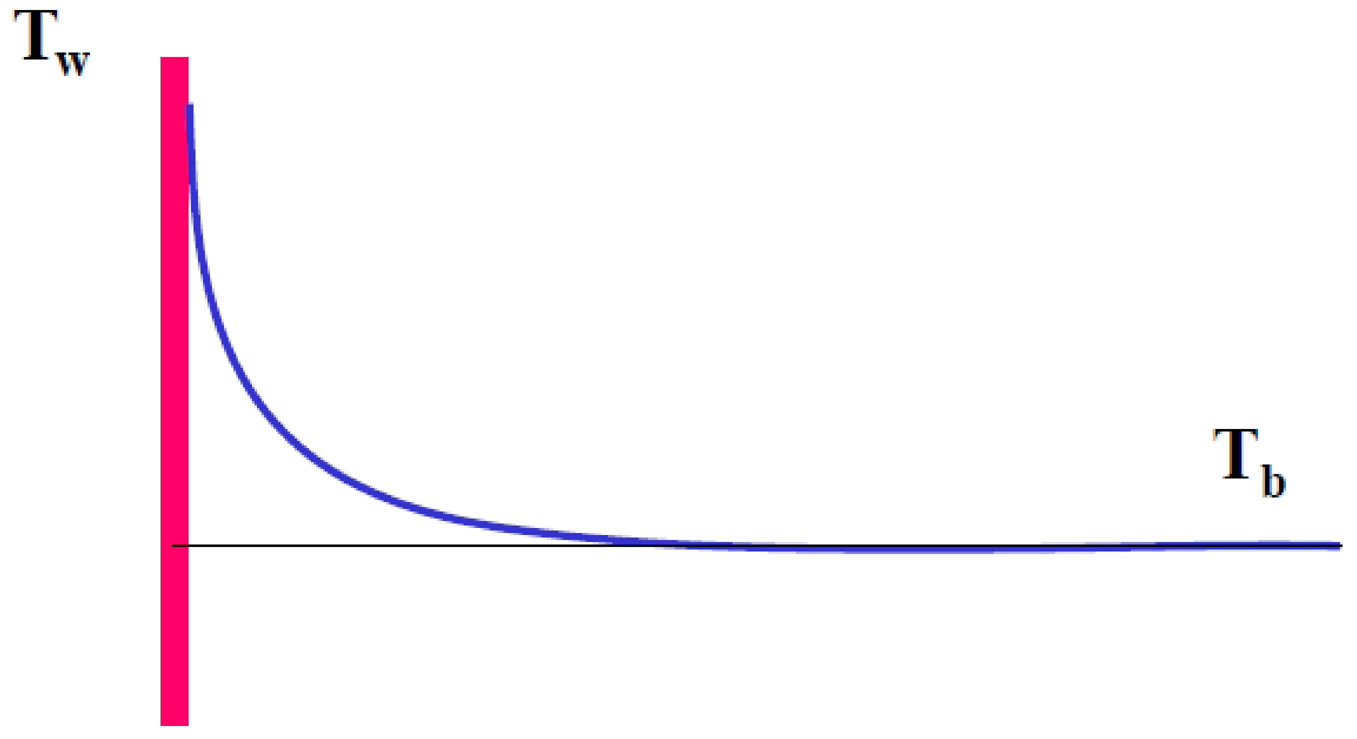
Chapter 14. Interphase Transport in Non-isothermal Systems

- Definitions of heat transfer coefficients(htc), h
- Analytical calculations of htc’s for forced convection through tubes and slits
- Htc’s for forced convection in tubes
- Htc’s for forced convection around submerged objects and for forced convection through packed beds
- Htc’s for free and mixed convection and for condensation of pure vapors on solid surface

