

# Optimum Design

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## Contents

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- ☑ **Ch. 1 Introduction to Optimum Design**
- ☑ **Ch. 2 Unconstrained Optimization Method: Gradient Method**
- ☑ **Ch. 3 Unconstrained Optimization Method: Enumerative Method**
- ☑ **Ch. 4 Constrained Optimization Method: Penalty Function Method**
- ☑ **Ch. 5 Constrained Optimization Method: LP (Linear Programming)**
- ☑ **Ch. 6 Constrained Optimization Method: SQP (Sequential Quadratic Programming)**
- ☑ **Ch. 7 Metaheuristic Optimization Method: Genetic Algorithms**
- ☑ **Ch. 8 Case Study of Optimal Dimension Design**
- ☑ **Ch. 9 Case Study of Optimal Route Design**
- ☑ **Ch. 10 Case Study of Optimal Layout Design**

# Ch. 1 Introduction to Optimum Design

- 1.1 Overview
- 1.2 Problem Statement of Optimum Design
- 1.3 Classification of Optimization Problems
- 1.4 Classification of Optimization Methods

## 1.1 Overview

## Indeterminate and Determinate Problems (1/2)

<p><b>Variables:</b> <math>x_1, x_2, x_3</math></p> <p><b>Equation:</b> <math>x_1 + x_2 + x_3 = 3</math></p> <ul style="list-style-type: none"> <li>✓ Number of variables: 3</li> <li>✓ Number of equations: 1</li> </ul> <p>Because the number of variables is larger than that of equations, this problem forms an <b>indeterminate system</b>.</p> <p><b>Solution for the indeterminate problem:</b></p> <p>We assume <u>two</u> unknown variables</p> <p style="margin-left: 20px;">↑</p> <p style="margin-left: 20px;">Number of variables (3) – Number of equations (1)</p> <p>Example) assume that <math>x_1 = 1, x_2 = 0</math></p> <p style="margin-left: 20px;">➔ <math>x_3 = 2</math></p>	<p><b>Equation of straight line</b></p> <p><math>y = a_0 + a_1x</math>    Where, <math>a_0, a_1</math> are given.</p> <ul style="list-style-type: none"> <li>✓ Number of variables: 2 <math>x, y</math></li> <li>✓ Number of equations: 1</li> </ul> <p>☞ We can get the value of <math>y</math> by assuming <math>x</math>.</p> <hr style="border-top: 1px dotted black;"/> <p><b>Finding intersection point (<math>x^*, y^*</math>) of two straight lines</b></p> <p><math>y = a_0 + a_1x</math>    Where, <math>a_0, a_1, b_0, b_1</math> are given.</p> <p><math>y = b_0 + b_1x</math></p> <ul style="list-style-type: none"> <li>✓ Number of variables: 2 <math>x, y</math></li> <li>✓ Number of equations: 2</li> </ul> <p>Because the number of variables is equals to that of equations, this problem forms an <b>determinate system</b>.</p>
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5


