Rock Mechanics & Experiment
암석역학 및 실험

Lecture 7. Stress solutions for openings in rock
Lecture 7. 암반 공동 (보어홀) 주변의 응력식

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Stress distribution

- Elastic case
  1) Internal pressure in the circular hole
  2) Circular hole under uniaxial and biaxial boundary stress
  3) Elliptical openings
  4) Temperature change in the circular hole
  5) Internal pressure + boundary stress + temperature change
  6) Hollow cylinder with internal pressure + confining pressure
  7) Spherical Cavity
- Elastoplastic case
  1) Circular hole under hydrostatic boundary stress

Introduction
Stability issues in underground opening

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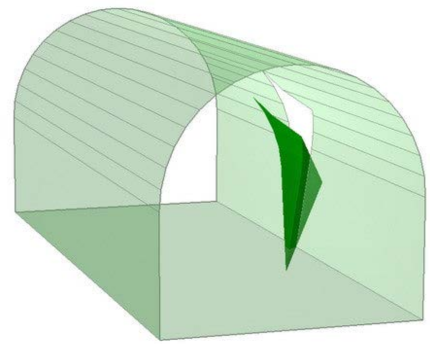
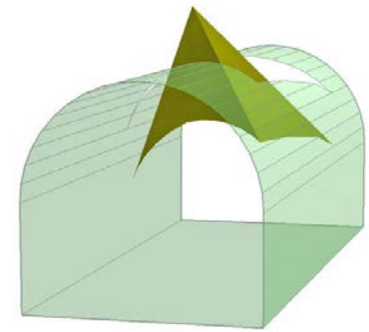
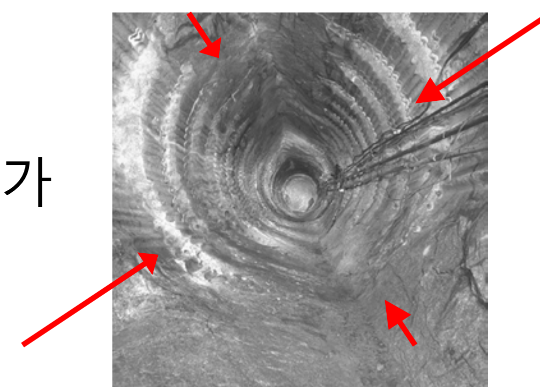
- Structurally-controlled instability (불연속면에 따른 불안정)
  - 절리등에 의해 불연속면이 형성되어 암반블록이 이탈하여 생기는 불안정성
  - 상대적으로 얕은 심도
- Stress-controlled instability (높은 응력에 따른 불안정)
  - 높은 응력에 의해 암반의 파괴가 발생하여 생기는 불안정성
  - 상대적으로 깊은 심도



Sliding wedge in sidewall



Falling wedge in the roof



V notched failure induced by high in situ stress, ~400 m deep, Winnipeg, Canada (Chandler, 2004)

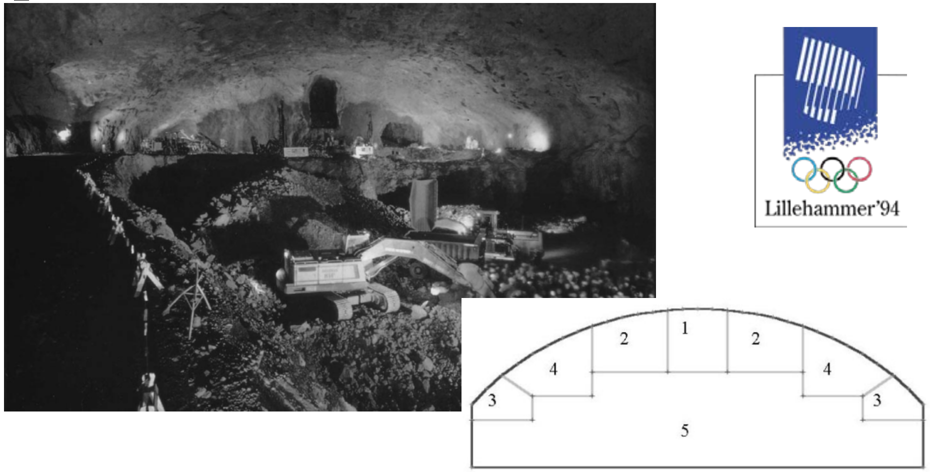
Fracturing at the tip of V-notch (Chandler, 2004)

Chandler, N., 2004, Developing tools for excavation design at Canada's Underground Research Laboratory, IJRMMS;41(8): 1229-1249.

Introduction

- Stress (and displacement) analysis important for underground openings (boreholes);
  - Planning the location
  - Dimensions
  - Shapes
  - Orientation
  - Selecting supports

Lillehammer (Norway), Gjøvik Olympic Mountain Hall
(62 m span, 25 m heigth, 91 m long, overburden 25-50 m, σ_H=3.5-4.5 MPa) (Scheldt et al. 2003)



- Road/Railway tunnel, hydroelectric power station, pressure headrace tunnel, oil storage cavern, mining eng, petroleum eng (borehole for reservoir), geothermal eng, geological repository for nuclear waste, …

Scheldt et al, 2003, Comparison of continuous and discontinuous modelling for computational rock mechanics, ISRM Congress, 1043-1048

Introduction
Stress in Cartesian, Polar & Cylindrical Coordinates

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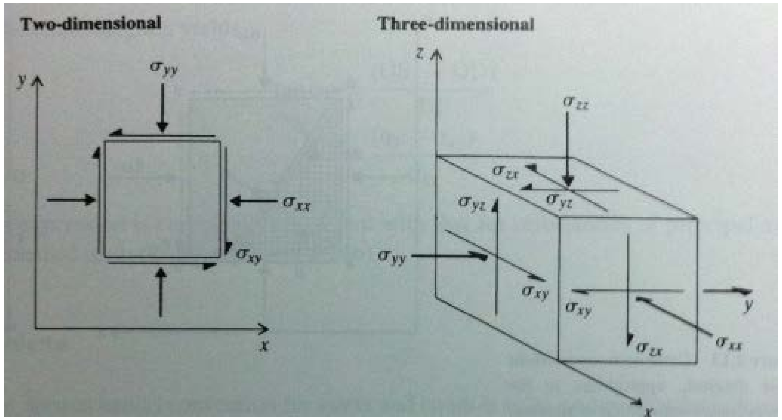
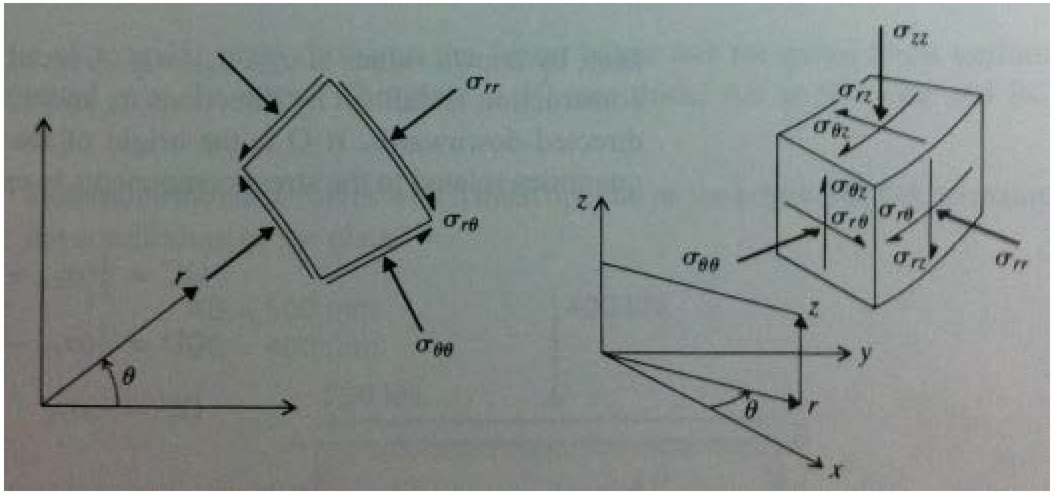
- 2D & 3D Cartesian Coordinates



- Polar & Cylindrical coordinates



Stress transformation from Cartesian to Cylindrical coordinates?

$$\begin{pmatrix} \sigma_r & \tau_{r\theta} \\ \tau_{r\theta} & \sigma_\theta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}^T$$

$$\sigma_r = \sigma_x \cos^2\theta + 2\tau_{xy}\sin\theta\cos\theta + \sigma_y \sin^2\theta$$

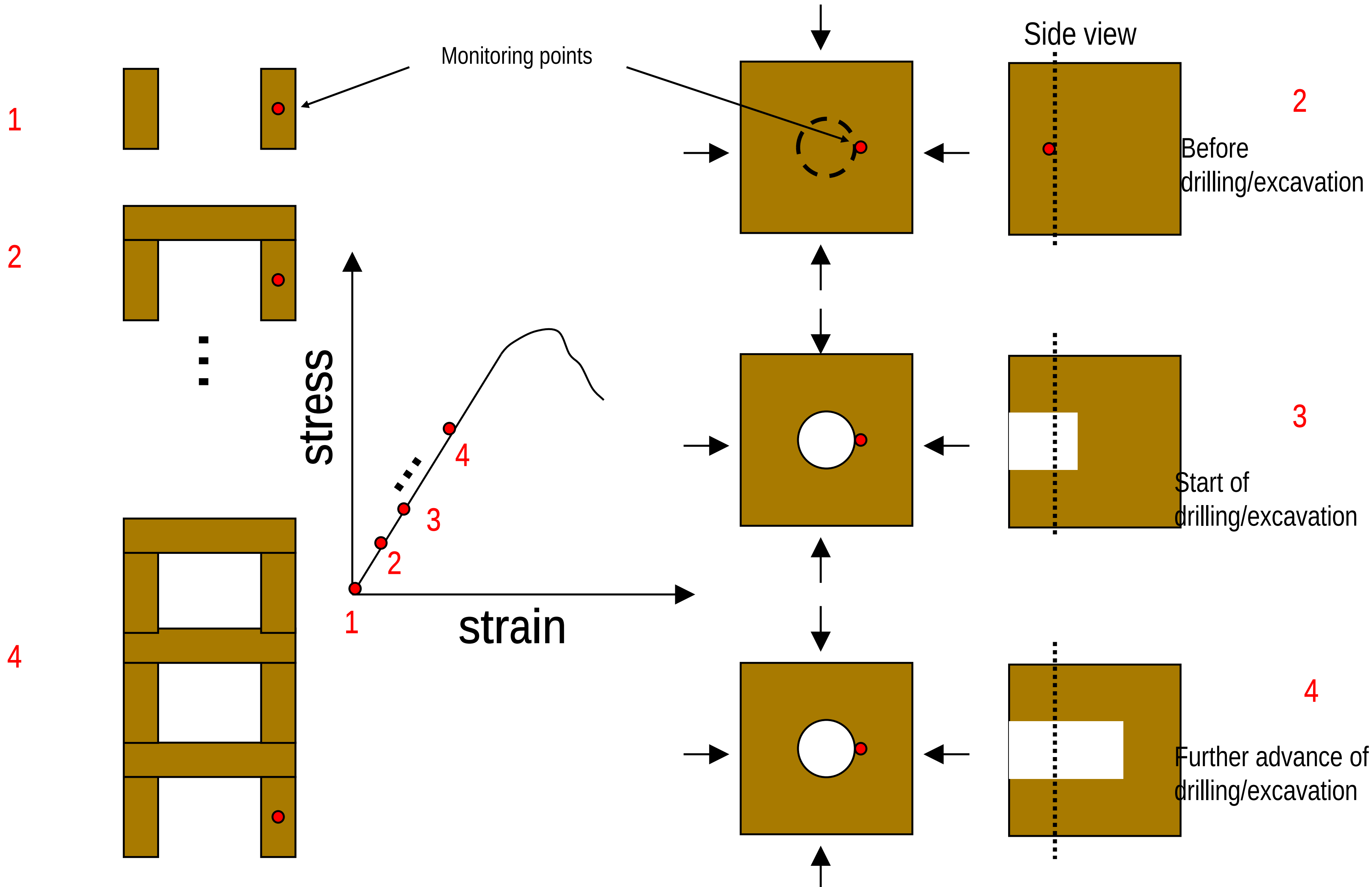
$$\sigma_\theta = \sigma_x \sin^2\theta - 2\tau_{xy}\sin\theta\cos\theta + \sigma_y \cos^2\theta$$

$$\tau_{r\theta} = \left(\sigma_y - \sigma_x\right)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

Nature of Underground Geomechanics

Civil structural problems:
Mechanics of "Addition"

Underground Geomechanics problems:
Mechanics of "Removal"



- Saint Venant's Principle
  - The stresses due to two statically equivalent loadings applied over a small area are significantly different only in the vicinity the area on which the loadings are applied, and at distances which are large in comparison with the linear dimensions of the area on which the loadings are applied, the effect due to these two loadings are the same
  - In other words, local change doesn't make a global change as long as the resultant forces are the same
  - Fundamentally, local stress tends to diffuses

