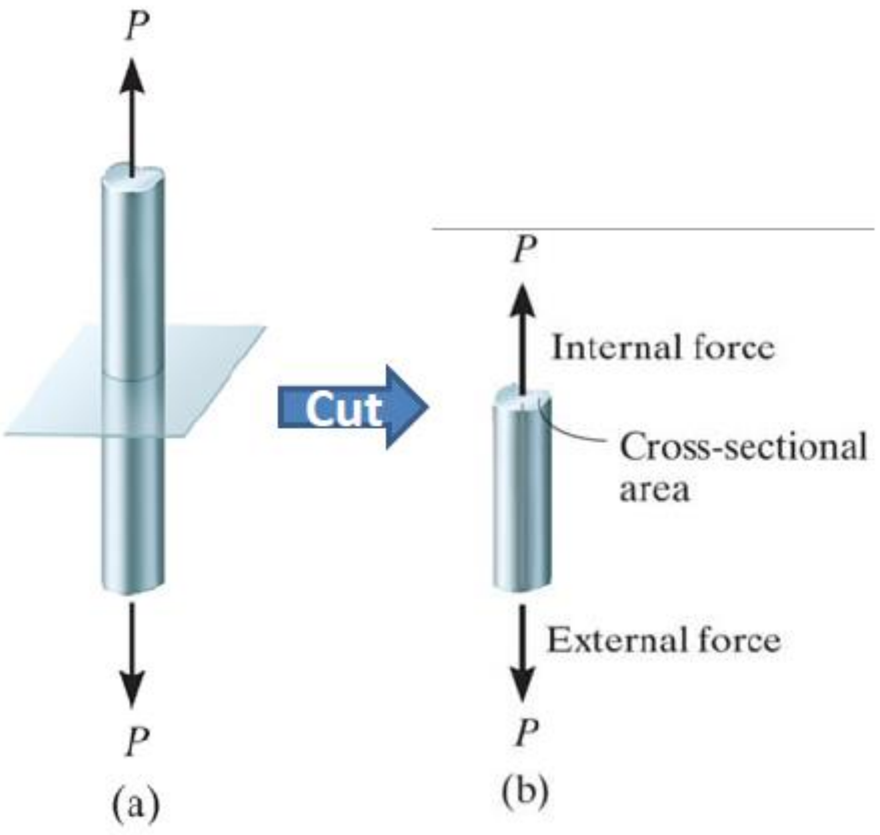
Normal stress

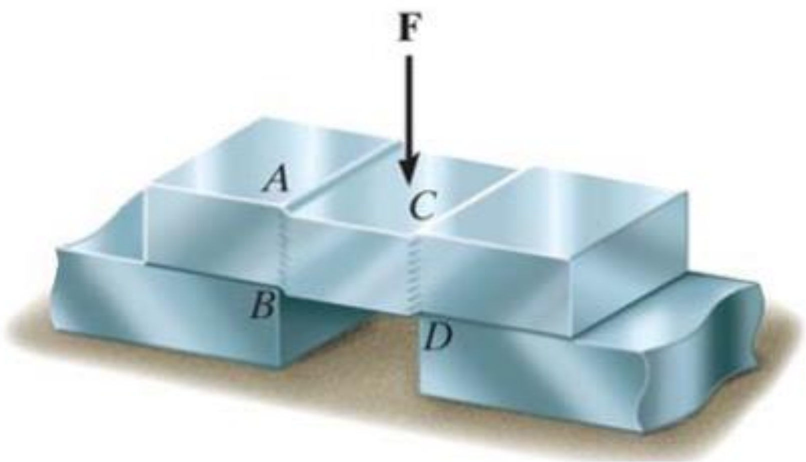
- A bar is subjected to an axial load.
- The "average" normal stress is

$$\boxed{\sigma = \frac{P}{A}}$$

- In general, the stress across the section is not uniform.
- Uniform stress can be assumed.



