Thermofluid Processes

Reversible Non-Flow Process

- Constant volume process => no work (as rigid vessel)

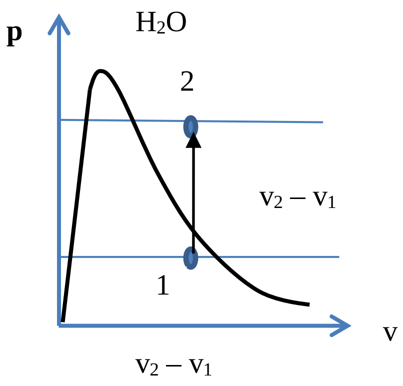
$$Q_{1-2} = (u_2 - u_1) + \underbrace{\cancel{W_{1-2}}}_{0} = u_2 - u_1 \qquad => (U_2 - U_1)$$

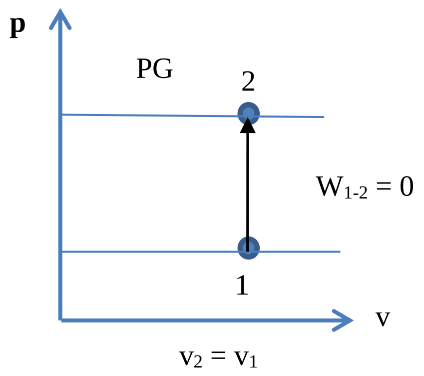
$$= m\,c_v\,(T_2 - T_1)$$

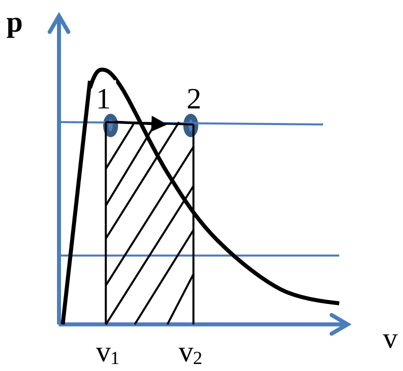
- Constant pressure process: boundary moves (isobaric)

$$W_{1-2} = \int_{v_2}^{v_1} p\,dv = p \int_{v_2}^{v_1} dv = p\,(v_2 - v_1)$$

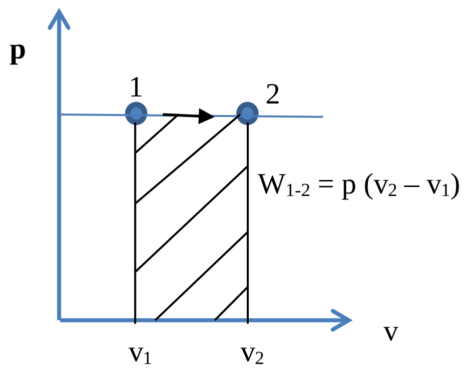
$$Q_{1-2} = (u_2 - u_1) + W_{1-2} = (u_2 - u_1) + p\,(v_2 - v_1)$$
$$= (u_2 - p\,v_2) - (u_1 + p\,v_1) = h_2 - h_1 = H_2 - H_1$$
$$= m\,C_p\,(T_2 - T_1)$$





