10^{-3}
10	0.23
100	12.7

if 150 km/hr= 41.7m/s, this suggests if you throw 100 ㎛ particles with 150 km/hr in still air, 100 ㎛ will travel 529 mm.
For 1 ㎛, the stopping distance is smaller than 1 mm
For 0.01 ㎛ (10 nm), $6.8 \times 10^{-6} \times 41.7$ mm

- Stokes number

$$Stk = \frac{x_p}{L_c} = \frac{U_0 \tau}{L_c} = \frac{\tau}{L_c / U_0} = \frac{\tau}{\tau_{flow}}$$

the ratio of relaxation time to the flow characteristic time

If $Stk << 1$, $\tau << \tau_{flow}$

particles quickly follow the change of flow.

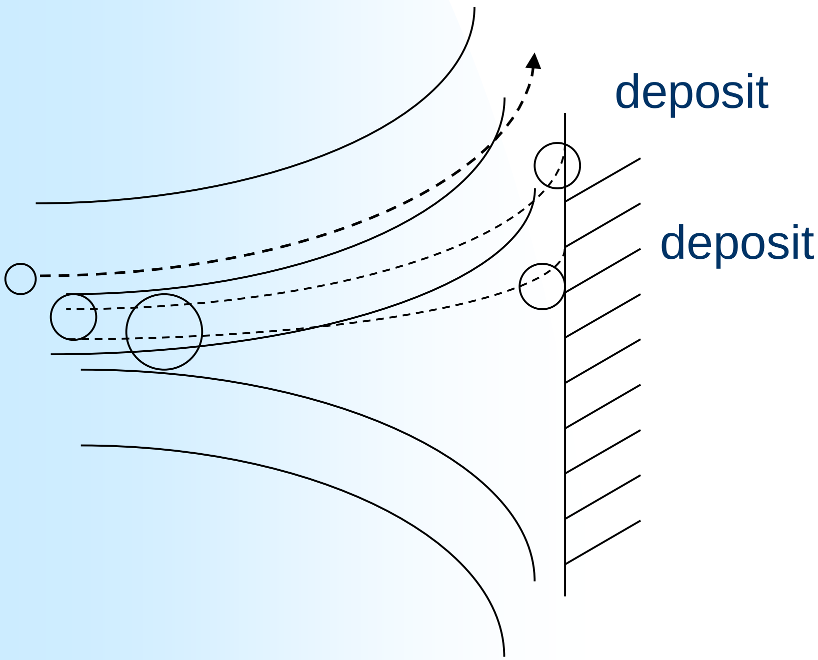
On the other hand,

if $Stk >> 1$, $\tau >> \tau_{flow}$

particles can easily deviate from the flow streamline → inertial effect becomes important.

"One can use LDV (Laser Doppler Velocimeter) to measure particle velocity by suspending seed particles in a flow. Therefore, it is important to use small particles ensuring small St*k* number to correctly measure the flow velocity. Otherwise, you may measure the particle velocity that deviates from the real flow velocity.

Sometimes, you may want to deviate your particles from flow. For example, to collect particles, you always need to deviate the particles from streamline. Otherwise, you can not deposit particle on the surface.



So, it is very difficult to collect small particles by utilizing inertial force.

But if you increase U_0, then you can increase St*k* no. or inertial effect for given particle size, then you can deposit even small particles. → Supersonic impactor can be used to collect nano particles. Or you can increase Cc by lowering the pressure. → Low Pressure impactor can be used to collect nanoparticles