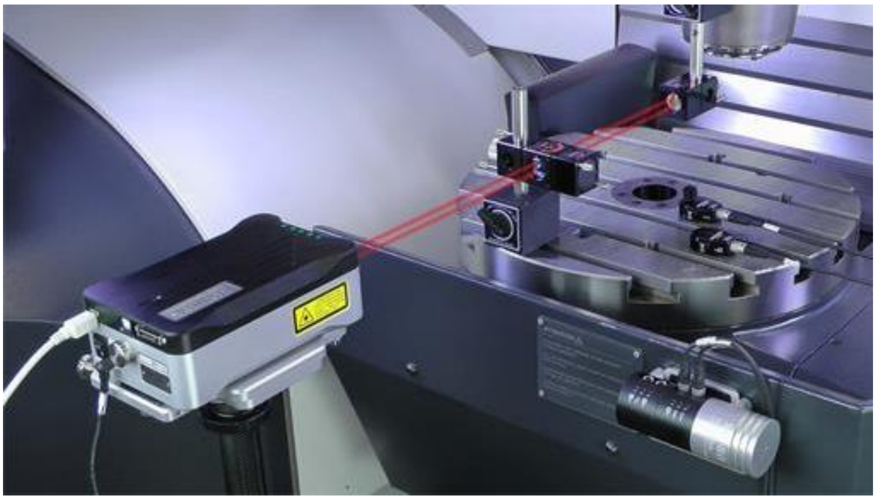
Precision Metrology –Positional error measurement

Positional error: Linear displacement accuracy, by the length measurement

<u>Commercially available instruments for positioning error measurement</u>

Device	Range	Resolution	Accuracy
Laser Interferometer	10m	2.5nm	0.1 ppm
Linear scale	5m	5nm	2um
Step Gauge	1.5m	10mm	0.1um



Laser Interferometer, Source:renishaw.co.kr

Linear scale, Source:renishaw.com

Step gauge; source:mitutoyo.com

Laser Interferometer

: a standard equipment for length measurement

Length standard based on the Kr lamp, that is the legal basis for International length metrology since 1967. Standard length bar of 1 metre length was used before 1967.

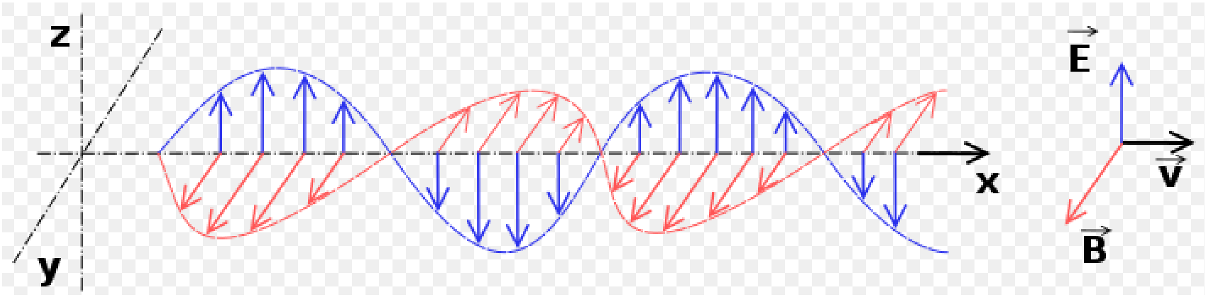
1m ≡ Distance travelled by light from Kr lamp in vacuum during a time scale of 1/299,792,458 sec.

This is realized by I_2 stabilized laser interferometer. The commercial laser interferometer is also acceptable for the standard length measurement, when it is environmentally compensated with traceability.