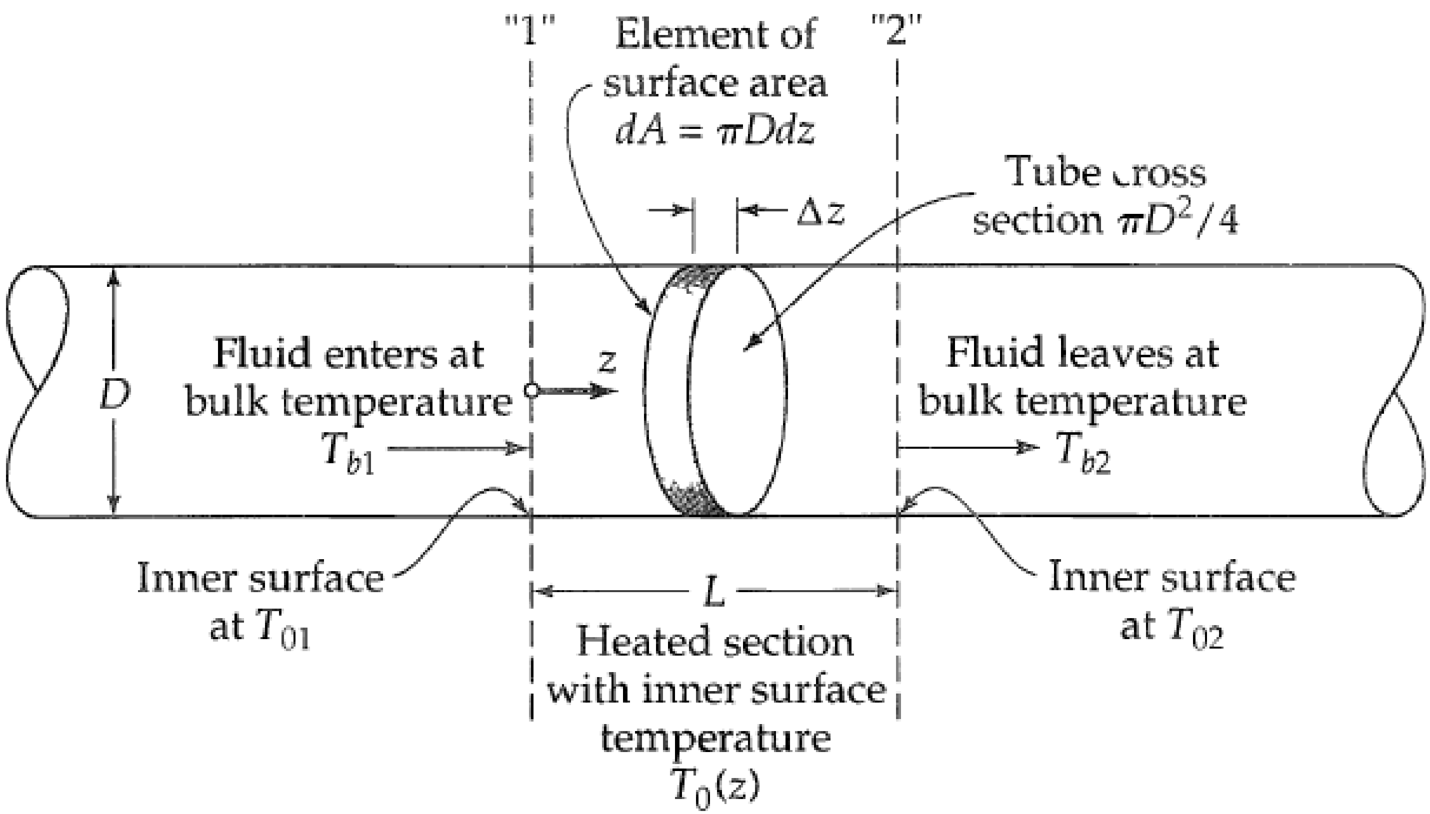
Definition of heat transfer coefficient

- Definition
  - Q = heat flow into the fluid $Q = hA\Delta T$
  - A = characteristic area
  - ΔT = characteristic temperature difference
- Two case are studied
  - Flow in conduits
  - Flow around submerged objects

Heat transfer coefficient



Flow in conduits



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Flow in conduits. Definitions for h

- Flow in a circular tube of diameter D with a heated wall section of length L.
  - Based on temperature difference at the inlet

$$Q = h_1(\pi DL)(T_{01} - T_{b1}) \equiv h_1(\pi DL)\Delta T_1$$

  - Based on arithmetic mean of terminal temperature difference

$$Q = h_a(\pi DL)\left(\frac{(T_{01} - T_{b1}) + (T_{02} - T_{b2})}{2}\right) \equiv h_a(\pi DL)\Delta T_a$$

  - Based on logarithmic mean temperature difference

$$Q = h_{\ln}(\pi DL)\left(\frac{(T_{01} - T_{b1}) - (T_{02} - T_{b2})}{\ln(T_{01} - T_{b1}) - \ln(T_{02} - T_{b2})}\right) \equiv h_{\ln}(\pi DL)\Delta T_{\ln}$$

  - Differential form

$$dQ = h_{\text{loc}}(\pi D dz)(T_0 - T_b) \equiv h_{\text{loc}}(\pi D dz)\Delta T_{\text{loc}}$$

Flow around submerged objects. Definitions for h

- Fluid around a sphere of radius R
  - Uniform surface temperature, T_0
  - Approaching fluid with temperature, T_∞

- Mean heat transfer coefficient

$$Q = h_m(4\pi R^2)(T_0 - T_\infty)$$

- Local heat transfer coefficient

$$dQ = h_{\text{loc}}(dA)(T_0 - T_\infty)$$

Typical values for h

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Table 14.1-1 Typical Orders of Magnitude for Heat Transfer Coefficients[a]

