#### • Electrical properties

① Coulomb's law and unit conversion

Electrostatic repulsive force  $F_E$  between two point charges (q, q') of like sign but separated by a distance R

$$F_E = K_E \frac{qq'}{R^2}$$

Where  $K_E$  is a unit conversion constant

For SI unit system

- unit of charge q is C [Coulomb]
- 1 A (ampere) is 1 C/s
- unit of potential difference is V (Volt)
- $1 V = 1 N \cdot m/C$



In this case 
$$K_E = \frac{1}{4\pi\varepsilon_0} = 9.0 \times 10^9 N \cdot m^2 / C^2$$

where  $\mathcal{E}_0$  is the permittivity of a vacuum :  $8.85 \times 10^{-12} C^2 / N \cdot m^2$ 

For cgs unit, the unit of change is stC (stat Coulomb)

the force unit: dyne

the potential difference is stV (stat Volt)

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In this case, K_F = 1
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2 Electric field

If electric field is applied near charged particles, those particles experience "electrostatic force".

We express the strength of such a field in terms of the magnitude of the force  $F_E$  produced per unit charge on the particle.



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# TABLE 15.1Conversion Factors and Constants for SI and cgs ElectrostaticUnits

Quantity	SI	cgs
Charge	1 C	$3.0 \times 10^9 \text{ stC}$
Current	1 A	$3.0 \times 10^9$ stA
Potential difference	1 V	0.0033 stV
K <sub>F</sub>	$9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$	La seloj p apresel se
Charge on an electron, e	$1.60 \times 10^{-19} \mathrm{C}$	$4.80 \times 10^{-10} \text{ stC}$
Electrons per unit charge	$6.3 \times 10^{18}/\text{C}$	$2.1 \times 10^9/\text{stC}$
Electrical mobility	$1 \text{ m}^2/\text{V} \cdot \text{s}$	$3.0 \times 10^6 \text{ cm}^2/\text{stV} \cdot \text{s}$
Field strength	1N/C or V/m	$3.3 \times 10^{-5}$ dyn/stC or stV/cm



The field strength or electric field intensity,

$$\vec{E} = \vec{F}_E / q [N / C]$$
 or  $[dyn / stC]$ 

: field strength is the vector having the same direction as the force  $F_E$ .

In other word, 
$$\ \vec{F}_{_E} \,=\, q\,\vec{E}\,=\,ne\,\vec{E}$$

Here, "e" is the elementary charge on an electron=  $1.6 \times 10^{-19} \text{ C}$ 

Once you know particle charge and electric field intensity, you can calculate the "electric force" exerted on the particle and determine particle motion subjected to the given electric field

$$q \vec{E} = f V_e$$

 Potential difference or voltage difference between two points is defined as the work required to move a unit charge between two points



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$$\Delta W = \Delta V = \frac{F_E \cdot \Delta x}{q}$$
 potential gradient  
$$\therefore E = \frac{\Delta V}{\Delta x} [V/m] = N/C$$

• The electric field around a single point charge **q** can be immediately obtained by the definition of Coulomb law

$$F_{E} = K_{E} \frac{qq}{R^{2}}$$
$$E = \frac{F_{E}}{q'} = \frac{K_{E}q}{R^{2}}$$

	•	•	•	٠	•
q́			R		q
/:	•				,

(imaginary charge q')





③ Electrical mobility

When a charged particle is placed in an electric field, the particle moves

$$F_E = neE = \frac{3\pi\mu d_p V_E}{C_c}$$

$$V_{E} = \frac{neEC_{c}}{3\pi\mu d_{p}} = BF_{E} = ZE \quad B: \text{ particle mobility}$$
  
Z: electrical mobility  $(m^{2} / V \cdot s)$ 

$$Z = \frac{neC_{c}}{3\pi\mu d_{p}}$$



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Particle Diameter	Electrical Mobility (m <sup>2</sup> /V · s) <sup>a</sup>			
(µm)	Singly Charged	Maximum Charge <sup>b</sup>		
Electron	$6.7 \times 10^{-2}$	odisce insing Casher		
Negative air ion	$1.6 \times 10^{-4}$	) (1		
Positive air ion	$1.4 \times 10^{-4}$	and the <u>n</u> strain		
0.01	$2.1 \times 10^{-6}$	$7.3 \times 10^{-4}$		
0.1	$2.7 \times 10^{-8}$	$9.3 \times 10^{-4}$		
1.0	1.1 × 10 <sup>-9</sup>	$(2.5 \times 10^{-3})^{c}$		
10	$9.7 \times 10^{-11}$	$(6.7 \times 10^{-3})^{c}$		
100	$9.3 \times 10^{-12}$	$(1.1 \times 10^{-2})^{c}$		
		and the second second second second second second		