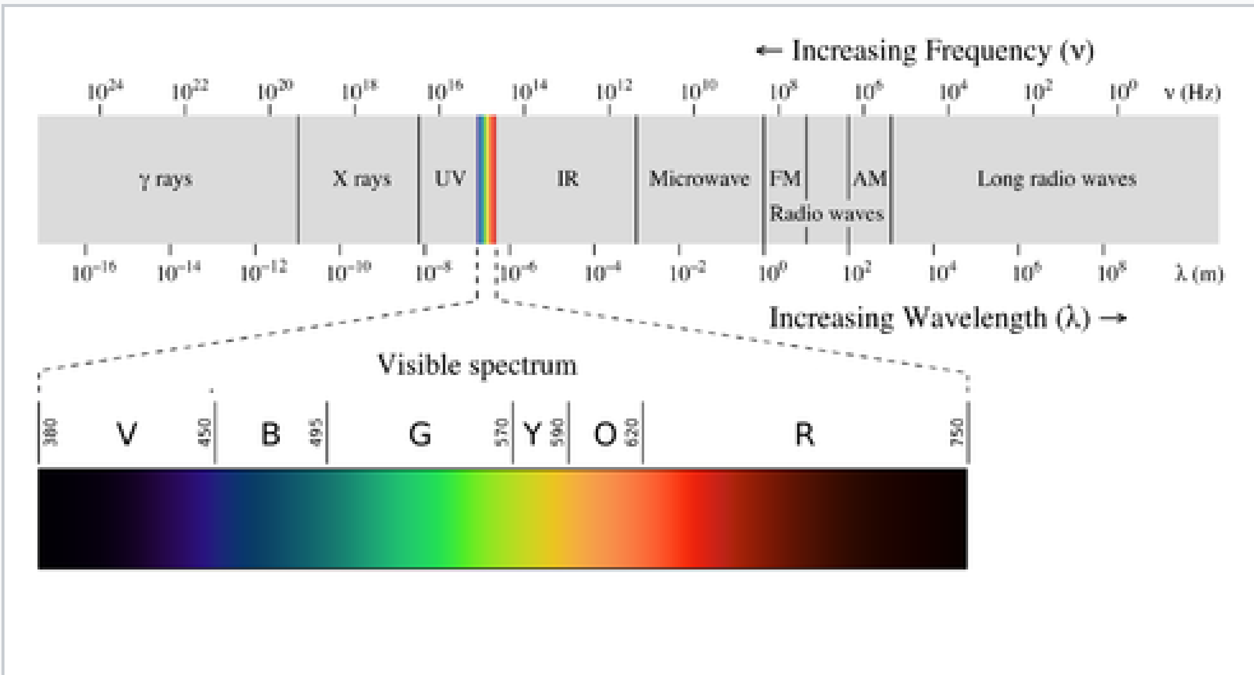
Wave equation for light propagation

In the past, light was considered as Particle (Newton), or Wave(Huygens), now understood as 'Electro-magnetic beam' since Clark Maxwell;

Light: Electro Magnetic Wave



Source:wikipedia



Electromagnetic spectrum with visible light highlighted

Source:wikipedia

1) In vacuum, the light can be described as vectors such as **E** (electric field), **B** (magnetic field)

2) When $\partial \mathbf{E}/\partial t=0$, $\partial \mathbf{B}/\partial t=0$; **E**, **B** are independent in the 3D space

3) When $\partial \mathbf{E}/\partial t\neq 0$, $\partial \mathbf{B}/\partial t\neq 0$; **E**, **B** are not independent in the 3D space, and can be described as the Maxwell's wave equation

Maxwell's equation:

Let **E**(x,y,z) be the Electric field, **M**(x,y,z) be the Magnetic field in 3D(x,y,z) space.

Maxwell's equations are;

$\nabla \times \mathbf{E}=-\partial \mathbf{B}/\partial t$

$\nabla \times \mathbf{B}=\mu_0\varepsilon_0\, \partial \mathbf{E}/\partial t$

$\nabla\cdot\mathbf{E} = 0$

$\nabla\cdot\mathbf{B} = 0$

Thus, the wave equations are

$\nabla^2\mathbf{E} = 1/C_0^2\cdot\partial^2\mathbf{E}/\partial t^2$

$\nabla^2\mathbf{B} = 1/C_0^2 \cdot \partial^2\mathbf{B}/\partial t^2$, where

μ_0=Permeability constant in vacuum

$=4\pi \cdot 10^{-7}$ H/m

ε_0=Permittivity constant in vacuum

$=8.854 \cdot 10^{-13}$ F/m

C_0=Speed of light in vacuum

$=1/\sqrt{\mu_0\varepsilon_0}$

=299,792,458 m/sec ≒ $3.0 \cdot 10^8$ m/sec

To solve the wave equations, let U(x,y,z,t) be the solution;

$\nabla^2U=\partial^2U/\partial x^2 + \partial^2U/\partial y^2 + \partial^2U/\partial z^2=1/C_0^2\partial^2U/\partial t^2$

For 1 Dimensional case, U=U(x,t)

$\partial^2U/\partial x^2 = 1/C_0^2 \, \partial^2U/\partial t^2$

$\therefore U(x,t)=U_0\cos(kx-\omega t)$=Harmonic function of x and t

Where k=propagation const=nk_0

k=propagation constant=$2\pi/\lambda$

λ= wavelength of light, and $1/\lambda$=wave number

n=refractive index of media=C_0/C_m

=speed of light in vacuum/speed of light in media

=1 (vacuum), 1.0002926(air), 1.33(water), 1.5-1.7 (glass)