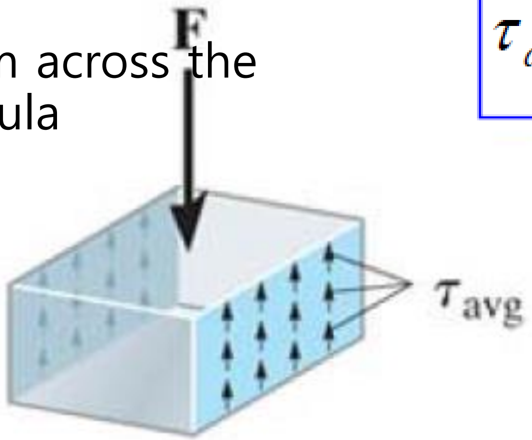
Shear stress

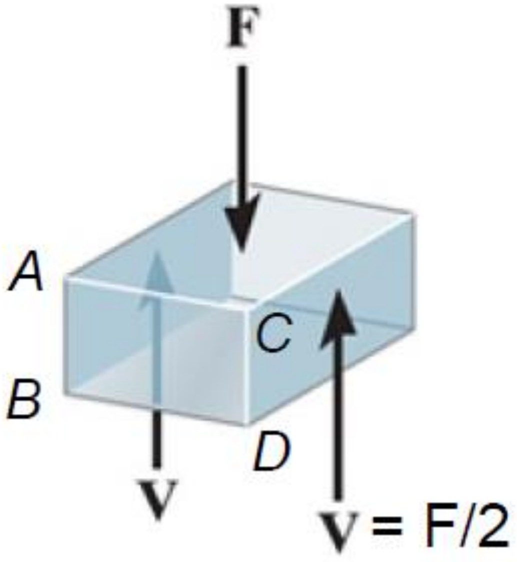


- Shear stress is the stress component that acts in the plane of the sectional area.
- If the supports are considered rigid and F is large enough, it will cause the material of the bar to fail along vertical planes (shear failure)
- The average shear stress (on each vertical plane) is

$$\boxed{\tau_{avg} = \frac{V}{A}}$$

- Note: Shear stress is actually not uniform across the section. We will derive shear stress formula





Deformation

- When a force is applied to a body, it will change the body’s shape and size. These changes are deformation.
- Example: The rubber strip (membrane) is subjected to tension.
- The vertical line is lengthened.
- The horizontal line is shortened.
- The inclined line changes its length and rotates.

Undeformed

Deformed due to tension

Engineering stress and strain

- Engineering or nominal stress = Applied load *P* divided by the specimen's original cross-sectional area (*A0*).
- Engineering or nominal strain = Change in the specimen's gauge length by the specimen's original gauge length (*L0*).

$$\boxed{\sigma = \frac{P}{A_0}} \qquad \boxed{\varepsilon = \frac{\delta}{L_0}}$$

- The plot of σ−ε based on the above definitions gives the conventional stress-strain diagram.
- If the actual cross-sectional area and specimen length are used, the plot is called the true stress-strain diagram.

