Chapter 6. Interphase Transport in Isothermal Systems

- Definition of friction factors
- Friction factors for flow in tubes
- Friction factors for flow around spheres
- Friction factors for packed beds

Definition of Friction factors

- Forces from fluid to solid surfaces

$$F_{f\rightarrow s} = F_s + F_k$$

  - F_s = Force exerted by the fluid even it were stationary
  - F_k = Force associated with the motion of the fluid

Definition of Friction factors

- Force caused by the motion

$$F_k = AKf$$

  - A = Characteristic area
  - K = Characteristic kinetic energy per unit volume
  - f = Friction factor

Definition for flow in conduits

- K is taken to be:

$$K = \frac{1}{2}\rho\langle v\rangle^2$$

- A is taken to be the wetted surface
- For circular tubes of radius R and length L, Fk then is

$$F_k = (2\pi RL)\left(\frac{1}{2}\rho\langle v\rangle^2\right) f$$

Definition for flow in conduits

- If pressure difference is measured, f (Fanning friction factor) may be written as:

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{P_0 - P_L}{\frac{1}{2}\rho\langle v\rangle^2}\right)$$

Definitions for flow around submerged objects

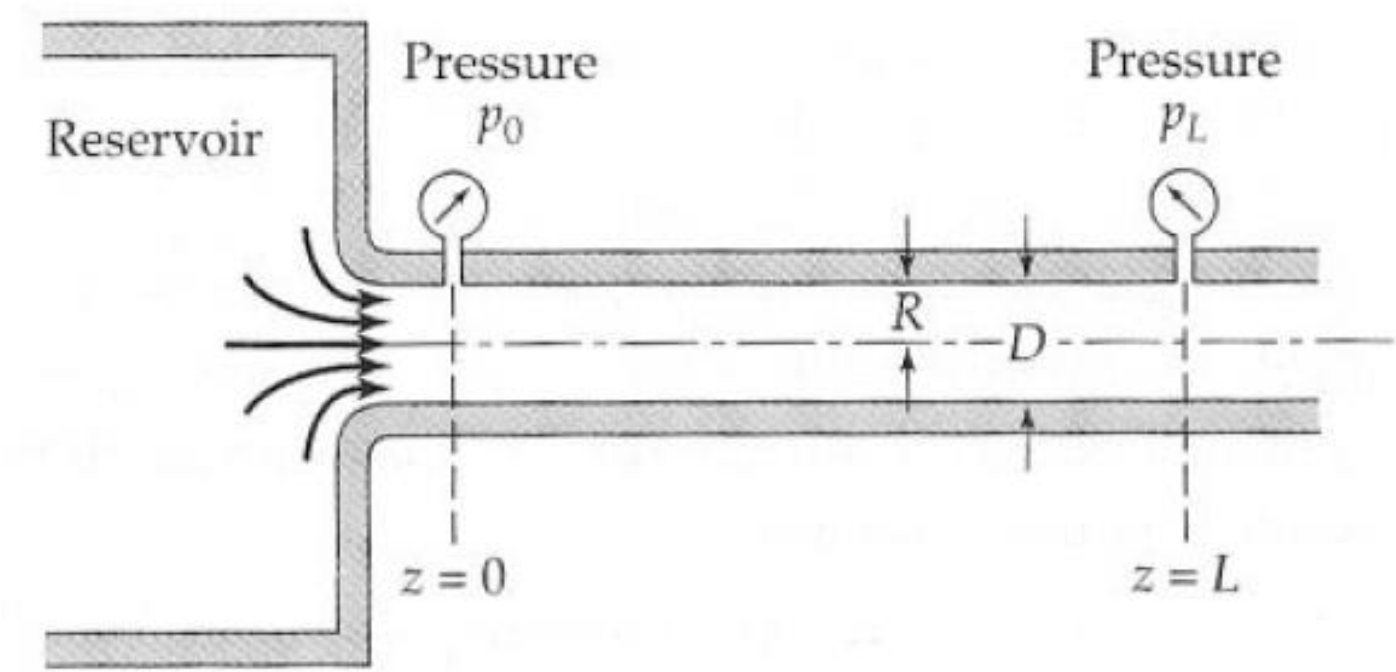
- K is taken to be:

$$K = \frac{1}{2}\rho {v_\infty}^2$$

- A is the projection of the solid onto a plane perpendicular to velocity of the approaching fluid
- For flow around a sphere of radius R

$$F_k = (\pi R^2)\left(\frac{1}{2}\rho {v_\infty}^2\right) f$$

Friction factors for flow in tubes. Dimensional analysis



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Equations for flow in tubes

- Force from fluid on the inner wall

$$F_k(t) = \int_0^L \int_0^{2\pi} \left(-\mu \frac{\partial v_z}{\partial r} \right) \Bigg|_{r=R} R d\theta dz$$