k_0=propagation constant in vacuum

ω=angular velocity=$2\pi f$

where f=frequency of light

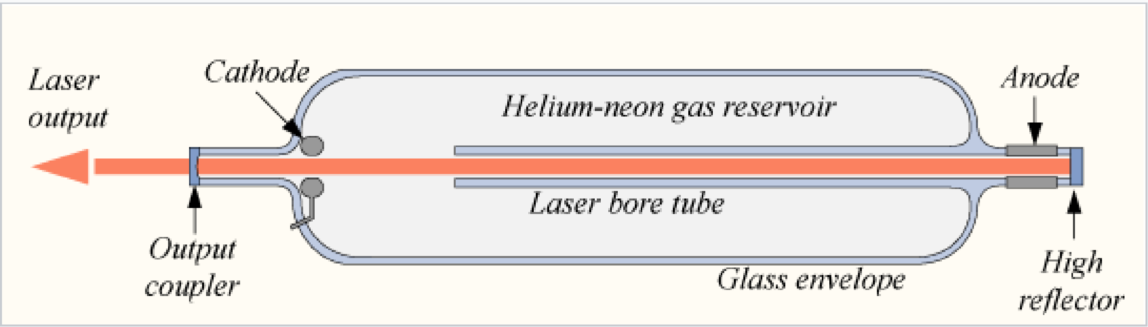
For 3D case, $U=U(x,y,z,t)$

$U(x,y,z,t)=U_0\cos(\mathbf{k}\cdot\mathbf{r}-\omega t)$

Where $\mathbf{k}=(k_x, k_y, k_z)$=propagation vector

$\mathbf{r}=(x, y, z)$=position vector

LASER: Light Amplification by Stimulated Emission of Radiation, that is, a line spectrum emitted from the two different energy levels in a specific atom.



Schematic diagram of a helium–neon laser

source:wikipedia

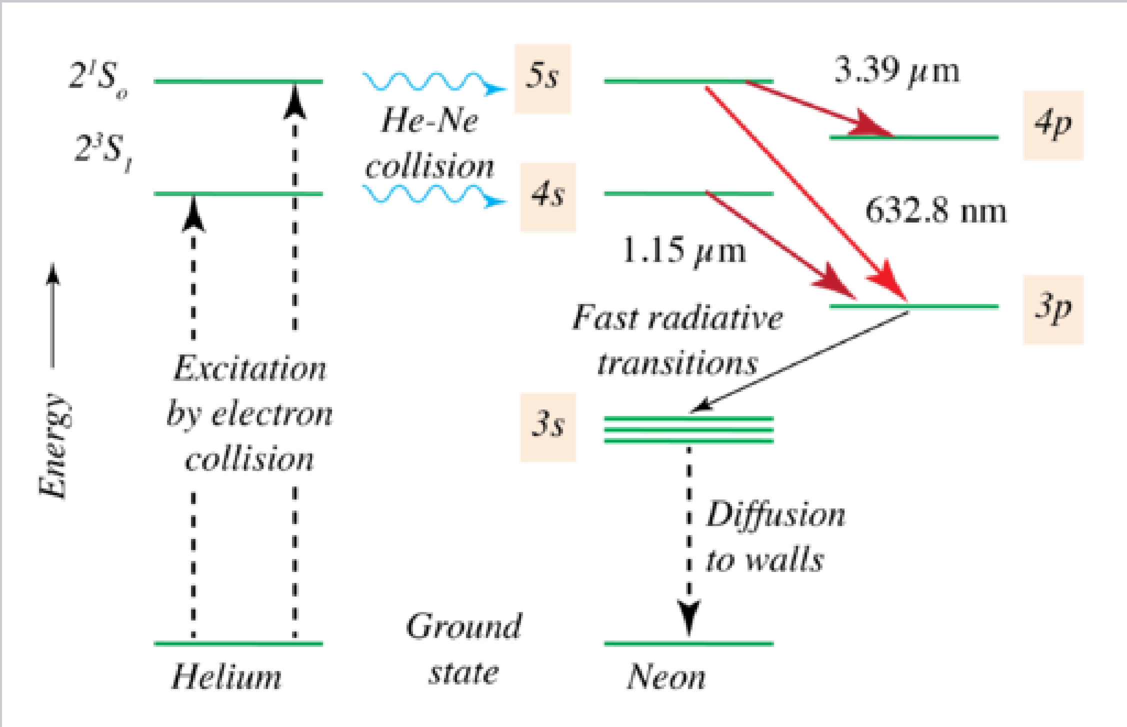
-Ionized atoms: ion, electrons carry the energy