System	h $(W/m^2 \cdot K)$ or $(kcal/m^2 \cdot hr \cdot C)$	h $(Btu/ft^2 \cdot hr \cdot F)$
Free convection		
Gases	3–20	1–4
Liquids	100–600	20–120
Boiling water	1000–20,000	200–4000
Forced convection		
Gases	10–100	2–20
Liquids	50–500	10–100
Water	500–10,000	100–2000
Condensing vapors	1000–100,000	200–20,000

Chilton-Colburn j-factor for heat transfer

- In function of dimensionless numbers

$$j_H = \frac{Nu}{RePr^{1/3}} \qquad j_{H,ln} = \frac{h_{ln}}{\langle \rho v \rangle \hat{C}_p} \left(\frac{\hat{C}_p \mu}{k} \right)^{2/3}$$

- Several analytical expressions may be obtained (e.g., §14.2)
- Analytical expressions for constant physical properties
- The properties are evaluated at "film" temperature,
  - For tubes and slits

$$T_f = \tfrac{1}{2}(T_{0a} + T_{ba}) \qquad T_{0a} = \tfrac{1}{2}(T_{01} + T_{02}) \qquad T_{ba} = \tfrac{1}{2}(T_{b1} + T_{b2})$$

T_{0a} is the arithmetic average of the surface temperatures at the two ends

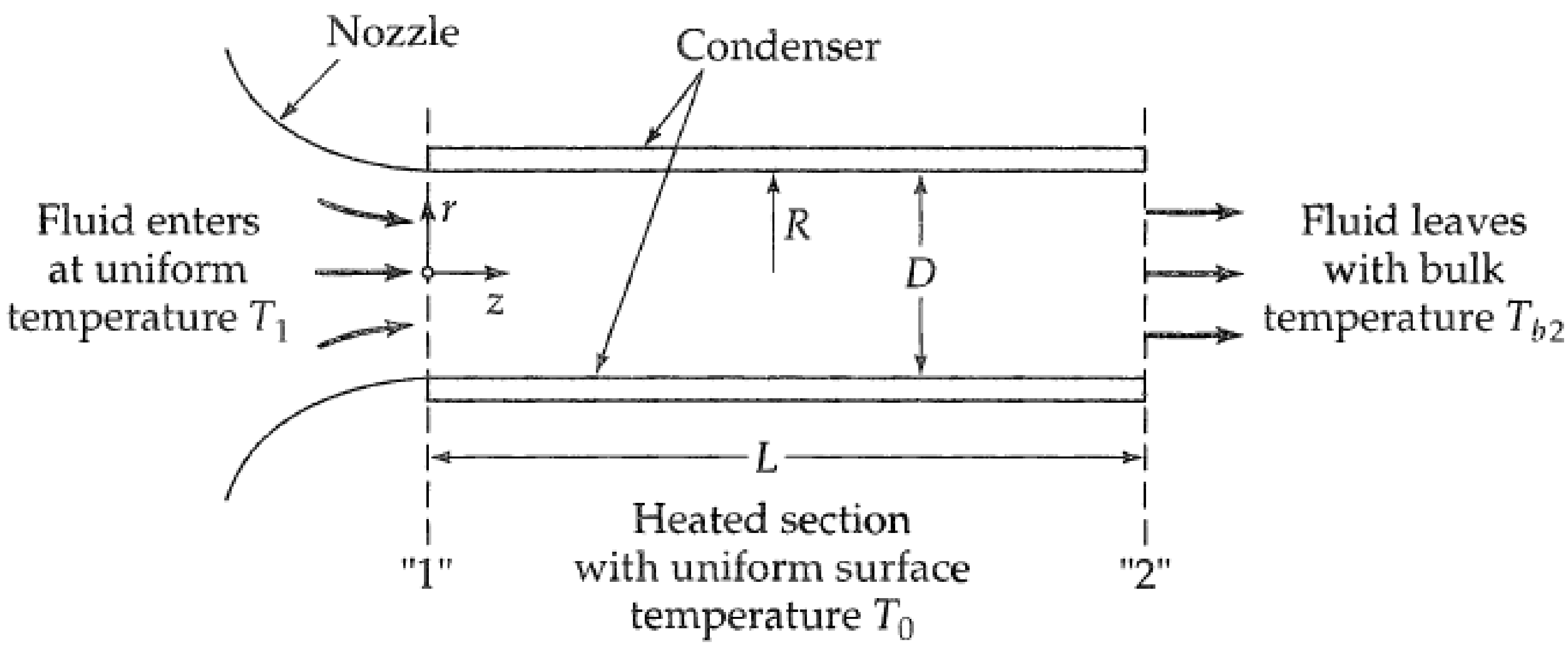
T_{ba} is the arithmetic average of the inlet and outlet bulk

