M2794.007700 Smart Materials and Design

Failure analysis of lamina and laminate

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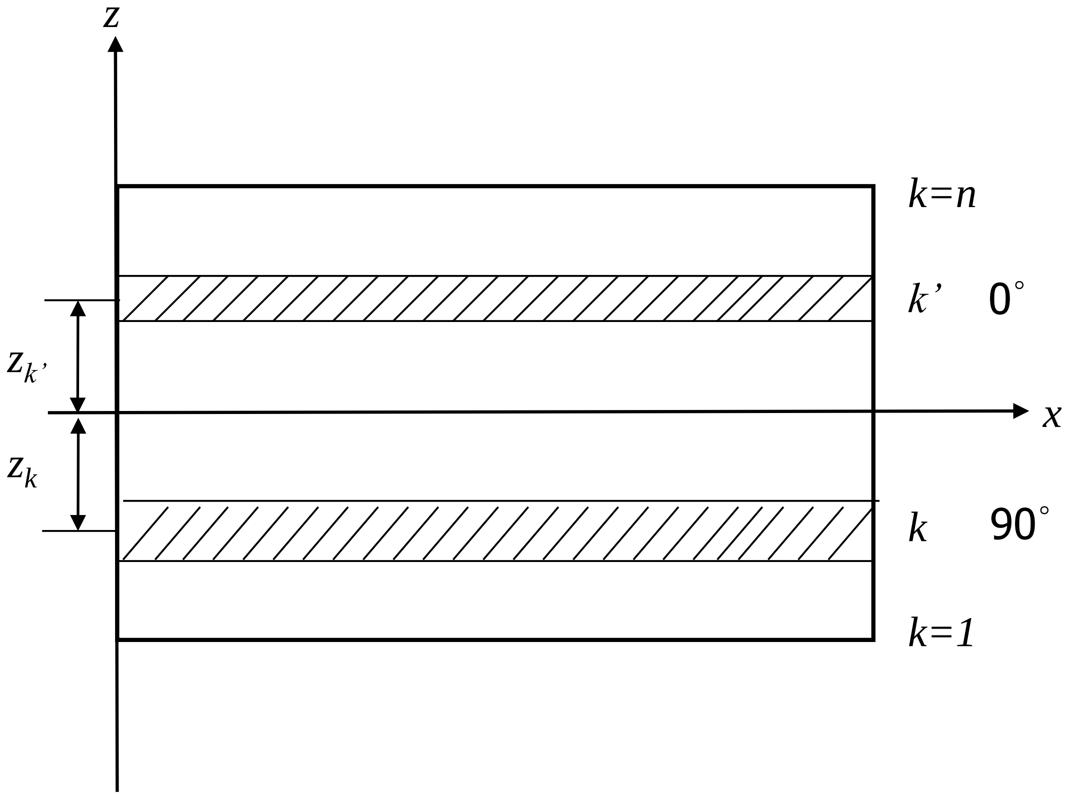
Anti symmetric cross-ply laminates

Consist of 0° and 90° plies arranged in such a way that for every 0° ply at a distance z from the mid-plane then is a 90° ply of the same material and thickness at a distance $-z$ from the mid-plane

⇒ even # of plies (total)

$$+z_k = -z_{k'}$$

$$t_k = t_{k'}$$



Anti symmetric angle-ply laminates

Consist of pairs of plies of $+\theta_i$ and $-\theta_i$
($0^\circ < \theta_i < 90^\circ$)

$$z_k = -z_{k'}$$

$$t_k = t_{k'}$$

$$\begin{cases} A_{is} = D_{is} = 0 \\ B_{xx} = B_{yy} = B_{xy} = B_{ss} = 0 \end{cases}$$

Balanced laminate (continue)

$$t_k = t_{k'} \qquad Q_k = -Q_{k'} \quad \Rightarrow \quad \boxed{A_{is} = 0}$$

$$i = x, y$$

$$\begin{cases} A_{xs} = 0 \\ A_{ys} = 0 \end{cases}$$

$$\left(\begin{aligned} & A_{is} = \sum_{k=1}^{n} Q_{is}^{k} t_k \\ & \qquad Q_{xs}(\theta) = m^3 \overset{\text{odd}}{n} (Q_{11} - Q_{12} - 2Q_{66}) + m \overset{\text{odd}}{n^3} (Q_{12} - Q_{22} + 2Q_{66}) \end{aligned} \right)$$

$$\Rightarrow Q_{is}(\theta) = -Q_{is}(-\theta)$$

Anti symmetric laminates

$$[\theta_1 / \theta_2 / -\theta_2 / -\theta_1]$$

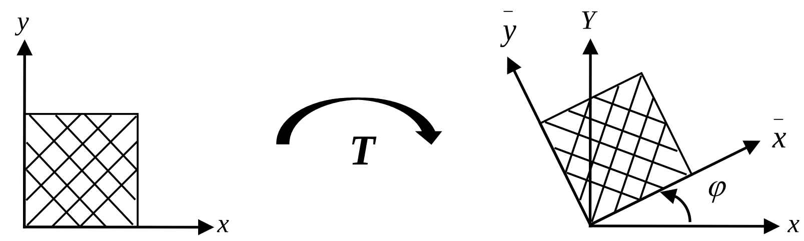
Balanced $\rightarrow$ $A_{is} = 0$

$$\boxed{D_{is} = 0}$$

$$(h^3 - h_{k-1}^3) = (h_{k'}^3 - h_{k-1}^3)$$

$$Q_{is}^{k} = -Q_{is}^{k'}$$

Transformation of laminate stiffness & compliance



For Balanced sym. laminate

Principal axes

↓

Homo/orthotropic
→orthotropic laminate

Symmetric $\longrightarrow \boxed{B_{ij} = 0}$

Balanced $\longrightarrow \boxed{A_{i\bar{s}} = 0}$

$$(i = \bar{x}, \bar{y})$$

$$\begin{pmatrix} A_{\bar{x}\bar{s}} = 0 \\ A_{\bar{y}\bar{s}} = 0 \end{pmatrix}$$

$$\begin{pmatrix} N_{\bar{x}} \\ N_{\bar{y}} \\ N_{\bar{s}} \end{pmatrix} = \begin{pmatrix} A_{\bar{x}\bar{x}} & A_{\bar{x}\bar{y}} & 0 \\ A_{\bar{y}\bar{x}} & A_{\bar{y}\bar{y}} & 0 \\ 0 & 0 & A_{\bar{s}\bar{s}} \end{pmatrix} \begin{pmatrix} \varepsilon_{\bar{x}}^{0} \\ \varepsilon_{\bar{y}}^{0} \\ \gamma_{\bar{s}}^{0} \end{pmatrix}$$

Quasi-Isotropic Laminates

- **Elastic properties are independent of orientation**

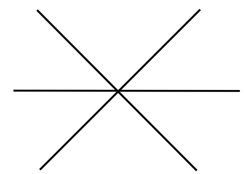
$$[A]_{\bar{x},\bar{y}} = [A]_{x,y} = \text{constant}$$

$$[a]_{\bar{x},\bar{y}} = [a]_{x,y} = \text{constant}$$

$$\text{Balanced} \Rightarrow A_{xs} = A_{ys} = 0$$

$$\text{Example} : [0/60/-60]_s$$

$$[0/\pm 45/90]_s$$

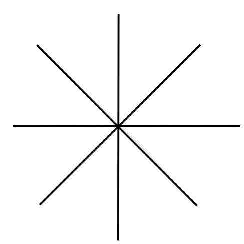


$n = 3 \quad 0/\dfrac{\pi}{3}/\dfrac{2\pi}{3}$

60 120

$$\text{General form} \left[0/\frac{\pi}{n}/\frac{2\pi}{n}/\ldots/\frac{n-1}{n}\pi \right]_s$$

$$or \quad \left[\frac{\pi}{n}/\frac{2\pi}{n}/\ldots/\pi \right]_s$$



$n > 2$

$n = 4 \quad 0/\dfrac{\pi}{4}/\dfrac{2\pi}{4}/\dfrac{3\pi}{4}$

Design consideration

"black Aluminum" – for metal designer

$B_{ij} = 0 \quad \Rightarrow$ symmetric layer

$\Rightarrow$ remove warpage from manufacturing

$\Delta T \downarrow$

$A_{is} = 0 \quad \Rightarrow$ balanced or crossply

$D_{is} = 0 \quad \Rightarrow$ antisymmetric or crossply

$\Rightarrow$ minimized by increasing the number of layers for the same overall laminate thickness

