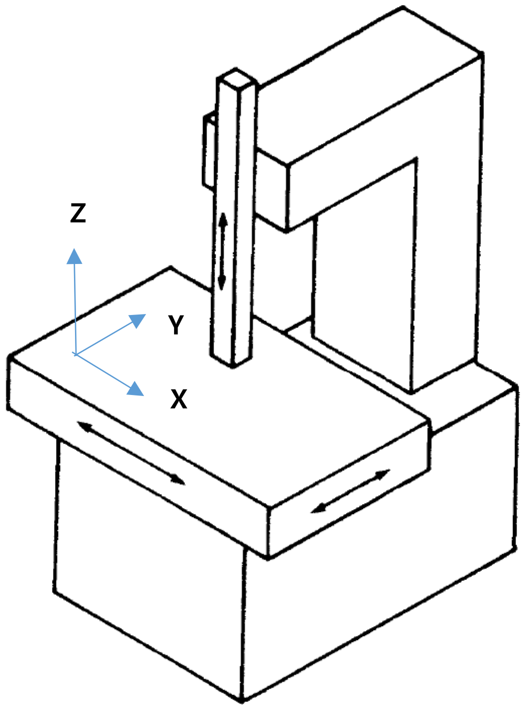
Precision Metrology 20 volumetric error for_Moving_table_type M/C

Column Type Machine or Moving Table Type Machine



Kinematic Chain:

Reference->Y slide->X slide (Work Piece Coordinate)

Reference->Z slide

Y slide motion:

Rotational transformation matrix, **T**y;

and Translational motion vector, **L**y

[$O_R X_R Y_R Z_R$]: Reference coordinate, $\mathbf{X}_R$

[$O_1 X_1 Y_1 Z_1$]: Moving coordinate fixed on the Y slide, $\mathbf{X}_1$

Two coordinates were initially the same.

The Y slide motion can be expressed as the reference coordinates

$\mathbf{X}_R=\mathbf{T}y\mathbf{X}_1+\mathbf{L}y$; eq(1)

Where

$$\mathbf{T}y=\begin{bmatrix} 1 & -Ez(y) & Ey(y) \\ Ez(y) & 1 & -Ex(y) \\ -Ey(y) & Ex(y) & 1 \end{bmatrix} \qquad \mathbf{L}y=\begin{bmatrix} \delta x(y) \\ Y+\delta y(y) \\ \delta z(y) \end{bmatrix}$$

<u>X slide motion</u>:

Rotational transformation matrix, $\mathbf{T}x$;

and Translational motion vector, $\mathbf{L}x$

[$O_1 X_1 Y_1 Z_1$]: Moving coordinate fixed on Y slide, $\mathbf{X}_1$

[$O_2 X_2 Y_2 Z_2$]: Moving coordinate fixed on the X slide, $\mathbf{X}_2$

Two coordinates were initially the same.

The X slide motion can be expressed in the Y slide coordinates.

$\mathbf{X}_1=\mathbf{T}x\mathbf{X}_2+\mathbf{L}x$; eq(2)

Where

$$\mathbf{T}x=\begin{bmatrix} 1 & -Ez(x) & Ey(x) \\ Ez(x) & 1 & -Ex(x) \\ -Ey(x) & Ex(x) & 1 \end{bmatrix} \qquad \mathbf{L}x=\begin{bmatrix} X+\delta x(x) \\ \delta y(x)-\alpha X \\ \delta z(x) \end{bmatrix}$$

Z slide motion:

Rotational transformation matrix, **T**z;

and Translational motion vector, **L**z

$[O_RX_RY_RZ_R]$: Reference coordinate, $\mathbf{X}_R$

$[O_3X_3Y_3Z_3]$: Moving coordinate fixed on the Z slide, $\mathbf{X}_3$

Two coordinates were initially the same.

The Z slide motion can be expressed in the reference coordinates.