  - For submerged objects

$$T_f = \tfrac{1}{2}(T_0 + T_\infty)$$

uniform surface temperature T_0

Heat transfer coefficients for forced convection in tubes



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Forced convection in tubes

- Same procedure as that used for friction factors
- Heat flow from the tube wall into the fluid

$$Q(t) = \int_0^L \int_0^{2\pi} \left(+k \frac{\partial T}{\partial r} \right) \Big|_{r=R} R \, d\theta \, dz$$

- From dimension analysis

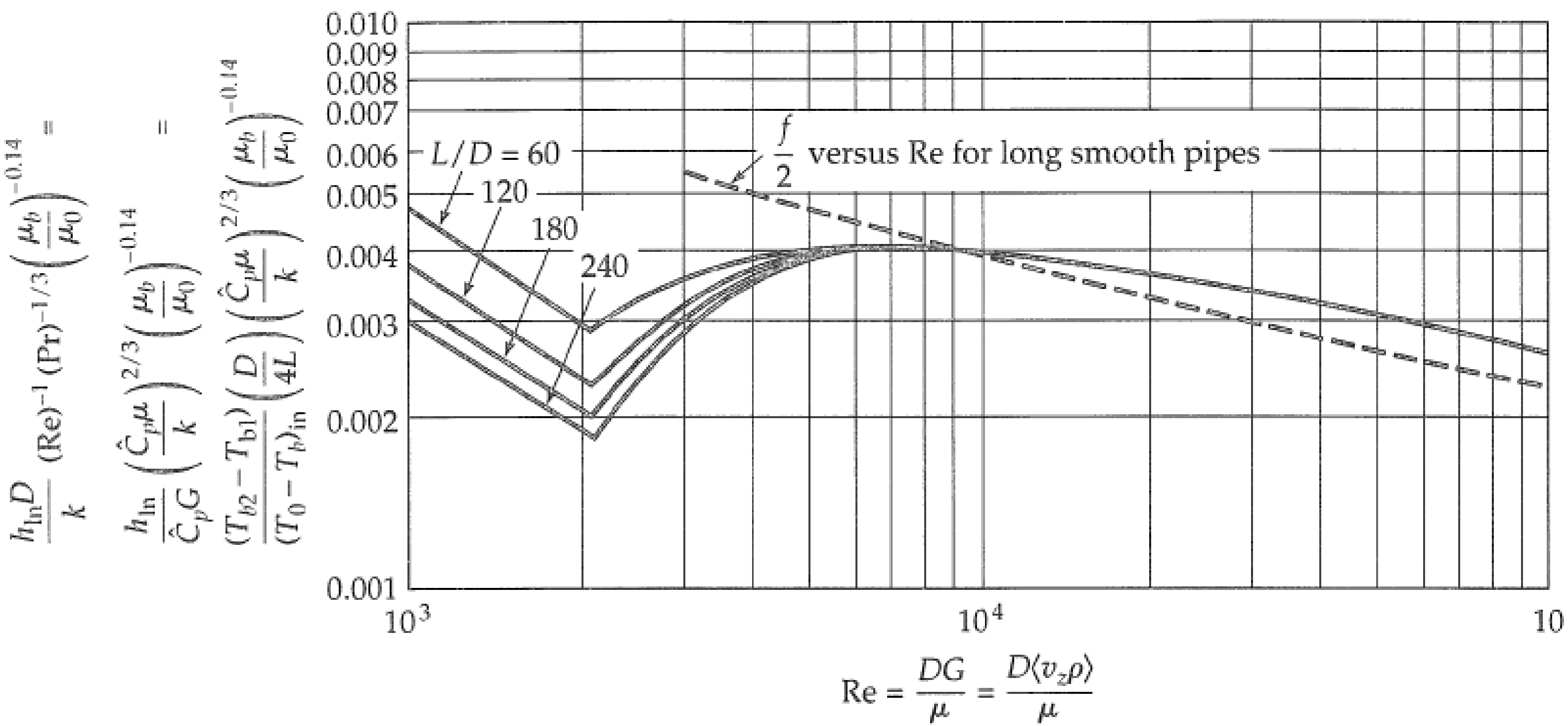
$$\mathrm{Nu}_1 = \mathrm{Nu}_1(\mathrm{Re}, \mathrm{Pr}, \mathrm{Br}, L/D)$$