Introduction
Saint Venant's Principle

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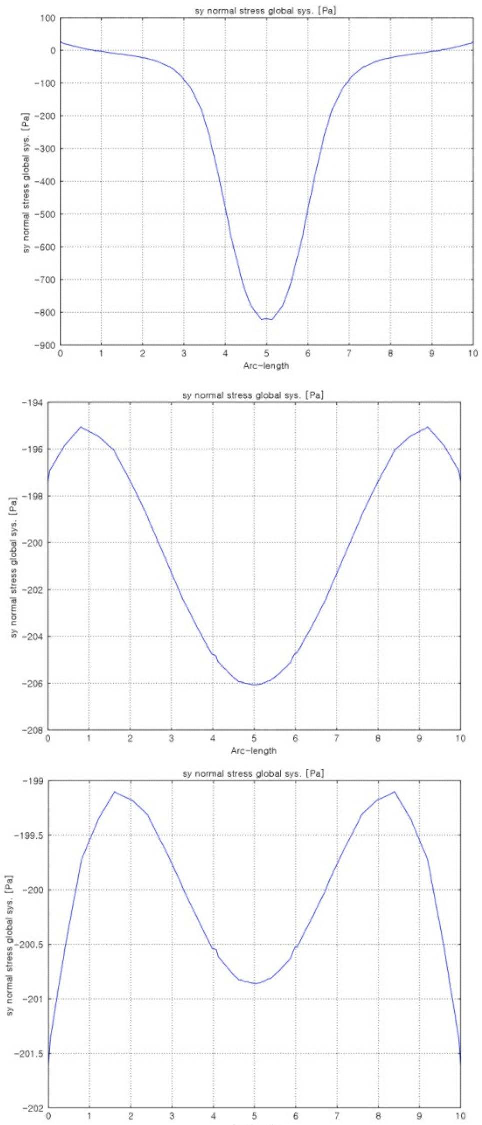
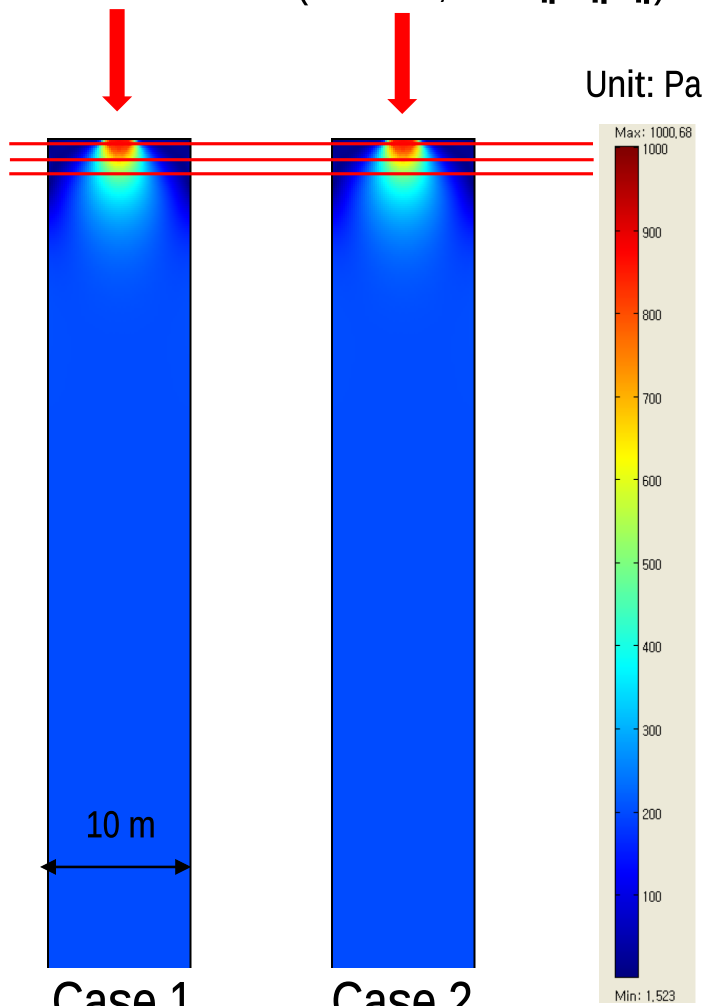
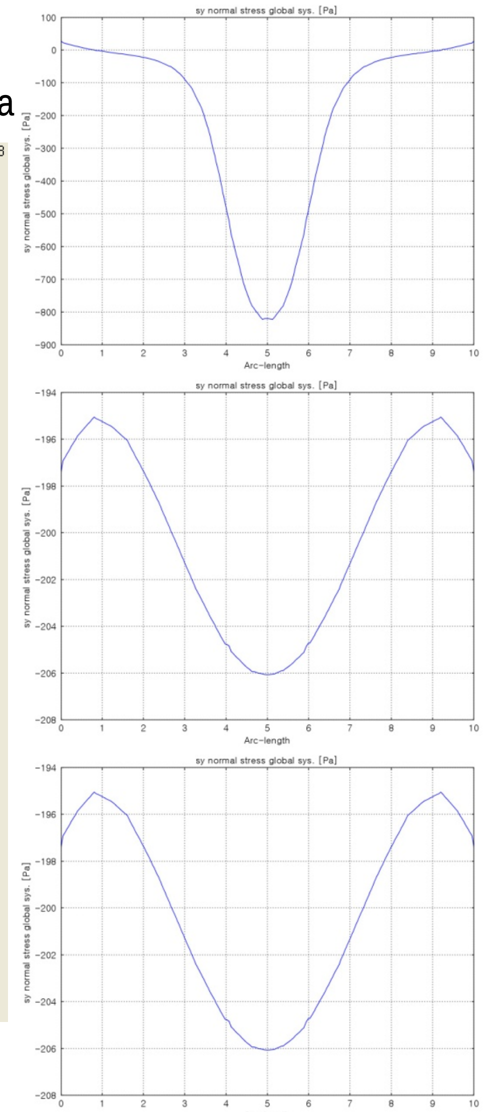
Concentrated Load (1000 Pa, 2 m에 대해)

Unit: Pa







Case 1
E=200 GPa

Case 2
E=2 MPa

σ_y distribution

Stresses distribution around a borehole
Stress equilibrium – Conservation of load

- Concept of “Stress Redistribution”

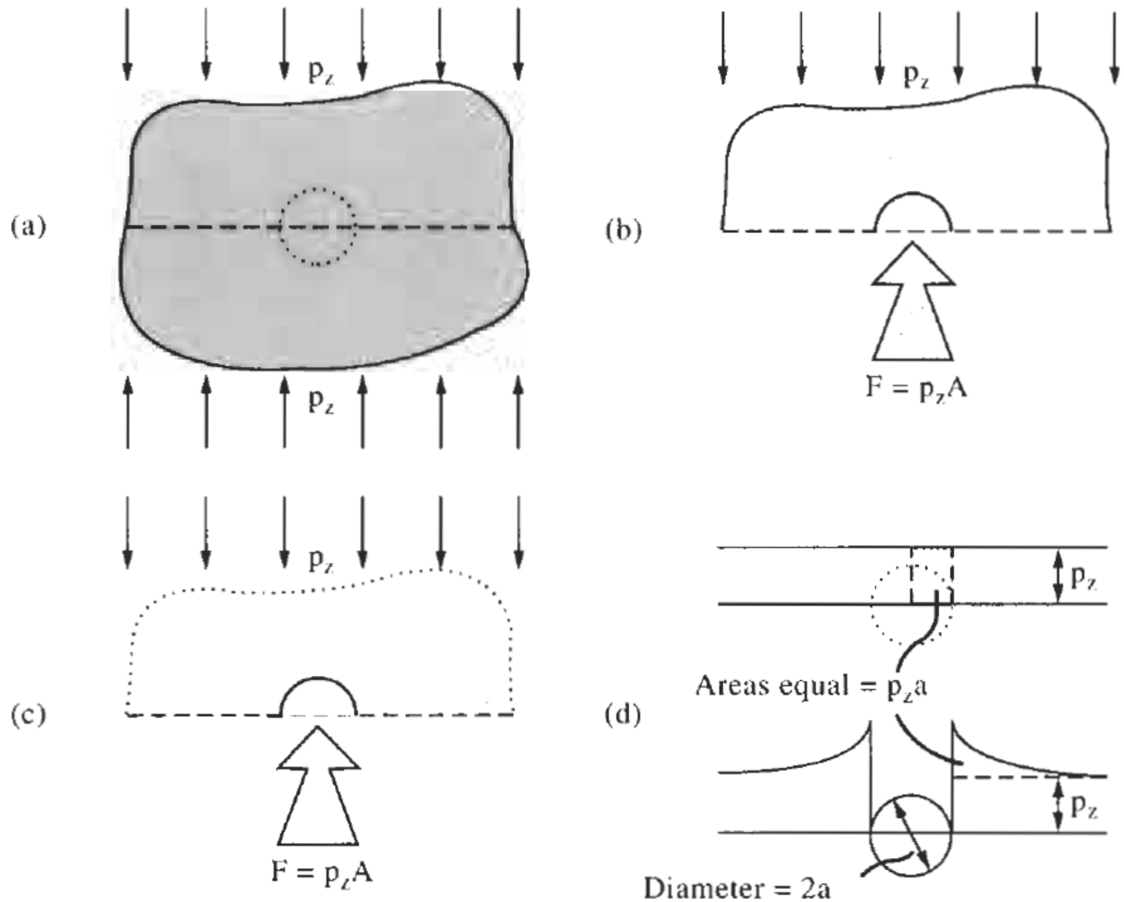


Figure 19.12 Principle of conservation of load before and after excavation.

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

Stresses distribution around a borehole
Requirements (2D)

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- Stress equilibrium Eq.

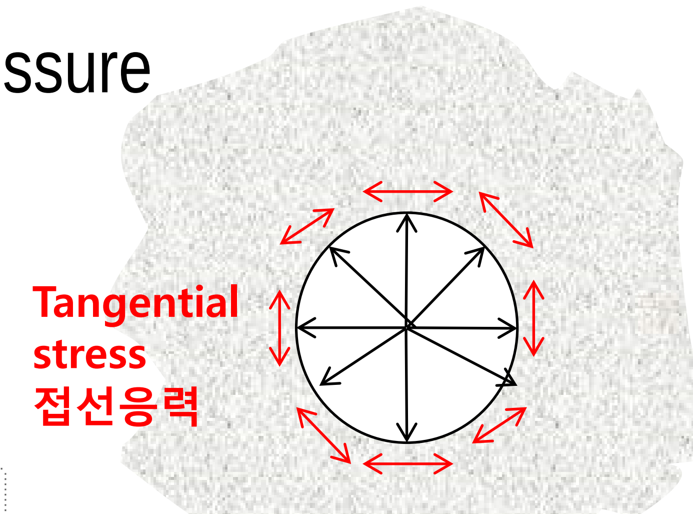
$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \rho b_x = 0$$

- Compatability Eq.

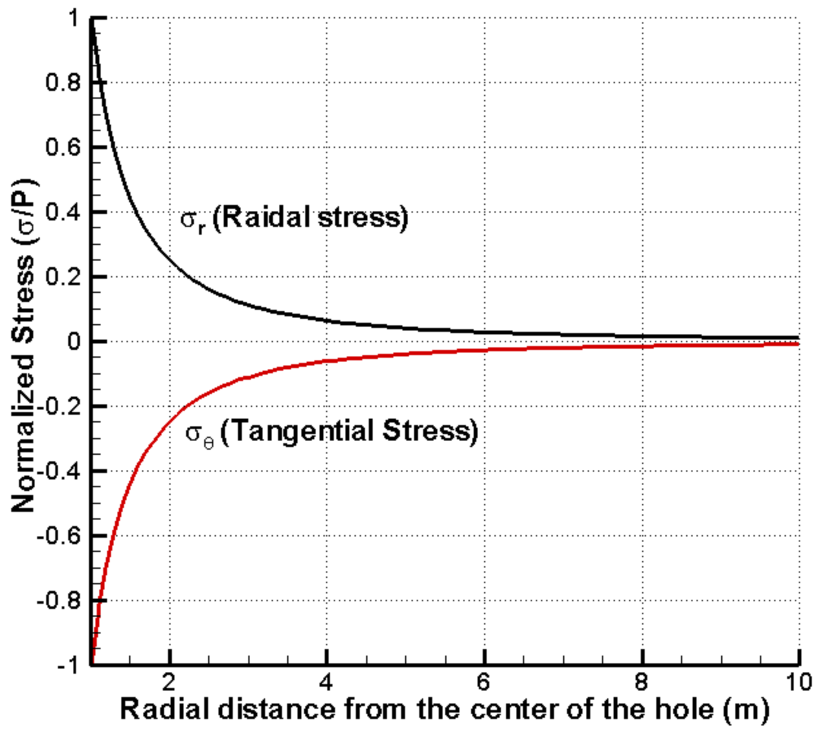
$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x \partial y}$$

- Boundary condition (inner hole, and/or far field (in situ) stress)
- Typically assume CHILE (Continuous, Homogeneous, Isotropic and Linearly-Elastic) materials
- DIANE (Discontinuous, Inhomogeneous, Anisotropic, and Non-Elastic) materials require more complex analysis

Borehole stability problem
1) internal pressure in the circular hole



- Increase of internal mud/hydraulic pressure
  - Water pressure
  - Mud pressure
  - Injection pressure



a:radius
r: distance from the center

$$\sigma_r = P_w \frac{a^2}{r^2}$$

$$\sigma_\theta = -P_w \frac{a^2}{r^2}$$

$$\tau_{r\theta} = 0$$

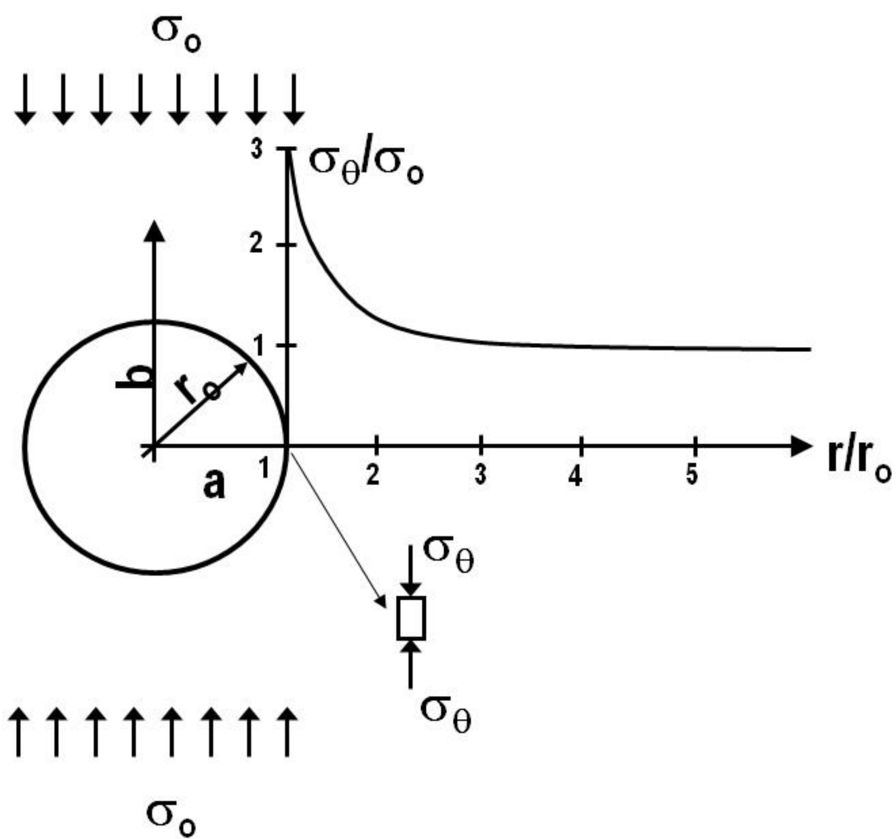
$$u_r = -\frac{P_w}{2G}\frac{a^2}{r}$$

$$v_\theta = 0$$

Stresses distribution around a borehole

2)Kirsch solution – circular hole under uniaxial/biaxial boundary stress

- Stress concentration factor due to uniaxial stress is ‘3’.
- Influence of borehole is within 2~3 times of radius