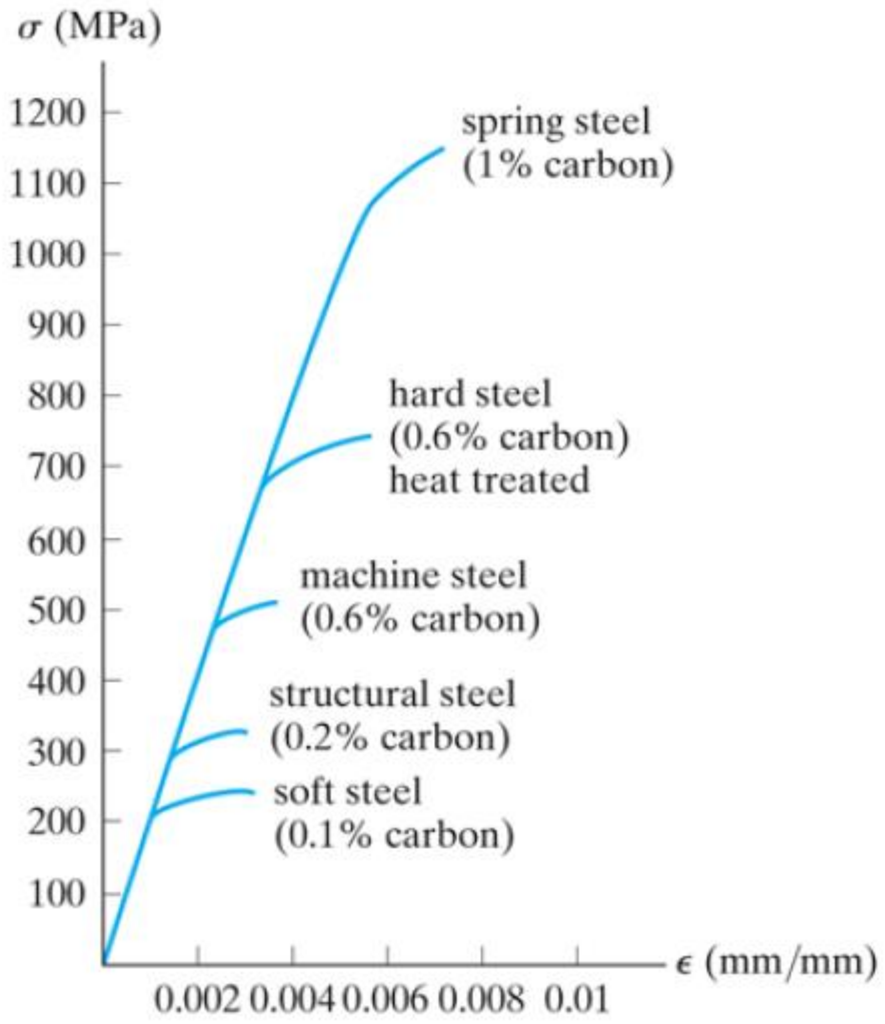
Hooke’s law

- In the elastic region, a linear relationship exists between stress and strain as follows.

$$\boxed{\sigma = E\,\varepsilon}$$

- σ = normal stress
- ε = normal strain
- E = modulus of elasticity or Young’s modulus

- Note that steels of different grades have different proportional limits, but their Young’s modulus values are practically the same.

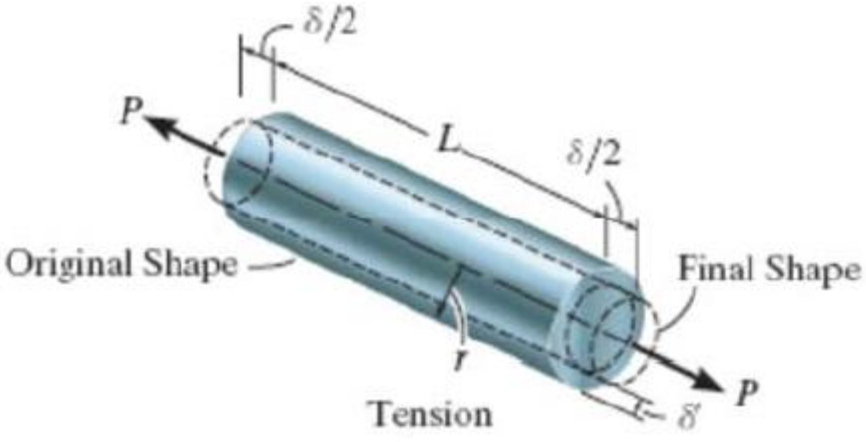


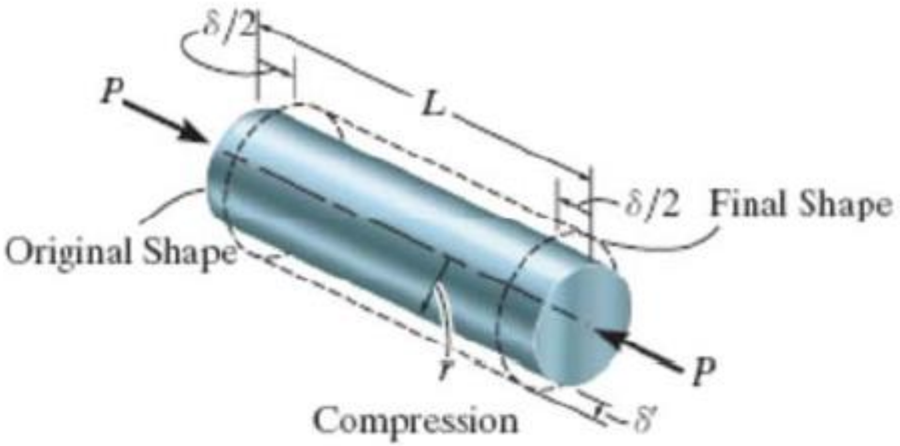
Poisson's ratio

- When a deformable body is subjected to a tensile force, not only does it elongates in the direction of force but it also contracts laterally.
- Likewise, a compressive force acting on a body causes it to contract in the direction of force but its sides expand laterally.
- The ratio of lateral strain to the longitudinal strain is called Poisson's ratio:

$$\boxed{\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}}$$

Poisson's ratio is dimensionless.
It ranges between 0 and 0.5.
Typical values are about 1/4 to 1/3.
(~0.5 for rubber & elastomer).