## Indeterminate and Determinate Problems (2/2)

<p style="text-align: center;"><b>Determinate problem</b></p> <p><b>Variables:</b> <math>x_1, x_2, x_3</math></p> <p><b>Equations:</b> <math>f_1(x_1, x_2, x_3) = 0</math></p> <p style="margin-left: 40px;"><math>f_2(x_1, x_2, x_3) = 0</math></p> <p style="margin-left: 40px;"><math>f_3(x_1, x_2, x_3) = 0</math></p> <p>If <math>f_1, f_2,</math> and <math>f_3</math> are <b>linearly independent</b>, then</p> <ul style="list-style-type: none"> <li>✓ Number of variables: 3</li> <li>✓ Number of equations: 3</li> </ul> <p>Since the number of equations is equal to that of variables, this problem can be solved.</p> <p>❓ What happens if <math>2 \times f_3 = f_2</math>?</p> <p>Then <math>f_2</math> and <math>f_3</math> are <b>linearly dependent</b>.</p> <p>Since the number of equations, which are linearly independent, is less than that of variables, this problem forms an <b>indeterminate system</b>.</p>	<p style="text-align: center;"><b>Indeterminate problem</b></p> <p><b>Variables:</b> <math>x_1, x_2, x_3</math></p> <p><b>Equations:</b> <math>f_1(x_1, x_2, x_3) = 0</math></p> <p style="margin-left: 40px;"><math>f_2(x_1, x_2, x_3) = 0</math></p> <p style="margin-left: 40px;"><math>f_3(x_1, x_2, x_3) = 0</math></p> <p>If <math>f_1</math> and <math>f_2</math> are only linearly independent, then</p> <ul style="list-style-type: none"> <li>✓ Number of variables: 3</li> <li>✓ Number of equations: 2</li> </ul> <p>Since the number of equations is less than that of variables, one equation should be added to solve this problem.</p> <table style="width: 100%; border-collapse: collapse; font-size: small;"> <tr> <td style="width: 33%;">Added Equation</td> <td style="width: 33%;">Solution</td> <td style="width: 33%;">We can obtain many sets of solutions by assuming different equations.</td> </tr> <tr> <td><math>f_1^1 = 0</math></td> <td><math>(x_1^1, x_2^1, x_3^1)</math></td> <td rowspan="3">◆ Indeterminate problem</td> </tr> <tr> <td><math>f_2^2 = 0</math></td> <td><math>(x_1^2, x_2^2, x_3^2)</math></td> </tr> <tr> <td style="text-align: center;">⋮</td> <td></td> </tr> </table> <div style="border: 1px solid black; padding: 5px; margin-top: 5px; font-size: small;"> <p>We need a certain criteria to determine the proper solution. By adding the criteria, this problem can be formulated as an <b>optimization problem</b>.</p> </div>	Added Equation	Solution	We can obtain many sets of solutions by assuming different equations.	$f_1^1 = 0$	$(x_1^1, x_2^1, x_3^1)$	◆ Indeterminate problem	$f_2^2 = 0$	$(x_1^2, x_2^2, x_3^2)$	⋮	
Added Equation	Solution	We can obtain many sets of solutions by assuming different equations.									
$f_1^1 = 0$	$(x_1^1, x_2^1, x_3^1)$	◆ Indeterminate problem									
$f_2^2 = 0$	$(x_1^2, x_2^2, x_3^2)$										
⋮											

## Example of a Design Problem

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**Esthetic\* Design of a Dress**



**Find (Design variables)**

- Size, material, color, etc.

**Constraints**

- There are some requirements, but it can be difficult to formulate them.
- By using the sense of a designer, the requirements are satisfied.

**Objective function (Criteria to determine the proper design variables)**

- There are many design alternatives.
- Among them, we should select the best one. How?
- Criteria: Preference, cost, etc.
- But it can be also difficult to formulate the objective function.

➔ Indeterminate, optimization problem

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## Mathematical Model for Determination of Optimal Principal Dimensions of a Ship - Summary ("Conceptual Ship Design Equation")

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<b>Find (Design variables)</b>	$L, B, D, C_B$ <small>length breadth depth block coefficient</small>	<b>Given (Owner's requirements)</b>	$DWT, CC_{req}, T_{max}(=T), V$ <small>deadweight Required cargo hold capacity maximum draft ship speed</small>
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**Physical constraint**

→ Displacement - Weight equilibrium (**Weight equation**) - Equality constraint

$$L \cdot B \cdot T \cdot C_B \cdot \rho_{sw} \cdot C_{\alpha} = DWT_{given} + LWT(L, B, D, C_B)$$

$$= DWT_{given} + C_s \cdot L^{1.6} (B + D) + C_o \cdot L \cdot B$$

$$+ C_{power} \cdot (L \cdot B \cdot T \cdot C_B)^{2/3} \cdot V^3 \dots(2.3)$$

**Economical constraints (Owner's requirements)**

→ Required cargo hold capacity (**Volume equation**) - Equality constraint

$$CC_{req} = C_{CH} \cdot L \cdot B \cdot D \dots(3.1)$$

- DFOC (Daily Fuel Oil Consumption)  
: It is related with the resistance and propulsion.

- Delivery date  
: It is related with the shipbuilding process.