$\mathbf{X}_R=\mathbf{T}z\mathbf{X}_3+\mathbf{L}z$; eq(3)

Where

$$\mathbf{T}z=\begin{bmatrix} 1 & -Ez(z) & Ey(z) \\ Ez(z) & 1 & -Ex(z) \\ -Ey(z) & Ex(z) & 1 \end{bmatrix} \qquad \mathbf{L}z=\begin{bmatrix} \delta x(z)-\beta_1 Z \\ \delta y(z)-\beta_2 Z \\ Z+\delta z(x) \end{bmatrix}$$

The volumetric error is defined in the work piece coordinates, $\mathbf{X}_2$. Thus from eq(2),

$\mathbf{X}_2=\mathbf{T}x^{-1}\{\mathbf{X}_1-\mathbf{L}x\}$, and from eq(1)

$\mathbf{X}_1=Ty^{-1}\{\mathbf{X}_R-\mathbf{L}y\}$, and from eq(3)

Therefore

$\mathbf{X}_2=\mathbf{T}x^{-1}[\mathbf{T}y^{-1}\{\mathbf{T}z\mathbf{X}_3+\mathbf{L}z-\mathbf{L}y\}-\mathbf{L}x]$

; Vector form of the volumetric error formulation

For a tool point (Xp,Yp,Zp) in the $\mathbf{X}_3$ coordinate;

$\mathbf{T}z\mathbf{X}_p+\mathbf{L}z-\mathbf{L}y=$

$$\begin{bmatrix}1 & -Ez(z) & Ey(z)\\ Ez(z) & 1 & -Ex(z)\\ -Ey(z) & Ex(z) & 1\end{bmatrix}\begin{bmatrix}Xp\\ Yp\\ Zp\end{bmatrix} + \begin{bmatrix}\delta x(z)-\beta_1 Z\\ \delta y(z)-\beta_2 Z\\ Z+\delta z(z)\end{bmatrix} - \begin{bmatrix}\delta x(y)\\ Y+\delta y(y)\\ \delta z(y)\end{bmatrix} =$$

$$\begin{bmatrix}\delta x(z)-\beta_1 Z-\delta x(y)+Xp-YpEz(z)+ZpEy(z)\\ \delta y(z)-\beta_2 Z-Y-\delta y(y)+XpEz(z)+Yp-ZpEx(z)\\ Z+\delta z(z)-\delta z(y)-XpEy(z)+YpEx(z)+Zp\end{bmatrix}$$

After ignoring over 2'nd order,

$$\mathbf{T}y^{-1}\{\mathbf{T}z\mathbf{X}_p+\mathbf{L}z-\mathbf{L}y\}-\mathbf{L}x=$$

$$\begin{bmatrix}1 & Ez(y) & -Ey(y)\\ -Ez(y) & 1 & Ex(y)\\ Ey(y) & -Ex(y) & 1\end{bmatrix}\begin{bmatrix}\delta x(z)-\beta_1 Z-\delta x(y)+Xp-YpEz(z)+ZpEy(z)\\ \delta y(z)-\beta_2 Z-Y-\delta y(y)+XpEz(z)+Yp-ZpEx(z)\\ Z+\delta z(z)-\delta z(y)-XpEy(z)+YpEx(z)+Zp\end{bmatrix} - \begin{bmatrix}X+\delta x(x)\\ \delta y(x)-\alpha X\\ \delta z(x)\end{bmatrix} =$$

$$\begin{bmatrix}-X-\delta x(x)+\delta x(z)-\beta_1 Z-\delta x(y)+Xp-YpEz(z)+ZpEy(z)+Ez(y)(-Y+Yp)-Ey(y)(Z+Zp)\\ -\delta y(x)+\alpha X+\delta y(z)-\beta_2 Z-Y-\delta y(y)+XpEz(z)+Yp-ZpEx(z)\ -XpEz(y)+(Z+Zp)Ex(y)\\ -\delta z(x)+Z+\delta z(z)-\delta z(y)-XpEy(z)+YpEx(z)+Zp+XpEy(y)-Ex(y)(-Y+Yp)\end{bmatrix}$$

$$\mathbf{T}x^{-1}[\mathbf{T}y^{-1}\{\mathbf{T}z\mathbf{X}_p+\mathbf{L}z-\mathbf{L}y\}-\mathbf{L}x]=$$

$$\begin{bmatrix} 1 & Ez(x) & -Ey(x) \\ -Ez(x) & 1 & Ex(x) \\ Ey(x) & -Ex(x) & 1 \end{bmatrix} [\mathbf{T}y^{-1}\{\mathbf{T}z\mathbf{X}_p+\mathbf{L}z-\mathbf{L}y\}-\mathbf{L}x]= \begin{bmatrix} Xa \\ Ya \\ Za \end{bmatrix}$$

Thus,

$Xa=-X-\delta x(x)+\delta x(z)-\beta_1 Z-\delta x(y)+Xp-YpEz(z)+ZpEy(z)+Ez(y)(-Y+Yp)$

