- Mid-term Exam 1: April 17 (Mon) 7:00 PM
- Mid-term Exam 2: May 15 (Mon) 7:00 PM
- Final Exam: June 2 (Fri) 7:00 PM
- No Class:
  - Confirmed: GLT(03/23), ACS(04/04, 04/06)

* 2. Text and References	Main Text: "Transport Phenomena" 2nd ed., R.B. Bird, W.E. Stewart, E.N. Lightfoot								
	References:								
	"Fundamentals of heat and mass transfer" Incropera, DeWitt, Bergman, Lavine								
	"Fundamentals of momentum, heat, and mass transfer" Welty, Wicks, Wilson, Rorrer								
	"Diffusion: mass transfer in fluid systems" Cussler								
	"Numerical Methods for Engineers" Chapra, Canale								
* 3. Evaluation		Mid-term1	Mid-term2	Final					
	5	25	25	45	0	0	0	0	
	Other:						※ Possib	le to change	

Week	Course Outline		
1	Introduction		
2	Fluid statics		
3	Control volume approach		
4	Conservation equations		
5	Laminar flow		
6	Turbulent flow		
7	Fundamentals of heat transfer		
8	Conduction (Mid-term exam)		
9	Convective heat transfer		
10	Boiling and condensation		
11	Radiation heat transfer		
12	Fundamentals of mass transfer		
13	Steady molecular diffusion		
14	Unsteady molecular diffusion		
15	Convective mass transfer (Final exam)		

# Chapter 1. Viscosity and the Mechanisms of Momentum Transport

- Newton's law of viscosity (molecular momentum transport)
- Generalisation of Newton's law of viscosity
- Pressure and temperature dependence of viscosity
- Molecular theory of the viscosity of gases at low density, of liquids, and of suspensions and emulsions
- Convective momentum transport

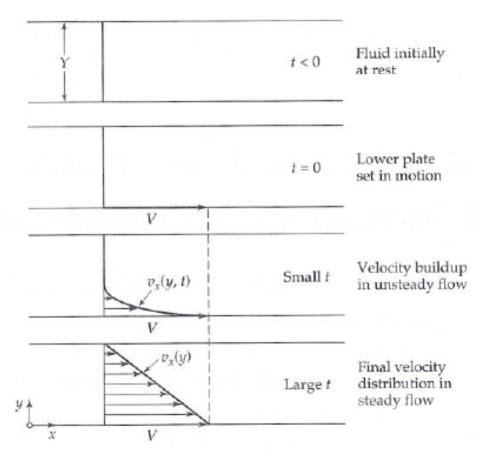
# Newton's law of viscosity (molecular transport of momentum)

At steady state

$$\frac{F}{A} = \mu \frac{V}{Y}$$

 Newton's law of viscosity

$$\tau_{yx} = -\mu \frac{dv_{x}}{dy}$$



"Transport Phenomena" 2nd ed., R.B. Bird, W.E. Stewart, E.N. Lightfoot

# Newton's law of viscosity

Molecular transport of momentum

$$\tau_{yx} = -\mu \frac{dv_{x}}{dy}$$

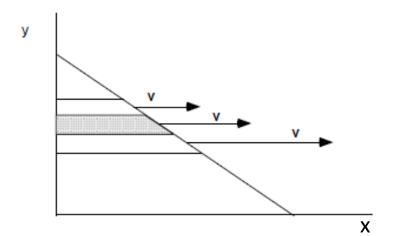
- $-\mu = Viscosity$
- $-\tau_{yx}$  = Shear stress in the x-direction at constant y
- Velocity gradient is the driving force for momentum transport

# Newton's law of viscosity

#### Another interpretation

- x-momentum is transmitted through the fluid in the y-direction
- $-\tau_{yx}$  = viscous flux of x-momentum in the positive y-direction.

$$\tau_{yx} = -\mu \frac{dv_{x}}{dy}$$



#### Newtonian fluids

 $\mu = \mu(T)$ 

- All gases and most simple liquids
  - Viscosity is **independent** of τ
  - Units: Pa·s
- Non-Newtonian fluids
  - Viscosity is dependent of τ
  - Polymeric liquids, pastes, slurries
- Kinematic viscosity
  - Units: m<sup>2</sup>/s