Min. Roll Period : e.g.,

$$T_R \geq 12 \text{ sec} \dots\dots(6)$$

**Regulatory constraint**

→ Freeboard regulation (ICLL 1966) - Inequality constraint

$$D \geq T + C_{FB} \cdot D \dots(4)$$

**Stability regulation (MARPOL, SOLAS, ICLL)**

$$GM \geq GM_{Required} \dots(5)$$

or  $GZ \geq GZ_{Required}$

**Objective function (Criteria to determine the proper principal dimensions)**

Building Cost =  $C_{PS} \cdot C_s \cdot L^{1.6} (B + D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{power} \cdot (L \cdot B \cdot T \cdot C_B)^{2/3} \cdot V^3$

**4 variables (L, B, D, C<sub>B</sub>), 2 equality constraints ((2.3), (3.1)), 3 inequality constraints ((4), (5), (6))**

➔ Optimization problem

8

## Determination of Optimal Principal Dimensions of a Ship

### Optimization Problem

➔ Minimize/maximize an objective function with constraints on design variables

**Find (Design variables)**  
 $x_1, x_2, x_3, x_4$

**Equality constraint**  
 $h(x_1, x_2, x_3, x_4) = 0$

**Inequality constraint**  
 $g(x_1, x_2, x_3, x_4) \leq 0$

**Objective function**  
 $f(x_1, x_2, x_3, x_4)$

### Engineering Design of Ship (Simplified)

Find (Design variables)

$L(= x_1), B(= x_2), D(= x_3), C_B(= x_4)$   
length      breadth      depth      block coefficient

Weight equation

$$L \cdot B \cdot T \cdot C_B \cdot \rho_{sw} \cdot C_\alpha = DWT_{given} + LWT(L, B, D, C_B)$$

$$x_1 \cdot x_2 \cdot x_4 \cdot C_1 = C_2 + h'(x_1, x_2, x_3, x_4)$$

$$x_1 \cdot x_2 \cdot x_4 \cdot C_1 - C_2 - h'(x_1, x_2, x_3, x_4) = h(x_1, x_2, x_3, x_4) = 0$$

Objective function (Criteria to determine the proper principal dimensions)

$$Building\ Cost = C_{PS} \cdot C_s \cdot L^{1.6} (B + D) + C_{PO} \cdot C_o \cdot L \cdot B + C_{PM} \cdot C_{ma} \cdot NMCR$$

$$f(x_1, x_2, x_3, x_4) = C_3 \cdot x_1^{1.6} \cdot (x_2 + x_3) + C_4 \cdot x_1 \cdot x_2 + C_5$$

-  $T, C_\alpha, \rho_{sw}, DWT_{given}, C_{PS}, C_s, C_{PO}, C_o, C_{PM}, C_{ma}, NMCR$  are Given

**Characteristics of the constraints**

- ✓ **Physical constraints** are usually formulated as equality constraints.  
(Example of ship design: Weight equation)
- ✓ **Economical constraints, regulatory constraints, and constraints related with politics and culture** are formulated as inequality constraints.  
(Example of ship design: Required cargo hold capacity (Volume equation), Freeboard regulation (ICLL 1966))