## TABLE 15.2Electrical Mobility of Electrons, Ions, andAerosol Particles at Standard Conditions

<sup>a</sup>For mobility in cm<sup>2</sup>/stV  $\cdot$  s, multiply the value shown by 3 × 10<sup>6</sup>. <sup>b</sup>Based on the ion limit. (See Section 15.6.) <sup>c</sup>Velocity (m/s) in a unit electric field, but because Re > 1.0, Eq. 15.22 does not hold.



• Millikan oil-drop experiment

$$neE = \frac{\pi}{6}d_p^3 \rho_p g$$

He found that the value of the charges on an oil droplet (submicro-meter) different sizes and charges was always multiples of a fundamental unit of charge (e, 2e, 3e, ....)







• Electrical migration (Most aerosols are charged)

Force on a particle carrying *i* elementary units of charge in an electric field E is

$$F = ieE$$
  $e : 1.6 \times 10^{-19} C$ 

$$C_e = \frac{ieE}{f}$$

$$ieE = fC_{e}$$

Migration velocity due to charge and electric field



Electrical mobility (Z)

$$C_e = ZE$$
  $Z = \frac{ie}{f}$ 

Charging mechanisms

- Direct ionization of the particles: radioactive rays, X-ray, etc. could ionize the particle directly. However, at the same time, these energetic rays can also ionize air surrounding the particles. Since the volume fraction of particle is small less than 10<sup>-4</sup>, therefore, air ionization may be more dominant than particle ionization.

Thus particle charging should result more from attachment of air ions than by direct ionization.

- Tribo electrification or frictional electrification charge is imparted to dry non-metallic particles when they come in contact with metals or with other particles



Although triboelectrification is a common charging mechanism, reasons for its occurrence still remains obscure.

#### - flame charging

Flame charging occurs when particles are formed in or pass through a flame. At the high temperature of the flame, direct ionization of gas molecules creates high concentrations of positive and negative ions and also thermionic emissions of electrons. Particles have net charge resulted from balance among those,

#### - Spray electrification

Surface forces (or surface tensions) in liquids of high dielectric constants increase the concentration of electrons or negative ions in the outer liquid surface. The disruption of these surfaces imparts a predominantly negative



### Table 12.2 Charge Preference in Frictional Charging

+ End Asbestos Mica Glass Calcite Quartz Magnesium Lead Gypsum Zinc Pyrite Copper Silver Silicon Sulfur Rubber - End



charge to the smaller droplets while larger ones will be neutral, positive or negative in approximately equal proportions.

Tide at the sea shore can generate negative ions or water fall  $\rightarrow$  These negative ions may comfort human beings.

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-Collisions with ions or ion clusters
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diffusion charging : random collision of ions due to thermal motion (even without E-field  $d_p < 1 \mu$ )

Field Charging : collisions of ion due to ion movement when  $d_p > 1 \mu m$ electrical field is applied

unipolar charging

bipolar charging Kernixed ion of positive and negative charge



(1) Diffusion Charging

: random collisions between ions and particles (generally, particle Brownian motion is negligible due to the difference in mobility)

Because both the particle and the ions are charged, the random thermal motion of the ions in the vicinity of a particle is influenced by an electrostatic force.

This force gives rise to a tendency of the ions to migrate away from the particle as the particle charge increases.



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So, ion concentration near the particle should decrease from ambient value as particle gets charges.

If the particle has a charge q=ne, the ion concentration very near the particle surface should be given by a Boltzmann expression. (White, 1963)

$$N_{i,s} = N_i \exp(-\frac{2ne^2}{d_p kT})$$

 $N_i$ : ambient ion concentration

We need know the effusion flux

$$=\frac{1}{4}N_{i,s}\bar{c}_i$$

$$\therefore \frac{dn}{dt} = N_i \exp(-\frac{2ne^2}{d_\rho kT} \frac{1}{4} \pi d_\rho^2) \times \sqrt{\frac{8kT}{\pi m_i}} \xrightarrow{kT} \overline{c_i}$$

where m<sub>i</sub> is the mass of ions.



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