Stresses distribution around a borehole

2)Kirsch solution – circular hole under uniaxial/biaxial boundary stress

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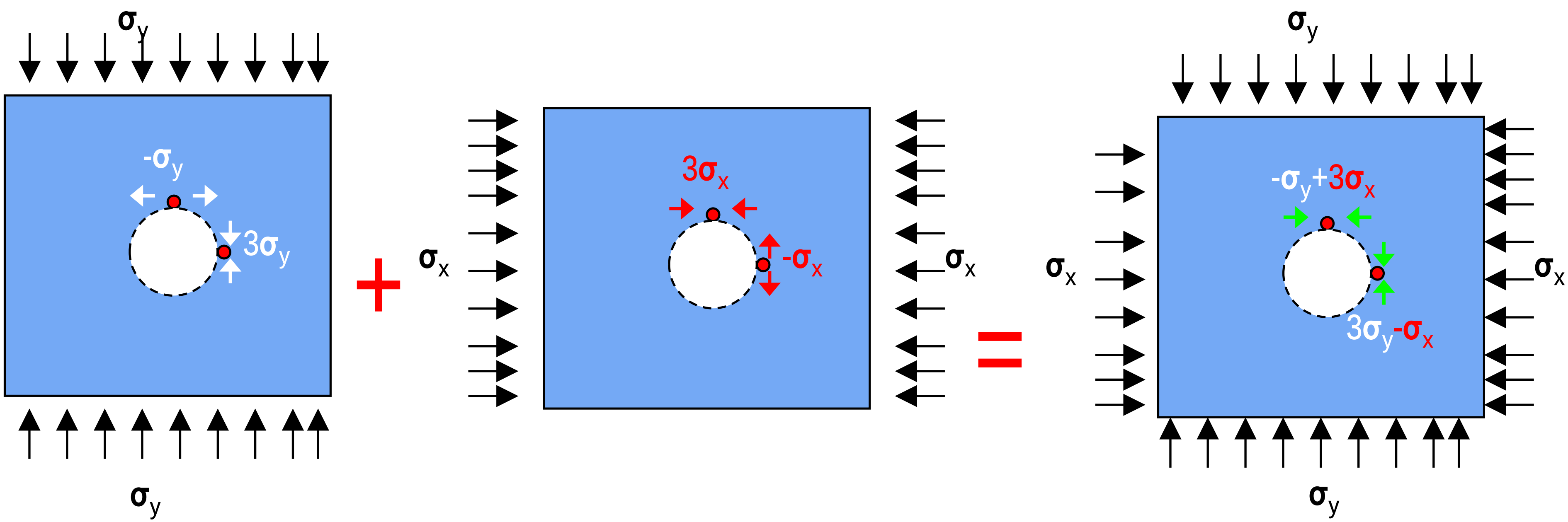

Under uniaxial stress condition

Maximum Stress concentration 3

Minimum Stress concentration -1

Biaxial Stress condition:

By superposition

Stresses distribution around a borehole

2)Kirsch solution – circular hole under uniaxial/biaxial boundary stress

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- Stress distribution around a circular borehole Kirsch (1898)
- Homogeneous rock under plane strain condition within elastic range

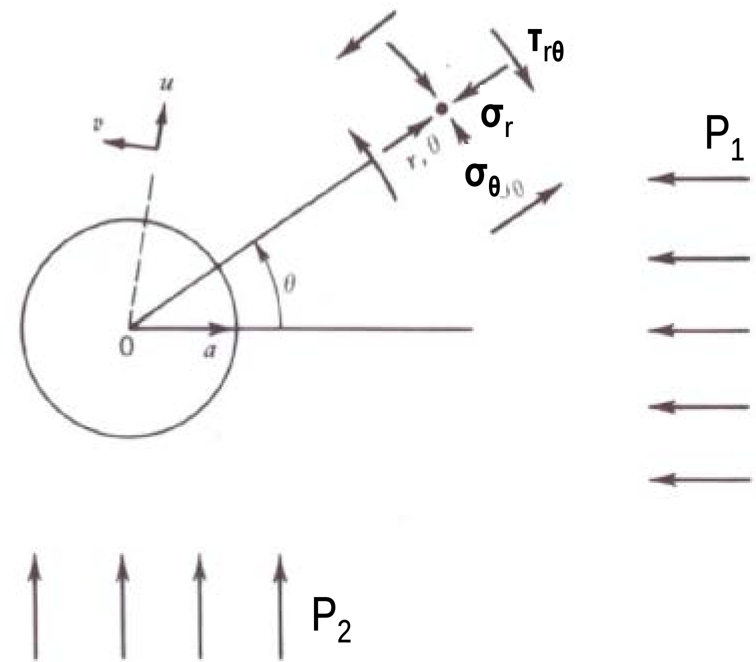
$$\sigma_r = \frac{P_1+P_2}{2}\left(1-\frac{a^2}{r^2}\right)+\frac{P_1-P_2}{2}\left(1-\frac{4a^2}{r^2}+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\sigma_\theta = \frac{P_1+P_2}{2}\left(1+\frac{a^2}{r^2}\right)-\frac{P_1-P_2}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

$$\tau_{r\theta} = \frac{P_1-P_2}{2}\left(1+\frac{2a^2}{r^2}-\frac{3a^4}{r^4}\right)\sin 2\theta$$

$$u_r = \frac{P_1+P_2}{4G}\frac{a^2}{r}+\frac{P_1-P_2}{4G}\frac{a^2}{r}\left(4(1-\nu)-\frac{a^2}{r^2}\right)\cos 2\theta$$

$$v_\theta = -\frac{P_1-P_2}{4G}\frac{a^2}{r}\left(2(1-2\nu)+\frac{a^2}{r^2}\right)\sin 2\theta$$



a: radius of a hole

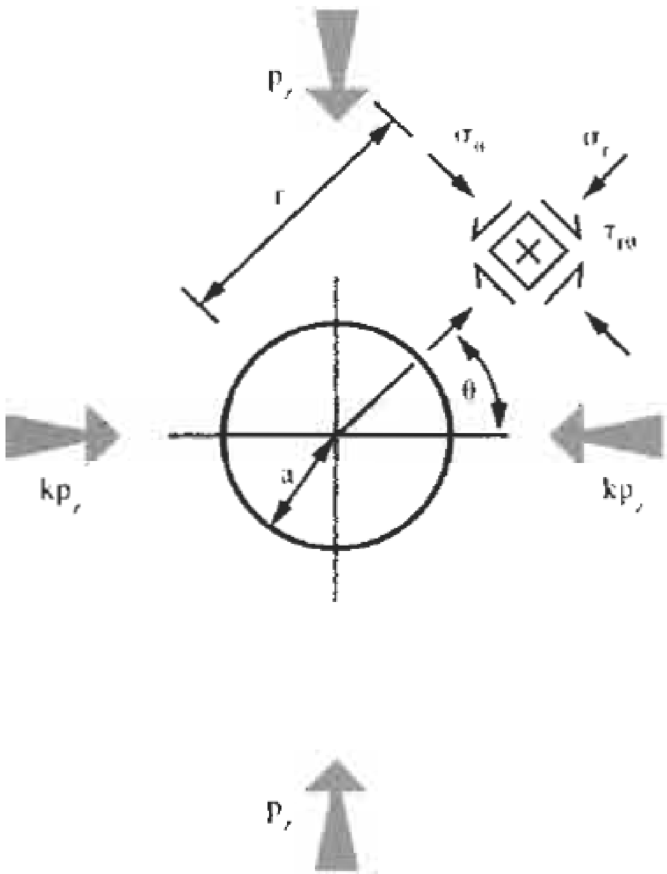
r: radial distance from the center of the hole

θ: measured from P_1

P_1 and P_2: boundary in situ stress

Stresses distribution around a borehole

2)Kirsch solution – circular hole under uniaxial/biaxial boundary stress



$$\sigma_r = \frac{1}{2}p_z\left\{(1+k)\left(1-\frac{a^2}{r^2}\right)-(1-k)\left(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\sigma_\theta = \frac{1}{2}p_z\left\{(1+k)\left(1+\frac{a^2}{r^2}\right)+(1-k)\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{1}{2}p_z\left\{(1-k)\left(1+2\frac{a^2}{r^2}-3\frac{a^4}{r^4}\right)\sin 2\theta\right\}$$

$$u_r = -\frac{p_z a}{4G}\left\{(1+k)-(1-k)\left(4(1-\nu)-\frac{a^2}{r^2}\right)\cos 2\theta\right\} \times \frac{a}{r}$$

$$u_\theta = -\frac{p_z a}{4G}\left\{(1-k)\left(2(1-2\nu)+\frac{a^2}{r^2}\right)\sin 2\theta\right\} \times \frac{a}{r}$$

Error in the textbook

Figure 19.10 Stresses and displacements induced around a circular excavation in plane strain (for a CHILE material).

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

- Some insights;
- Stresses distribution around a borehole is
  - independent of size of radius
  - independent of Elastic modulus of rocks
  - Poisson's ratio has some influence on vertical stress distribution (in vertical hole)

Stresses distribution around a borehole

2) Kirsch solution – at boundaries

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- At boundaries (by putting a=r)

$$\sigma_\theta = \frac{S_{H\max} + S_{h\min}}{2}\left(1+\frac{a^2}{r^2}\right) - \frac{S_{H\max} - S_{h\min}}{2}\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta$$

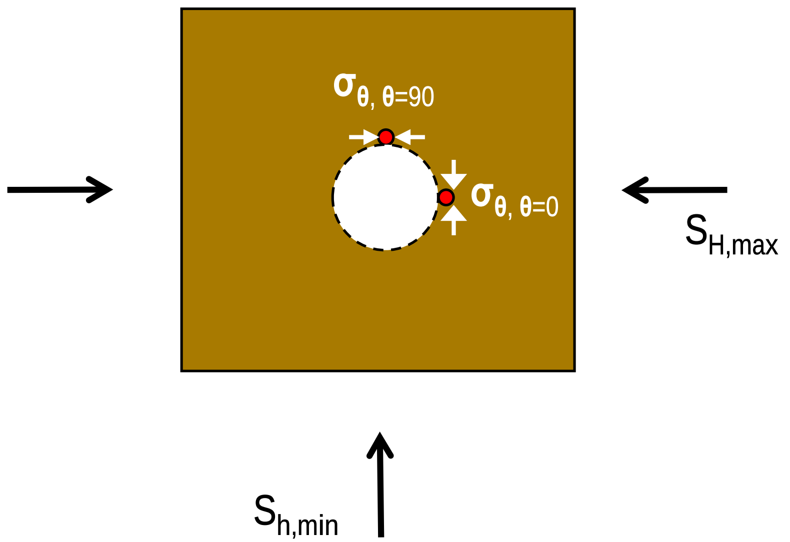
$$\sigma_r = 0$$

$$\boxed{\sigma_\theta = S_{H\max} + S_{h\min} - 2\left(S_{H\max} - S_{h\min}\right)\cos 2\theta}$$