## Classification of Optimization Problems and Methods - Summary

	Unconstrained optimization problem		Constrained optimization problem		
	Linear	Nonlinear	Linear	Nonlinear	
Objective function (example)	Minimize $f(x)$ $f(x) = x_1 + 2x_2$	Minimize $f(x)$ $f(x) = x_1^2 + x_2^2 - 3x_1x_2$	Minimize $f(x)$ $f(x) = x_1 + 2x_2$	Minimize $f(x)$ $f(x) = x_1^2 + x_2^2 - 3x_1x_2$	Minimize $f(x)$ $f(x) = x_1^2 + x_2^2 - 3x_1x_2$
Constraints (example)	None	None	$h(x) = x_1 + 5x_2 = 0$ $g(x) = -x_1 \leq 0$	$h(x) = x_1 + 5x_2 = 0$ $g(x) = -x_1 \leq 0$	$g_1(x) = \frac{1}{6}x_1^2 + \frac{1}{6}x_2^2 - 1.0 \leq 0$ $g_2(x) = -x_1 \leq 0$
Optimization methods for continuous value	① Gradient method - Steepest descent method - Conjugate gradient method - Newton method - Davidon-Fletcher-Powell (DFP) method - Broyden-Fletcher-Goldfarb-Shanno (BFGS) method  ② Enumerative method - Hooke & Jeeves method - Nelder & Mead method - Golden section search method		Linear Programming (LP) method is usually used.  Simplex Method (Linear Programming)	Penalty function method: Converting the constrained optimization problem to the unconstrained optimization problem by using the penalty function, the problem can be solved using unconstrained optimization method.  Quadratic programming (QP) method	Sequential Linear Programming (SLP) method First, linearize the nonlinear problem and then obtain the solution to this linear approximation problem using the linear programming method. And then, repeat the linearization.  Sequential Quadratic Programming (SQP) method First, approximate a quadratic objective function and linear constraints, find the search direction and then obtain the solution to this quadratic programming problem in this direction. And then, repeat the approximation.
Optimization methods for discrete value	Integer programming: ① Cut algorithm ② Enumeration algorithm ③ Constructive algorithm				
Metaheuristic optimization	Genetic algorithm (GA), Ant algorithm, Simulated annealing, etc.				

## 1.2 Problem Statement of Optimum Design

### General Formulation of an Optimization Problem

*Minimize*

$$f(\mathbf{x})$$

Objective Function

*Subject to*

$$g_j(\mathbf{x}) \leq 0, j = 1, \dots, m$$

: Inequality Constraint

$$h_k(\mathbf{x}) = 0, k = 1, \dots, p$$

: Equality Constraint

$$\mathbf{x}_l \leq \mathbf{x} \leq \mathbf{x}_u$$

: Upper and Lower Limits of Design Variables

*Where*

$$\mathbf{x} = (x_1, x_2, \dots, x_n)$$

Design Variables

## Components of an Optimization Problem (1/3)

### ☑ Design variable

- A set of variables that describe the system such as size and position, etc.
- It is also called 'Free variable' or 'Independent variable'.
- Cf. Dependent Variable
  - : A variable that is dependent on the design variable (independent variable)

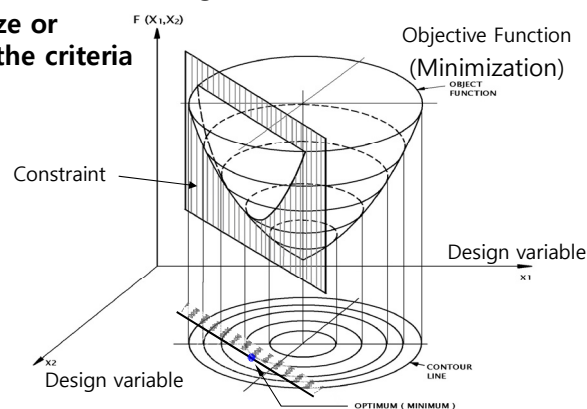
### ☑ Constraint

- A certain set of specified requirements and restrictions placed on a design
- It is a function of the design variables.
- Inequality Constraint (' $\leq$ ' or ' $\geq$ '), Equality Constraint ('=')

## Components of an Optimization Problem (2/3)

### ☑ Objective function

- A criteria to compare the different design and determine the proper design such as cost, profit, weight, etc.
- It is a function of the design variables.
- To minimize or maximize the criteria



## Components of an Optimization Problem (3/3)

Determination of the optimal design considering the objective function (maximization) and constraints

(a) Unconstrained optimization, Unimodal case

(b) Unconstrained optimization, Multimodal case

(c) Constrained optimization, Unimodal case

(d) Constrained optimization, Multimodal case

15

## Example of the Formulation of an Optimization Problem

$\text{Minimize } f = -4x_1 - 5x_2$	$\text{Minimize } f(\mathbf{x})$	Objective Function
$\text{Subject to } \begin{aligned} -x_1 + x_2 &\leq 4 \\ x_1 + x_2 &\leq 6 \\ 5x_1 + x_2 &= 10 \\ 0 &\leq x_1, x_2 \end{aligned}$	$\text{Subject to } \begin{aligned} g_j(\mathbf{x}) &\leq 0, j = 1, \dots, m \\ h_k(\mathbf{x}) &= 0, k = 1, \dots, p \\ \mathbf{x}_l &\leq \mathbf{x} \leq \mathbf{x}_u \end{aligned}$	Constraints
	: Inequality constraint : Equality constraint : Constraint about limits of design variables	