Crossply $\quad \Rightarrow B_{ij}, A_{ij}, D_{is} = 0$

$$\begin{cases} \text{Recommended : symmetric \& balanced lay-up with fine ply interdispersion} \\ D_{is} \neq 0 \quad \text{: bending - torsion coupling} \\ \quad \text{Eg) X-29 to achieve aeroelastic stability} \end{cases}$$

For symmetric balanced laminate

$$\bar{E}_x = \frac{1}{ha_{xx}} \qquad \bar{E}_y = \frac{1}{ha_{yy}} \qquad , \bar{G}_{xy} = \frac{1}{ha_{ss}}$$

Ex)

①$[\pm\theta]$ **Anti symmetric angle ply laminate**

$A_{xs}=A_{ys}=0$

$D_{xs}=D_{ys}=0$

$B_{xx}=B_{xy}=B_{yy}=B_{ss}=0$

②$[0/\pm 45]$ **balanced laminate**

$A_{xs}=A_{ys}=0$

③$[0/90_2]_s$ **symmetric cross ply laminate**

$B_{ij}=0$, $A_{xs}=A_{ys}=D_{xs}=D_{ys}=0$

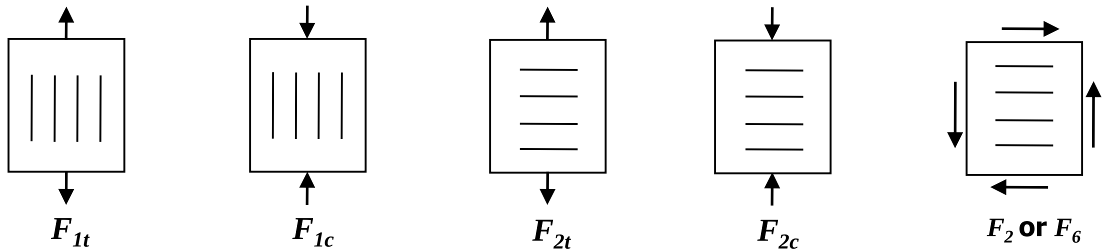
④$[0/45/90/-45]$ **Balanced laminate**

$A_{xs}=A_{ys}=0$

Failure – strength of lamina

- **Failure theories**
  - Max. stress
  - Max strain
  - Tsai – Hill
  - Tsai - Wu

- **Parameters**



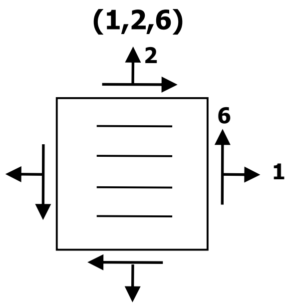
Failure – strength of lamina

- **Maximum stress theory at the principal material axes**

$$\sigma_1 = \begin{cases} F_{1t} & , \quad \sigma_1 > 0 \\ -F_{1t} & , \quad \sigma_1 < 0 \end{cases}$$

$$\sigma_2 = \begin{cases} F_{2t} & , \quad \sigma_2 > 0 \\ -F_{2t} & , \quad \sigma_2 < 0 \end{cases}$$

$$|\sigma_6| = |\tau_3| = F_6$$



- **Maximum strain theory - includes *Poisson's* effect**

$$\varepsilon_1 = \begin{cases} \varepsilon_{1t}^u & , \quad \varepsilon_1 > 0 \\ \varepsilon_{1c}^u & , \quad \varepsilon_1 < 0 \end{cases}$$

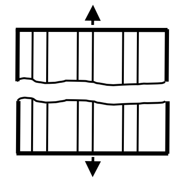
$$\varepsilon_2 = \begin{cases} \varepsilon_{2t}^u & , \quad \varepsilon_2 > 0 \\ \varepsilon_{2c}^u & , \quad \varepsilon_2 < 0 \end{cases}$$

$$|\gamma_3| = 2|\varepsilon_{12}| = \gamma_3^u$$

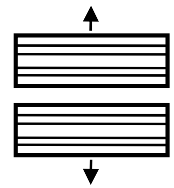
ε^u – ultimate strain

Failure – strength of lamina

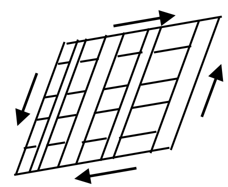
- **Tension along the fiber**
  - fiber breakage & matrix failure



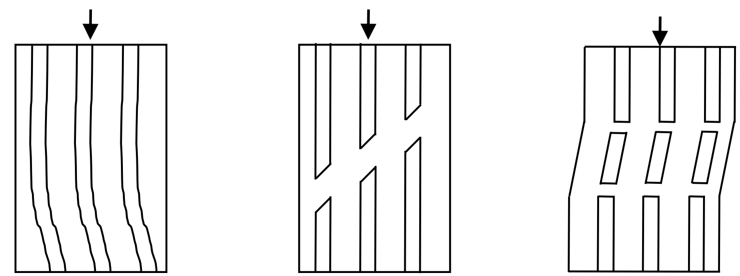
- **Tension normal to the fiber**
  - matrix cracking



- **Shear**
  - matrix shear out

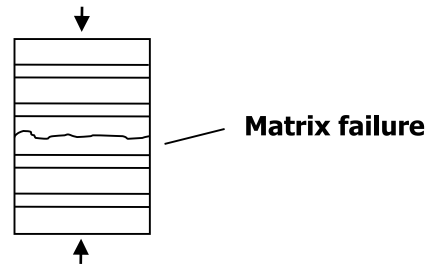


- **Compression along the fiber**



Micro buckling | **shear out in fibers** | **kinking**

- **Compression normal to the fiber**



Failure – strength of lamina

- **Tsai – Hill (Modified Von Mises criterion)**

Von Mises criterion $$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \sigma_Y^2$$

Tsai – Hill theory $$A\sigma_1^2 + B\sigma_2^2 + C\sigma_1 \sigma_2 + D\tau_3 = 1 \qquad \tau_3 = \sigma_6$$