$$\tau_{r\theta} = 0$$

$$\theta = 0, \quad \sigma_\theta = -S_{H\max} + 3S_{h\min}$$

$$\theta = 90, \quad \sigma_\theta = 3S_{H\max} - S_{h\min}$$



$$k = \frac{S_{H\max}}{S_{h\min}}$$

$$\sigma_\theta = P\left\{(1+k) + 2(1-k)\cos 2\theta\right\}$$

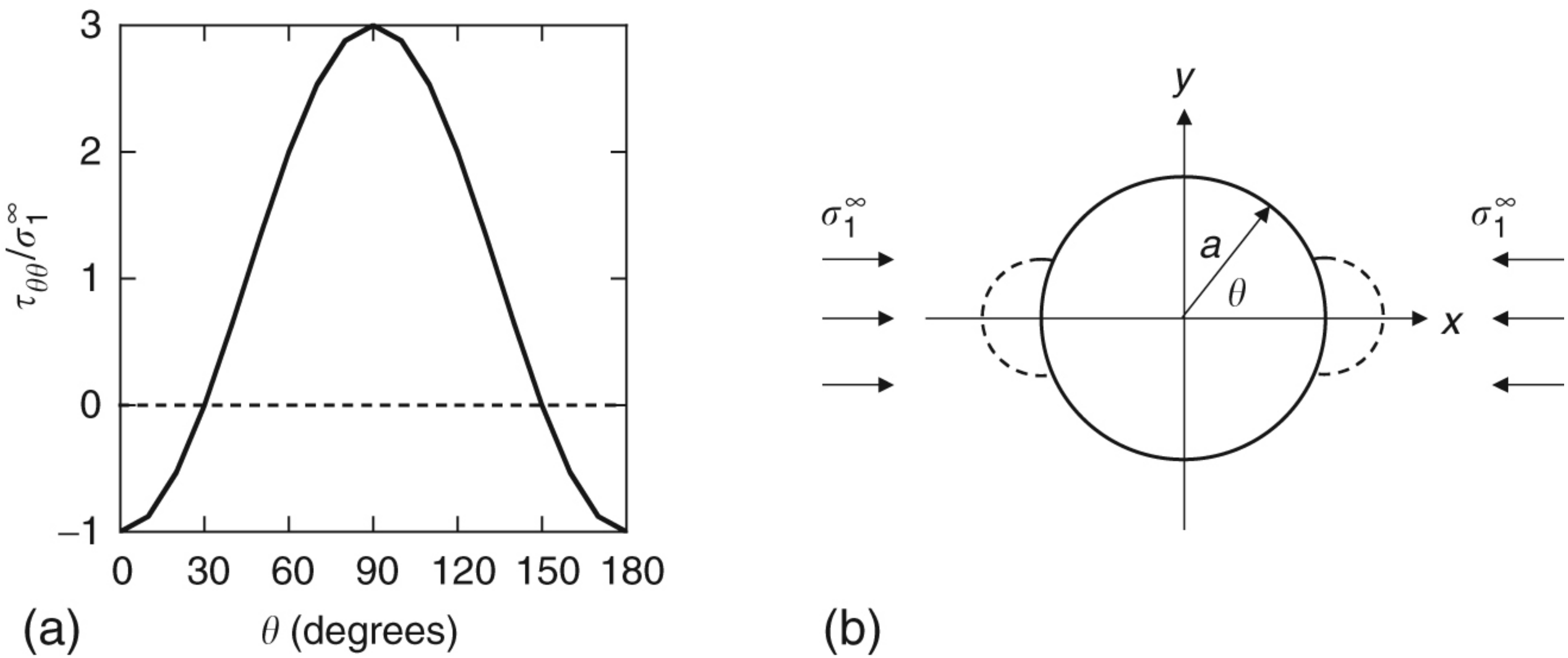
$$\theta = 0, \quad \sigma_\theta = P(-k+3)$$

$$\theta = 90, \quad \sigma_\theta = P(3k-1)$$

Stresses distribution around a borehole
2) Kirsch solution – at boundaries

- Tangential stress changes along the boundaries under uniaxial stress

$$\sigma_\theta = P\left(1 - 2\cos 2\theta\right)$$
$$P = \sigma_1^\infty$$

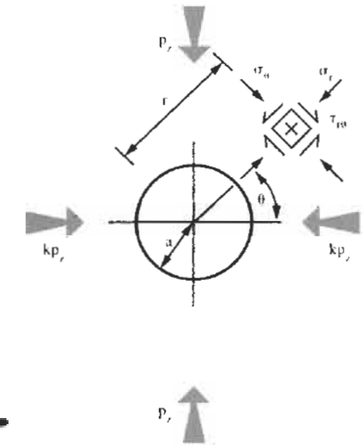


(a) (b)

Jaeger, Cook & Zimmerman, 2007, Fundamentals of Rock Mechanics, Blackwell Publishing

Stresses distribution around a borehole
2) Kirsch solution – effect of stress ratio, k

- Effect of stress ratio, k



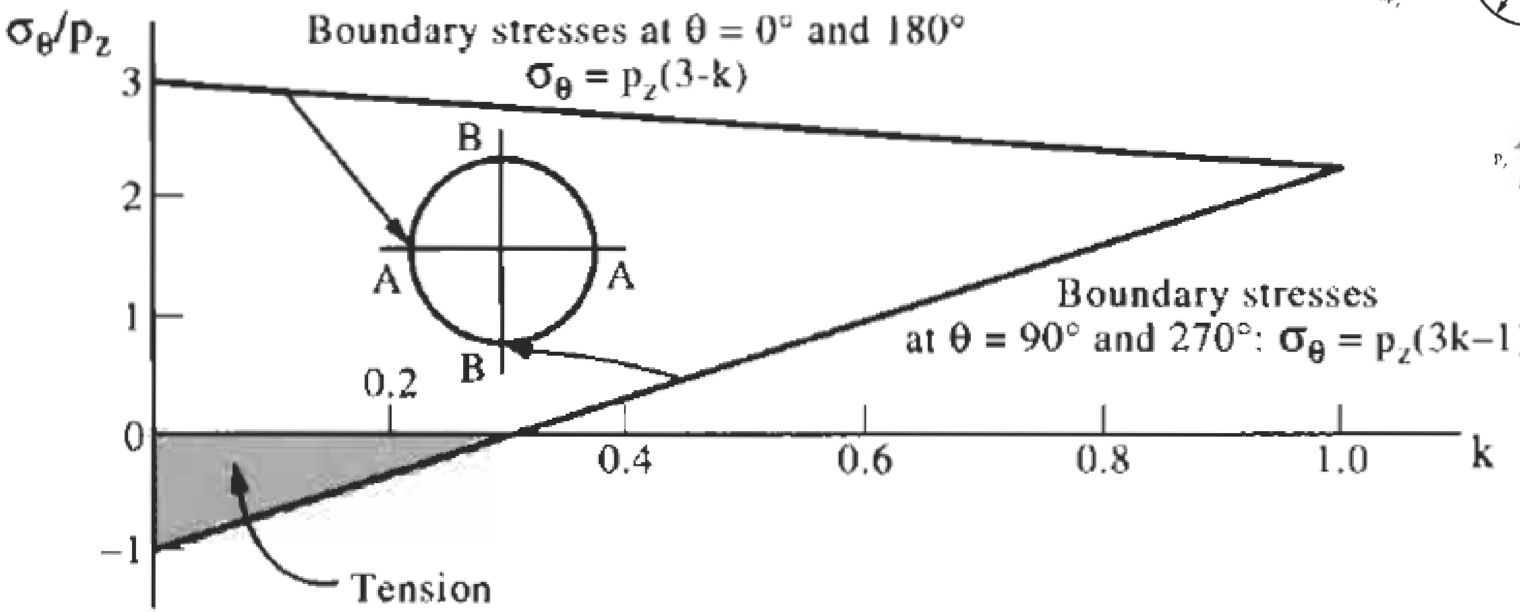


Figure 19.11 Stress concentration factors due to a circular opening.

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

Stresses distribution around a borehole
2) Kirsch solution – hydrostatic case (k=1)

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- Hydrostatic boundary stress (k=1)

$$\sigma_r = P\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = P\left(1 + \frac{a^2}{r^2}\right)$$

  – Stress concentration?

- Hydrostatic boundary stress (k=1) + internal pressure

$$\sigma_r = P\left(1 - \frac{a^2}{r^2}\right) + P_w \frac{a^2}{r^2}$$

$$\sigma_\theta = P\left(1 + \frac{a^2}{r^2}\right) - P_w \frac{a^2}{r^2}$$

  – If P=Pw?

Stresses distribution around a borehole
borehole breakout and hydraulic fracturing

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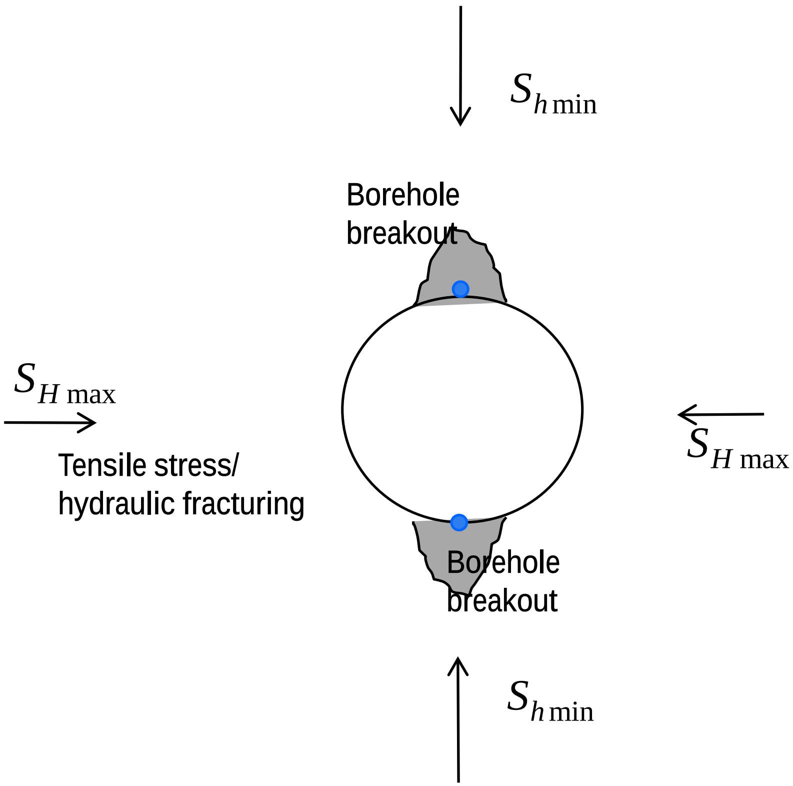
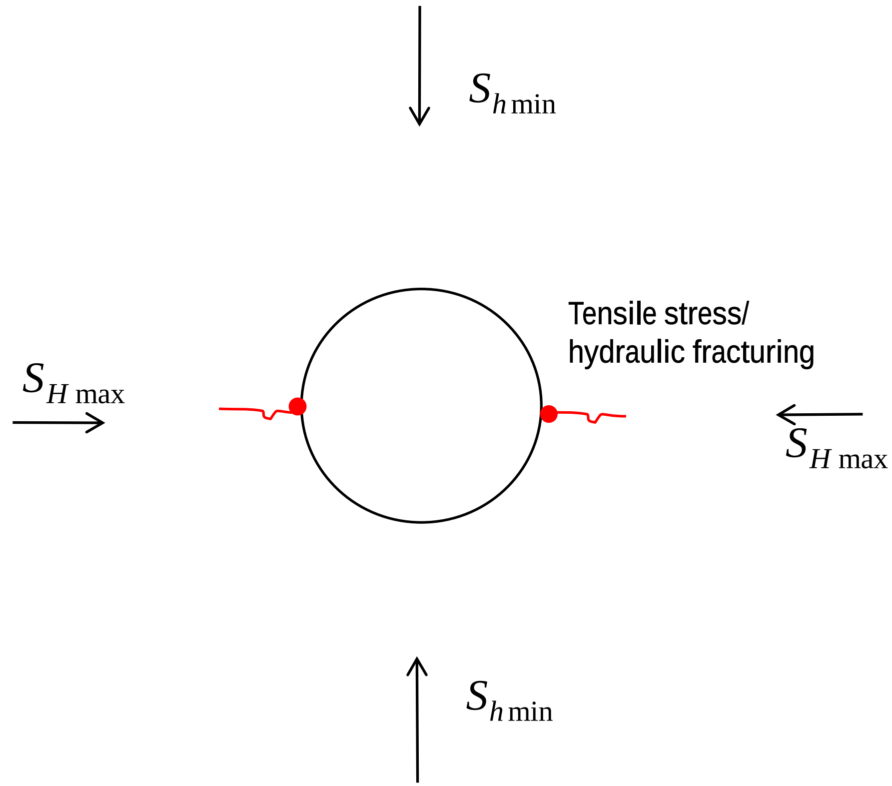
Borehole breakout



$$3S_{h\max} - S_{h\min} > \sigma_c$$

Hydraulic fracturing



$$P_w > 3S_{h\min} - S_{H\max} + T_0$$

Assumption: Impermeable reservoir & impermeable borehole wall

Stresses distribution around a borehole
borehole breakout

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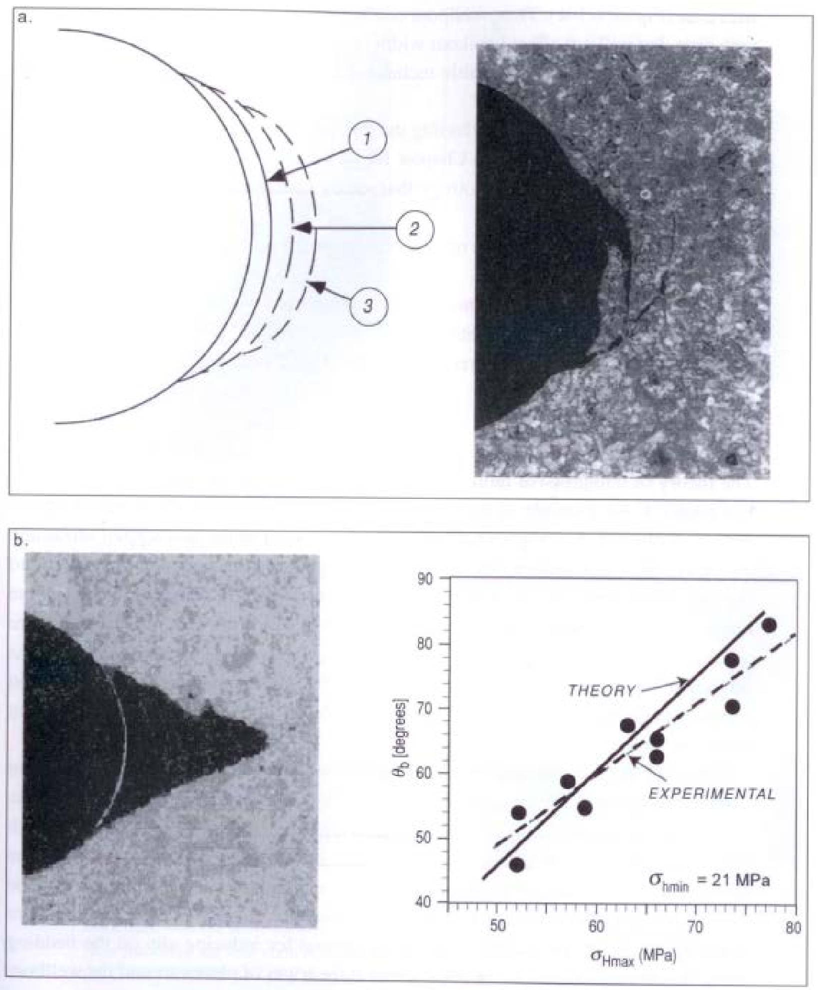
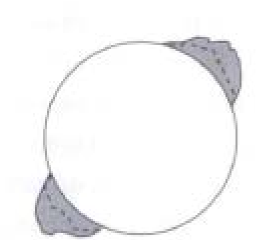


