

## Bipolar Charging

An important example of bipolar charging is the atmospheric aerosol that is exposed to both positive and negative ions. These ions are generated in the atmosphere by cosmic rays and the radioactive decay of radon and thoron gases emanating from the soil. Air ion concentrations normally range around 500 per cc at ground level. The ions are believed to consist of singly charged molecules surrounded by a cluster of a few neutral molecules. The ratio of the concentrations of the positive to negative ions is about 1.2; negative ion mobilities are somewhat higher than the positive ion mobilities. Of special interest are (a) the number of charges assumed by particles in the presence of the bipolar ions, as a function of particle size, and (b) the fraction of the particles that become charged. These are calculated in the analysis that follows, which has application not only to atmospheric particles but also to aerosol instrumentation.

To simplify the calculations, it is assumed that the concentrations, mobilities, and other properties of the positive and negative ions are equal and that the concentrations of the ions and charged particles have reached a steady state. We consider a group of particles of uniform size; no coagulation occurs, so a polydisperse aerosol can be treated as a set of uncoupled monodisperse particles. The rates at which ions of both signs attach to particles are assumed to be independent of each other. In the steady state, ions are generated and destroyed at the same rate by attachment to particles. Calculations indicate that ion recombination is not an important mechanism for ion loss in the atmosphere (Bricard and Pradel, 1966).

Let  $N_i$  be the concentration of particles carrying  $i$  charges, all of like sign, and let  $N_0$  be the concentration of electrically neutral particles. The symmetry of the problem requires that the concentration of particles with charge  $-i$  is also  $N_i$ . Particles carrying a charge  $i$  increase their charge to  $i + 1$  at a rate  $\beta_i N_i$ , where  $\beta_i$  is the rate of successful collisions of positively charged ions with particles carrying  $i$  charges. Particles of charge  $i + 1$  collide with negative ions to join the  $i$  class at a rate  $\beta'_i N_{i+1}$ . The steady-state assumption is satisfied by equating the two rates, leading to the following series of equations:

$$\beta_0 N_0 = \beta'_1 N_1; \quad \beta_1 N_1 = \beta'_2 N_2; \quad \dots; \quad \beta_{i-1} N_{i-1} = \beta'_i N_i \quad (2.48)$$

where  $N_i$  is the steady-state concentration of particles with  $i$  charges. Equations (2.48) have the form of the detailed balance relationships that appear in classical equilibrium theory. However, this is not an equilibrium system because the process is driven by the rate that ions are generated by cosmic rays and radon decay.

In the continuum regime ( $d_p \gg 0.1 \mu\text{m}$ ), the ion fluxes  $\beta_i$  and  $\beta'_i$  can be estimated from steady-state solutions to the ion diffusion equation (2.43) in the presence of a Coulomb force field surrounding the particles; image forces are neglected (Fuchs and Sutugin, 1971):

$$\beta_i = \frac{2\pi D d_p \lambda_i n_{i\infty}}{\exp \lambda_i - 1} \quad (2.49a)$$

$$\beta'_i = \frac{2\pi D d_p \lambda_i n_{i\infty}}{1 - \exp(-\lambda_i)} \quad (2.49b)$$

$$\beta_0 = 2\pi D d_p n_{i\infty} \quad (2.49c)$$

where  $D$  is the ion diffusion coefficient,  $n_{i\infty}$  is the concentration of ions in the gas, and  $\lambda_i = |2ie^2/d_p kT|$ . It follows from these relationships that

$$\beta_i/\beta'_i = \exp(-\lambda_i) \quad (2.50)$$

Multiplying the equalities (2.48) and rearranging

$$N_i = \frac{\beta_{i-1}}{\beta'_i} \frac{\beta_{i-2}}{\beta'_{i-1}} \dots \frac{\beta_1}{\beta'_2} \frac{\beta_0}{\beta'_1} N_0 \quad (2.51a)$$

$$= \frac{\beta_0}{\beta_i} \left( \frac{\beta_i}{\beta'_i} \right) \left( \frac{\beta_{i-1}}{\beta'_{i-1}} \right) \dots \left( \frac{\beta_1}{\beta'_1} \right) N_0 \quad (2.51b)$$

Substituting (2.50) in (2.51b), we obtain

$$N_i = N_0 \frac{\exp \lambda_{i-1}}{\lambda_i} \exp \left( - \sum_1^i \lambda_k \right) \quad (2.52)$$

and substituting for  $\lambda_i$ , we obtain

$$\exp \left[ - \sum_1^i \lambda_k \right] = \exp \left( - \frac{2e^2}{d_p k T} \sum_1^i k \right) \quad (2.52a)$$