-Electrons collide with gas atoms; exciting to various energy status

-Light emitting of radiation @632.8nm wavelength

He(2):$1s^2$

Ne(10):$1s^2$ $2s^2$ $2p^3$ $3s^2$ $3p^3$ $3d^4$



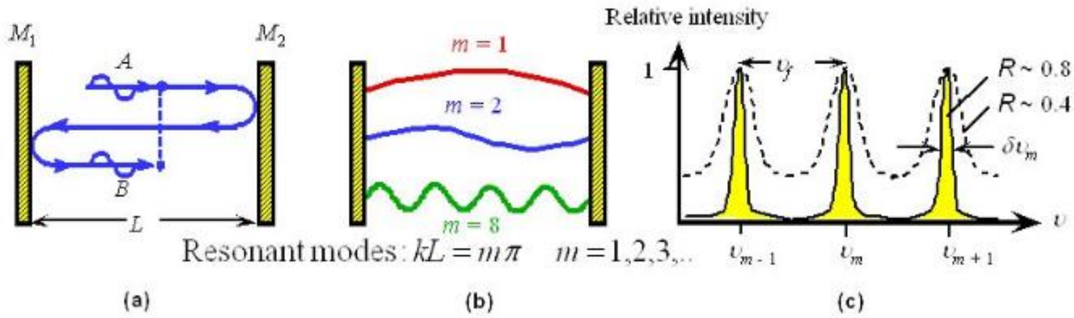
Energy levels in a He-Ne Laser

source:wikipedia

The emitted laser has very broad width, thus is tuned by an optical resonator such as Fabry Perot resonator. Two mirrors are on a well-defined position, and only several modes are in resonant state, giving relative high intensity.

The wavelength stability is very important, and thus Iodine(I_2) stabilized environment is used for laser tube and optical resonator, making highly stable environment toward the wavelength stability. The Zeeman effect is also implemented to reduce the broad width of light.

Fabry-Perot Resonator



Schematic illustration of the Fabry-Perot optical cavity and its properties. (a) Reflected waves interfere. (b) Only standing EM waves, *modes*, of certain wavelengths are allowed in the cavity. (c) Intensity vs. frequency for various modes. *R* is mirror reflectance and lower *R* means higher loss from the cavity.

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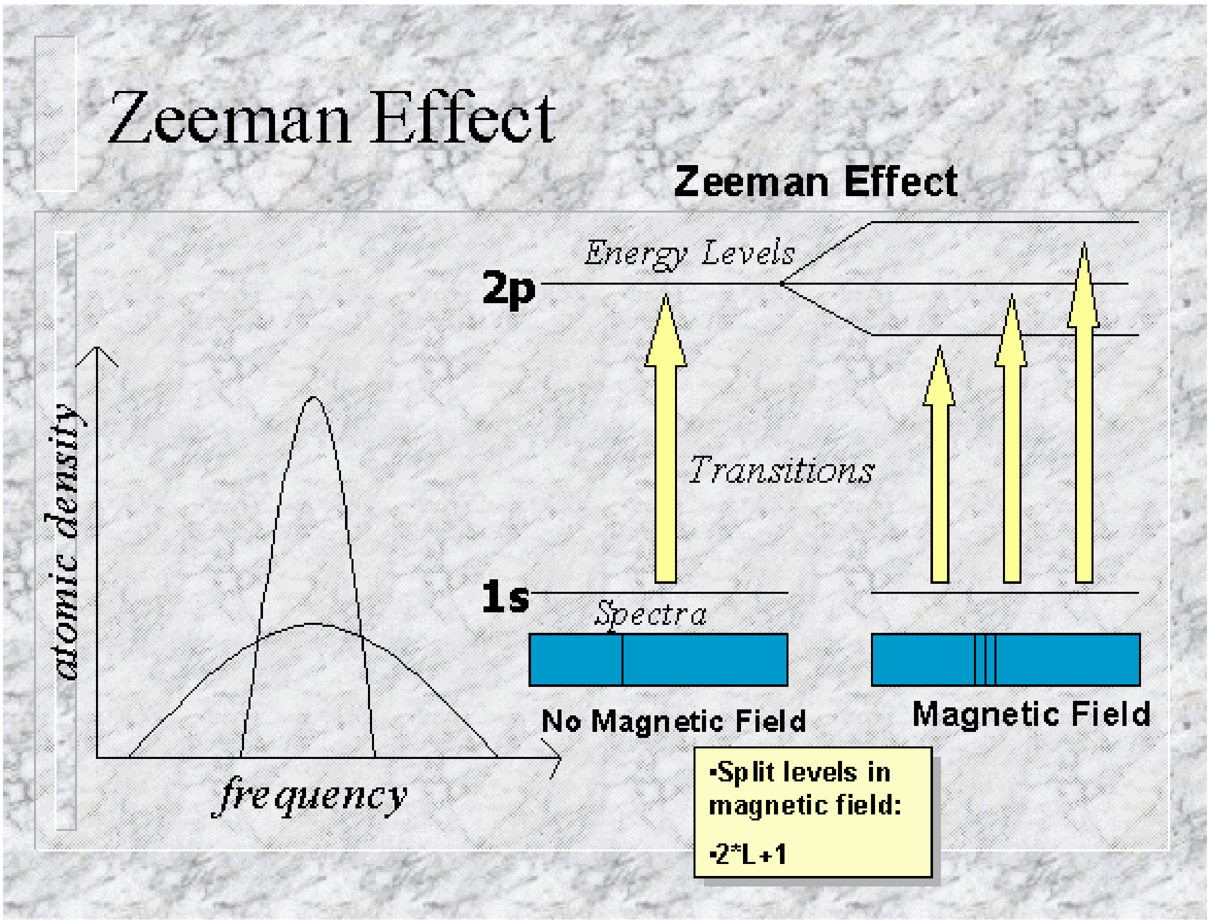
R: reflectance of the optical intensity, *k*: optical wavenumber