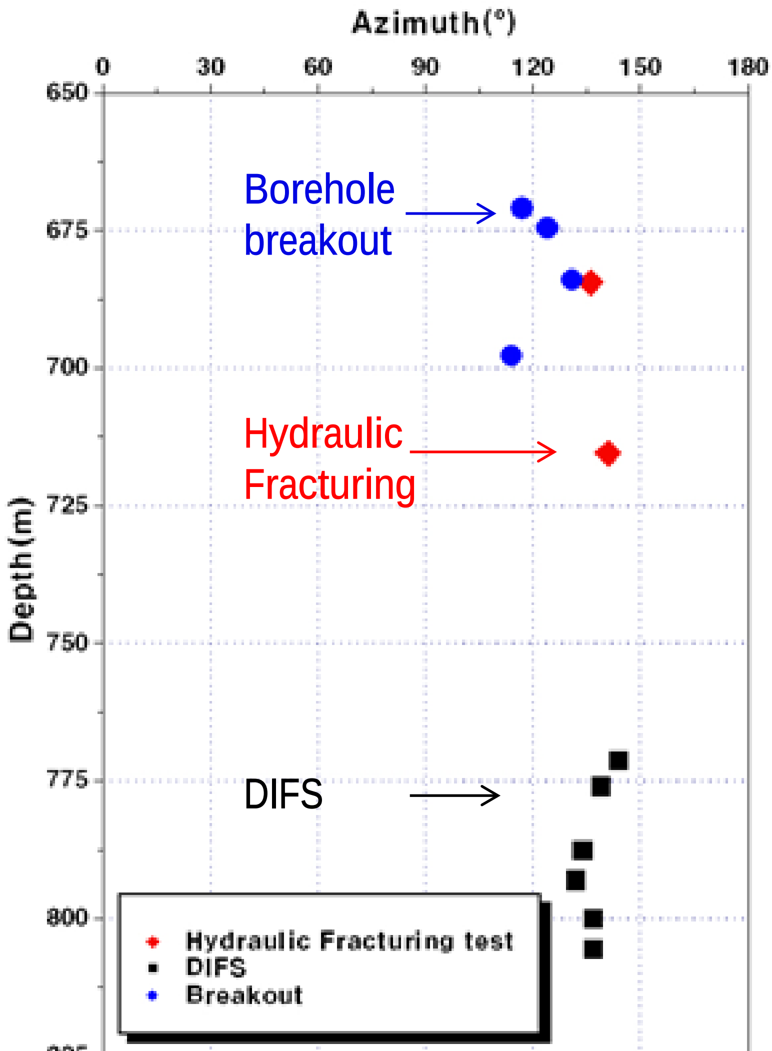
Figure 6.15. After the formation of wellbore breakouts, they are expected to increase in depth, but not width. This is as shown theoretically in (a) after Zoback, Moos *et al.* (1985) and confirmed by laboratory studies (Haimson and Herrick 1989). It can be seen photographically that breakouts in laboratory experiments deepen but do not widen after formation. A shown in (b), measured breakout widths compare very well with those predicted by the simple thoery presented in Zoback, Moos *et al.* (1985) which form the basic for the breakout shapes illustreated in Figures 6.2 and 6.3.

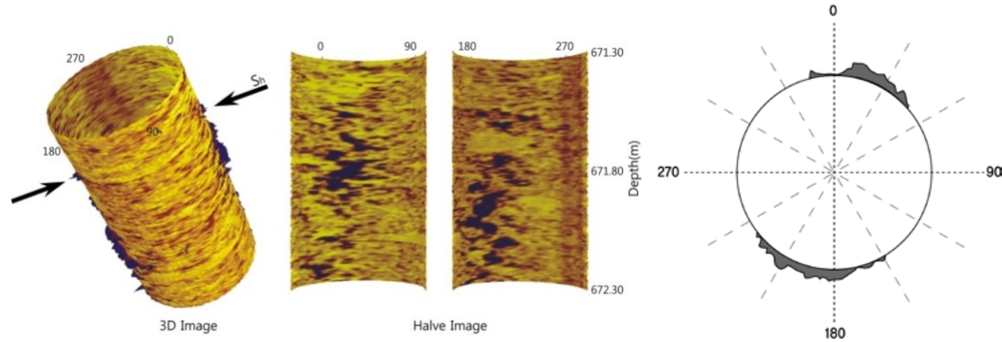
- wellbore enlargements caused by stress-induced failure of a well occurring 180 degree apart
- Induced by compressive (shear) failure
- Occur in the direction of minimum horizontal stress



Zoback, 2007, Reservoir Geomechanics, Cambridge Univ Press

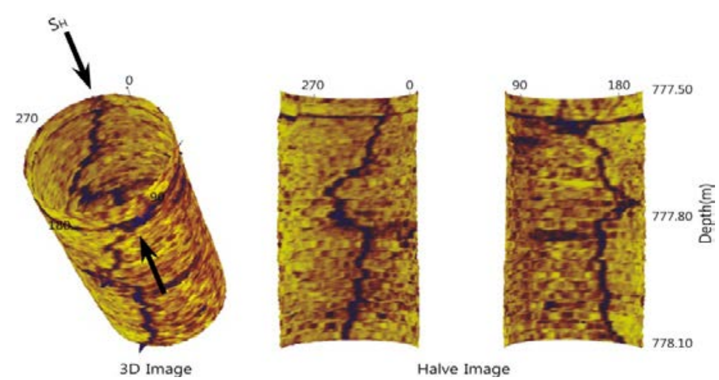
Stresses distribution around a borehole
borehole breakout (Pohang EGS site)





Borehole breakout (670 – 900 m)

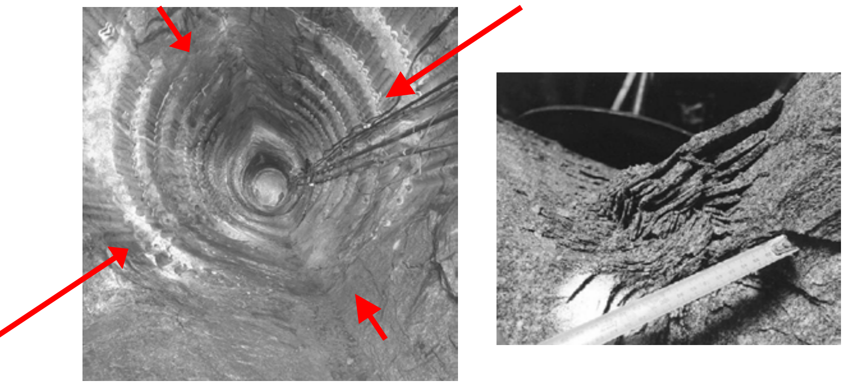
by borehole acoustic scanner



Drilling induced fractures (DIFS) (770 – 810 m)

Stresses distribution around a borehole
borehole breakout – rock spalling

- Similar observation can be found in underground construction

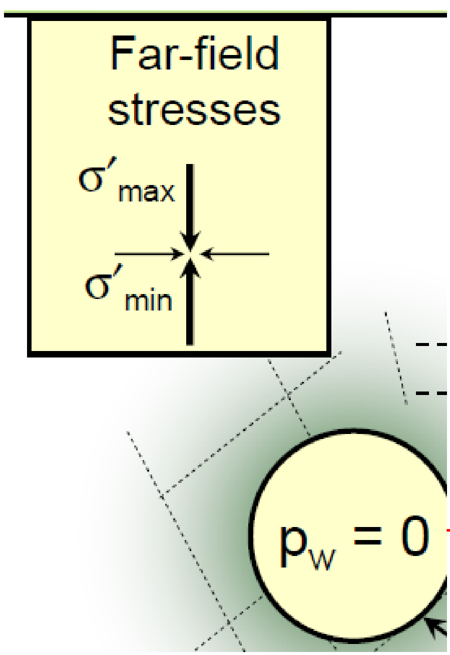


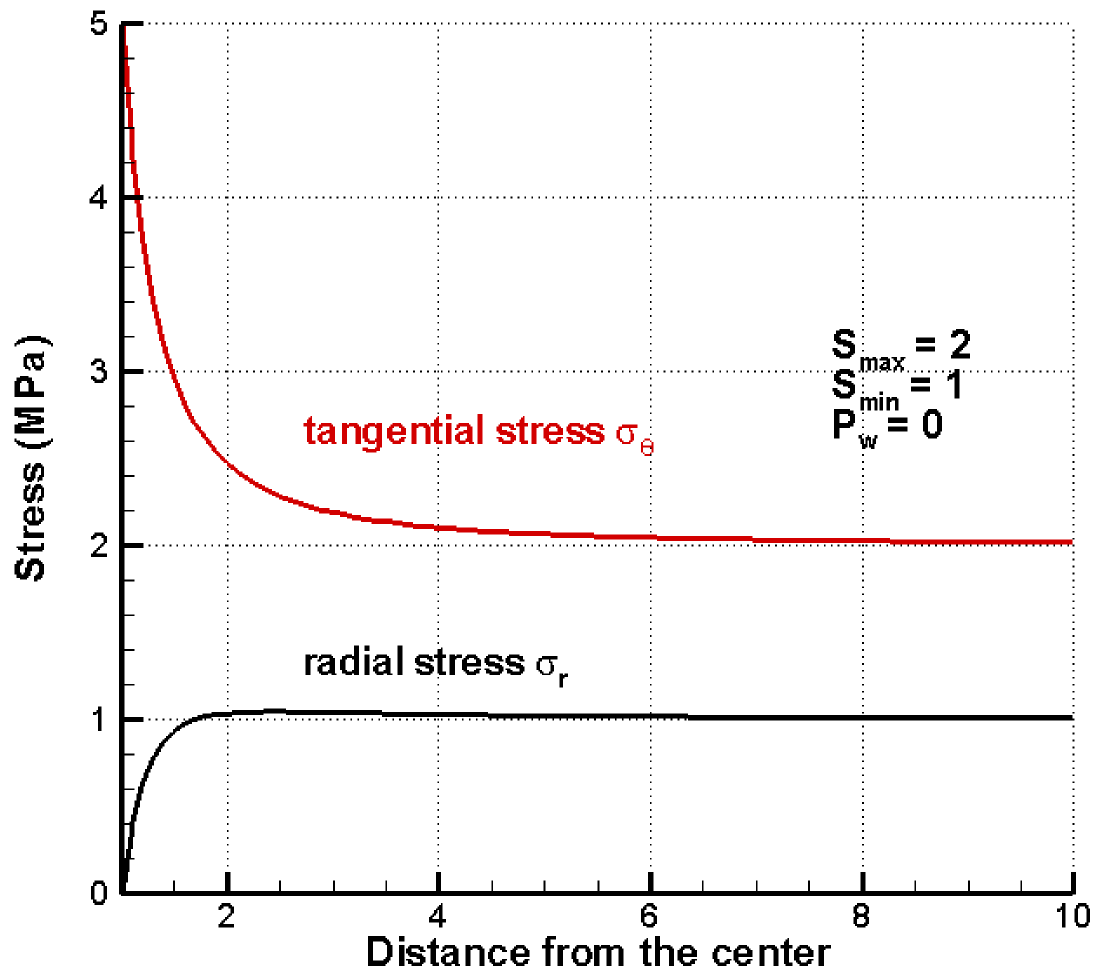
V notched failure due to high in situ stress
(400 m, Winnipeg, Canada, Chandler, 2004)

Winnipeg, Canada (Min, 2002)

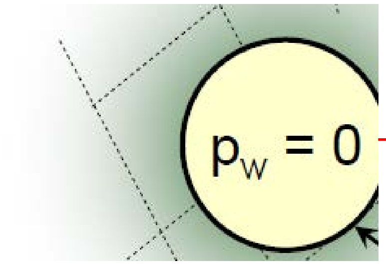
Chandler, N., 2004, Developing tools for excavation design at Canada's Underground Research Laboratory, IJRMMS;41(8): 1229-1249.