16



## 1.3 Classification of Optimization Problems

### Classification of Optimization Problems (1/4)

#### Existence of constraints

##### ■ Unconstrained optimization problem

- Minimize the objective function  $f(x)$  without any constraints on the design variables  $x$ .

Minimize	$f(x)$
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##### ■ Constrained optimization problem

- Minimize the objective function  $f(x)$  with some constraints on the design variables  $x$ .

Minimize	$f(x)$
Subject to	$h(x)=0$
	$g(x)\leq 0$

## Classification of Optimization Problems (2/4)

### ☑ Number of objective functions

#### ■ Single-objective optimization problem

$$\begin{array}{ll} \text{Minimize} & f(x) \\ \text{Subject to} & h(x)=0 \\ & g(x)\leq 0 \end{array}$$

#### ■ Multi-objective optimization problem

- Weighting Method, Constraint Method, etc.

$$\begin{array}{ll} \text{Minimize} & f_1(x), f_2(x), f_3(x) \\ \text{Subject to} & h(x)=0 \\ & g(x)\leq 0 \end{array}$$

## Classification of Optimization Problems (3/4)

### ☑ Linearity of objective function and constraints

#### ■ Linear optimization problem

- The objective function ( $f(x)$ ) and constraints ( $h(x)$ ,  $g(x)$ ) are linear functions of the design variables  $x$ .

$$\begin{array}{ll} \text{Minimize} & f(\mathbf{x}) = x_1 + 2x_2 \\ \text{Subject to} & h(\mathbf{x}) = x_1 + 5x_2 = 0 \\ & g(\mathbf{x}) = -x_1 \leq 0 \end{array}$$

#### ■ Nonlinear optimization problem

- The objective function ( $f(x)$ ) or constraints ( $h(x)$ ,  $g(x)$ ) are nonlinear functions of the design variables  $x$ .

$$\begin{array}{ll} \text{Minimize} & f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2 \\ \text{Subject to} & h(\mathbf{x}) = x_1 + 5x_2 = 0 \\ & g(\mathbf{x}) = -x_1 \leq 0 \end{array}$$

$$\begin{array}{ll} \text{Minimize} & f(\mathbf{x}) = x_1^2 + x_2^2 - 3x_1x_2 \\ \text{Subject to} & g_1(\mathbf{x}) = \frac{1}{6}x_1^2 + \frac{1}{6}x_2^2 - 1.0 \leq 0 \\ & g_2(\mathbf{x}) = -x_1 \leq 0 \end{array}$$

## Classification of Optimization Problems (4/4)

### ☑ Type of design variables

#### ■ Continuous optimization problem

- Design variables are continuous in the optimization problem.

#### ■ Discrete optimization problem

- Design variables are discrete in the optimization problem.
- It is also called a 'combinatorial optimization problem'.
- Example) Integer programming problem