- For negligible viscous dissipation heating, the Brinkman number can be omitted.
- Heat transfer coefficients are then defined as

$$\mathrm{Nu}_a = \mathrm{Nu}_a(\mathrm{Re}, \mathrm{Pr}, L/D)$$
$$\mathrm{Nu}_{\mathrm{ln}} = \mathrm{Nu}_{\mathrm{ln}}(\mathrm{Re}, \mathrm{Pr}, L/D)$$
$$\mathrm{Nu}_{\mathrm{loc}} = \mathrm{Nu}_{\mathrm{loc}}(\mathrm{Re}, \mathrm{Pr}, L/D)$$

$\mathrm{Nu}_a = h_a D/k$, $\mathrm{Nu}_{\mathrm{ln}} = h_{\mathrm{ln}} D/k$, and $\mathrm{Nu}_{\mathrm{loc}} = h_{\mathrm{loc}} D/k$

Heat transfer coefficient for fully developed flow in smooth tubes



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Forced convection in tubes. Special cases

- For very large temperature differences

$$\mathrm{Nu} = \mathrm{Nu}(\mathrm{Re}, \mathrm{Pr}, L/D, \mu_b/\mu_0)$$

- For highly turbulent flow For Re > 20 000

$$\mathrm{Nu}_{\ln} = 0.026\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{\mu_b}{\mu_0}\right)^{0.14}$$

- For laminar flow

$$\mathrm{Nu}_{\ln} = 1.86\left(\mathrm{RePr}\,\frac{D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_0}\right)^{0.14}$$

- Remarks Colburn analogy

$$j_{H,\ln} \approx \tfrac{1}{2}f \qquad (\mathrm{Re} > 10{,}000)$$

$$j_{H,\ln} = \frac{\mathrm{Nu}_{\ln}}{\mathrm{RePr}^{1/3}} = \frac{h_{\ln}}{\langle\rho v\rangle\hat{C}_p}\left(\frac{\hat{C}_p\mu}{k}\right)^{2/3}$$

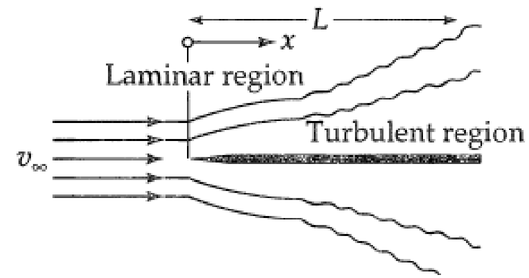
Heat transfer coefficients for forced convection around submerged objects

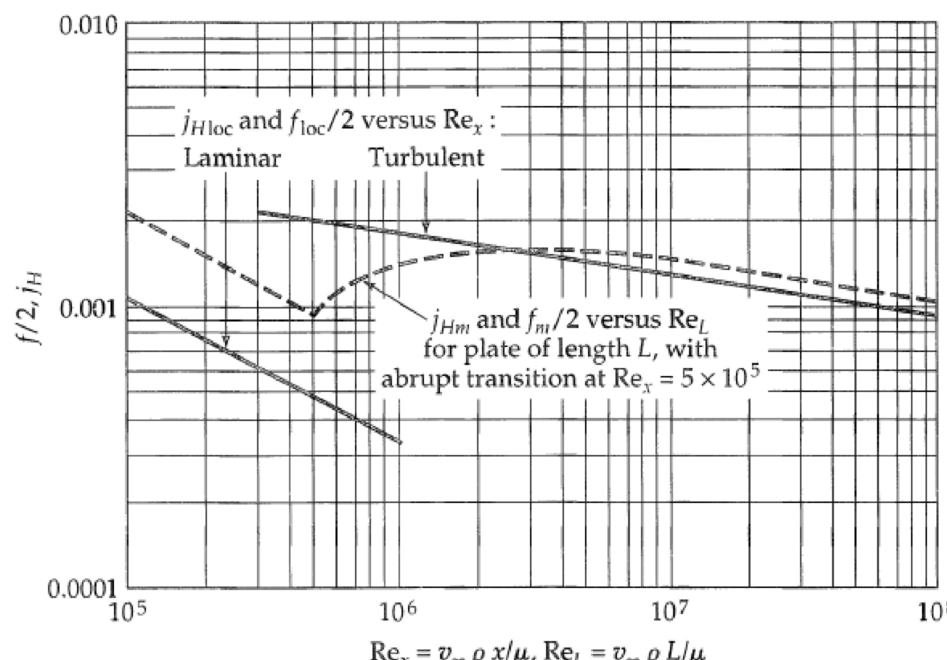
- Flow along a flat plate (parallel to the flow)

$$\tfrac{1}{2} f_{\text{loc}} = 0.332\ \text{Re}_x^{-1/2}$$

- Colburn analogy:

$$j_{H,\text{loc}} = \tfrac{1}{2} f_{\text{loc}} = 0.332\ \text{Re}_x^{-1/2}$$





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Other submerged objects

- Flow around a sphere with constant temperature at the surface

$$\mathrm{Nu}_m = 2 + 0.60\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}$$