$-Ey(y)(Z+Zp)+Ez(x)(-Y+Yp)-Ey(x)(Z+Zp)$

$Ya=-\delta y(x)+\alpha X+\delta y(z)-\beta_2 Z-Y-\delta y(y)+XpEz(z)+Yp-ZpEx(z)$

$-XpEz(y)+(Z+Zp)Ex(y)-Ez(x)(-X+Xp)+Ex(x)(Z+Zp)$

$Za= -\delta z(x)+Z+\delta z(z)-\delta z(y)-XpEy(z)+YpEx(z)+Zp+XpEy(y)-Ex(y)(-Y+Yp)$

$+Ey(x)(-X+Xp)-Ex(x)(-Y+Yp)$

Therefore,

$\Delta X=Xa-(-X+Xp)=-\delta x(x)+\delta x(z)-\beta_1 Z-\delta x(y)-YpEz(z)+ZpEy(z)+Ez(y)(-Y+Yp)$

$-Ey(y)(Z+Zp)+Ez(x)(-Y+Yp)-Ey(x)(Z+Zp)$

$=-\delta x(x)-\delta x(y)+\delta x(z)-\beta_1 Z-Z(Ey(x)+Ey(y))-Y(\underline{Ez(x)}+Ez(y))$

$+Yp(Ez(x)+Ez(y)-E(z))-Zp(Ey(x)+Ey(y)-Ey(z))$

$\Delta Y = Ya-(-Y+Yp) = -\delta y(y) - \delta y(x) + \delta y(z) + \alpha X - \beta_2 Z + Z(Ex(x)+Ex(y)) + XEz(x)$

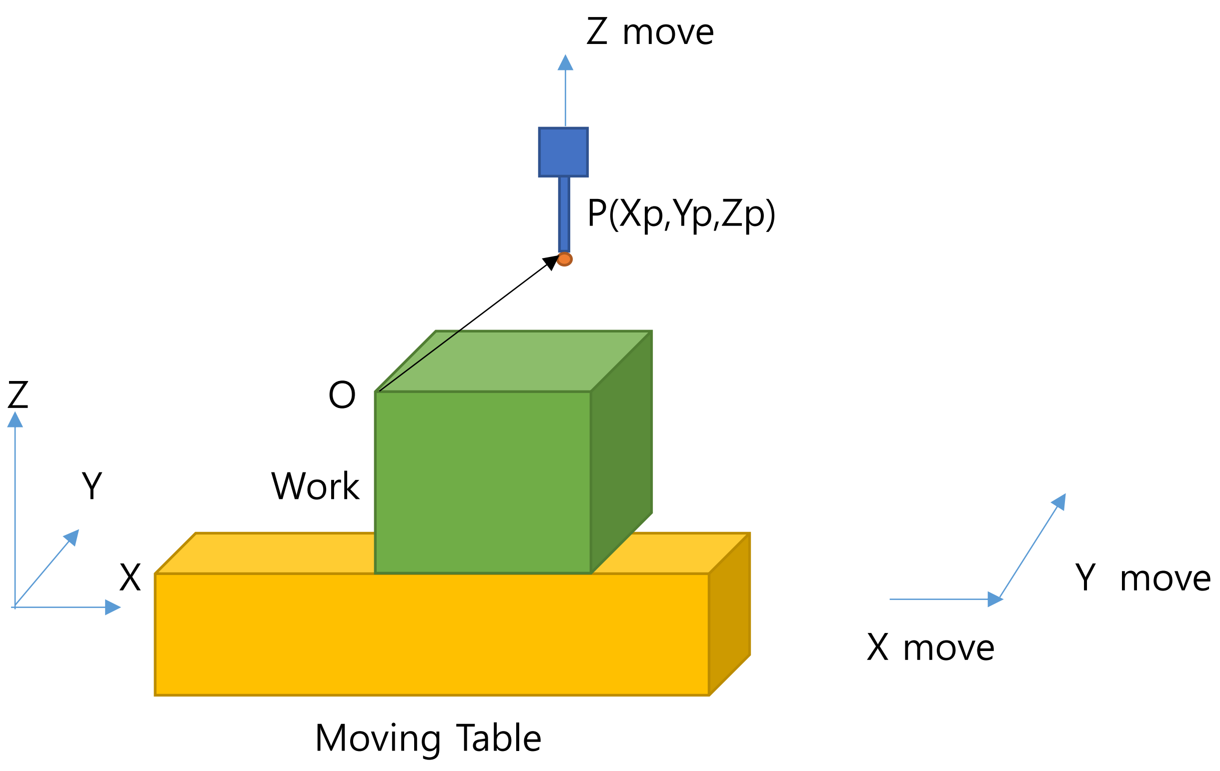
$-Xp(Ez(x)+Ez(y)-Ez(z)) + Zp(Ex(x)+Ex(y)-Ex(z))$