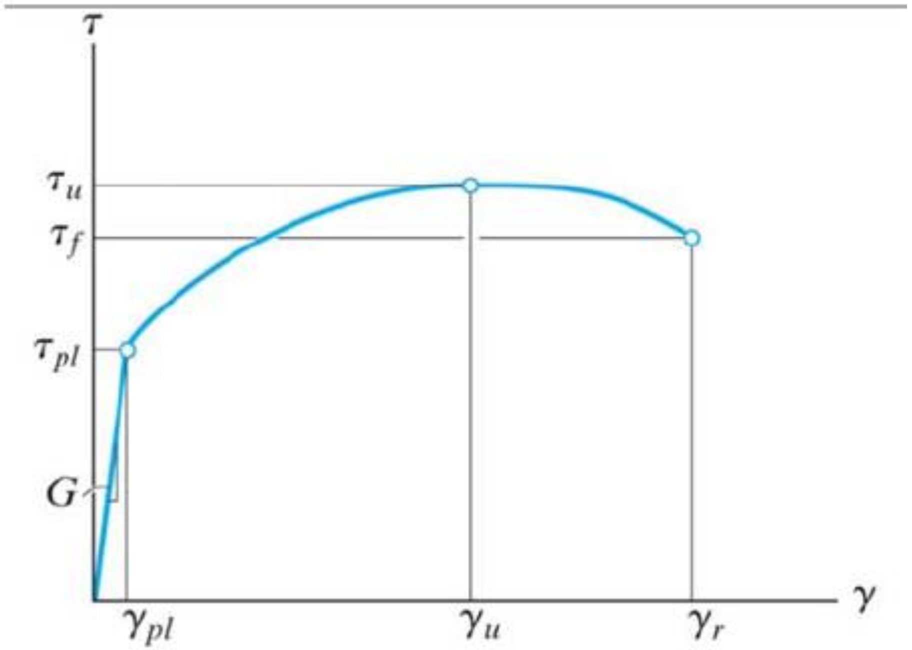


Shear stress-strain relation

When subjected to shear, most engineering materials will exhibit linear-elastic behaviour in shear distortion until a proportional limit.

Hooke's law applies for shear stress-strain relation in the elastic range.

$$\boxed{\tau = G\gamma}$$



τ = shear stress

γ = (engineering) shear strain

G= shear modulus of elasticity

E and G are related through Poisson's ratio:

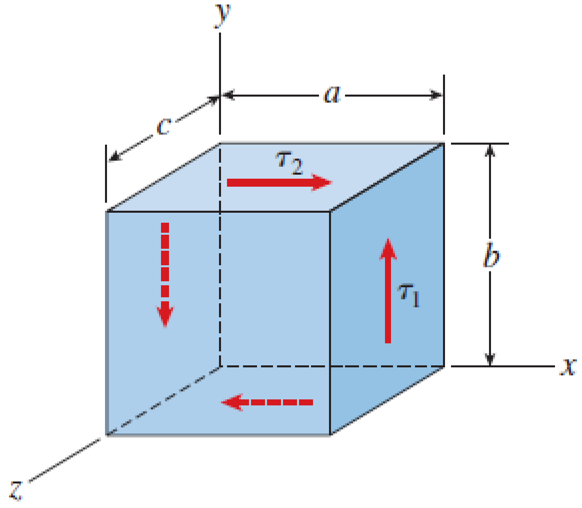
$$\boxed{G = \frac{E}{2(1+\nu)}}$$

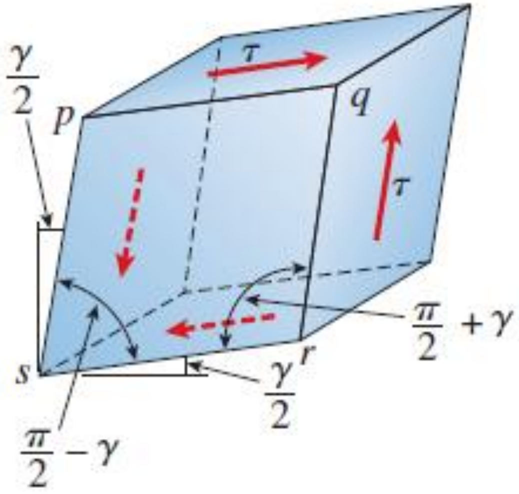
Shear strain

Shear strains cause a change in its shape (distortion)