## Generalisation of Newton's law of viscosity

Velocity is a vector

$$v_x = (x, y, z, t); \quad v_y = (x, y, z, t); \quad v_z = (x, y, z, t);$$

• Stress is a tensor with 9 components

$$\boldsymbol{\tau}_{ij} = \begin{bmatrix} \boldsymbol{\tau}_{11} & \boldsymbol{\tau}_{12} & \boldsymbol{\tau}_{13} \\ \boldsymbol{\tau}_{21} & \boldsymbol{\tau}_{22} & \boldsymbol{\tau}_{23} \\ \boldsymbol{\tau}_{31} & \boldsymbol{\tau}_{32} & \boldsymbol{\tau}_{33} \end{bmatrix}$$

# Generalisation of Newton's law of viscosity

 A half of the cube is removed. Forces on the free surface:

$$\mathbf{p} \cdot \mathbf{\delta}_{\mathbf{x}}$$
  $\mathbf{\tau}_{\mathbf{x}} = \mathbf{\tau}_{\mathbf{x}\mathbf{x}} + \mathbf{\tau}_{\mathbf{x}\mathbf{y}} + \mathbf{\tau}_{\mathbf{x}\mathbf{z}}$ 

- $\mathbf{p} \cdot \mathbf{\delta}_{x}$  Associated to the pressure (perpendicular to the surface)
  - τ<sub>x</sub> Associated to the viscous forces. For a plane perpendicular to x-direction

### Generalisation of Newton's law of viscosity

#### · Pressure and viscous forces

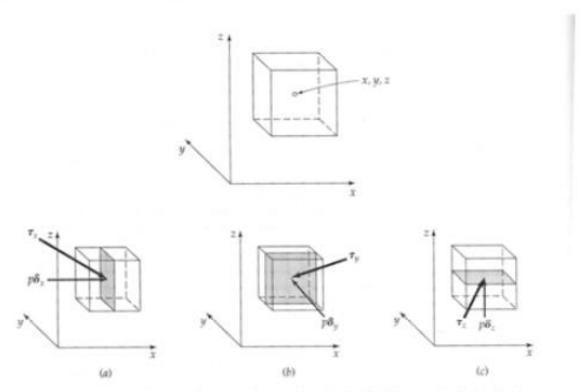


Fig. 1.2-1 Pressure and viscous forces acting on planes in the fluid perpendicular to the three coordinate systems. The shaded planes have unit area.

"Transport Phenomena" 2nd ed., R.B. Bird, W.E. Stewart, E.N. Lightfoot

#### Molecular stress

#### Molecular Stress

$$\pi_{ij} = p \cdot \delta_{ij} + \tau_{ij}$$

- Normal stress  $\pi_{ij(i=j)}$
- Shear Stress  $\pi_{ij(i\neq j)}$

#### Interpretations

- $\pi_{ij}$  Force in j-direction on a unit area perpendicular to i-direction
- $\pi_{ij}$  Flux of j-momentum in the positive i-direction

#### Stress tensor

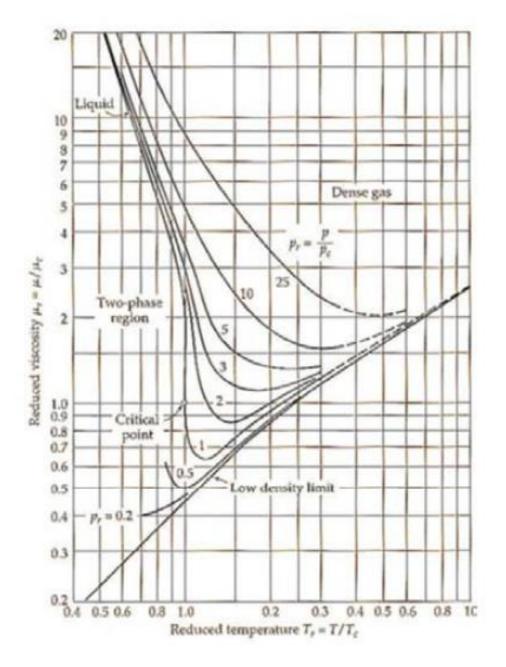
$$\tau_{ij} = -\mu \!\! \left( \frac{\partial v_j}{\partial x_i} \! + \! \frac{\partial v_i}{\partial x_j} \right) \! + \! \left( \frac{2}{3} \mu \! - \! \kappa \right) \!\! \left( \frac{\partial v_x}{\partial x} \! + \! \frac{\partial v_y}{\partial y} \! + \! \frac{\partial v_z}{\partial z} \right) \! \delta_{ij}$$

$$\tau = -\mu \left(\nabla \mathbf{v} + (\nabla \mathbf{v})^{+}\right) + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathbf{v})\delta$$

- $\mu = V_{iscosity}$
- $\kappa$  = Dilatational viscosity, commonly no used

## Viscosity of gases and liquids

- Viscosity is a function of  $\mu = \mu(T, p)$
- Viscosity may be obtained from
  - Molecular theories
  - Empirical correlations
  - Correlation based in molecular theories
- Reference
  - The Properties of Gases and Liquids by Poling, Prausnitz, and O'Connell, 5th Ed.