- Force from fluid on the inner wall in function of f

$$F_k(t) = (2\pi RL) \left(\frac{1}{2} \rho \langle v \rangle^2 \right) f$$

Equations for flow in tubes

- Equating both equations, f may be obtained

$$f(t) = \frac{\int_0^L \int_0^{2\pi} \left(-\mu \frac{\partial v_z}{\partial r}\right)\Big|_{r=R} R d\theta dz}{(2\pi RL)\left(\frac{1}{2}\rho \langle v \rangle^2\right)}$$

Using dimensionless variables

$$\tilde{r} = \frac{r}{D} \qquad \tilde{z} = \frac{z}{D} \qquad \tilde{v} = \frac{v}{\langle v_z \rangle} \qquad \tilde{t} = \frac{\langle v_z \rangle t}{D}$$

$$\tilde{P} = \frac{P - P_o}{\rho \langle v_z \rangle^2} \qquad Re = \frac{D \langle v_z \rangle \rho}{\mu}$$

$$f(\tilde{t}) = \frac{D \int_0^{L/D} \int_0^{2\pi} \left(-\mu \frac{\partial \tilde{v}_z}{\partial r} \right) \Big|_{\tilde{r} = \frac{1}{2}} R d\theta d\tilde{z}}{\pi \, L \, Re}$$

Friction factors for flow in smooth circular tubes

- In general $$f = f(Re, \frac{L}{D})$$
- For fully developed flow $$f = f(Re)$$
- Laminar flow

$$f = \frac{16}{Re} \quad \begin{cases} Re < 2100 \ \ stable \\ Re > 2100\, usually\ unstable \end{cases}$$