-simple eq.
-no distinction of tension, comp

$$\boxed{\frac{\sigma_1^2}{F_1^2} + \frac{\sigma_2^2}{F_2^2} + \frac{\tau_3^2}{F_6^2} - \frac{\sigma_1 \sigma_2}{F_1^2} = 1}$$

$$F_1 = \begin{cases} F_{1t} , & \sigma_1 > 0 \\ F_{1t} , & \sigma_1 < 0 \end{cases}$$

$$\mathrm{F}_2 = \begin{cases} F_{2t} , & \sigma_2 > 0 \\ F_{2t} , & \sigma_2 < 0 \end{cases}$$

$$|\sigma_6| = |\tau_3| = F_6$$

Failure – strength of lamina

- **Tsai – Wu (Tensor polynomial)**
  : incorporate tensile/compression difference

$$\boxed{f_1\sigma_1 + f_2\sigma_2 + f_{11}\sigma_1^2 + f_{22}\sigma_2^2 + f_{66}\tau_3^2 + 2f_{12}\sigma_1\sigma_2 = 1}$$

where

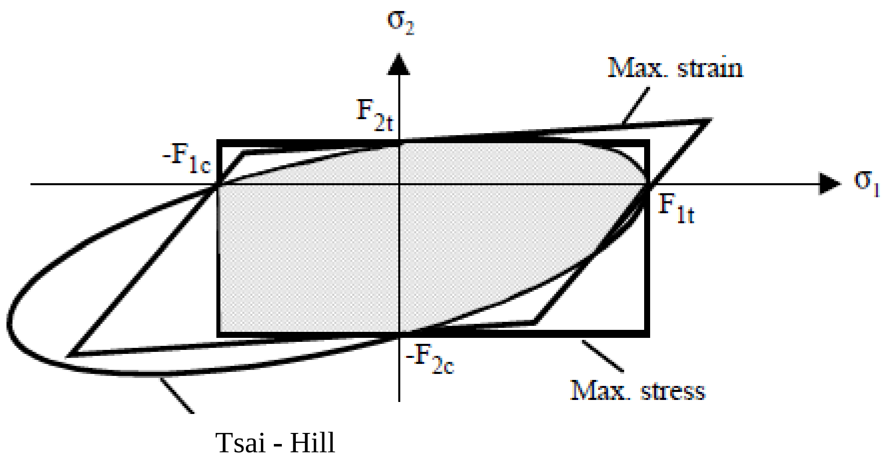
$$f_1 = \frac{1}{F_{1t}} - \frac{1}{F_{1c}} \quad , \qquad f_{11} = \frac{1}{F_{1t}F_{1c}}$$

$$f_2 = \frac{1}{F_{2t}} - \frac{1}{F_{2c}} \quad , \qquad f_{22} = \frac{1}{F_{2t}F_{2c}} \ , \ f_{66} = \frac{1}{F_6^2}$$

$$f_{12} \cong -\frac{1}{2}(f_{11}f_{22})^{\frac{1}{2}}$$

Failure – strength of lamina

- **Failure Envelope**
  - Conservative approach



- **Note**
  - Hashin failure criterion (combination of difference modes)
    - Tensile – fiber failure

$$\left(\frac{\sigma_1}{F_{1t}}\right)^2 + \left(\frac{\sigma_6}{F_6}\right)^2 = 1 \qquad \sigma_1 > 0$$

    - Compressive – fiber

$$|\sigma_1| = F_{1c} \qquad \sigma_1 < 0$$

    - Tensile – matrix cracking

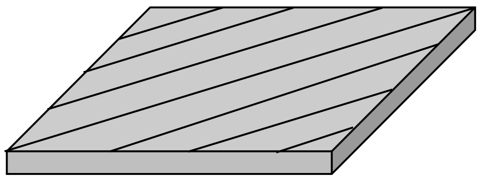
$$\left(\frac{\sigma_2}{F_{2t}}\right)^2 + \left(\frac{\sigma_6}{F_6}\right)^2 = 1 \qquad \sigma_2 > 0$$

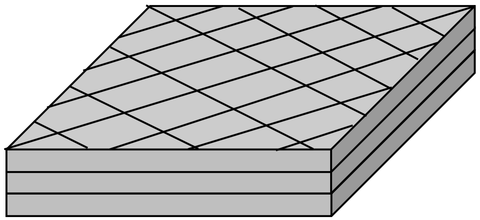
    - Compressive – matrix

$$\left(\frac{\sigma_2}{2F_6}\right)^2 + \left[\left(\frac{F_{2c}}{2F_6}\right)^2 - 1\right]\frac{\sigma_2}{F_{2c}} + \left(\frac{\sigma_6}{F_6}\right)^2 = 1$$

Failure analysis of multidirectional laminate

- **Failure criteria**





- Lamina failure
  - Max σ
  - Max ε
  - Tsai-Wu
  - Tasi-Hill

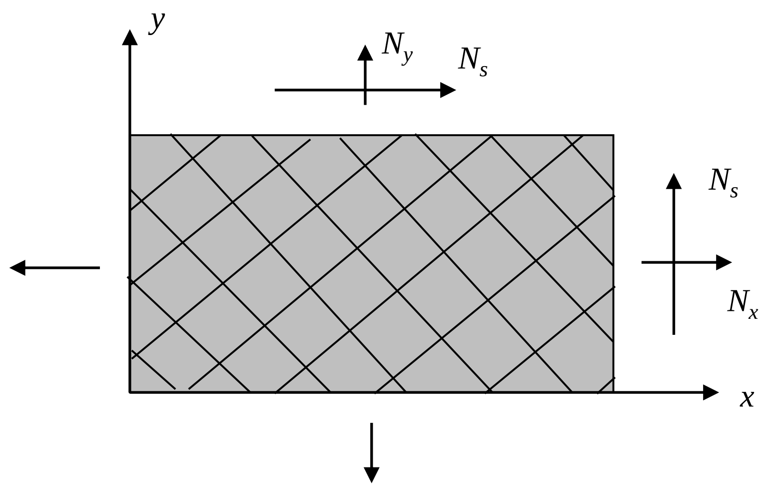
Apply

- Laminate failure
  - First-ply failure
  - Ultimate laminate failure
  - Interlaminar failure

→ Design of primary structure

Failure analysis of multidirectional laminate

- **In-plane loading of symmetric laminate**



$$\begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_s^0 \end{pmatrix} = \begin{bmatrix} & & \\ & a & \\ & & \end{bmatrix} \begin{pmatrix} N_x \\ N_y \\ N_s \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \frac{1}{2}\gamma_3 \end{pmatrix} = \begin{bmatrix} & & \\ & T & \\ & & \end{bmatrix}_k \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_s^0 \end{pmatrix}$$

Failure analysis of multidirectional laminate

- **Stress-strain curve of multidirectional laminate under uniaxial tension showing progressive failure**

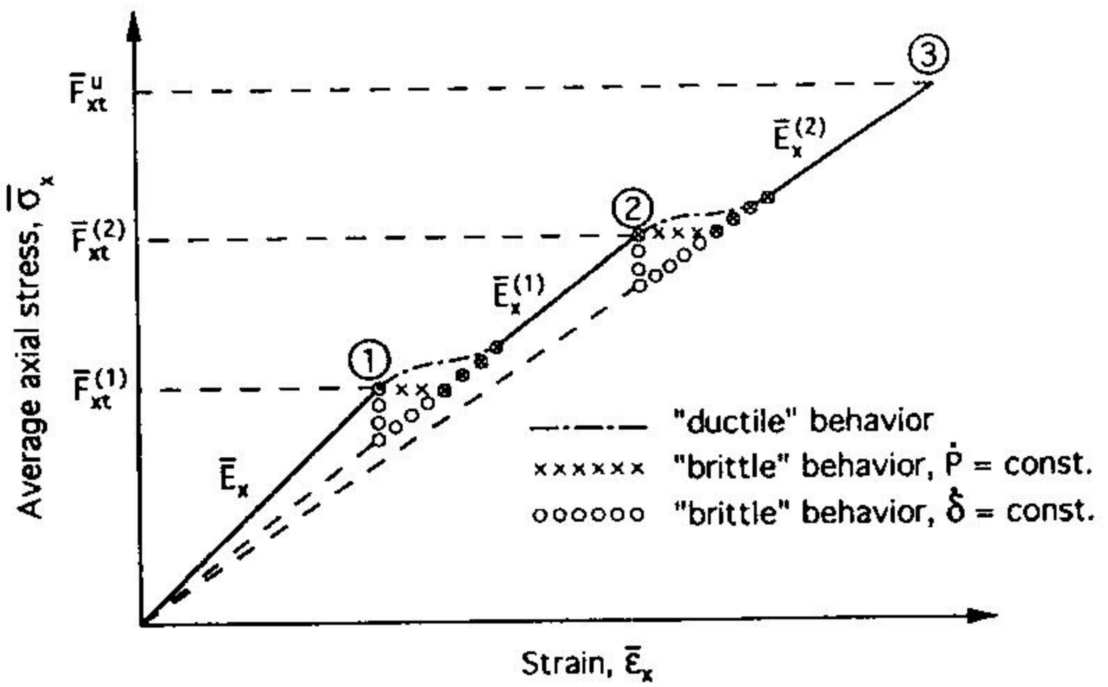


Fig. 7.17 Stress–strain curve of multidirectional laminate under uniaxial tension showing progressive failure ($\dot{P}$ = constant, load rate control; $\dot{\delta}$ = constant, strain rate control).