Remember the “slip” behavior of atoms

Internal force equilibrium condition





$$\tau_1 = \tau_2$$

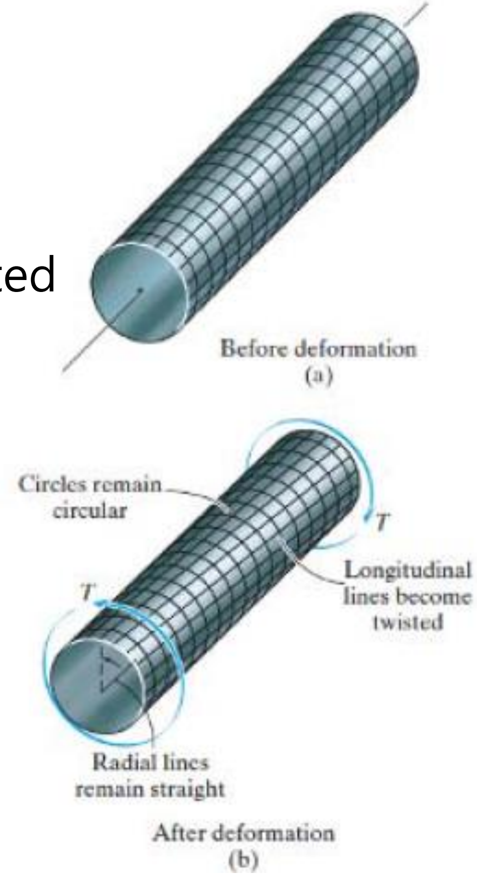
Torsion

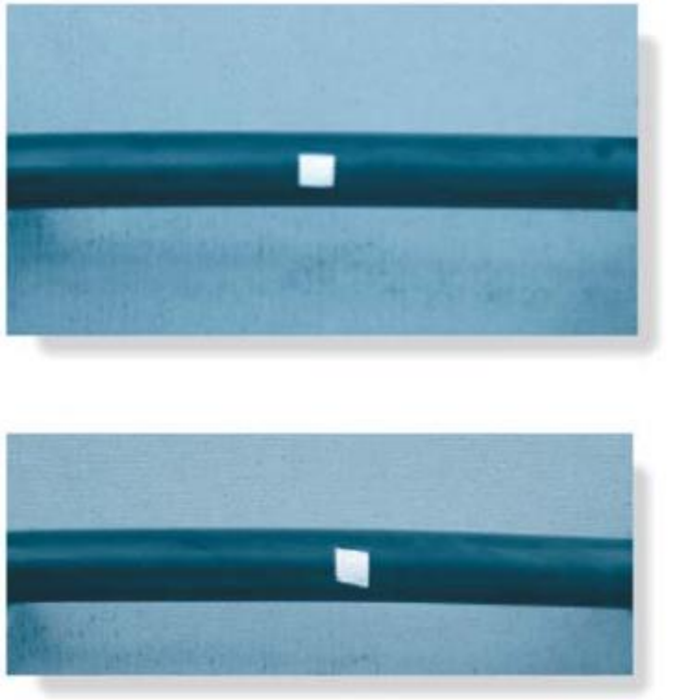
Torque is a moment that twists a member about its longitudinal axis.

Assumptions:

– Linear and elastic deformation

– Plane section remains plane and undistorted





Notice the deformation of the rectangular element when this rubber bar is subjected to a torque.

Static Review

Static equilibrium of a body requires **balance of forces** to prevent the body from translating, and **balance of moments** to prevent the body from rotating.

Equilibrium equations in vector form:

In scalar form, there are 6 equilibrium equations for 3D problems:

Vector equations: $\Sigma \underline{\mathbf{F}} = \underline{\mathbf{0}} \quad ; \quad \Sigma \underline{\mathbf{M}} = \underline{\mathbf{0}}$

Scalar equations in 3D: $\Sigma F_x = 0 \;;\quad \Sigma F_y = 0 \;;\quad \Sigma F_z = 0$

$$\Sigma M_x = 0 \;;\; \Sigma M_y = 0 \;;\; \Sigma M_z = 0$$

Free Body Diagram (FBD):