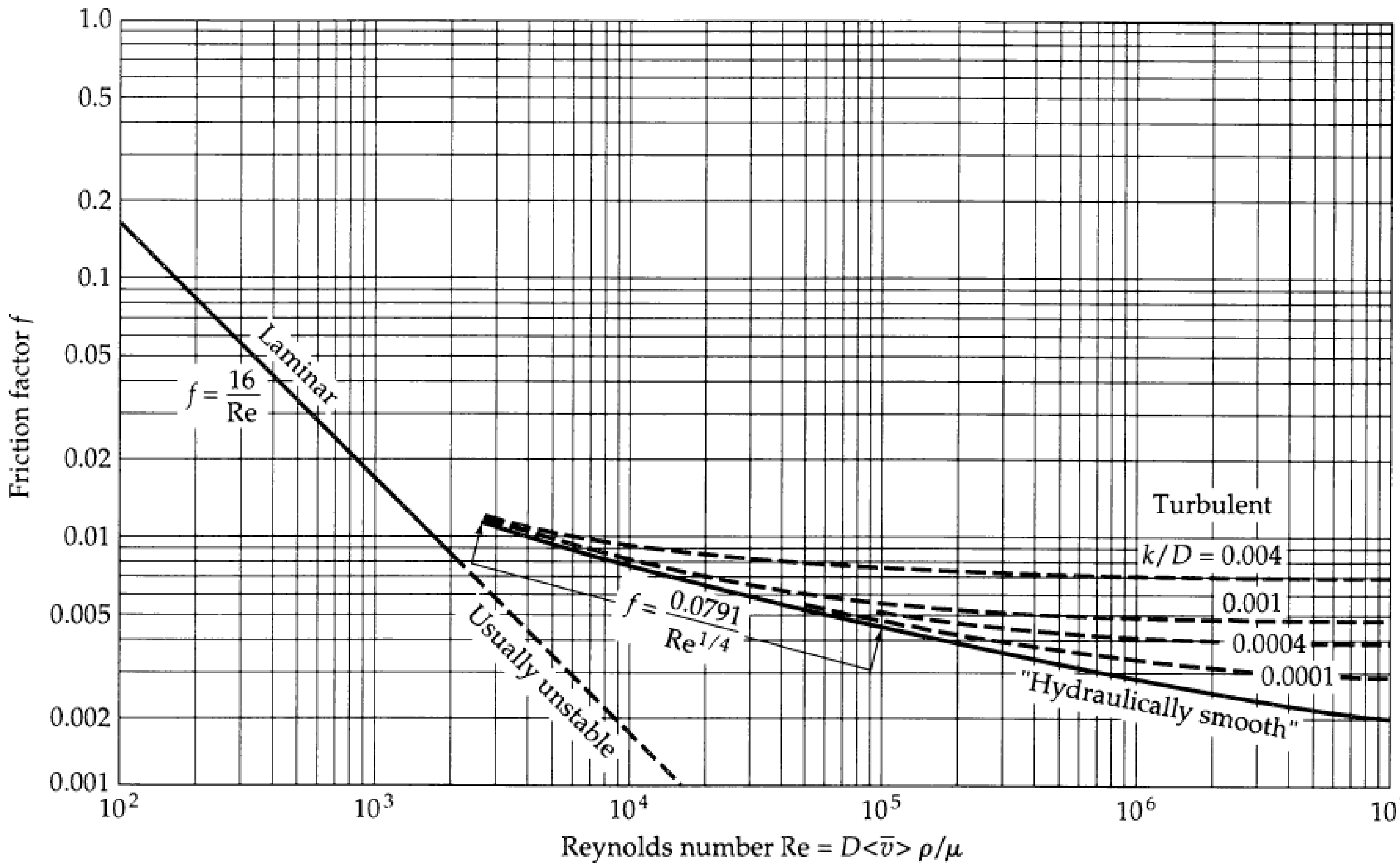
Friction factors for flow in tubes

- For turbulent flow, several empirical expressions
- Blasius formula

$$f = \frac{0.0791}{Re^{1/4}} \qquad 2.1 * 10^3 < Re < 10^5$$

- Others:
  - Prandtl formula and Barenblatt formula
  - Haaland equation for rough pipes

Friction factors for flow in tubes



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Mean Hydraulic radius

- For non circular tubes
- Mean hydraulic radius is defined
  - S = cross section
  - Z = wetted perimeter

$$R_h = \frac{S}{Z}$$

- Re in function of hydraulic radius

$$Re_h = \frac{4R_h\langle v_z\rangle\rho}{\mu}$$

- Not applicable for laminar flow

Friction factors for flow around spheres

- Following the same procedure used for flow in tubes
- The total force acting in z-direction on the sphere:

$$F_k = F_{form} + F_{friction} = (F_n - F_s) + F_t$$

$$F_n = total\ normal\ force$$

$$F_s = stationary\ force$$

$$F_t = tangential\ force$$

Friction factors for flow around spheres

$$F_{\text{form}}(t) = \int_0^{2\pi} \int_0^{\pi} (-\mathcal{P}|_{r=R} \cos \theta) R^2 \sin \theta \, d\theta \, d\phi$$

$$F_{\text{friction}}(t) = \int_0^{2\pi} \int_0^{\pi} \left(-\mu \left[r \frac{\partial}{\partial r} \left(\frac{v_\theta}{r} \right) + \frac{1}{r} \frac{\partial v_r}{\partial \theta} \right] \Bigg|_{r=R} \sin \theta \right) R^2 \sin \theta \, d\theta \, d\phi$$