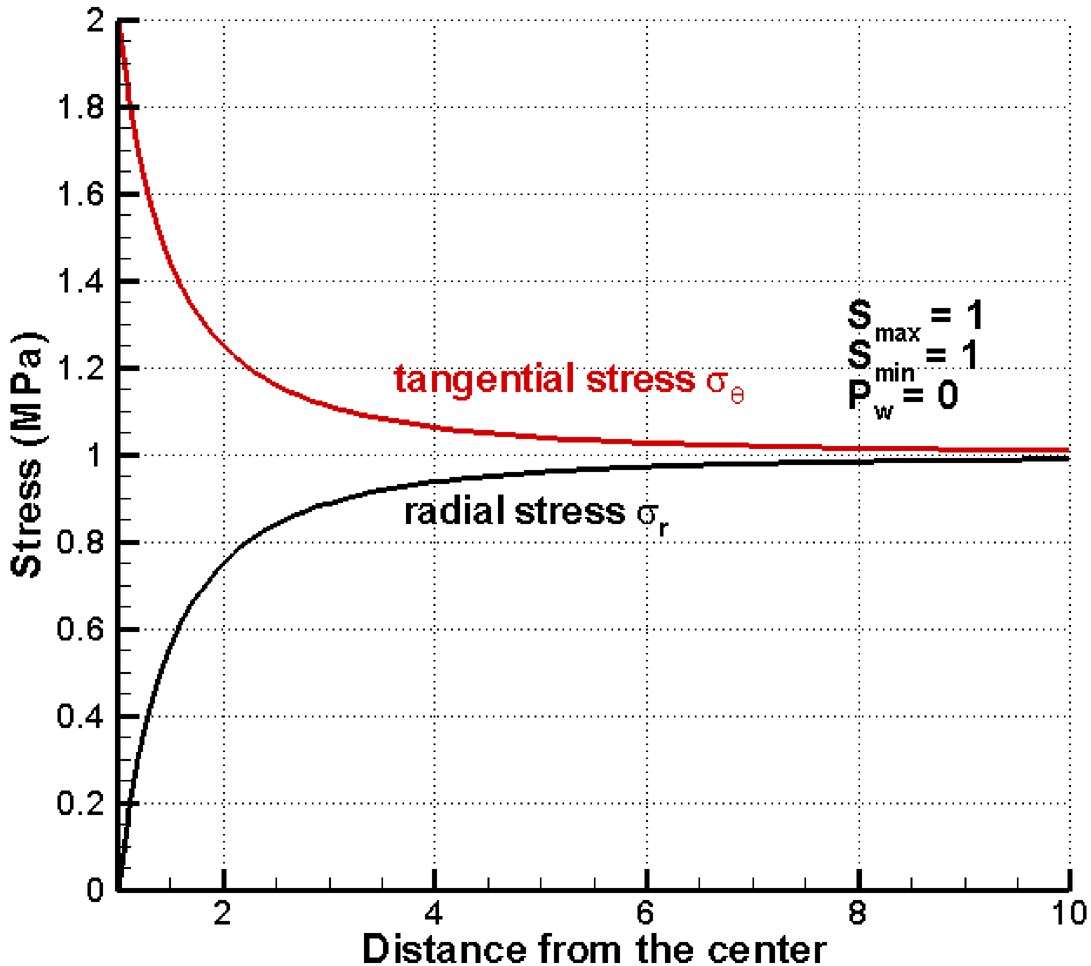
Stresses distribution around a borehole
Anisotropic boundary stress



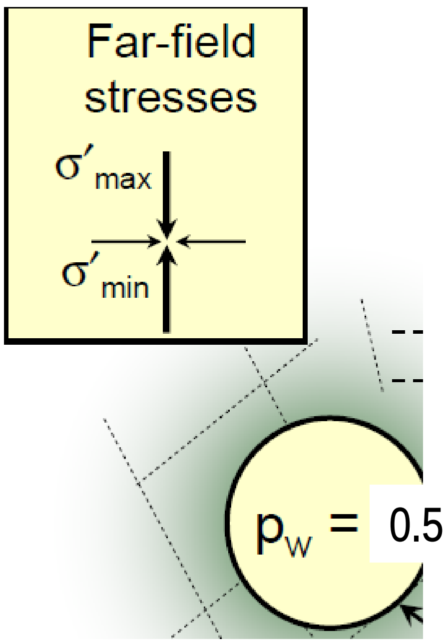


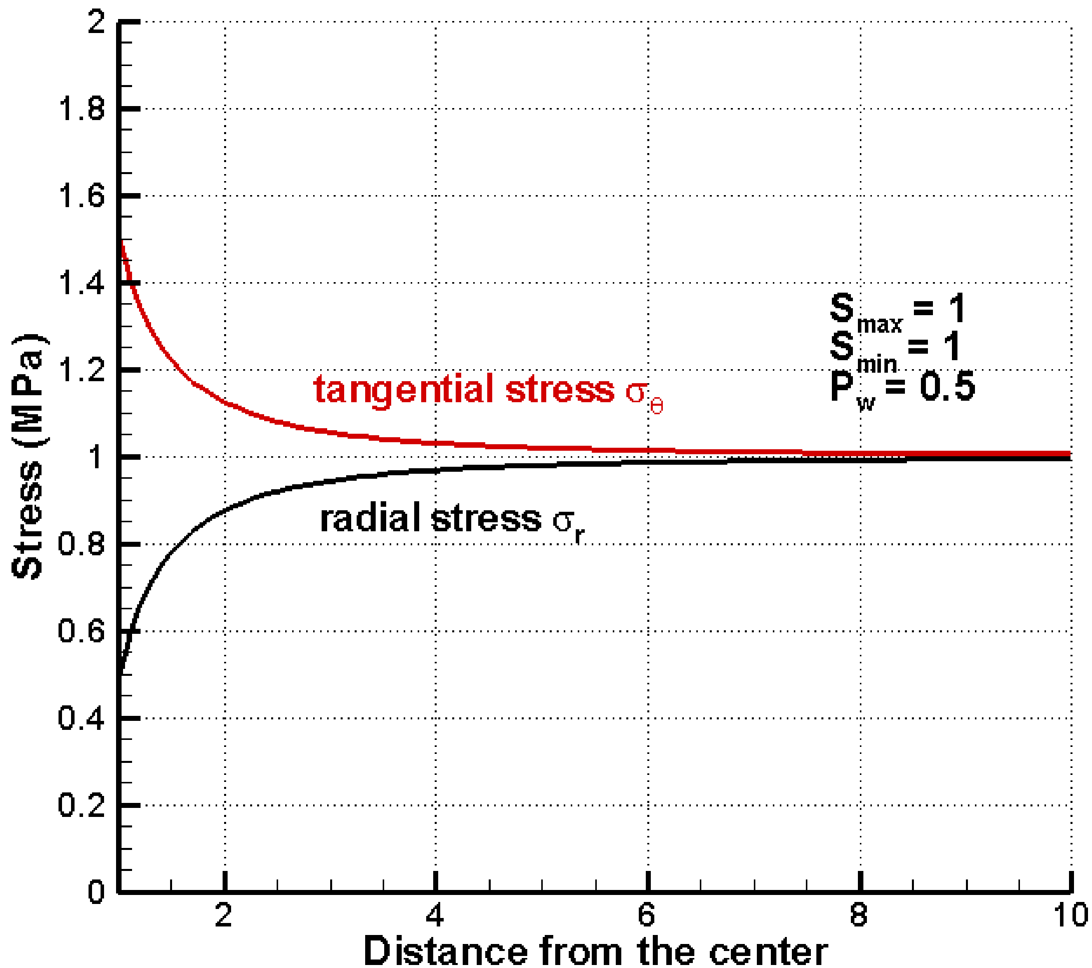
Stresses distribution around a borehole
Isotropic boundary stress



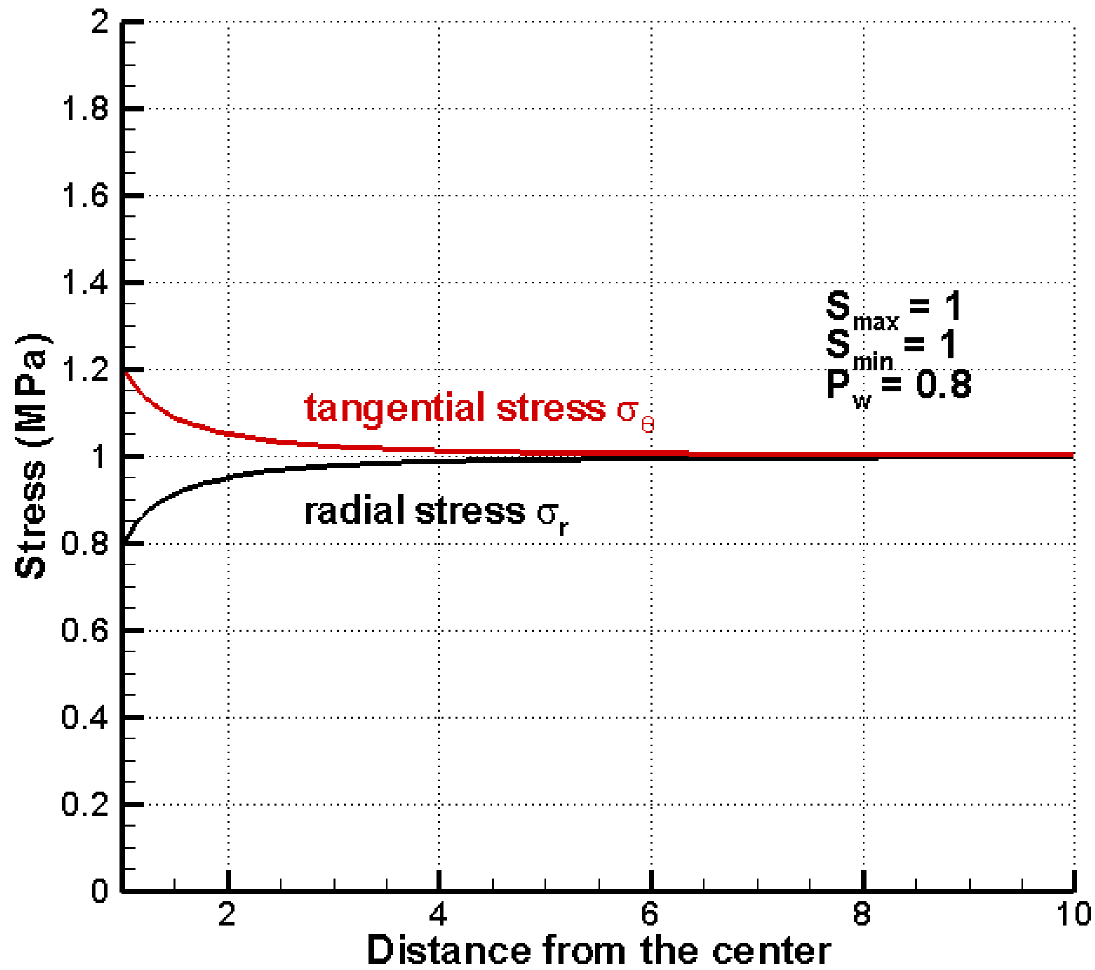


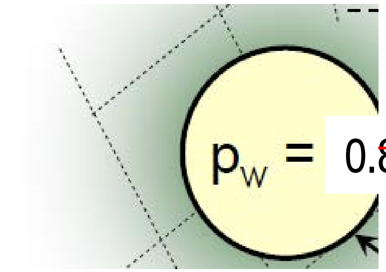
Stresses distribution around a borehole
Internal pressure from the opening/borehole





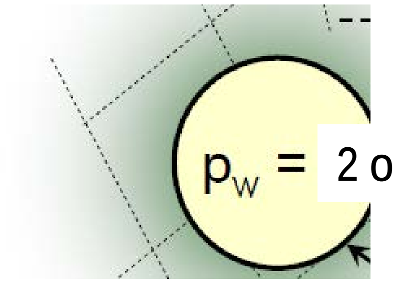
Stresses distribution around a borehole
Internal pressure from the opening/borehole

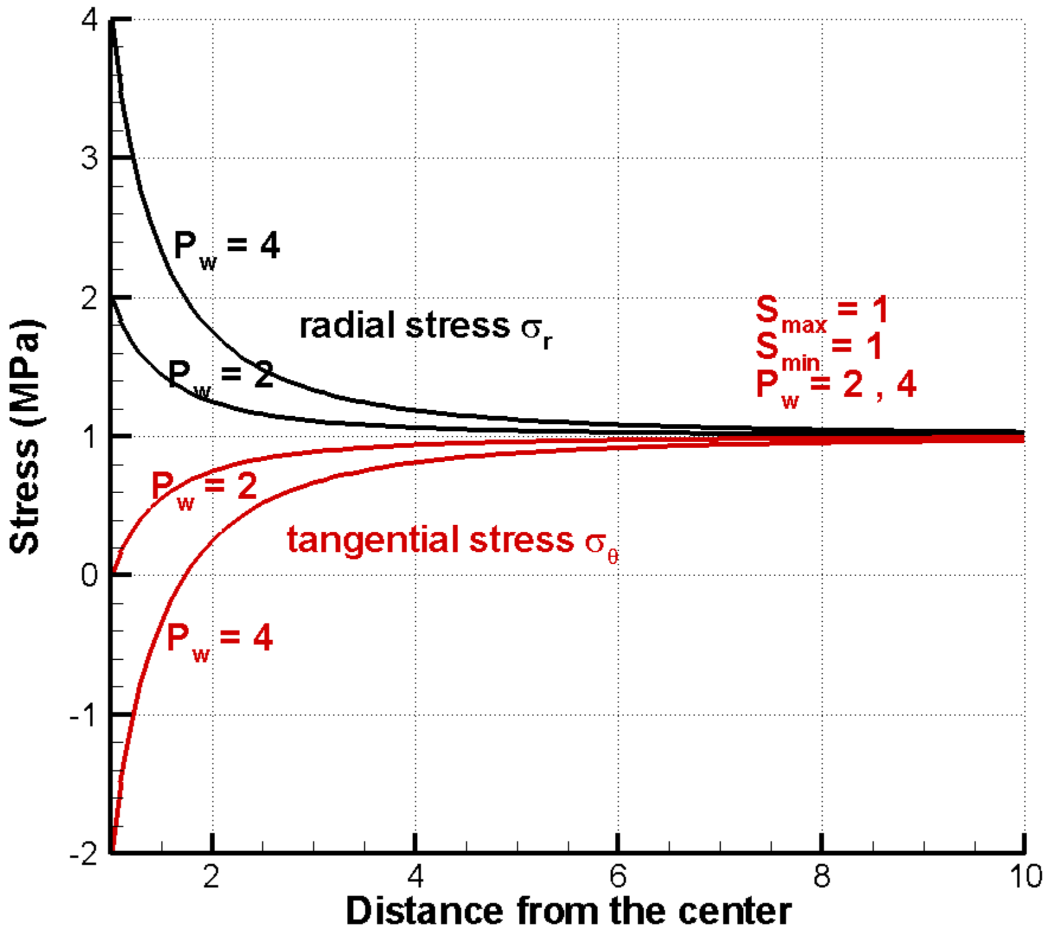




Stresses distribution around a borehole
Internal pressure from the opening/borehole

- Hydraulic fracturing occur when internal pressure is large.
- The location of the initiation of fracture can be decided depending on the boundary stress





Stresses distribution around an opening
3) Elliptical opening

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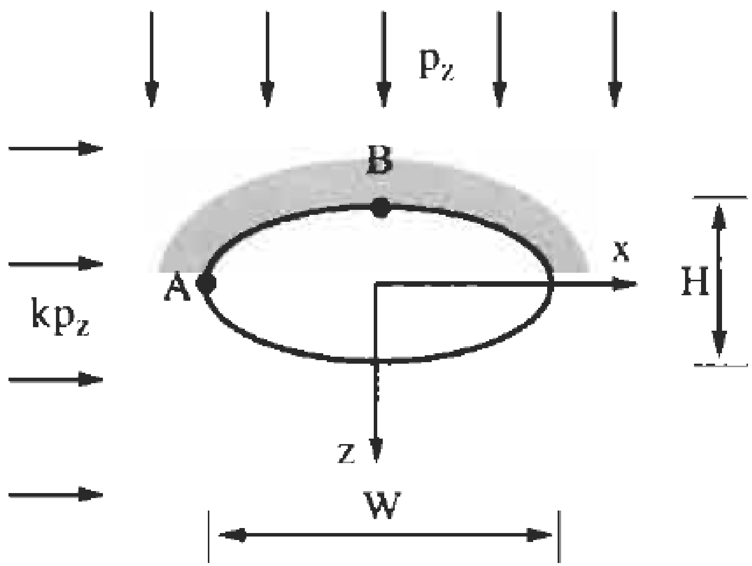
- Tangential stress concentration at peripheries

$$\sigma_A = P\left(1 - k + 2\frac{W}{H}\right) \qquad \sigma_A = P\left(k - 1 + 2k\frac{H}{W}\right)$$



$$\sigma_A = p(1 - k + 2q) = p\left(1 - k + \sqrt{\frac{2W}{\rho_A}}\right),$$

$$\sigma_B = p\left(k - 1 + \frac{2k}{q}\right) = p\left(k - 1 + k\sqrt{\frac{2H}{\rho_B}}\right)$$

where, for an ellipse, the radii of curvature are

$$\rho_A = \frac{H^2}{2W} \text{ and } \rho_B = \frac{W^2}{2H}$$

Figure 19.16 Stresses induced on the boundary of an elliptical excavation (aligned with axes parallel and perpendicular to the principal stresses) in plane strain for a CHILE material (after Bray, 1977; from Brady and Brown, 1985).