Failure analysis of multidirectional laminate

- **Stress in each ply (k_{th} ply)**

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \tau_3 \end{pmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{33} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_3 \end{pmatrix}_k$$

- **Safety factor** $S_{fk}(\sigma_1, \sigma_2, \tau_3)$

- **Apply Tsai-Wu (could be other criteria)**

$$f_1 S_{fk}\sigma_{1k} + f_2 S_{fk}\sigma_{2k} + f_{11} S_{fk}^2 \sigma_{1k}^2 + f_{22} S_{fk}^2 \sigma_{2k}^2 + f_{66} S_{fk}^2 \tau_{3k}^2 + 2 f_{12} S_{fk}^2 \sigma_{1k}\sigma_{2k} = 1$$

- **First-ply failure (FPF)**

$$S_f = (S_{fk})_{\min}$$

Failure analysis of multidirectional laminate

- **Progressive & ultimate laminate failure (ULF)**
  - After FPF, failure process continues to ULF

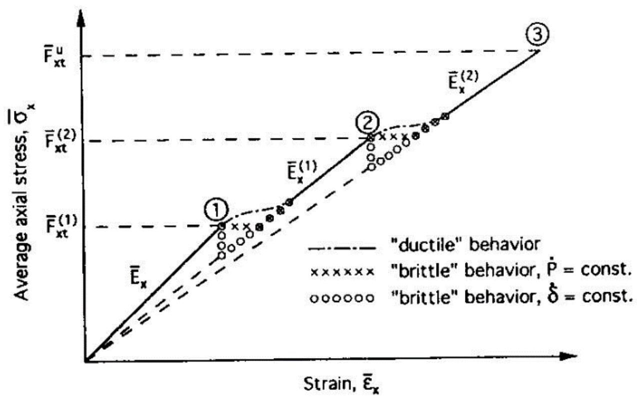


Fig. 7.17 Stress–strain curve of multidirectional laminate under uniaxial tension showing progressive failure ($\dot{P}$ = constant, load rate control; $\dot{\delta}$ = constant, strain rate control).

- **Analysis**

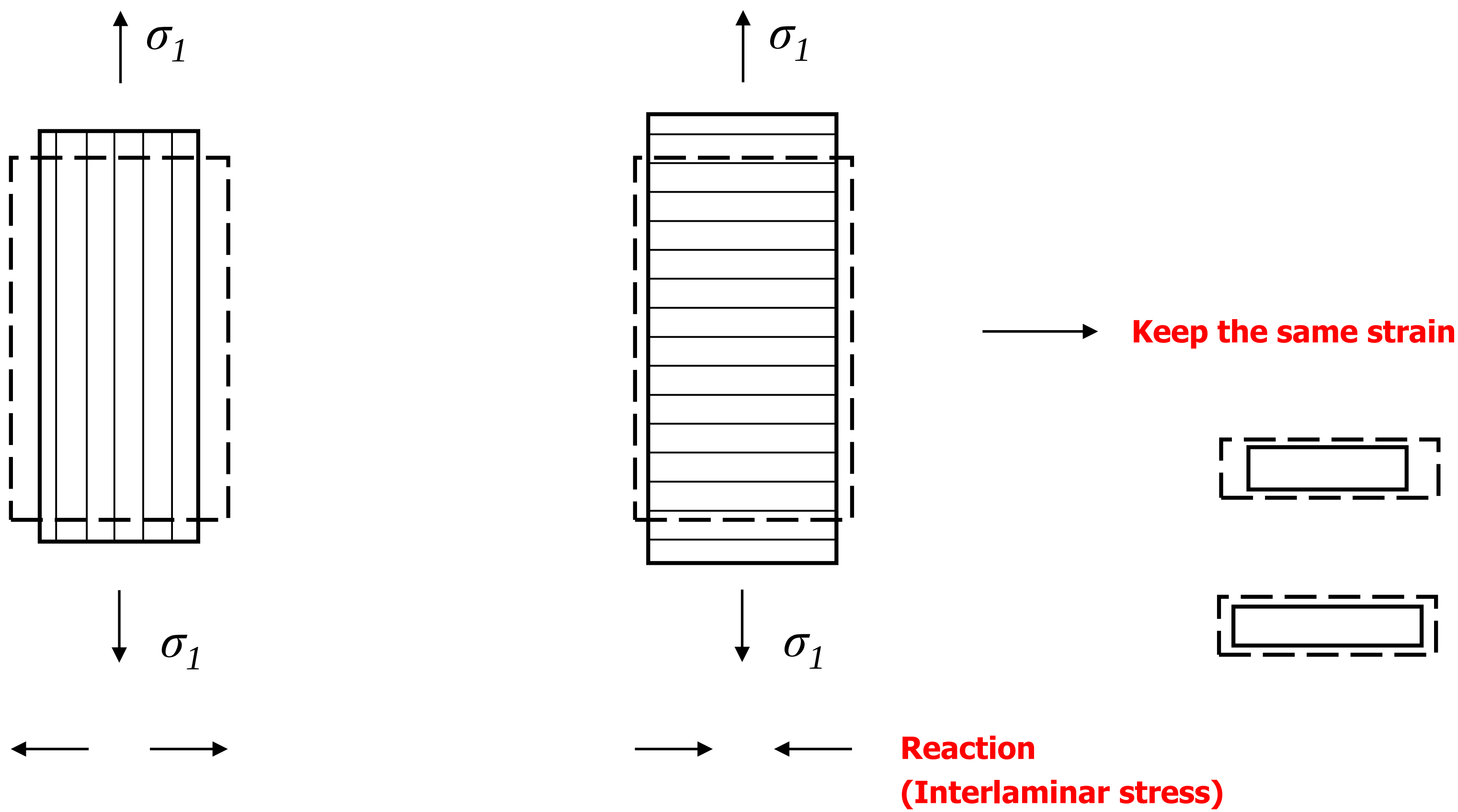
1) FPF
2) For failed lamina, either reduce stiffness or ply discount
   → New [A], [B], [D]
3) Recalculate stress → FPF
4) Repeat 1), 2), 3) until maximum stress is reached