$$f = f_{\text{form}} + f_{\text{friction}}$$

From the definition $\quad F_k = (\pi R^2)(\tfrac{1}{2}\rho v_\infty^2) f$

Dimensionless variables

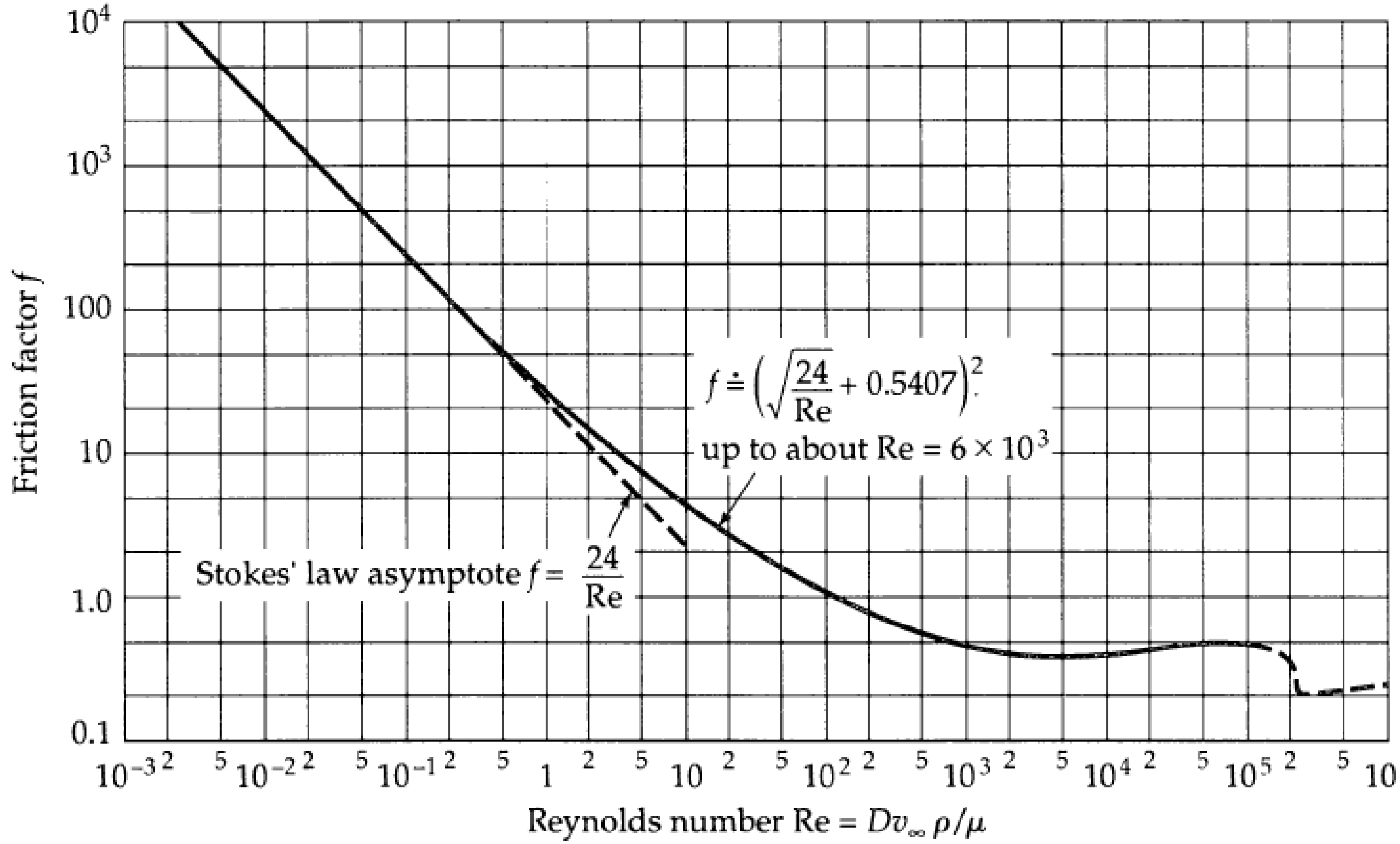
$$\breve{\mathcal{P}} = \frac{\mathcal{P}}{\rho v_\infty^2} \qquad \breve{v}_\theta = \frac{v_\theta}{v_\infty} \qquad \breve{r} = \frac{r}{R} \qquad \breve{t} = \frac{v_\infty t}{R}$$

$$f_{\text{form}}(\breve{t}) = \frac{2}{\pi} \int_0^{2\pi} \int_0^{\pi} (-\breve{\mathcal{P}}\,|_{\breve{r}=1} \cos\theta) \sin\theta \, d\theta \, d\phi$$

$$f_{\text{friction}}(\breve{t}) = -\frac{4}{\pi} \frac{1}{\text{Re}} \int_0^{2\pi} \int_0^{\pi} \left[\breve{r} \frac{\partial}{\partial \breve{r}} \left(\frac{\breve{v}_\theta}{\breve{r}} \right) \right] \Bigg|_{\breve{r}=1} \sin^2\theta \, d\theta \, d\phi$$

$$f = f(\text{Re})$$

Friction factors for flow around spheres



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Friction factors for flow around spheres

- For creeping flow around a sphere

$$f = \frac{24}{\mathrm{Re}} \qquad \text{for Re} < 0.1$$

- For turbulent flow (Newton's resistance law)

$$f \approx 0.44 \qquad \text{for } 5 \times 10^2 < \mathrm{Re} < 1 \times 10^5$$

- Other

$$f = \left(\sqrt{\frac{24}{\mathrm{Re}}} + 0.5407\right)^2 \qquad \text{for Re} < 6000$$