Noting that  $\sum_1^i k = i(i + 1)/2$  and substituting in (2.52a) gives

$$\exp \left( - \sum_1^i \lambda_k \right) = \exp \left( - \frac{e^2 i^2}{d_p k T} \right) \exp \left( - \frac{e^2 i}{d_p k T} \right) \quad (2.53a)$$

$$= \exp \left( - \frac{e^2 i^2}{d_p k T} \right) \exp \left( - \frac{\lambda_i}{2} \right) \quad (2.53b)$$

Substituting (2.53b) in (2.52) gives

$$N_i = N_0 \exp \left( -i^2 e^2 / d_p k T \right) \quad (2.54)$$

Thus, the particle charge distribution is approximated by the Boltzmann equation. This expression holds best for particles larger than about  $1 \mu\text{m}$ . For smaller particles, the flux terms (2.49) based on continuum transport theory must be modified semiempirically. The results of calculations of the fraction of charged particles are given in Table 2.2. The fraction refers to particles of charge of a given sign.

The number of particles per unit volume carrying  $n$  charges of both signs is twice that given in Eq. 12.32, assuming the numbers of positive and negative particles are equal.

The total number of positively charged particles  $c_+$  or negatively charged particles  $c_-$  per unit volume is

$$c_+ = c_- = \sum c_1 + c_2 + c_3 + \dots$$

and the total number of particles per unit volume is

$$c_T = c_0 + c_+ + c_-$$

The fraction of particles having  $n$  units of charge of one sign, denoted  $f(n)$ , is

$$f(n) = \frac{c_n}{c_T} = \frac{c_0 \exp[-n^2 e^2 / (dkT)]}{c_0 + \sum_1^{\infty} 2c_0 \exp[-n^2 e^2 / (dkT)]}$$



*Center for Nano Particle Control*

*Seoul National U., Mechanical & Aerospace Eng.*

or

$$f(n) = \frac{\exp[-n^2 e^2 / (dkT)]}{\sum_{-\infty}^{\infty} \exp[-n^2 e^2 / (dkT)]}$$



*Center for Nano Particle Control*

*Seoul National U., Mechanical & Aerospace Eng.*



$$f_n = \left( \frac{e^2}{\pi d_p kT} \right)^{1/2} \exp\left( \frac{-n^2 e^2}{d_p kT} \right) \quad (15.31)$$

Approximate for  
Particles larger  
than 0.5  $\mu\text{m}$

## Boltzmann Equilibrium Charge Distribution

$$f_n = \frac{\exp(-n^2 e^2 / d_p kT)}{\sum_{n=-\infty}^{\infty} \exp(-n^2 e^2 / d_p kT)} \quad (15.30)$$



**TABLE 12.5 Equilibrium Charge Distribution Fraction of Charge of Either Sign**

$d$	Number of charges on particle								
	0	1	2	3	4	5	6	7	8
0.05	0.602	0.385	0.013	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.426	0.482	0.087	0.005	0.000	0.000	0.000	0.000	0.000
0.20	0.301	0.453	0.193	0.046	0.006	0.000	0.000	0.000	0.000
0.50	0.190	0.340	0.241	0.137	0.062	0.022	0.006	0.001	0.000
1.00	0.135	0.254	0.214	0.161	0.108	0.065	0.035	0.017	0.007
2.00	0.095	0.185	0.170	0.147	0.121	0.093	0.068	0.047	0.031
5.00	0.060	0.119	0.115	0.109	0.100	0.091	0.080	0.069	0.058
10.00	0.043	0.085	0.083	0.081	0.078	0.074	0.069	0.064	0.059



*Center for Nano Particle Control*

*Seoul National U., Mechanical & Aerospace Eng.*

A common approach is to use a radioactive source such as polonium-210 or krypton-85 in a thin stainless steel tube to ionize air molecules inside a chamber through which the aerosol flows. The volume of the chamber should be designed such that it provides sufficient residence time for the aerosol to attain the Boltzmann equilibrium charge distribution. A value of  $6 \times 10^{+12}$  ion/s/m(+3) is required for complete neutralization. Usually, 2 sec is needed. In the atmosphere where  $10^{+9}$  ions / m(+3) exists, the same effect is achieved in 100 min..



*Center for Nano Particle Control*

*Seoul National U., Mechanical & Aerospace Eng.*