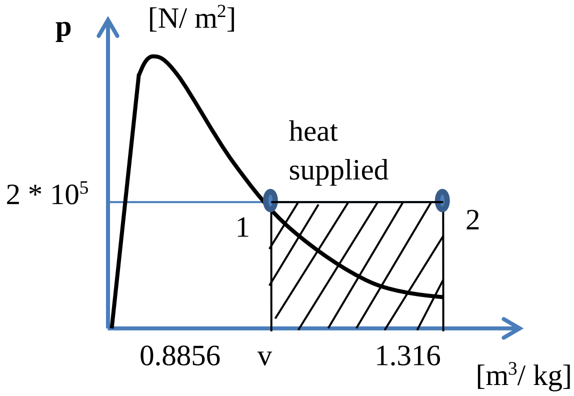


$p_2 - p_1$



.05 kg heated at 2 bar until $V_2 = .0658\ m^3$

(a) Steam initially dry saturated



$h_1 = h_g = 2707$ kJ/kg at 2 bar

$v_2 = V_2/ m = .0658/ .05 = 1.316\ m^3/ kg$

$p_2 = 2 * 10^5\ N/m^2$

⇨ $T_2 = 300°C$, $h_2 = 3072$ kJ/kg

$Q_{1\text{-}2} = H_2 – H_1 = m(h_2 - h_1)$

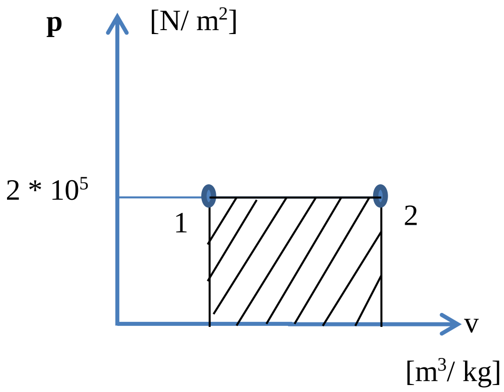
$= .05\ (3072 - 2707) = 18.25$ kJ

$W_{1\text{-}2} = p(v_2 – v_1) = 2 * 10^5(1.316 - .8856)$ Nm/kg

$= .05*2 * 10^5(1.316 - .8856)* 10^{-3}$

= 4.304 kJ

(b) Air initially at 130°C



$T_2 = (p_2 V_2)/ (mR)$

$= (2 * 10^5 * .0658) / (.05 * .287 * 10^3) = 917K$

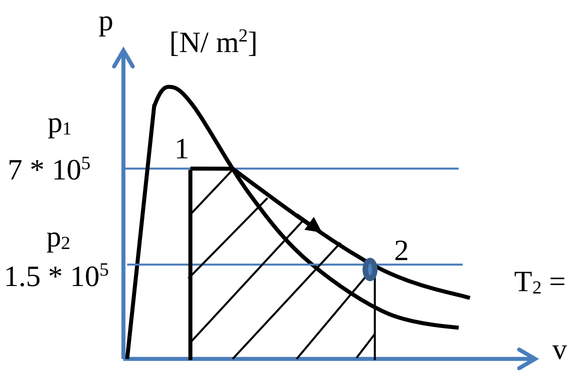
$Q_{1\text{-}2} = mC_p(T_2 - T_1)$

$= .05 * 1.005(917 - 403) = 25.83$ kJ

$W_{1\text{-}2} = p\ (v_2 – v_1)\ R(T_2 - T_1) = 0.287 *(917 - 403)$ kJ/kg

$= .05 * .287 * 514 = 7.38$ kJ

- Const temperature (isothermal) process



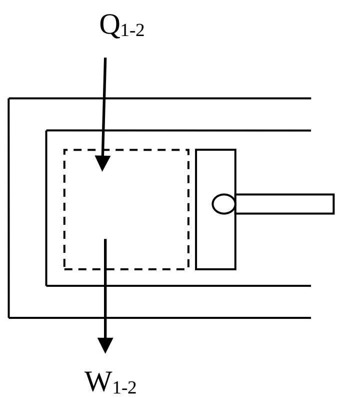
Steam @ 7 bar, x = .9, (T), rev→ 1.5 bar

$\Delta u, \Delta h, W_{1\text{-}2}$?

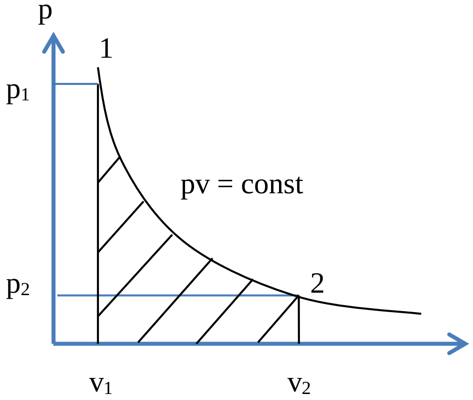
$Q_{1\text{-}2} = (u_2 – u_1) + W_{1\text{-}2}$

T_{set} @ 7 bar

$T_2 = T_1 = 165°C$



- Isothermal process for a perfect gas: $pv = RT$



$p_1v_1 = p_1v_1 = C$

$$W = C\int_1^2 \frac{dv}{v} = C\ln\frac{v_2}{v_1}$$

$$= p_1v_1 \ln\frac{v_2}{v_1}$$

$$= p_2v_2 \ln\frac{v_2}{v_1}$$

$$W = p_1V_1 \ln\frac{v_2}{v_1}$$

$$\frac{v_2}{v_1} = \frac{p_1}{p_2}, \qquad W = p_1V_1 \ln\frac{p_1}{p_2}$$

$$p_1v_1 = RT, \qquad w = RT\ln\frac{p_1}{p_2}$$

$$= W = mRT\ln\frac{p_1}{p_2}$$

Joule's Law: $U_2 - U_1 = m\,c_v\,(T_2 - T_1) = 0$ for (T) $\rightarrow$ isothermal process