Hudson & Harrison, 1997, Engineering Rock Mechanics, Elsevier

Stresses distribution around a borehole
4) Temperature change in the circular hole

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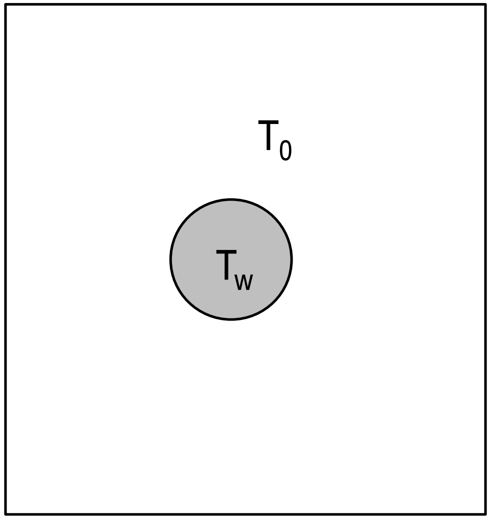
- Thermally induced hoop stress at the wall of cylindrical borehole

$$\sigma_r = 0$$

$$\sigma_\theta = \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

$$\tau_{r\theta} = 0$$



α: linear thermal expansion coefficient
E: Elastic Modulus
T_w: well temperature
T_0: reservoir temperature

Stresses distribution around a borehole
4) Temperature change in the circular hole

- Linear thermal expansion coefficient (unit: /K)

$$\frac{\Delta l}{l} = \alpha\left(T - T_0\right)$$

- Thermal stress ← thermal expansion + mechanical restraint
  - Thermal stress in 1D

$$\sigma_T = \alpha E\left(T - T_0\right)$$

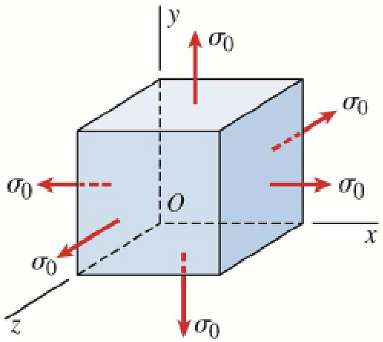
  - Thermal stress when a rock is completely (in all directions) restrained

$$\sigma_T = 3\alpha K\left(T - T_0\right) = \frac{E}{1-2\nu}\alpha\left(T - T_0\right)$$

Triaxial Stress
Hydrostatic Stress

- Hydrostatic Stress :
  - when three normal stresses are equal $\sigma_x = \sigma_y = \sigma_z = \sigma_0$
  - Any plane cut through the element will be subjected to the same normal stress σ_0
  - Normal Strain $\varepsilon_0 = \dfrac{\sigma_0}{E}(1-2\nu)$
  - Unit volume change

$$e = 3\varepsilon_0 = \frac{3\sigma_0}{E}(1-2\nu) = \frac{\sigma_0}{K}$$



  - Bulk modulus (of elasticity), K $K = \dfrac{E}{3(1-2\nu)} = \dfrac{1}{\beta}$ **Compressibility, β**
  - Uniform pressure in all directions: **<u>Hydrostatic</u>**

    An object submerged in water or deep rock within the earth

4) Temperature change in the circular hole

- Thermal stress due to cold mud/water injection is important for geothermal project
  - Ex) Injecting water T = 25°C, reservoir T = 75°C, Elastic modulus = 50 GPa, v=0.25, α = 1×10^{-5}/°C → hoop stress = 33 MPa ← big influence!!!

Stresses distribution around borehole
5) General solution

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- Stresses distribution around a borehole with;
  - Principal in situ stress boundary
  - Internal pore pressure (could be mud pressure)
  - Temperature change

$$\sigma_r = \frac{S_{H\max} + S_{h\min}}{2}\left(1 - \frac{R^2}{r^2}\right) + \frac{S_{H\max} - S_{h\min}}{2}\left(1 - \frac{4R^2}{r^2} + \frac{3R^4}{r^4}\right)\cos 2\theta + P_w \frac{R^2}{r^2}$$

$$\sigma_\theta = \frac{S_{H\max} + S_{h\min}}{2}\left(1 + \frac{R^2}{r^2}\right) - \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{3R^4}{r^4}\right)\cos 2\theta - P_w \frac{R^2}{r^2} + \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

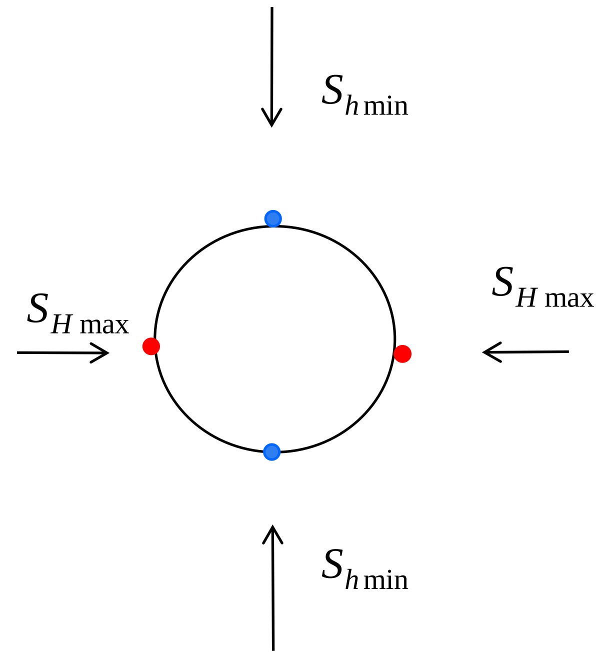
$$\tau_{r\theta} = \frac{S_{H\max} - S_{h\min}}{2}\left(1 + \frac{2R^2}{r^2} - \frac{3R^4}{r^4}\right)\sin 2\theta$$

Stresses distribution around borehole
5) General solution

– At the borehole wall (r = R), maximum and minimum hoop stresses are;

$$\sigma_{\theta,\min} = 3S_{h\min} - S_{H\max} - P_w + \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

$$\sigma_{\theta,\max} = 3S_{h\max} - S_{h\min} - P_w + \frac{E}{1-\nu}\alpha\left(T_w - T_0\right)$$

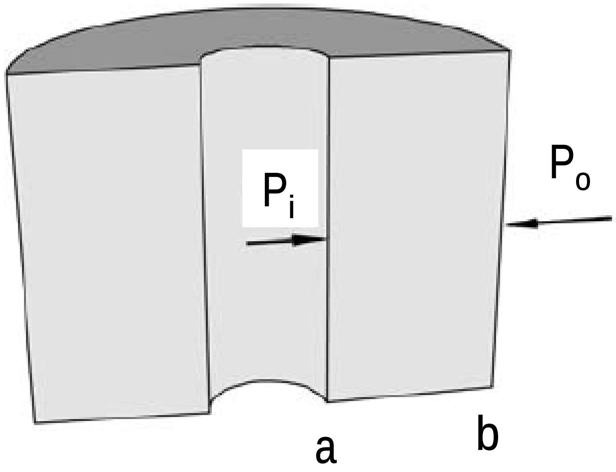


– Without considering temperature change,

$$\sigma_{\theta,\min} = 3S_{h\min} - S_{H\max} - P_w$$

$$\sigma_{\theta,\max} = 3S_{h\max} - S_{h\min} - P_w$$

Stresses distribution around borehole
6) Hollow cylinder

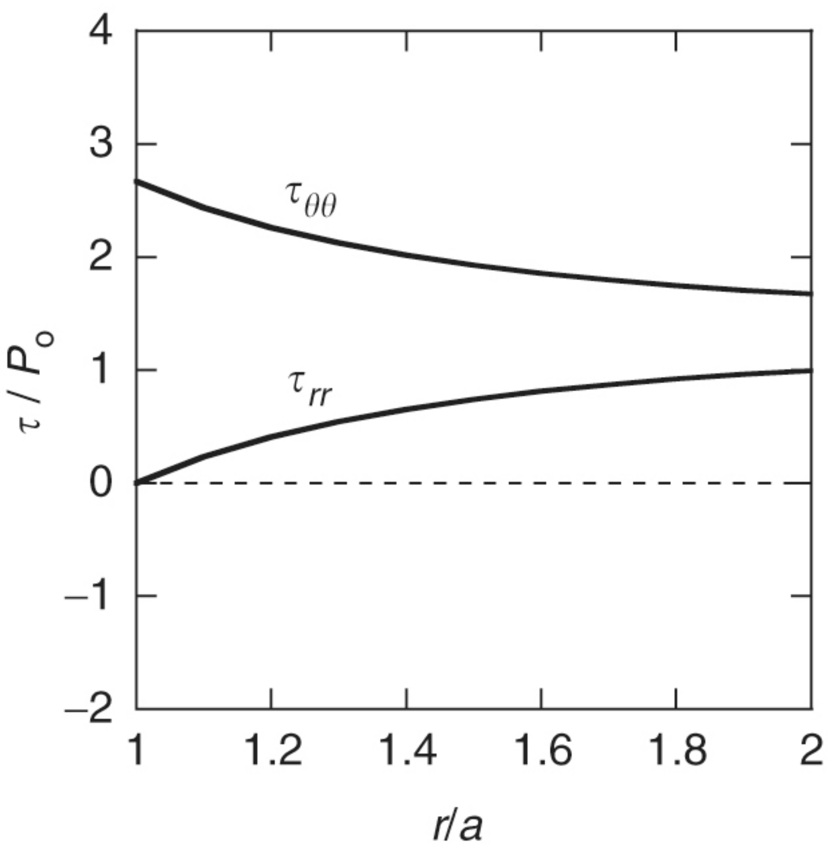


$$\sigma_r = \frac{(b^2 P_o - a^2 P_i)}{(b^2 - a^2)} + \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2}$$

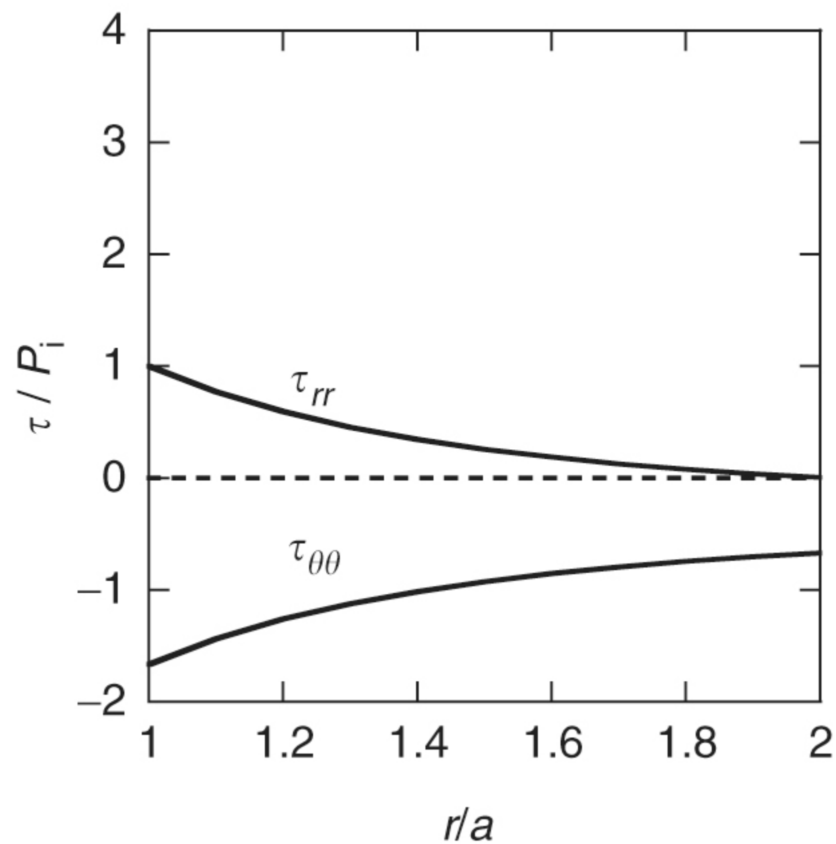
$$\sigma_\theta = \frac{(b^2 P_o - a^2 P_i)}{(b^2 - a^2)} - \frac{a^2 b^2 (P_i - P_o)}{(b^2 - a^2) r^2}$$