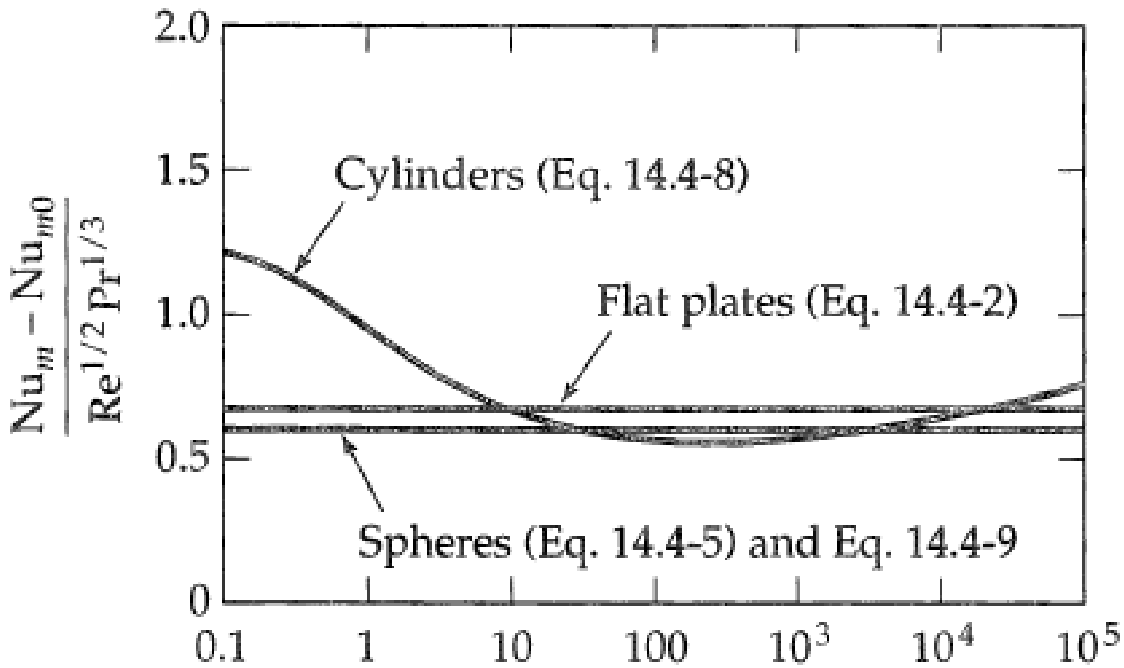
- Remarks.
  - Nu = 2, for a sphere in a stationary fluid
- Flow around a cylinder

$$\mathrm{Nu}_m = (0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3})\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_0}\right)^{1/4}$$

$$1.0 < \mathrm{Re} < 1.0 \times 10^5 \qquad 0.67 < \mathrm{Pr} < 300 \qquad 0.25 < \mu_\infty/\mu_0 < 5.2$$

- Remark. Cylinders in a stationary fluid of infinite extent do not admit a S-S solution

Nusselt numbers for flow around plates, spheres, and cylinders



$Nu_{m,0}$ is the mean Nusselt number at zero Reynolds number

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Heat transfer coefficients for forced convection through packed beds

- Defining the heat transfer coefficient, h_{loc}

$$dQ = h_{\text{loc}}(aS\,dz)(T_0 - T_b)$$

a is the outer surface area of particles per unit bed volume

- For gases and liquids (from experimental data)

$$j_H = 2.19\ \text{Re}^{-2/3} + 0.78\ \text{Re}^{-0.381}$$

- where

$$j_H = \frac{h_{\text{loc}}}{\hat{C}_p G_0}\left(\frac{\hat{C}_p \mu}{k}\right)^{2/3} \qquad \text{Re} = \frac{D_p G_0}{(1-\varepsilon)\mu\psi} = \frac{6G_0}{a\mu\psi}$$

ψ is a particle-shape factor

  - Physical parameters evaluated at the film temperature

Heat transfer coefficients for free and mixed convection. Some special cases

- Free convection near a vertical flat plate, the area mean Nusselt number, in general

$$\mathrm{Nu}_m = C(\mathrm{Gr}\mathrm{Pr})^{1/4}$$

$$\mathrm{Nu}_m = hH/k = q_{\mathrm{avg}}H/k(T_0 - T_1).$$

$$\mathrm{Ra} = \mathrm{Gr}\mathrm{Pr} \quad \textit{Rayleigh number}$$

C was found to be a weak function of Pr

- Only conduction, for no buoyant forces or forced convection

$$\mathrm{Nu}_m^{\mathrm{cond}} = K(\mathrm{shape}) \qquad \mathrm{Nu}_m^{\mathrm{cond}} = 2$$