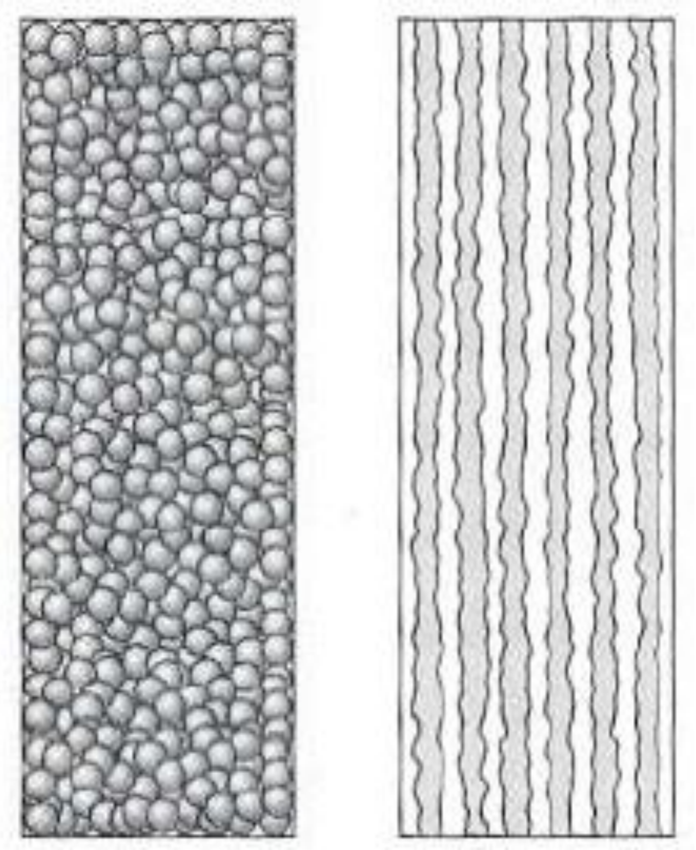
Friction factors for flow around spheres. Remarks

- No well defined the laminar-turbulent transition
- Contributions to f from both friction and form drag
- There is a kink in the f vs. Re curve associated with a shift in the separation zone

Friction Factors for Packed Columns

- Packing
  - Spheres, cylinders, Berl saddles, and so on
- It is assumed that:
  - Packing is statistically uniform. No Channelling
  - Small diameter of the packing particles in comparison to the column diameter
- Two approaches
  - Bundle of tangled tubes of weird cross section
  - Collection of submerged objects

Bundle of tangled tubes of weird cross section



- Reality. Cylindrical tube packed with spheres
- Model. The packed column modelled as a “tube bundle”

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Friction Factors for Packed Columns

- Definition of friction factor for packed columns
  - See Eq 6.1-4

$$f = \frac{1}{4}\left(\frac{D_p}{L}\right)\left(\frac{\mathscr{P}_0 - \mathscr{P}_L}{\frac{1}{2}\rho v_0^2}\right)$$

- D_p is the effective particle diameter
- v_0 is the superficial velocity (volume flow rate divided by the empty cross column section)

Friction Factors for Packed Columns as a tube bundle

- Introducing the pressure difference for a tube bundle.

$$\mathcal{P}_0 - \mathcal{P}_L = \tfrac{1}{2}\rho\langle v\rangle^2\left(\frac{L}{R_h}\right)f_{\text{tube}}$$

- Then, the friction factor for the packed beds is

$$f = \frac{1}{4}\frac{D_p}{R_h}\frac{\langle v\rangle^2}{v_0^2}f_{\text{tube}} = \frac{1}{4\epsilon^2}\frac{D_p}{R_h}f_{\text{tube}}$$

Hydraulic radius for packed columns

$$R_h = \left(\frac{\text{cross section available for flow}}{\text{wetted perimeter}}\right)$$

$$= \left(\frac{\text{volume available for flow}}{\text{total wetted surface}}\right)$$

$$= \frac{\left(\dfrac{\text{volume of voids}}{\text{volume of bed}}\right)}{\left(\dfrac{\text{wetted surface}}{\text{volume of bed}}\right)} = \frac{\varepsilon}{a}$$