1st Law: $q = (u_2 - u_1) + w = W$

- Reversible adiabatic non-flow process

$\cancel{q} = (u_2 - u_1) + w \qquad \rightarrow W = u_1 - u_2$

0 as adiabatic

Perfect gas: $dq = du + dw, \quad dw = pdv$

$dq = du + pdv = 0$ adiabatic

$h = pv + u, \quad dh = du + pdv + vdp$

$dq = dh - vdp = 0$

$p = RT/v, \quad du + RTdv/v = 0$

$u = C_v T \quad du = C_v\,dT$

$$\int\left[C_v dT + \frac{RTdv}{v}\right] = 0$$

$C_v \ln T + R\ln v = \text{const}$

$$\ln\frac{pv}{R} + \frac{R}{C_v}\ln v = \text{const}$$

$$C_v = \frac{R}{\gamma-1} \text{ or } \frac{R}{C_v} = \gamma-1$$

$$\ln\frac{pv}{R} + (\gamma-1)\ln v = \text{const}$$

$$\ln\frac{pv}{R} + \ln v^{\gamma-1} = \text{const}$$

$$\ln\frac{pv\,v^{\gamma-1}}{R} = \text{const}$$

$$\ln\frac{p\,v^{\gamma}}{R} = \text{const}$$

$$\frac{p\,v^{\gamma}}{R} = e^{\text{const}} = \text{const}$$

$$p\,v^{\gamma} = \text{const}$$

$$p = \frac{RT}{v}$$

$$RTv^{\gamma-1} = \text{const}$$

$$Tv^{\gamma-1} = \text{const}$$

$$v = \frac{RT}{p} \Rightarrow pv^{\gamma} = p\left(\frac{RT}{p}\right)^{\gamma} = \text{const}$$

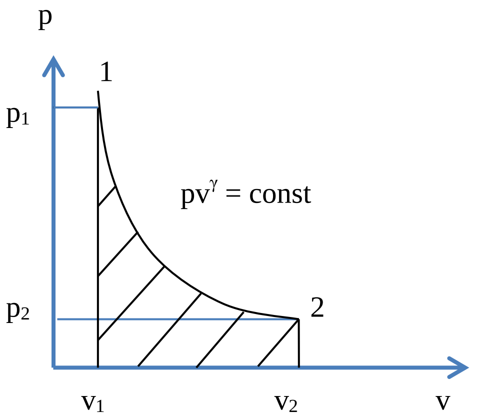
$$\frac{T^{\gamma}}{p^{\gamma-1}} = \text{const} \text{ or } \frac{T}{p^{1-\frac{1}{\gamma}}} = \text{const}$$

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^{\gamma}, \quad \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1}, \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{1-\frac{1}{\gamma}}$$

$$W = u_1 - u_2 = C_v(T_1 - T_2) \qquad C_v = \frac{R}{\gamma - 1}$$

$$= \frac{R(T_1 - T_2)}{\gamma - 1} \qquad pv = RT$$

$$\therefore W = \frac{p_1v_1 - p_2v_2}{\gamma - 1}$$



$$W = \int_1^2 pdv = \int_1^2 \frac{Cdv}{v^\gamma} = C\left[\frac{v^{1-\gamma}}{1-\gamma}\right]_{v_1}^{v_2}$$

$$= C\frac{v_2^{1-\gamma} - v_1^{1-\gamma}}{1-\gamma} = C\frac{v_1^{1-\gamma} - v_2^{1-\gamma}}{\gamma - 1}$$

$$= \frac{p_1v_1^2v_1^{1-\gamma} - p_2v_2^2v_2^{1-\gamma}}{\gamma - 1} \qquad \left(C = p_1v_1^\gamma = p_2v_2^\gamma\right)$$

$$\therefore W = \frac{p_1v_1 - p_2v_2}{\gamma - 1}$$

Polytropic Process:

$$pv^n = \text{const} \qquad w = \int_1^2 pdv$$

$$= C\int_1^2 \frac{dv}{v^n} = C\left[\frac{v^{1-n}}{1-n}\right]_{v_1}^{v_2} = C\left(\frac{v_2^{1-n} - v_1^{1-n}}{1-n}\right)$$

$$= \frac{p_1v_1^nv_1^{1-n} - p_2v_2^nv_2^{1-n}}{n-1}$$

$$= \frac{p_1v_1 - p_2v_2}{n-1}$$

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n$$

In a polytropic process the index n depends only on the heat and work quantities during the process.