Failure analysis of multidirectional laminate

- $\nu_{12} = 0.28 \rightarrow \nu_{21} = 0.024$

strain direction

loading direction



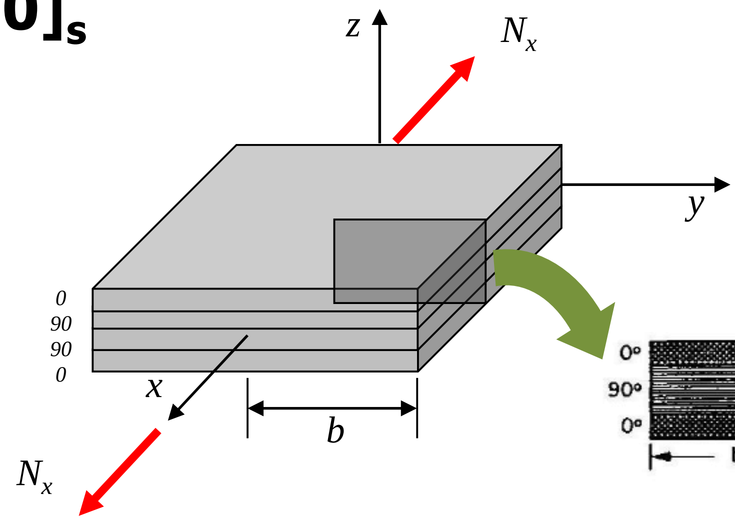
Failure analysis of multidirectional laminate

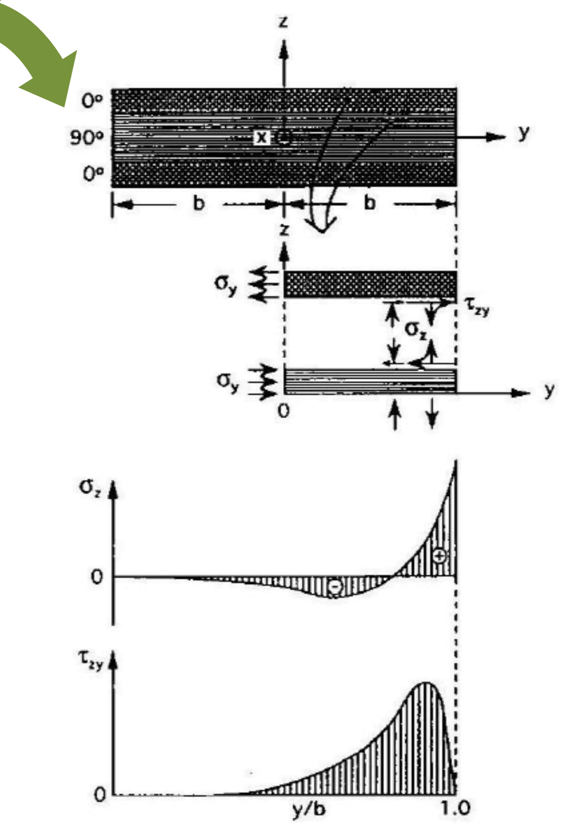
- **Interlaminar stress : edge effect**
  - Previous analysis (classical lamination theory)

    $\sigma_z = \tau_{xz} = \tau_{yz} = 0$ (plane stress assumption)
  - Near free edge, these stresses exist and cause delamination (interlaminar separation)
  - Stacking sequence affect greatly

Failure analysis of multidirectional laminate

- **Ex) $[0/90]_s$**





Greater near edge

Fig. 7.23 Distribution of interlaminar normal stress σ_z and interlaminar shear stress τ_{zy} in $[0/90]_s$ laminate under axial tension.

Failure analysis of multidirectional laminate

- **Cf) Angle ply**

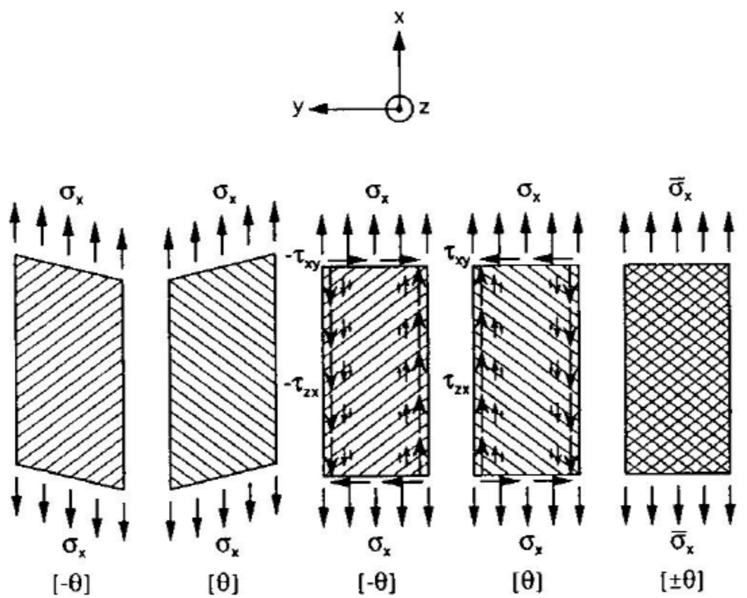


Fig. 7.19 Illustration of generation of interlaminar and intralaminar shear stresses in angle-ply laminate under axial tension.

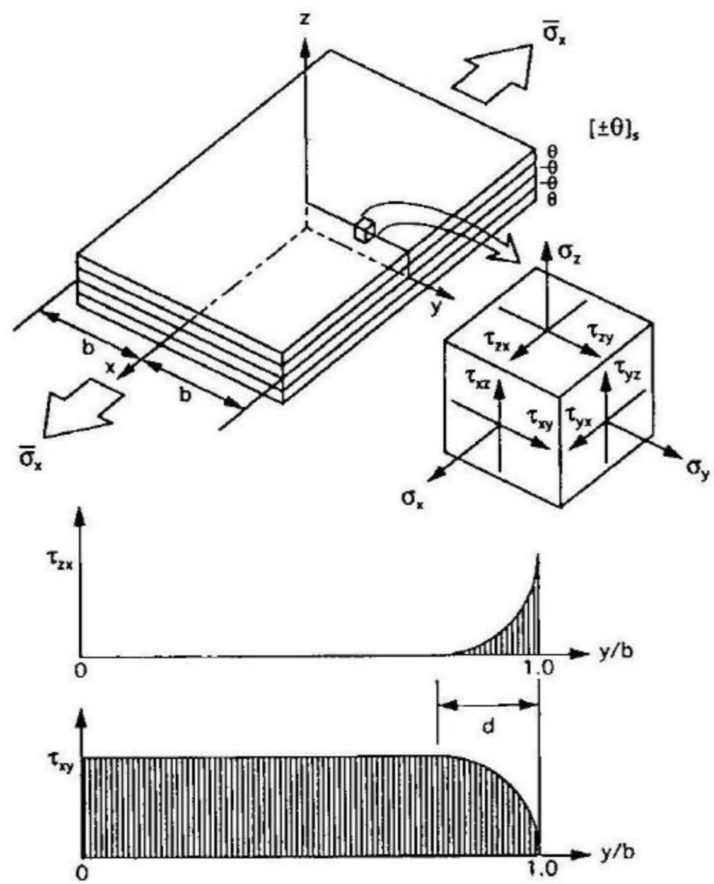
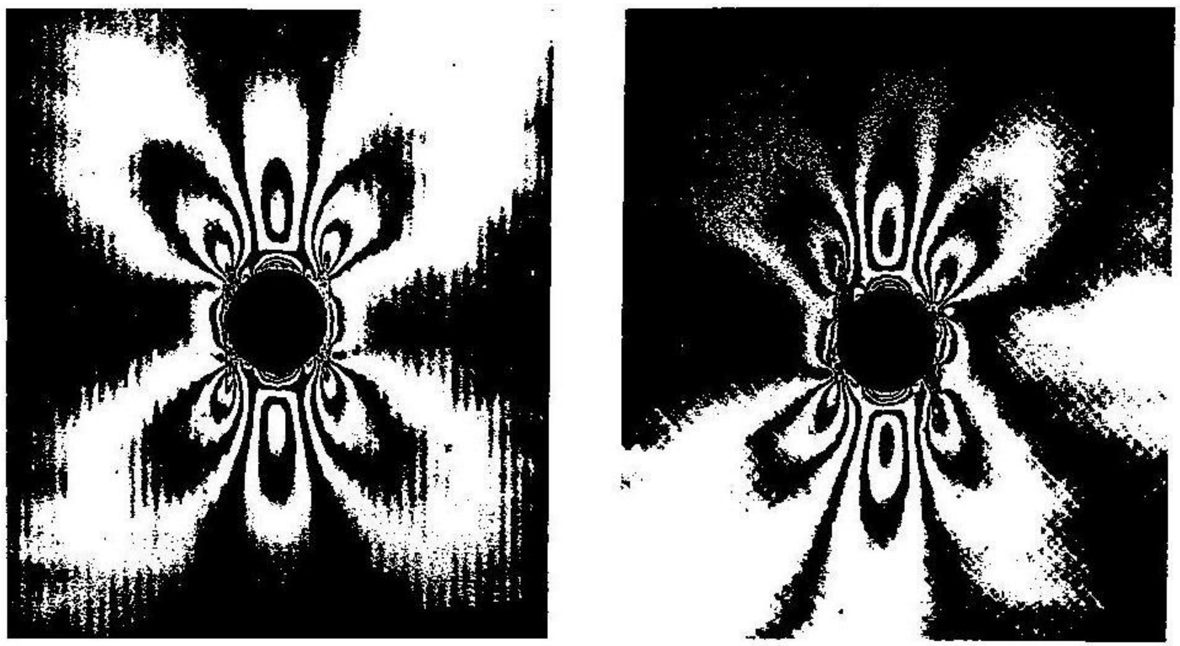