$\Delta Z = Za-(Z+Zp) = \delta z(z) - \delta z(x) - \delta z(y) - XEy(x) + Y(Ex(x)+Ex(y))$

$+Xp(Ey(x)+Ey(y)-Ey(z)) - Yp(Ex(x)+Ex(y)-Ex(z))$

This is the 3D volumetric error equation for the column type machine.

Column Type Machine



Nominal Position Vector, **OP**=(-X+Xp, -Y+Yp, Z+Zp)

Actual Position Vector, **OP**=(Xa, Ya, Za)

Thus $\Delta X = Xa-(-X+Xp)$, $\Delta Y = Ya-(-Y+Yp)$, $\Delta Z = Za-(Z+Zp)$

Application of the Volumetric error

(1) Volumetric error calibration

: To calibrate a machine in 3D working volume

For a nominal point (X,Y,Z); the volumetric error ($\Delta X,\Delta Y,\Delta Z$) can be calculated, thus the actual point will be ($X+\Delta X$, $Y+\Delta Y$, $Z+\Delta Z$);

The full relationship is evaluated such as

$\Delta X=\Delta X(X,Y,Z)$, $\Delta Y=\Delta Y(X,Y,Z)$, $\Delta Z=\Delta Z(X,Y,Z)$

(2) Volumetric Error Correction

: To do numerical correction via controller/processor

Use the corrected control command

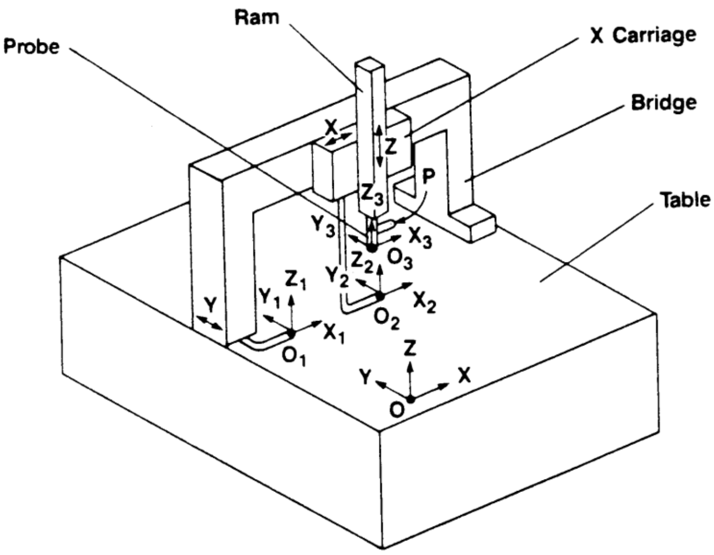
($X_T-\Delta X$, $Y_T-\Delta Y$, $Z_T-\Delta Z$) instead of (X_T,Y_T,Z_T)

(3) Performance simulation in the 3D working volume of machine

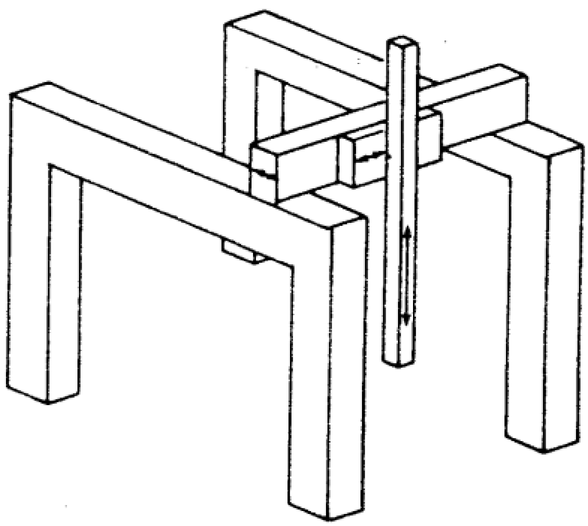
: Cylinder, Circle, Square, Freeform, Length, etc.