$n = 0 \quad pv^0 = p = \text{const}$: isobaric

$n = \gamma \quad pv^\gamma = \text{const}$ or $p^{-\gamma}v = \text{const}$, $v = \text{const}$

$n = 1 \quad pv = \text{const}$ or $T = \text{const}$: isothermal $\left(\because \frac{pv}{T} = \text{const}\right)$

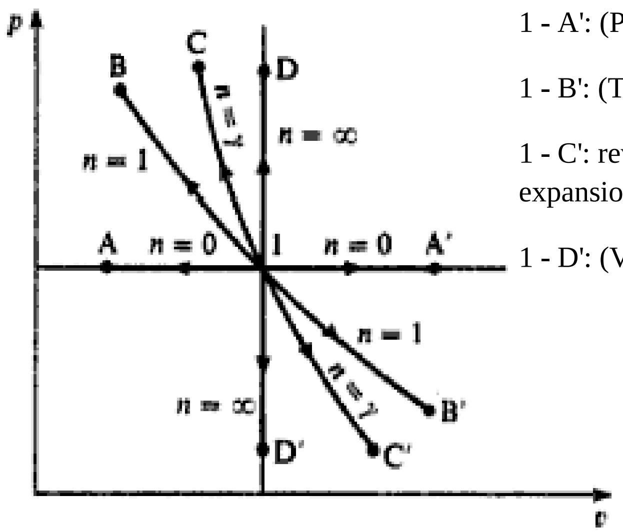
$n = \gamma \quad pv^\gamma = \text{const}$: reversible, adiatic

1 - A: (P) cooling

1 – B: (T) compression

1 - C: rev. adiabatic compression

1 - D': (V) heating



1 - A': (P) heating

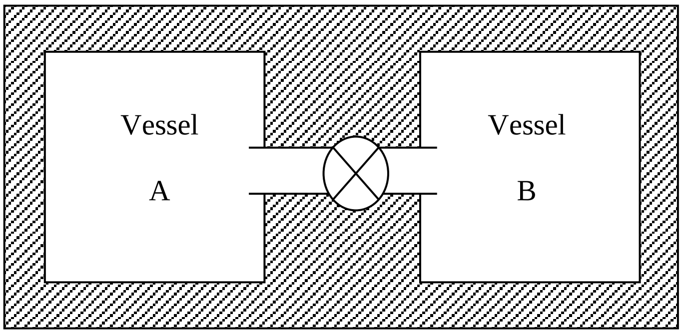
1 - B': (T) expansion

1 - C': rev. adiabatic expansion

1 - D': (V) cooling

Irreversible processes

lagged (perfectly thermally insulated)



t = 0: A filled with a fluid at a certain pressrue

B is evacuated

t > 0: A & B filled at a lower pressure

(1) Unrestricted, or free, expansion

⇨ Irreversible: extenal work must be done to restore the fluid to its initial condition

$q = (u_2 - u_1) + w$ $\qquad$ $q = 0 \because$ adiabatic

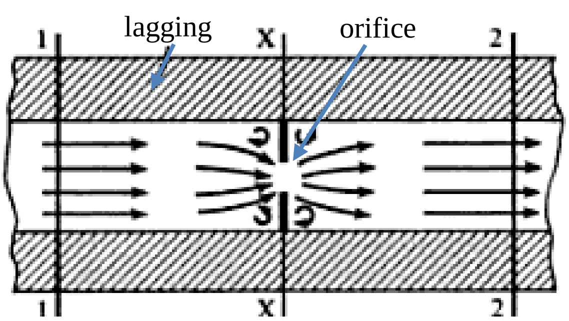
$w = 0$ $\because$ the system boundary does not move

The process is adiabatic, but irreversible. $u_2 = u_1$

Perfect gas: $u = C_v T$ $\quad$ $C_vT_1 = C_vT_2$ or $T_1 = T_2$

For a perfect gas undergoing a free expansion

(2) Throttling



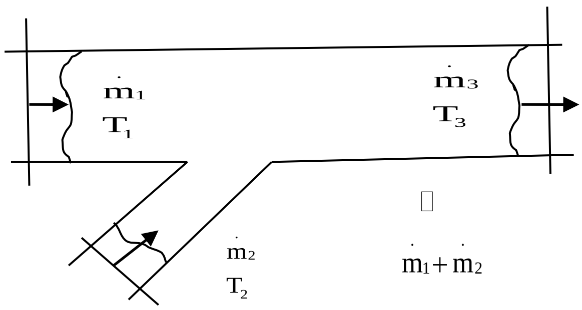
$$h_1 + \frac{V_1^2}{2} + \underbrace{\cancel{q}}_{0} = h_2 + \frac{V_2^2}{2} + \underbrace{\cancel{w}}_{0}$$