Source: cold-atoms.physics.lsa.umich.edu

Metrological application of Light wave

There are 4 major metrological applications of the light wave: Interference, Reflection, Refraction, and Diffraction

1.Interference

When the two waves meet with the same amplitude, frequency, velocity, but with different phase, or two waves meet from the same source but with different optical path, the interference phenomena is observed such as destructive (or dark) interference and constructive (or bright) interference. It is very strong advantage to use the light source as the metrological application.

2. Reflection

When the light is incident to a mirror, then the light is reflected from the mirror, giving the incident angle equal to the reflected angle (law of reflection).

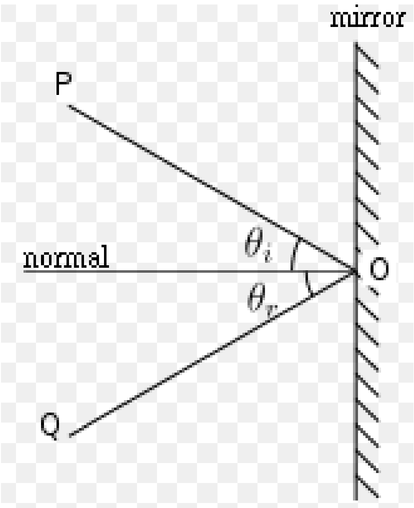
Law of reflection

$\theta_i=\theta_r$

where θ_i = the incident angle

θ_r = reflected angle, and

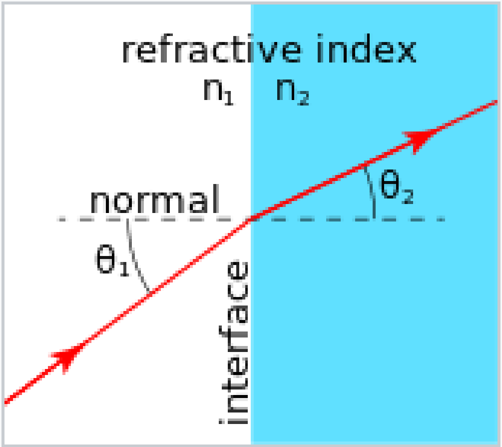
the angles are defined in the opposite direction from the normal to the mirror.



Source:Wikipedia

3.Refraction

When the light is passing through the interface from the first media to the second media, the light path is determined by the Snell's law, or law of refraction.



Refraction of a light ray

Source:Wikipedia

Law of Refraction, or Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

where n_1, n_2 are the refractive indices of materials, respectively,

θ_1 = angle of incidence

θ_2 = angle of refraction

4.Diffraction

When the light wave is passing through one or multiple slits, the light wave propagates downstream as though the light wave starts from the light source located at the slit.

For the Fraunhoffer diffraction or Far-field diffraction with the single slit;

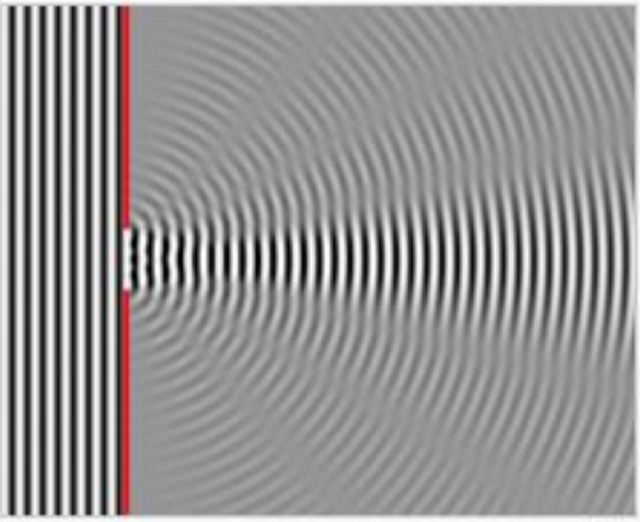
$D \cdot \sin\theta_m = m\lambda$, $m = \pm 1, \pm 2 \ldots$