## 1.4 Classification of Optimization Methods

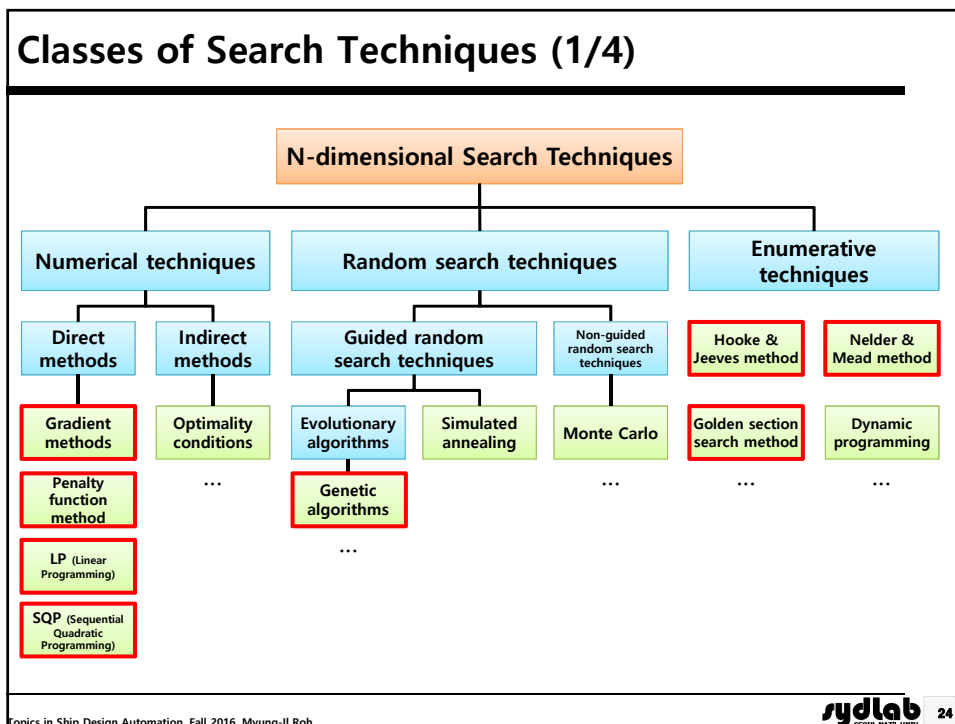
## Classification of Optimization Method

- ☑ Global Optimization Method
  - Advantage
    - It is useful for solving the global optimization problem which has many local optima.
  - Disadvantage
    - It needs many iterations (much time) to obtain the optimum.
  - Genetic Algorithms (GA), Simulated Annealing, etc.
  
- ☑ Local Optimization Method
  - Advantage
    - It needs relatively few iterations (less time) to obtain the optimum.
  - Disadvantage
    - It is only able to find the local optimum which is near to the starting point.
  - Sequential Quadratic Programming (SQP), Method of Feasible Directions (MFD), Multi-Start Optimization Method, etc.

\* Local optimum: The best near or in narrow region  
 \* Global optimum: The best in whole region

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23



## Classes of Search Techniques (2/4)

- Numerical techniques (Classical or calculus based techniques)**
  - Use deterministic approach to find best solution.** That is, Use a set of necessary and sufficient conditions to be satisfied by the solutions of an optimization problem.
  - Require knowledge of gradients or higher order derivatives.**
  - Indirect methods**
    - Search for local extremes by solving the usually nonlinear set of equations resulting from setting the gradient of the objective function to zero.
    - The search for possible solutions (function peaks) starts by restricting itself to points with zero slope in all directions.
    - **Methods using optimality conditions (e.g., Kuhn-Tucker condition)**
  - Direct methods**
    - Seek extremes by hopping around the search space and assessing the gradient of the new point, which guides the search.
    - This is simply the notion of hill climbing, which finds the best local point by climbing the steepest permissible gradient.
    - These techniques can be used only on a restricted set of well behaved functions.
    - **Gradient methods, Penalty function method, LP, SQP, etc.**

## Classes of Search Techniques (3/4)

- Guided random search techniques (Stochastic techniques)**
  - Based on enumerative, stochastic techniques but use additional information to guide the search.**
  - Two major subclasses are evolutionary algorithms and simulated annealing that both can be seen as evolutionary processes.**
  - Evolutionary algorithms**
    - Use **natural selection principles.**
    - This form of search evolves throughout generations, improving the features of potential solutions by means of biological inspired operations.
    - Genetic algorithms, Evolutionary Strategies (ES), etc.
  - Simulated annealing**
    - Uses a **thermodynamic evolution process** to search minimum energy states.

## Classes of Search Techniques (4/4)

### Enumerative techniques

- Search every point related to the function's domain space (finite or discretized), one point at a time.
- At each point, all possible solutions are generated and tested to find optimum.
- They are very simple to implement but usually require significant computation.
- These techniques are not suitable for applications with large domain spaces.
- Dynamic programming, Hooke and Jeeves method, Nelder and Mead method, golden section method, etc.