1.3 Pressure and temperature dependence of viscosity

 Reduced viscosity in function of the reduced temperature and reduced pressure

"Transport Phenomena" 2nd ed., R.B. Bird, W.E. Stewart, E.N. Lightfoot

### Critical pressure and temperature

- Critical pressure and temperature
- The critical temperature of a liquid is that temperature beyond which the liquid cannot exist, no matter how much pressure is placed on it.
- The pressure that is needed to cause the gas to condense at the critical temperature is the *critical* pressure.

# Pressure and temperature dependence of viscosity

 Viscosity of a gas at low density increases with increasing temperature

Viscosity of a liquid decreases with increasing temperature

# 1.4 Molecular theory of the viscosity of gases at low density

 At low pressure, average velocity (random) and mean free path λ

$$\overline{\mathbf{u}} = \sqrt{\frac{8 \cdot \mathbf{\kappa} \cdot \mathbf{T}}{\pi \cdot \mathbf{m}}} \qquad \qquad \lambda = \frac{1}{\sqrt{2\pi d^2 n}}$$

Viscosity

$$\mu = \frac{2}{3\pi} \frac{\sqrt{\pi m \kappa T}}{\pi d^2}$$

## Rigorous kinetic theory. Monoatomic gases

• Lennard-Jones potential  $\varphi(r) = 4\varepsilon \left| \left( \frac{\sigma}{r} \right)^{12} - \left( \frac{\sigma}{r} \right)^{6} \right|$ 

 Viscosity for a pure mono-atomic gas (it can be used for poly-atomic gases as well) in term of Lennard-Jones parameters

$$\mu = \frac{5}{16} \frac{\sqrt{\pi m \kappa T}}{\pi \sigma^2 \Omega_{\mu}}$$

### Convective momentum transport

- Momentum may be transported by the bulk flow of the fluid. Convective transport
  - Momentum flux across the shaded area are

$$v_x(\rho \vec{v}) \quad v_y(\rho \vec{v}) \quad v_z(\rho \vec{v})$$

Convective momentum-flux tensor (second-order)

- Combined momentum-flux

$$\rho vv = \sum_{i} \sum_{j} \delta_{i} \delta_{j} \rho v_{i} v_{j}$$

$$\phi = \pi + \rho \overline{v} \overline{v}$$

$$\pi = \mathbf{p} \cdot \mathbf{\delta} + \mathbf{\tau}$$

# Convective momentum transport. Convective momentum flux tensor

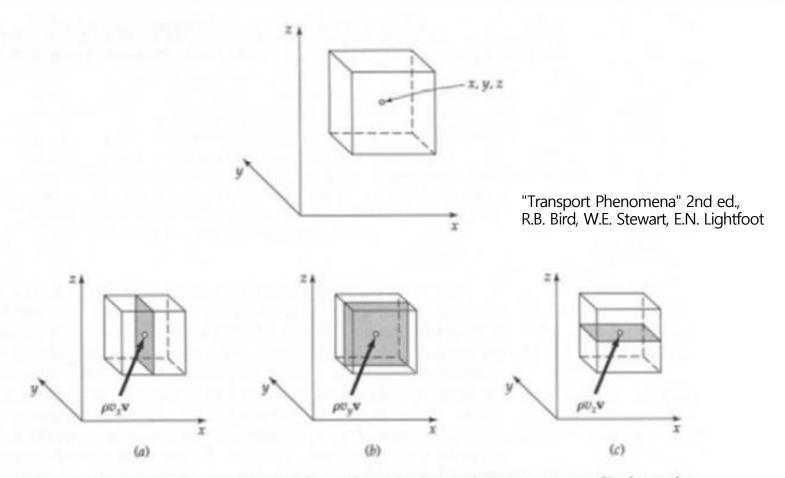


Fig. 1.7-1 The convective momentum fluxes through planes of unit area perpendicular to the coordinate directions.