Fig. 7.20 Distribution of interlaminar (τ_{zx}) and intralaminar (τ_{xy}) shear stresses in θ-layer of $[\pm\theta]_s$ angle-ply laminate under axial tension.

Failure analysis of multidirectional laminate



$[0_2/\pm 45/\bar{0}]_s$ $[\pm 45/0_2/\bar{0}]_s$

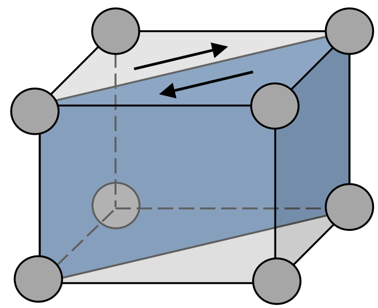
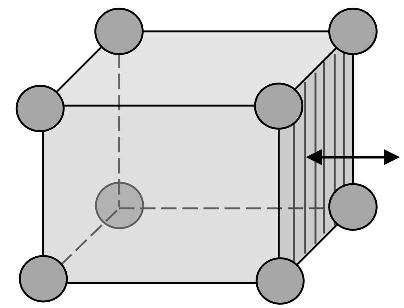
Fig. 7.25 Isochromatic fringe patterns in photoelastic coating around hole in boron/epoxy specimens of two different stacking sequences ($\bar{\sigma}_x = 392$ MPa [56.8 ksi]).[16]

- **$[0_2/\pm 45/\bar{0}]_s$**
  - Patterns near failure → symmetric
  - Catastrophic failure
  - Fail at 426 MPa
- **$[\pm 45/0_2/\bar{0}]_s$**
  - Skewed with higher stress concentration
  - Non catastrophic manner
  - Fail at 527 MPa

So, General multidirectional laminates → interlaminar stresses σ_z , τ_{zx} , τ_{zy}

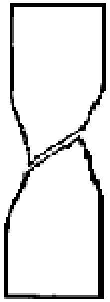
Fracture

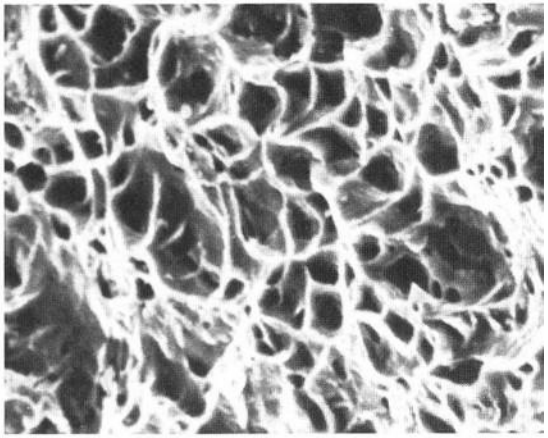
- **Metal fracture**
  - Two fracture modes

	Shear	Cleavage
	 BCC	 BCC
Movement	Sliding	Snapping apart
Occurrence	Gradual	Sudden
Behavior	Ductile	Brittle
	↓	↓
Temperature	High	Low
Load	Torsion	Tension or compression
Pressure	High	Low

Fracture - loading

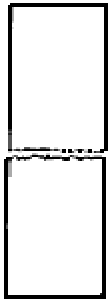
- **Tension**
  - Ductile metal
    - $\tau > \tau_s$
    - Slip : millions of micro-scale planes
    - Lateral deformation → necking

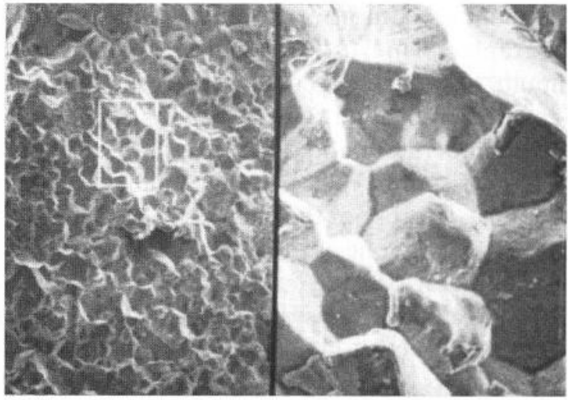




Dimple

  - Brittle metal
    - $\sigma > \sigma_{st}$

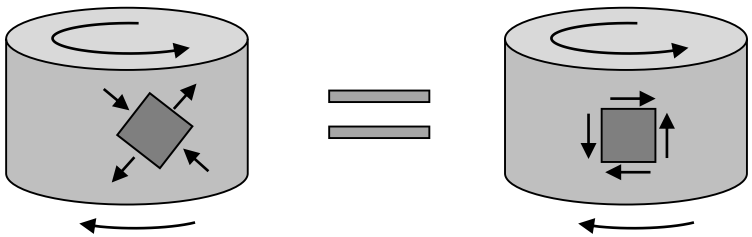




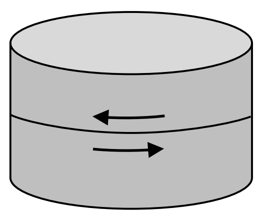
Cleavage

Fracture – loading (continued)