where D= width of slit, or size of aperture

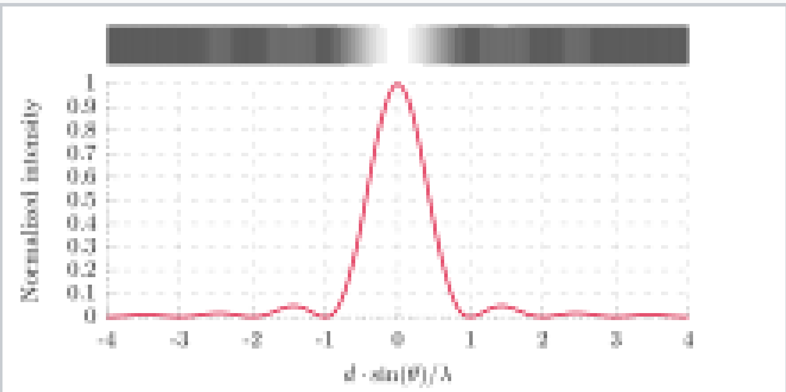
θ_m= Minimum angle for the m'th destructive (Dark) interference

λ=wavelength of light

(cf. Fresnel diffraction or Near-field diffraction)



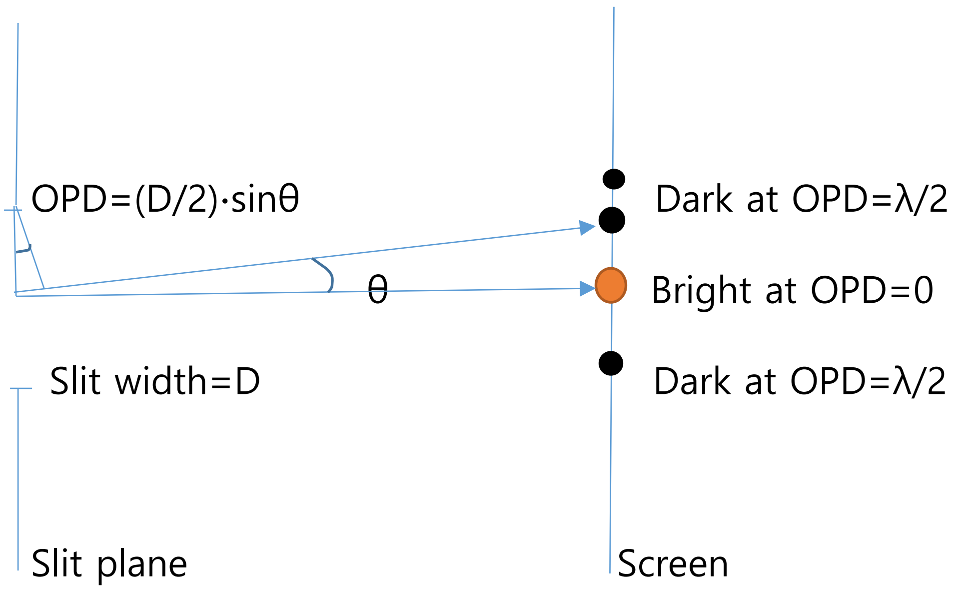
Numerical approximation of diffraction pattern from a slit of width four wavelengths with an incident plane wave. The main central beam, nulls, and phase reversals are apparent.



Graph and image of single-slit diffraction.

Source:Wikipedia

<u>Angle for the Dark Interference</u>



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Dark Interference occurs at angle θ when

$(D/2)\cdot\sin\theta = \lambda/2 \therefore \theta = \sin^{-1}(\lambda/D)$

And also at every $\pm\lambda/2$ OPD

$(D/2)\cdot\sin\theta_m = \pm m\cdot\lambda/2 \therefore \theta_m = \pm\sin^{-1}(m\lambda/D)$, m=1,2..

This is the case of the Fraunhoffer (Far-field) diffraction.

The Far-field can be assumed if $D^2/L\lambda \ll 1$,

where L=distance of screen from the slit plane

When the slit is located in the horizontal plane, the diffraction pattern is located in the vertical plane.