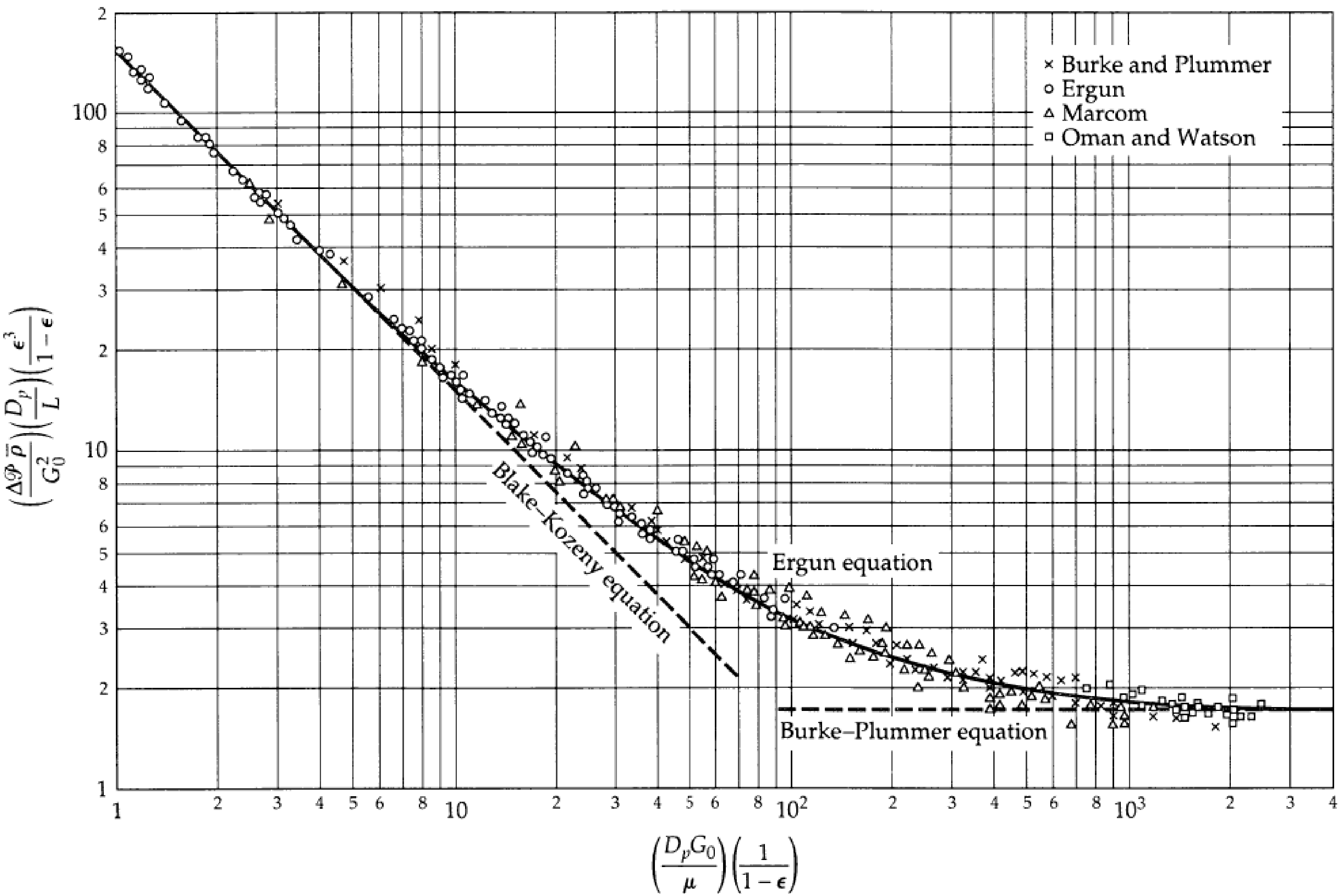
Semi-empirical relationships

- Laminar flow. Blake-Kozeny equation

$$\frac{\mathscr{P}_0 - \mathscr{P}_L}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1-\varepsilon)^2}{\varepsilon^3}$$

- For highly turbulent flow. Burke-Plummer equation

$$\frac{\mathscr{P}_0 - \mathscr{P}_L}{L} = \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right)\frac{1-\varepsilon}{\varepsilon^3}$$



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For the transition zone

- The Ergun Equation

$$\frac{\mathcal{P}_0 - \mathcal{P}_L}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right)\frac{1-\varepsilon}{\varepsilon^3}$$

- Dimensionless

$$\left(\frac{(\mathcal{P}_0 - \mathcal{P}_L)\rho}{G_0^2}\right)\left(\frac{D_p}{L}\right)\left(\frac{\varepsilon^3}{1-\varepsilon}\right) = 150\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right) + \frac{7}{4}$$