Various Machine Configuration

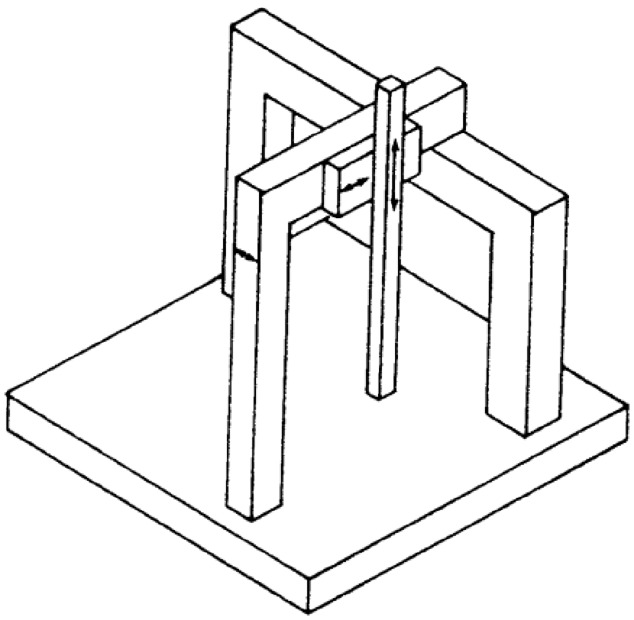
Moving Bridge Type Machine



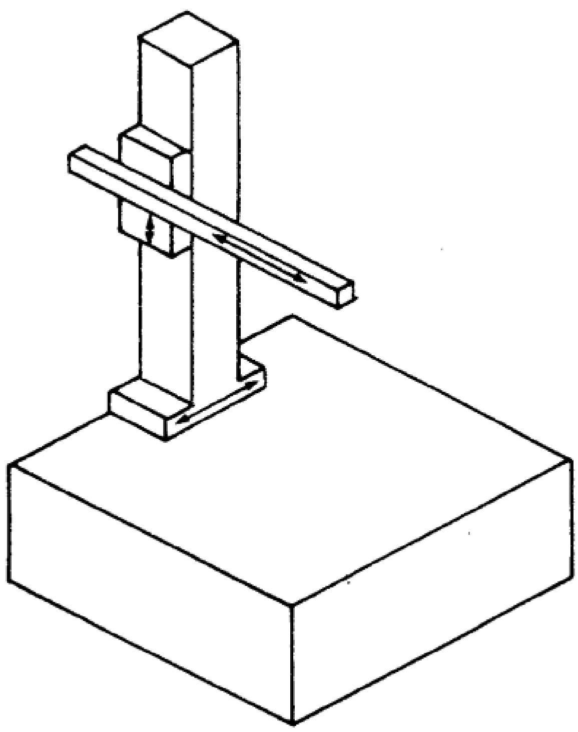
Gantry Type Machine



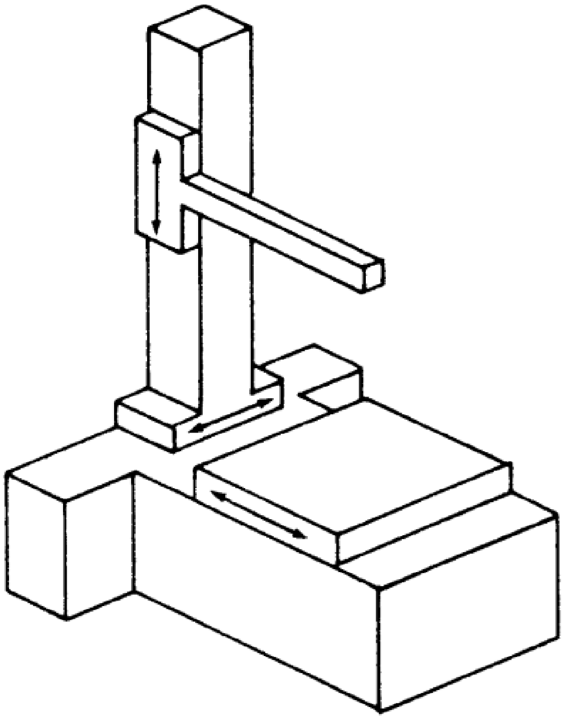
L-shaped Type Machine



Moving Horizontal Arm Type Machine



Fixed Horizontal Arm Type Machine



Fixed Bridge Type Machine