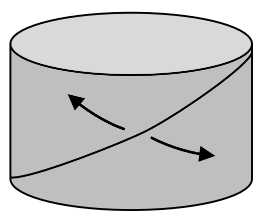
- **Torsion : stress max at surface**



  - Ductile : shear

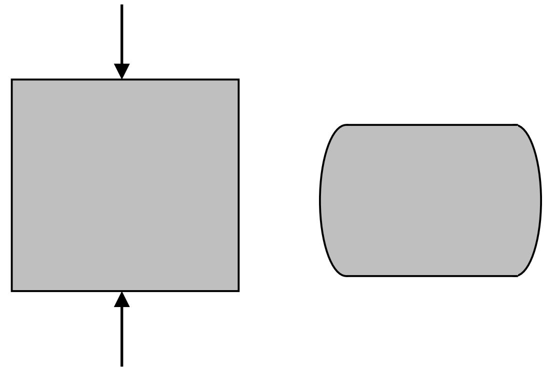


  - Brittle : torsion

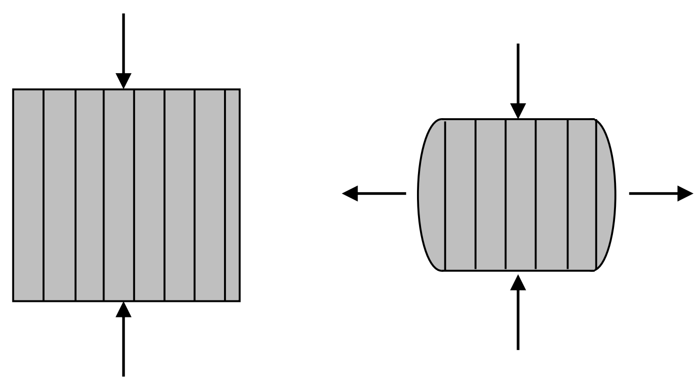


Fracture – loading (continued)

- **Compression**
  - Ductile : opposite to tension



  - Brittle

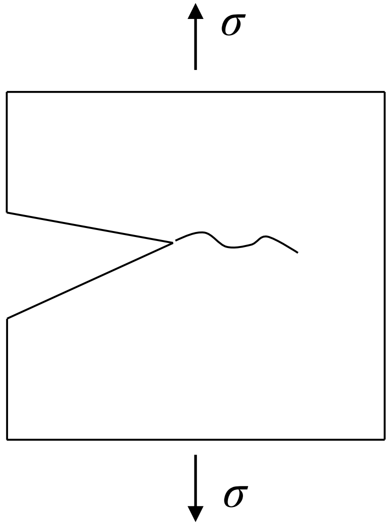


Tensile stress :
split along the loading direction

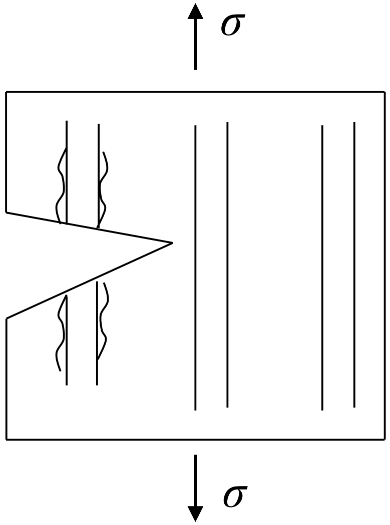
Fracture

- **(Failure) Delamination**
  - Strength analysis
  - Fracture mechanism approach
    - Assumption : presence of an initial cracklike flaw
    - Q : what loading will propagate crack?

  - Griffith's fracture



Isotropic



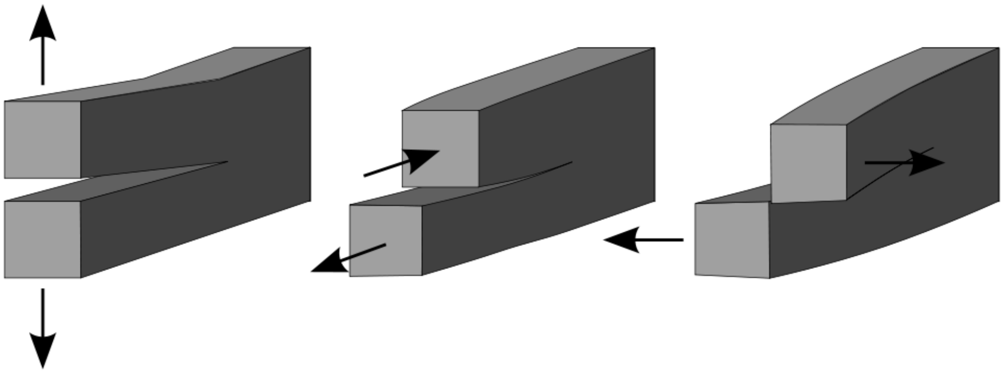
Composite

quite different mechanism

Fractures

- **For delamination, applying fracture theory to composite works**

- **3 modes of crack propagation**



Mode I
(Opening)

Mode II
(Shearing)

Mode III
(Tearing)

Fracture

- **Energy approach**
  - Crack extension occurs when the energy available for crack growth is sufficient to overcome the resistance of the material

$$W = U + \Gamma$$

Work done to the structure

Strain energy in the structure

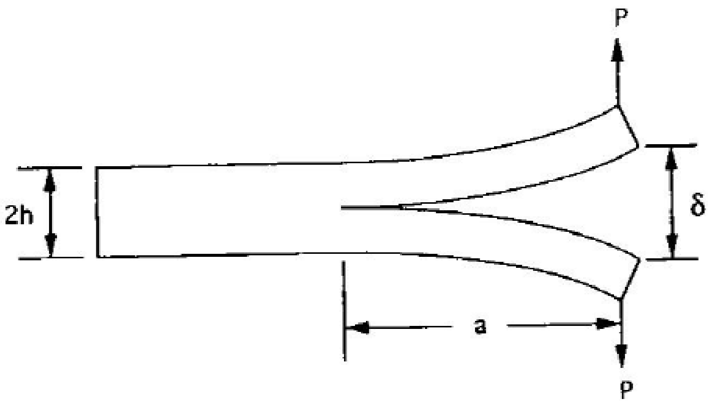
Fracture energy : dissipated as crack propagation

$$\frac{dW}{da} = \frac{dU}{da} + \frac{d\Gamma}{da} = \frac{dU}{da} + BG$$

a = length of crack

B = thickness of structure

G = fracture energy per unit length of crack



Fracture

- **Linear elastic** $\rightarrow$ $$U = \frac{1}{2} P\delta$$

$$\delta = CP \qquad \text{Where, } C = \text{Compliance}$$

$$\therefore \frac{dU}{da} = \frac{d}{da}\left(\frac{1}{2}P\delta\right) = \frac{d}{da}\left(\frac{1}{2}CP^2\right) = CP\frac{dP}{da} + \frac{P^2}{2}\frac{dC}{da}$$

1. Fixed grip

$$\frac{dW}{da} = 0 \longrightarrow \frac{dP}{da} = \frac{-P}{C}\frac{dC}{da}$$

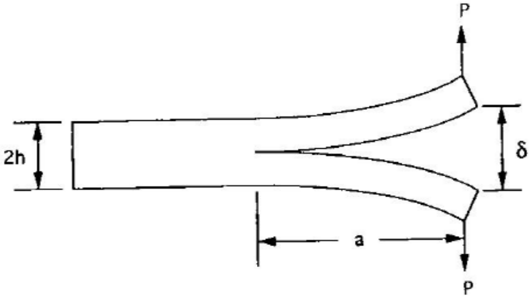
2. Fixed load

$$\frac{dW}{da} = P\frac{d\delta}{da} = P\left(C\frac{dP}{da} + P\frac{dC}{da}\right) \longrightarrow G = \frac{P^2}{2B}\frac{dC}{da}$$