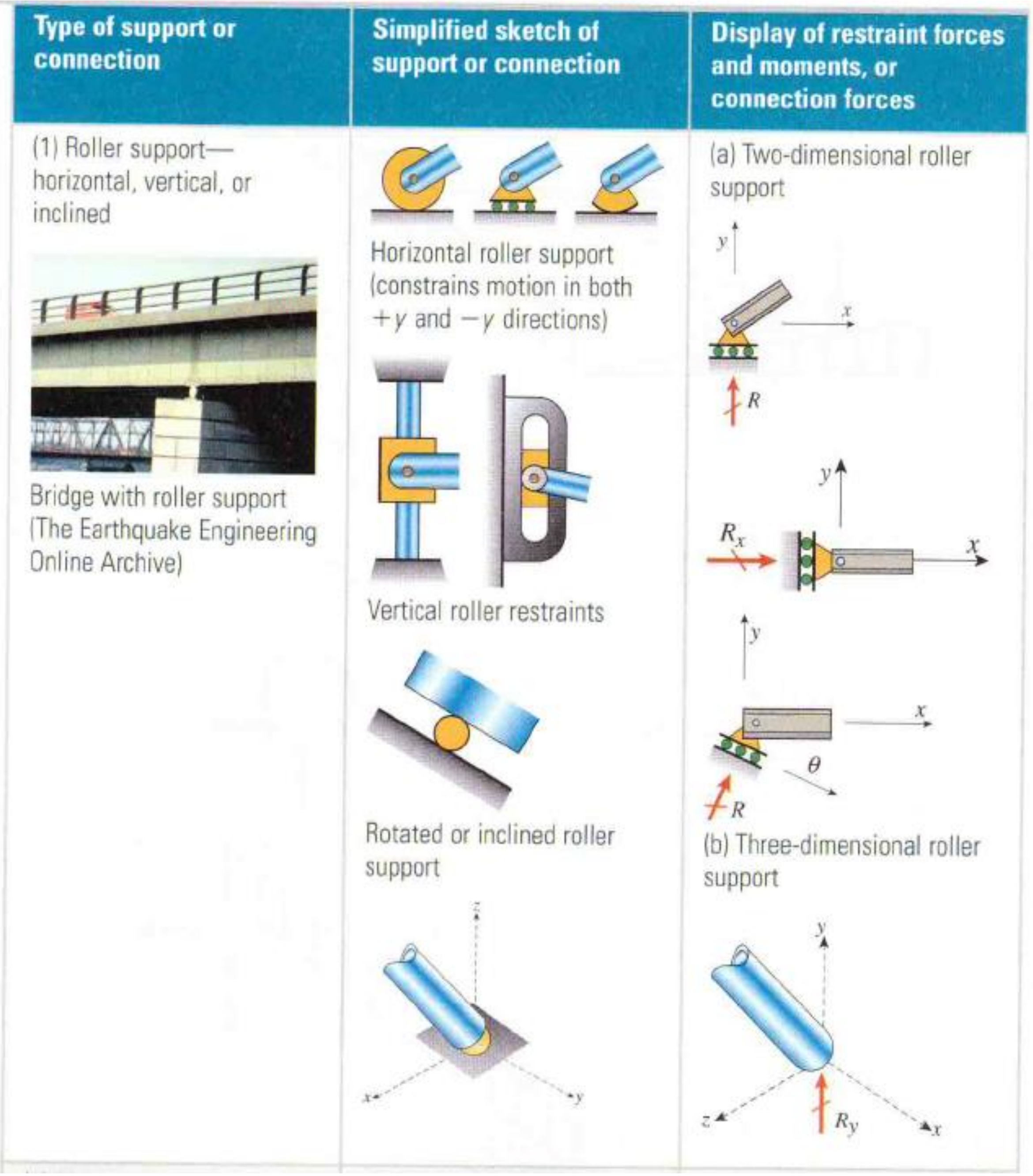
This is a very important and useful tool in most engineering disciplines!

Pictorial representation to analyze forces acting on a body of interest – which could be part of the structure or even an “infinitesimal” element (with dimension -> 0).

Use equilibrium to relate forces and moments acting on the FB.