When V_1 & V_2 are small or close to each other, then $h_1 = h_2$

Perfect gas: $h = C_pT$

$C_pT_1 = C_pT_2$ or $T_1 = T_2$

(3) Adiabatic mixing



q = 0, w = 0

$H_1 + H_2 = H_3$

neglecting KE

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \left(\dot{m}_1 + \dot{m}_2\right) h_3$$

$$\text{pG: } h = C_p T, \qquad \dot{m}_1 C_p T_1 + \dot{m}_2 C_p T_2 = \left(\dot{m}_1 + \dot{m}_2\right) C_p T_3$$

$$\dot{m}_1 T_1 + \dot{m}_2 T_2 = \left(\dot{m}_1 + \dot{m}_2\right) T_3$$

-highly irreversible due to the large amount of eddying and churning of the fluid.

Reversibele flow processess

$$h_1 + \frac{V_1^2}{2} + \underbrace{\cancel{q}}_{0} = h_2 + \frac{V_2^2}{2} + \underbrace{\cancel{w}}_{0} \quad \Rightarrow w = \left(h_1 - h_2\right) + \frac{V_1^2 - V_2^2}{2}$$

Non-steady flow processes

E: total energy of the system within the boundary

δm_1: mass enetering the system during a small time interval

δm_2: mass leaving the system during a small time interval

δq: heat transferred during the same time

δw: work done during the same time

$\delta m_1 \, p_1 v_1$: work done at inlet

$\delta m_2 \, p_2 v_2$: work done at outlet

$u_1 + V_1^2/2 + gz_1$: energy at inlet

$u_2 + V_2^2/2 + gz_2$: energy at outlet

1st law:

(Energy entering the system) – (Energy leaving the system) = (Change in energy of the system)

$\delta m_1 p_1 v_1 + \delta m_1(u_1 + V_1^2/2 + gz_1) + \delta Q - \delta w - \delta m_2 p_2 v_2 - \delta m_2 (u_2 + V_2^2/2 + gz_2) = \delta E$

$Q = \sum \delta Q$: total heat transferred during a finite time

$W = \sum \delta w$: total work done during a finite time

m': initial mass within the system boundaries

u': initial internal energy within the system boundaries

m": final mass within the system boundaries

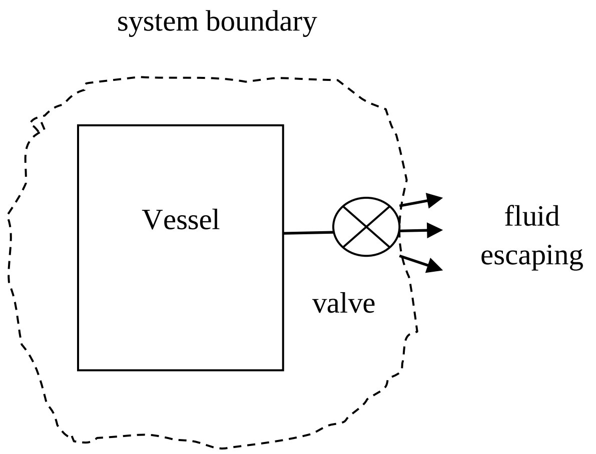
u": final internal energy within the system boundaries

$$\sum \delta E = m''u'' - m'u' + (m''u'' - m'u')$$

$$\sum \delta m_1 \left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + gz_1 \right) + Q = \sum \delta m_2 \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + gz_2 \right) + W$$

Continuity of mass: (Mass entering) – (Mass leaving) = (change in mass of the system)

$$\sum \delta m_1 - \sum \delta m_2 = m'' - m'$$



$W = 0$, $\delta m_1 = 0$ (no mass enter)

$gz_2 = 0$ (negligiable)

$$Q = \delta m_2 \left(h_2 + \frac{V_2^2}{2} \right) + (m''u'' - m'u')$$

where $h_2 = u_2 + p_2 v_2$