Fracture

- **For double cantilever beam test (DCB)**

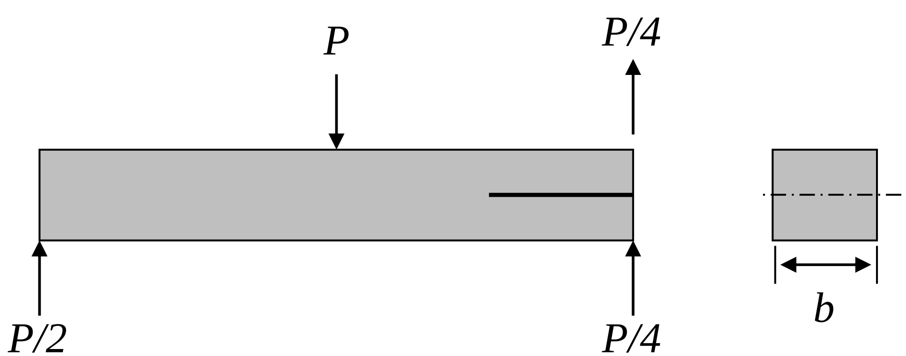
$$\delta = 2\left(\frac{Pa^3}{3EI}\right) \longrightarrow C = \frac{2a^3}{3EI}$$



$$\frac{dC}{da} = \frac{2a^2}{EI} \qquad \therefore G = \frac{P^2a^2}{BEI} \longrightarrow G_{IC}$$

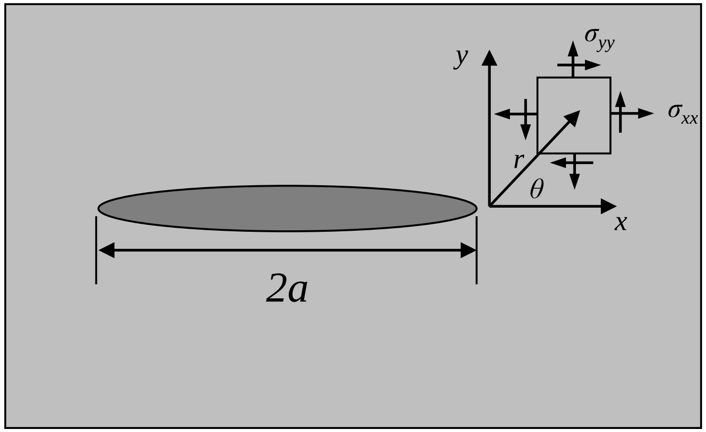
- **For End-notched flexure test (ENF)**

$$G = \frac{9P^2a^2}{16Eb^2h^3} \longrightarrow G_{IIC}$$



Fracture

- **Stress intensity approach**



$$K_I = \sigma\sqrt{\pi a}$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}}\cos\left(\frac{\theta}{2}\right)\left[1-\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{3\theta}{2}\right)\right]$$

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}}....$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}}....$$

- **Fracture at K_I = K_{IC}**

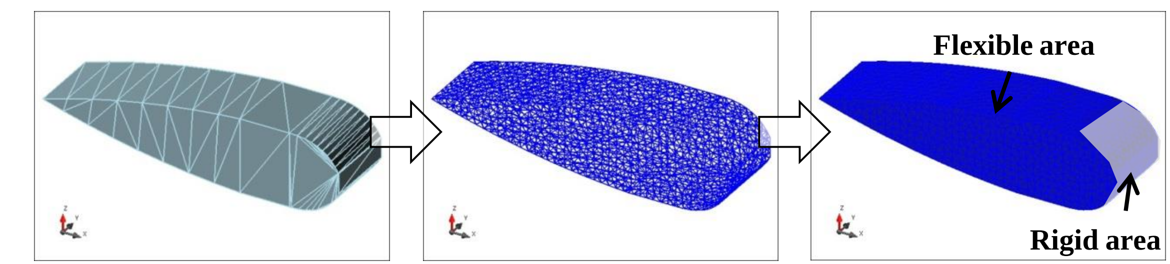
$\searrow$ *Measured mechanical property*

- **Relationship between K_I & G**

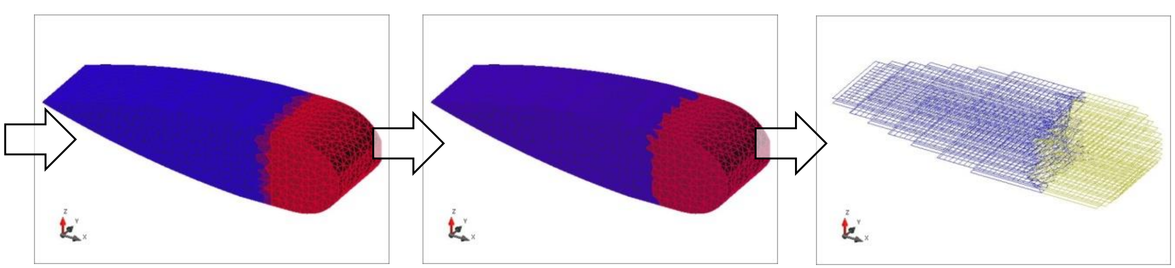
$$G = \frac{K_I^2}{E} \qquad \text{Mode I}$$

Functionally Graded Material (FGM)

- Design of functionally graded material and heterogeneous material compositions
- Optimization of material compositions by FEM analysis
- Process planning for fabrication of FGM by using layered manufacturing process

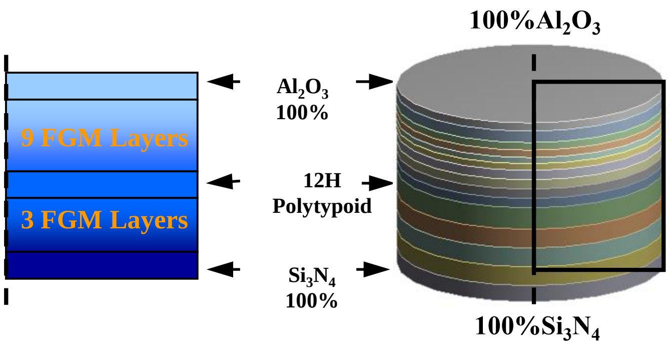


STL model **Mesh model (tetrahedron)** **Material design; boundary**

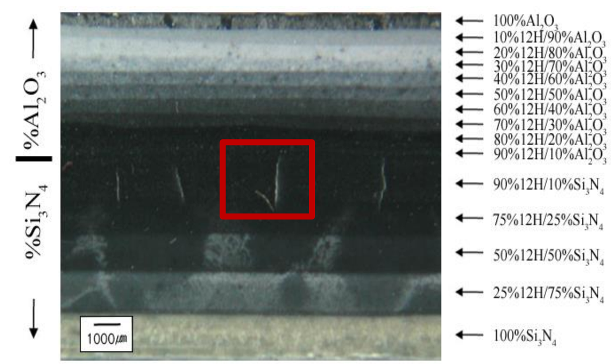


Material design; heterogeneity **Material design; gradient, 2** **Tool path; deposition only**

< FGM material modeling and process planning >



< Analysis model of FGM >



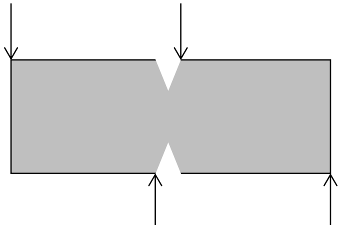
< Fabricated FGM part >

ASTM index for composite

- **Specimen preparation (ASTM 3039-76)**
  - Diamond saw
  - Tab
  - Hole
  - Hinge and Gig
  - Strain gage
- **Fiber volume fraction (ASTM D3171)**
  - Chemical matrix digestion
  - Photo micrographic
- **Tensile properties (ASTM D3039-76)**
  - Straight specimen

ASTM index for composite

- **Shear properties**
  - Various methods
  - [±45]ns tension test - In-Plane Shear Response (ASTM D3518)
  - V-notched beam test (ASTM D5379)



- **Compressive properties (ASTM D3410-87)**
  - Using fixture
  - 4 point bending of sandwich
- **Shear strength of joint (ASTM D3163-73)**
  - Multiple specimens from a panel
- **Fracture toughness (ASTM D5528-94a)**
  - Open gap → close → measure → open → repeat