Theory of Interference for Phase measurement

Two waves, Y_1, Y_2:

$Y_1=A\cos(kx-\omega t)$

$Y_2=A\cos(kx-\omega t-\varphi)$

where $k=2\pi/\lambda$, ω=angular velocity, φ=phase difference

The superposition, or sum, of two waves, $Y=Y_1+Y_2$

$=A\cos(kx-\omega t)+A\cos(kx-\omega t-\varphi)$

$=2A\cos(\varphi/2)\cdot\cos(kx-\omega t-\varphi/2)$

Intensity$=|Y_1+Y_2|^2 = 4A^2\cos^2(\varphi/2)\cdot\cos^2(kx-\omega t-\varphi/2)$

(1) Constructive Interference

$\cos^2(\varphi/2)=1$; $\varphi/2=0, \pm\pi, \pm 2\pi\ldots.$

Intensity$=4A^2$ =maximum='Bright'

Path difference $\Delta x=\varphi/k=\pm 2\pi\cdot\lambda/2\pi=\pm\lambda, \pm 2\lambda\ldots$

$\therefore$ OPD(Optical Path Difference)

=Even No. times $\lambda/2$

(2) <u>Destructive Interference</u>

$\cos^2(\varphi/2)=0$; $\varphi/2=\pm\pi/2$, $\pm 3\pi/2$....

Intensity=0=minimum='Dark'

Path difference $\Delta x=\varphi/k=\pm\lambda/2, \pm 3\lambda/2...$

$\therefore$ OPD(Optical Path Difference)

=Odd No. times $\lambda/2$

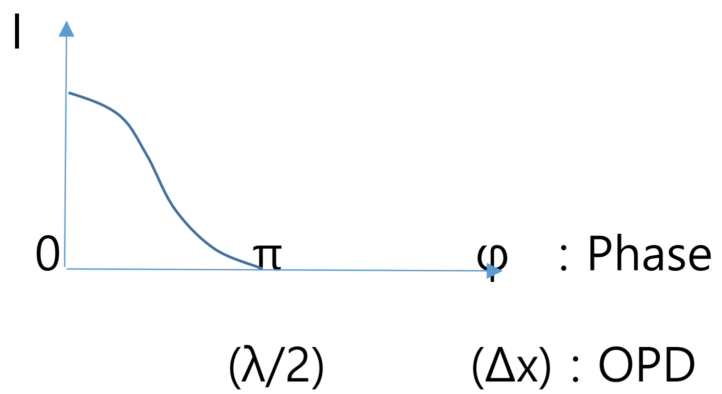
Therefore at every $\lambda/2$ path difference,

'Dark' and 'Bright' occurs alternatively.

Optical Path Difference=No. of Count times $\lambda/2$

Thus, physical resolution is $\lambda/2$, but the computation resolution is typically $\lambda/2$ divided by 2^n

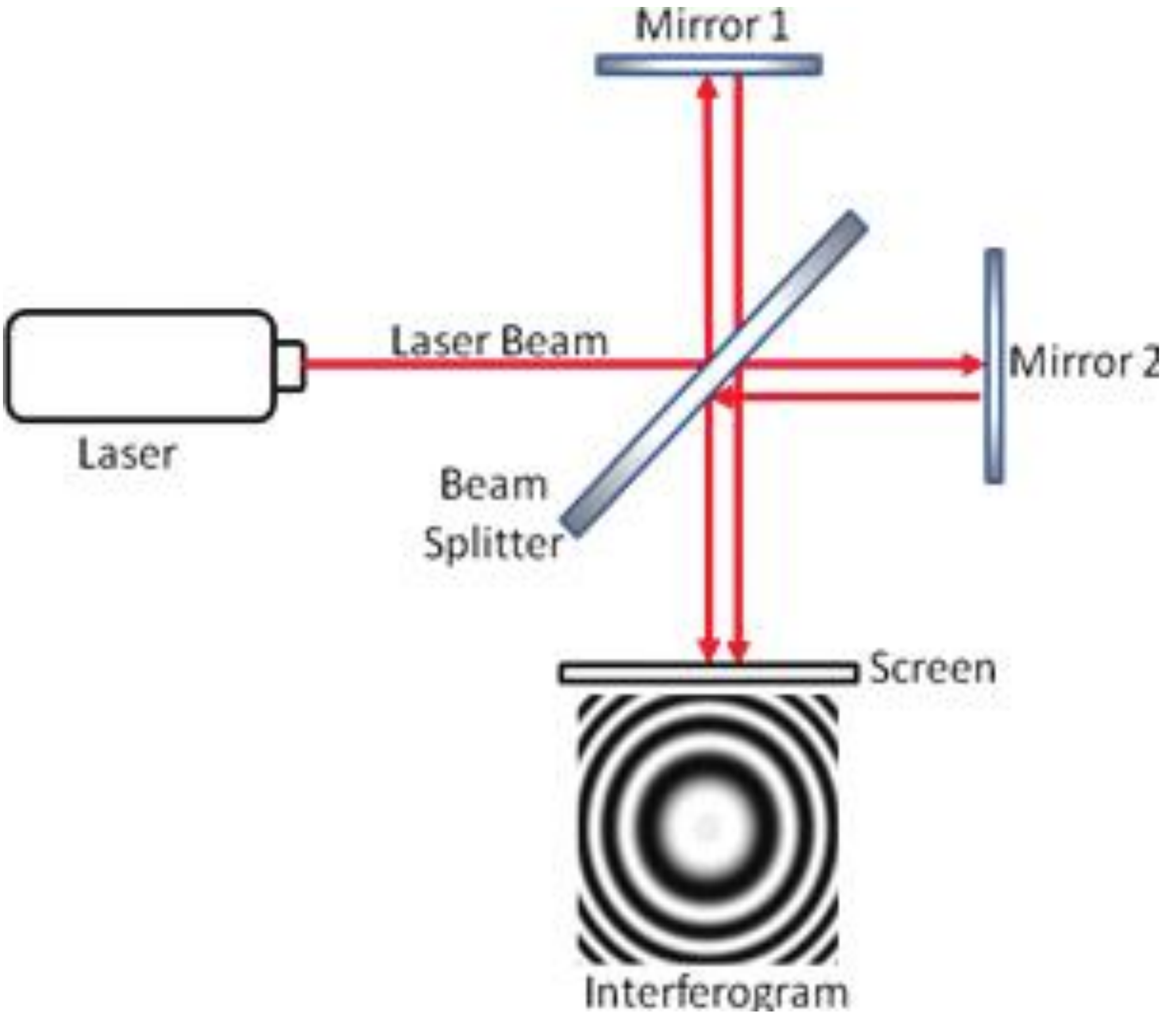
<u>Graph of $\cos^2(\varphi/2)$ = $(1+\cos\varphi)/2$ vs φ</u>