Stresses distribution around borehole

6) Hollow cylinder



With external pressure (P_0) alone



With internal pressure (P_i) alone

Stress distribution in a hollow cylinder with b = 2a

Jaeger, Cook & Zimmerman, 2007, Fundamentals of Rock Mechanics, Blackwell Publishing

Stresses distribution around borehole
7) Spherical cavity

- Spherical cavity with internal pressure

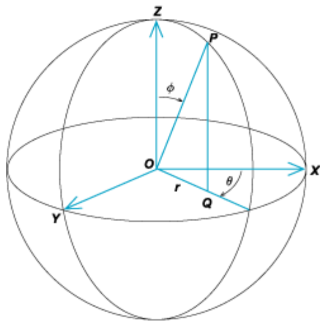
$$\sigma_r = P\left(\frac{a^3}{r^3}\right)$$

$$\sigma_\theta = \sigma_\phi = -\frac{P}{2}\left(\frac{a^3}{r^3}\right)$$

$$\sigma_{r\theta} = \sigma_{r\phi} = \sigma_{\theta\phi} = 0$$

$$u_r = -\frac{P}{4G}\frac{a^3}{r^2}$$

$$u_\theta = u_\phi = 0$$



Stresses distribution around borehole
7) Spherical cavity

- Spherical cavity in a 3D hydrostatic stress field

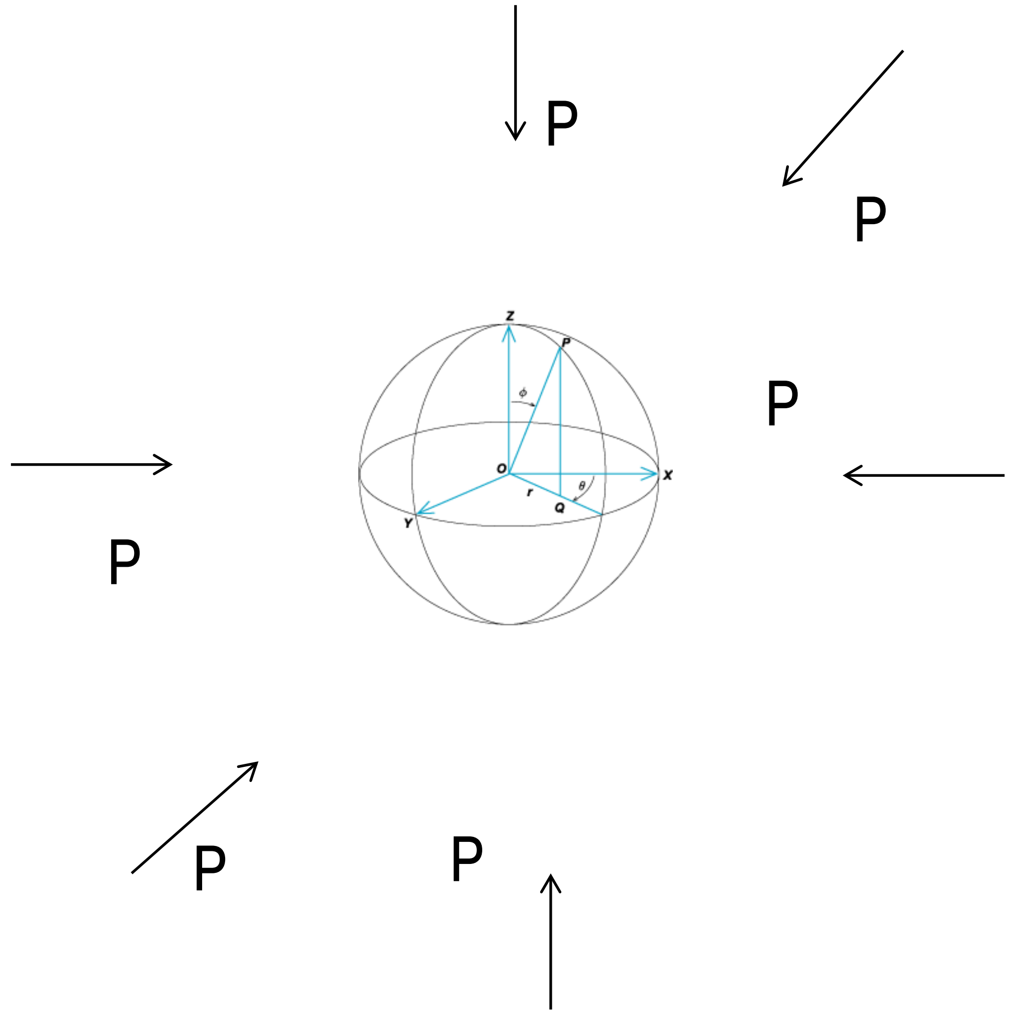
$$\sigma_r = P\left[1-\left(\frac{a^3}{r^3}\right)\right]$$

$$\sigma_\theta = \sigma_\phi = P\left[1+\left(\frac{a^3}{2r^3}\right)\right]$$

$$\sigma_{r\theta} = \sigma_{r\phi} = \sigma_{\theta\phi} = 0$$

$$u_r = \frac{P}{4G}\frac{a^3}{r^2}$$

$$u_\theta = u_\phi = 0$$



Stresses distribution around borehole
7) Spherical cavity

- Comparison of 2D circular opening and 3D spherical opening (internal pressure P)

$$\sigma_\theta = -P_w \frac{a^2}{r^2}$$

$$\sigma_\theta = \sigma_\phi = -\frac{P}{2}\left(\frac{a^3}{r^3}\right)$$

$$u_r = -\frac{P_w}{2G}\frac{a^2}{r}$$

$$u_r = -\frac{P}{4G}\frac{a^3}{r^2}$$

Stresses distribution around borehole
7) Spherical cavity

- Comparison of 2D circular opening and 3D spherical opening (hydrostatic case)

$$\sigma_\theta = P\left(1+\frac{a^2}{r^2}\right)$$

$$\sigma_\theta = \sigma_\phi = P\left[1+\left(\frac{a^3}{2r^3}\right)\right]$$

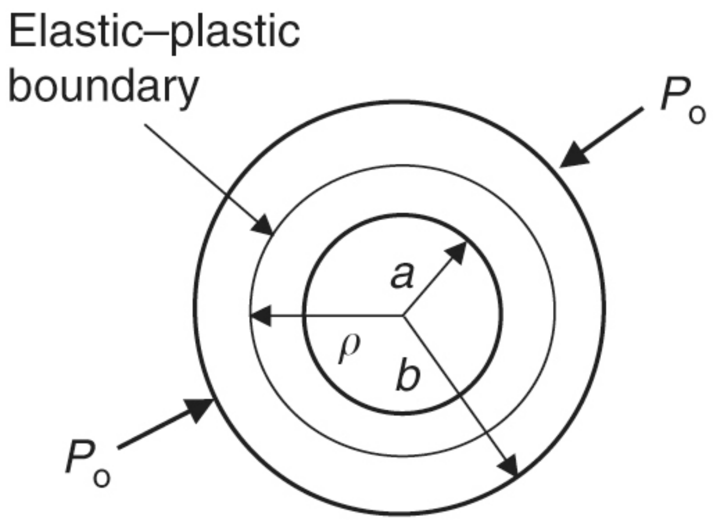
$$u_r = \frac{P}{2G}\frac{a^2}{r}$$

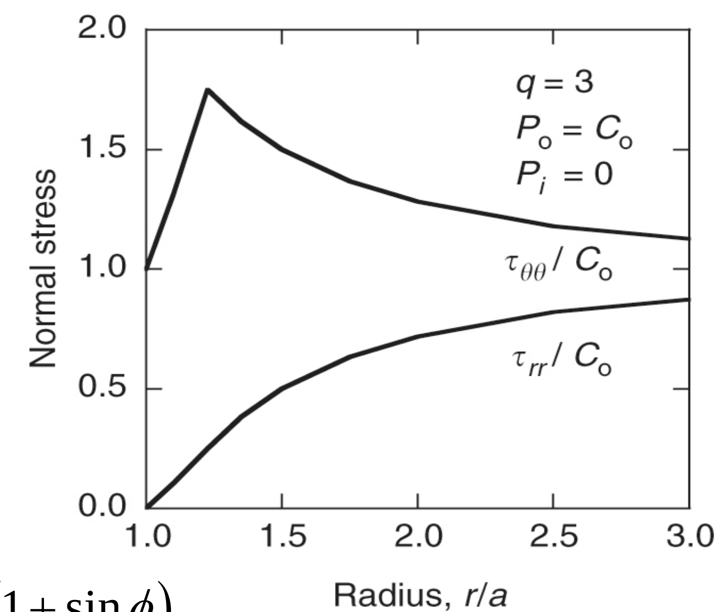
$$u_r = \frac{P}{4G}\frac{a^3}{r^2}$$

Stresses distribution around borehole
Elasto-Plastic Solution with circular hole

- A more general case around the borehole can be considered for inelastic case using failure criteria such as Mohr-Coulomb failure criteria.
- Analytical solution exists only for hydrostatic boundary stress condition.

Borehole stability problem
Elasto-Plastic Solution with circular hole





Failure criterion: $\sigma_1 = C_0 + q\sigma_3 = 2S_0 \tan\beta + \sigma_3 \dfrac{(1+\sin\phi)}{(1-\sin\phi)}$

$$\frac{\rho}{a} = \left\{ \frac{2\left[P_0(q-1)+C_0\right]}{\left[P_i(q-1)+C_0\right](q+1)} \right\}^{1/(q-1)}$$

$a < r < \rho$

$$\sigma_r = \left[P_i + (C_0/2)\right](r/a)^2 - (C_0/2)$$

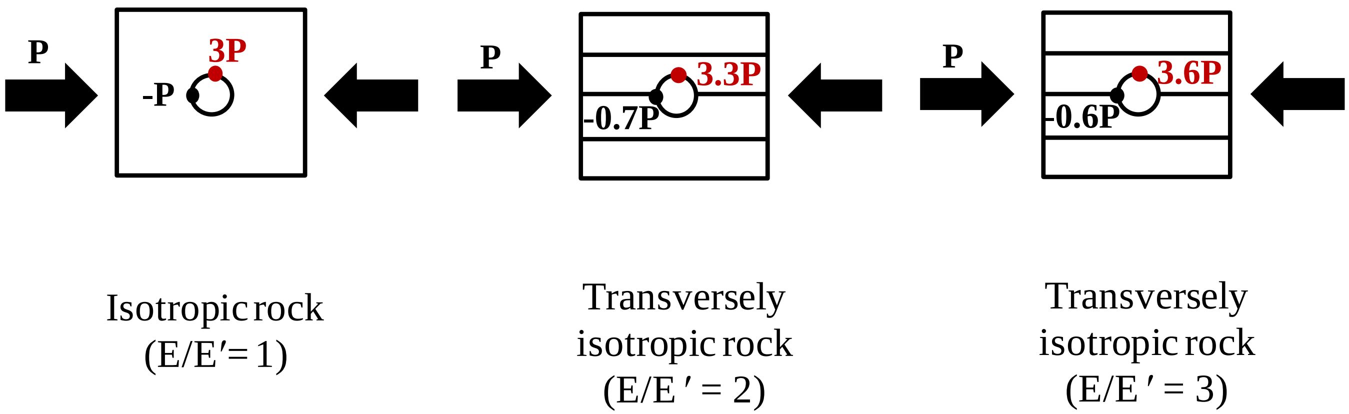
$$\sigma_\theta = 3\left[P_i + (C_0/2)\right](r/a)^2 - (C_0/2)$$

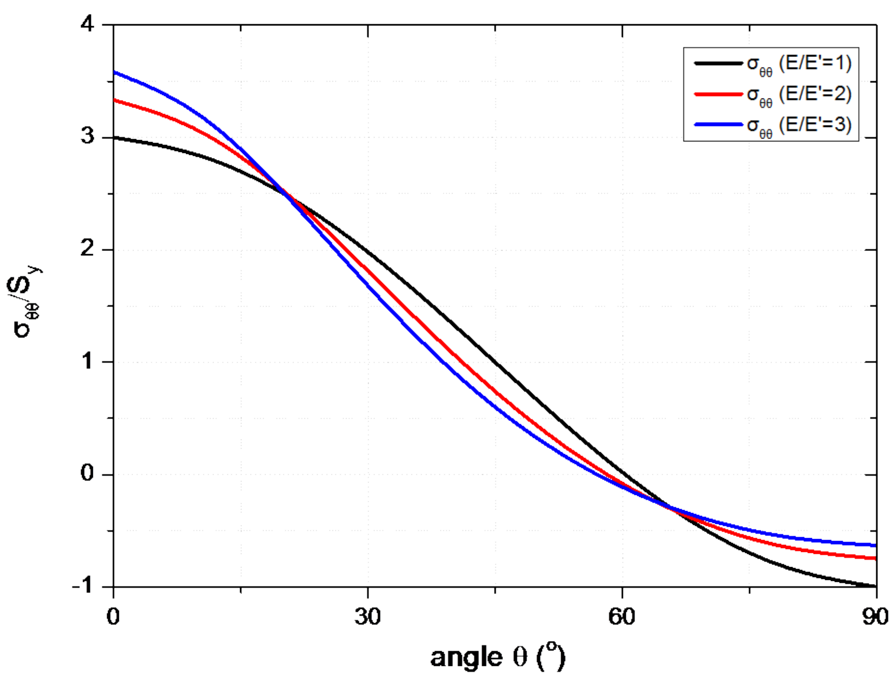
$\rho < r < b$

$$\sigma_r = P_0 - \frac{1}{2}\left[P_0 + (C_0/2)\right](r/a)^2$$

$$\sigma_\theta = P_0 + \frac{1}{2}\left[P_0 + (C_0/2)\right](r/a)^2$$

Stresses distribution around borehole
Effect of Anisotropy





- Stress concentration differs depending on the anisotropy of elastic modulus

Hanna Kim, 2012, MSc Thesis, SNU