  - K equal to zero for all objects with at least one infinite dimension (e.g., infinite cylinder).
  - K for spheres is equal 2 (D as characteristic length)

- Thin laminar boundary layers (e.g., isothermal vertical flat plate)

$$\mathrm{Nu}_m^{\mathrm{lam}} = C(\mathrm{Pr}, \mathrm{shape})(\mathrm{Gr}\mathrm{Pr})^{1/4}$$

Heat transfer coefficients for free and mixed convection. Some special cases

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$$\mathrm{Nu}_m = hH/k = q_{\mathrm{avg}}H/k(T_0 - T_1).$$

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- Thin laminar boundary layers (e.g., isothermal vertical flat plate)

$$\mathrm{Nu}_m^{\mathrm{lam}} = C(\mathrm{Pr}, \mathrm{shape})(\mathrm{GrPr})^{1/4}$$

Heat transfer coefficients for free and mixed convection. Some special cases

- Turbulent boundary layers
  - The effects of turbulence increases gradually
  - Laminar and turbulent contributions are commonly combined

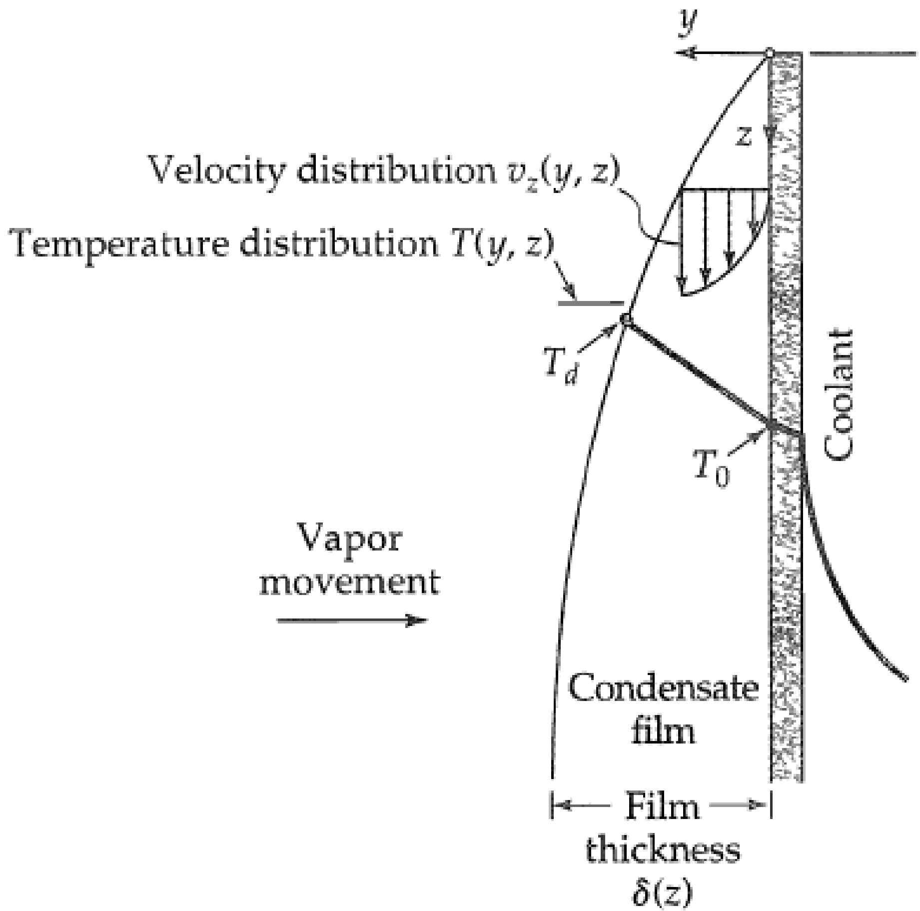
$$\mathrm{Nu}_m^{\mathrm{free}} = [(\mathrm{Nu}_m^{\mathrm{comb}})^m + (\mathrm{Nu}_m^{\mathrm{turb}})^m]^{1/m}$$

  - The values of m are heavily geometry-dependent. For the vertical isothermal flat plate, m=6

- Mixed Free and forced convection

$$\mathrm{Nu}_m^{\mathrm{total}} = [\mathrm{Nu}_m^{\mathrm{free}})^3 + (\mathrm{Nu}_m^{\mathrm{forced}})^3]^{1/3}$$