Static Review

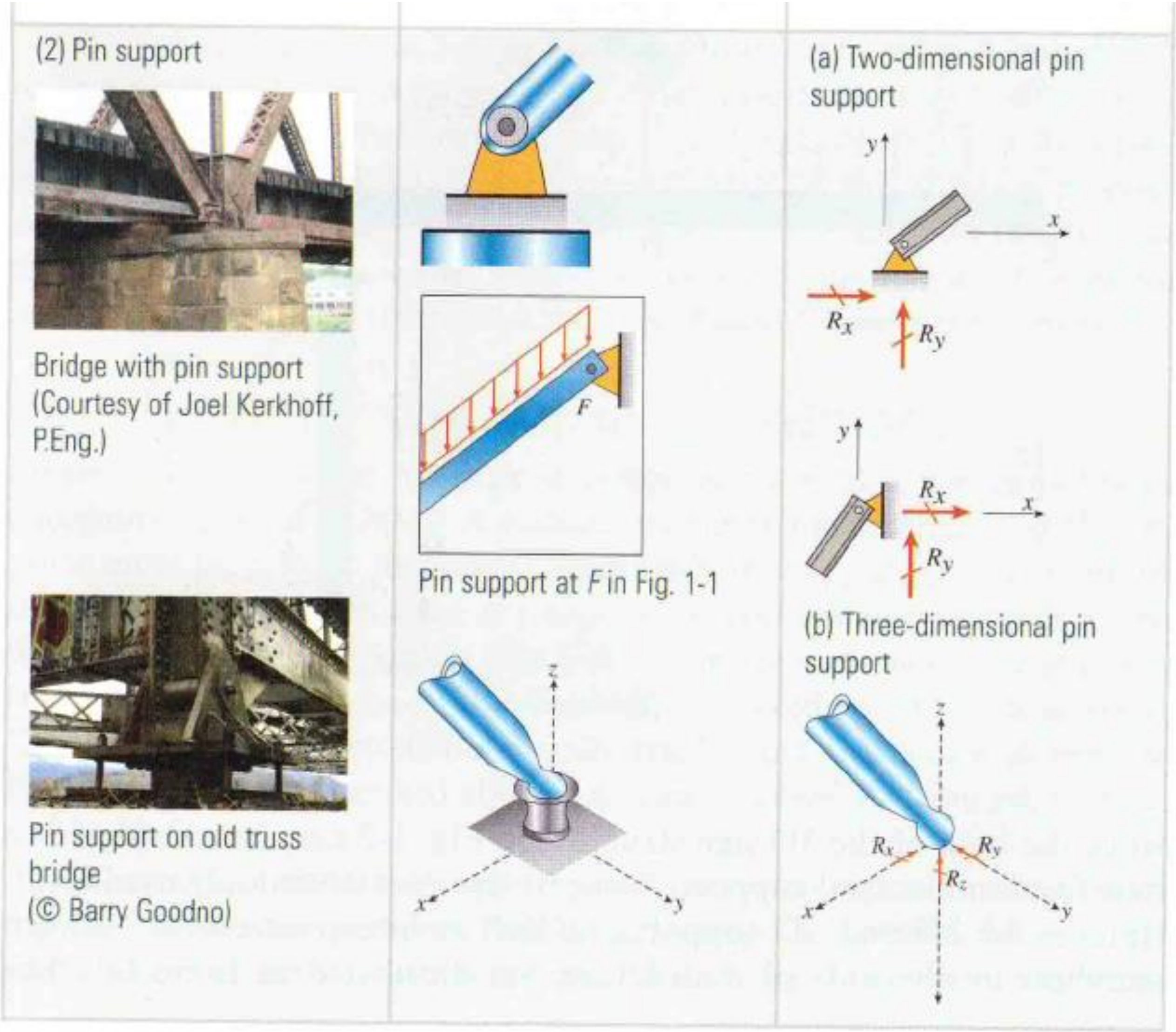
Roller support



Type of support or connection	Simplified sketch of support or connection	Display of restraint forces and moments, or connection forces
(1) Roller support—horizontal, vertical, or inclined Bridge with roller support (The Earthquake Engineering Online Archive)	Horizontal roller support (constrains motion in both $+y$ and $-y$ directions) Vertical roller restraints Rotated or inclined roller support	(a) Two-dimensional roller support (b) Three-dimensional roller support