A typical application is the Michelson Interferometer

(For the Michelson interferometer, bright to dark occurs at every λ/4 movement of mirror 2. Why?)

Michelson Interferometer



Source:Digital processing techniques for fringe analysis by M.Mora-Gonzales

Beat Phenomena

Another application of interference is beat phenomena.

When two waves meet with slight different frequencies, the beat phenomena occurs.

At a fixed point, the waves are only function of time, thus, Y_1=Acos($2\pi f_1 t$), Y_2=Acos($2\pi f_2 t$)

The superposed wave, $Y=Y_1+Y_2$

$=A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$

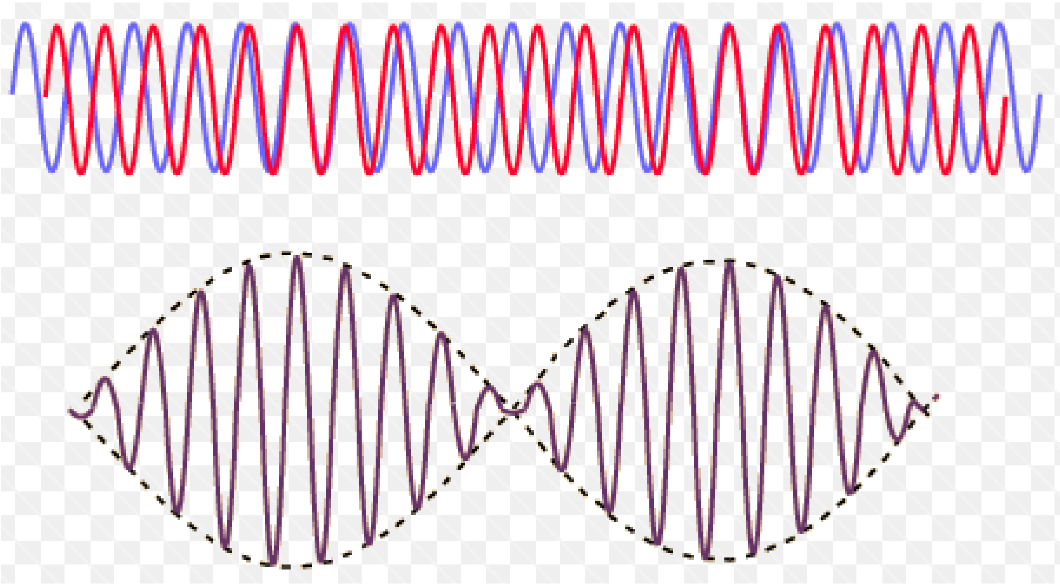
$=2A\cos(2\pi f_1 t-2\pi f_2 t)/2\cdot\cos(2\pi f_1 t+2\pi f_2 t)/2$

$=2A(2\pi f_{amplitude})t\cdot\cos(2\pi f_{waveform})t$

Where

$f_{amplitude}= (f_1-f_2)/2$, i.e. frequency of amplitude

$f_{waveform}=(f_1+f_2)/2$, i.e. frequency of waveform



Source:Hyperphysics by R.Nave