Heat transfer coefficients for condensation of pure vapours on solid surfaces



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Some relationships

- Heat transfer coefficient for condensation of a pure vapor on a solid surface of area A

$$Q = h_m A(T_d - T_0) = w\Delta\hat{H}_{vap}$$

  - T_d is the dew point of the vapour

- Laminar non-rippling condensate flow.
  - Film condensation on a horizontal tube

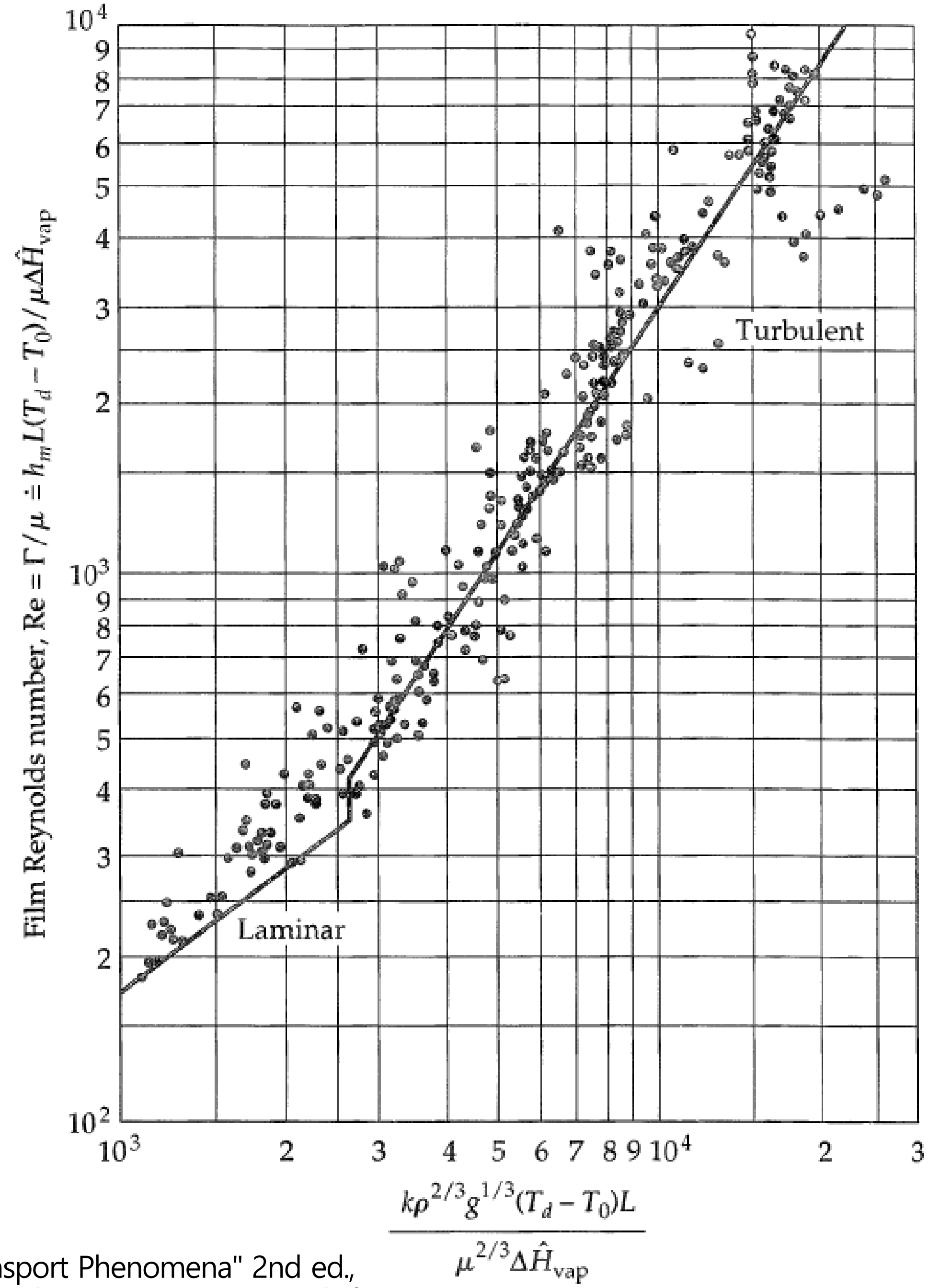
$$h_m = 0.954\left(\frac{k^3\rho^2 gL}{\mu w}\right)^{1/3}$$

  - Vertical tubes or vertical walls of height L

$$h_m = \frac{4}{3}\left(\frac{k^3\rho^2 g}{3\mu\Gamma}\right)^{1/3}$$

  - Γ is total rate of condensate flow from the bottom per unit width

Correlation of heat transfer data for film condensation of pure vapors on vertical surfaces



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