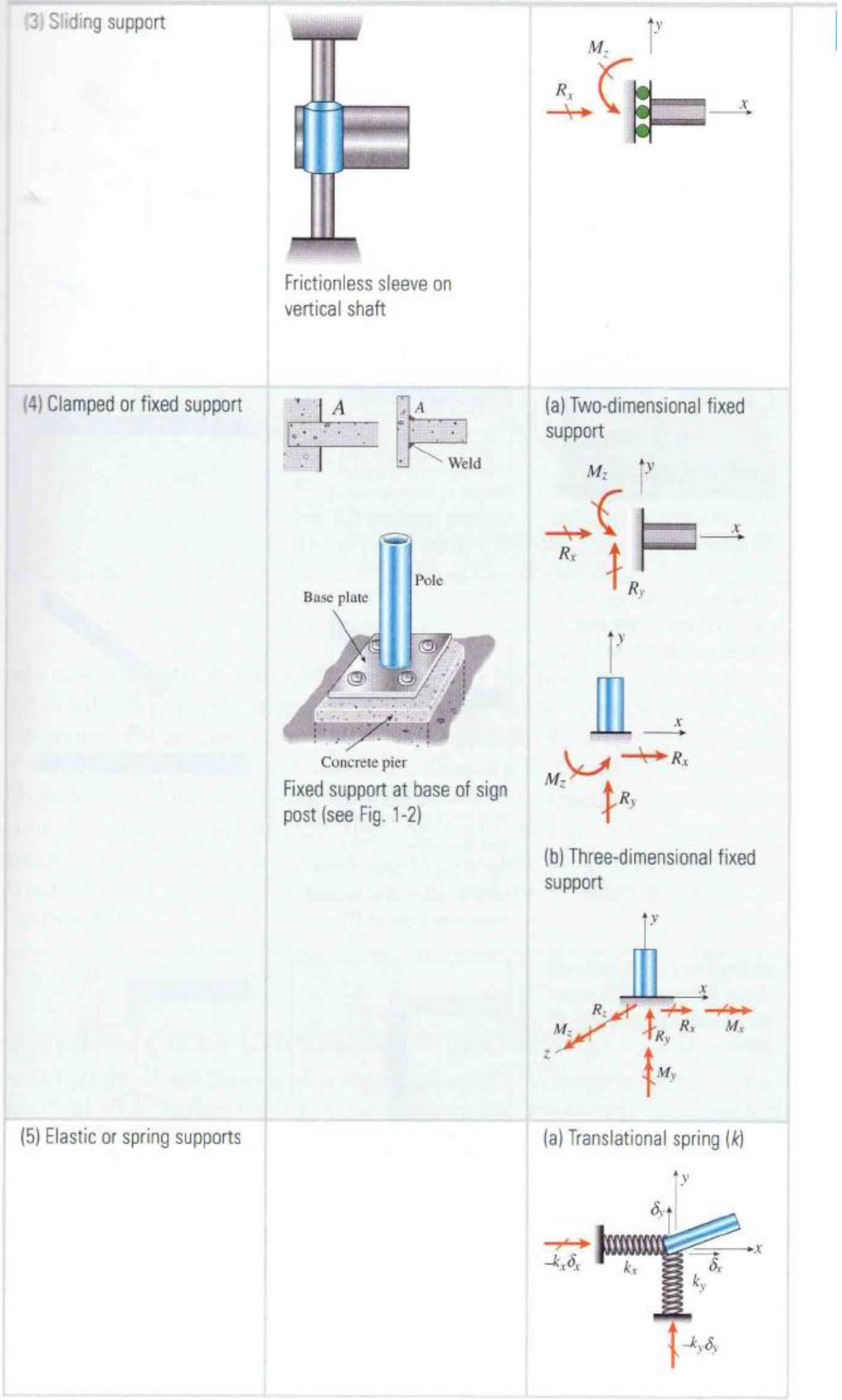
Static Review

Pin support



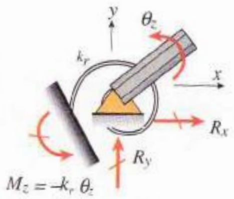
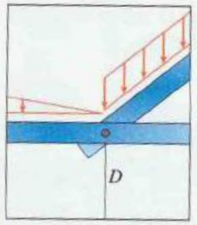
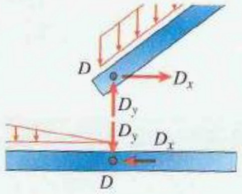
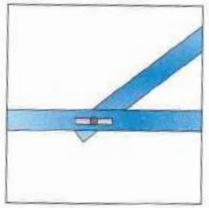
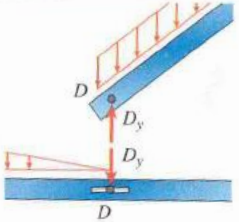
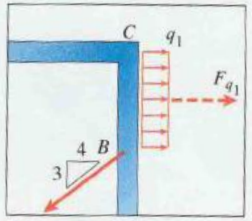
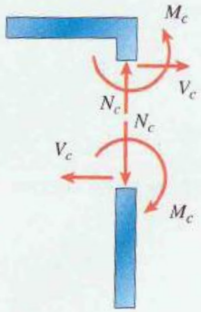
Static Review

Other supports



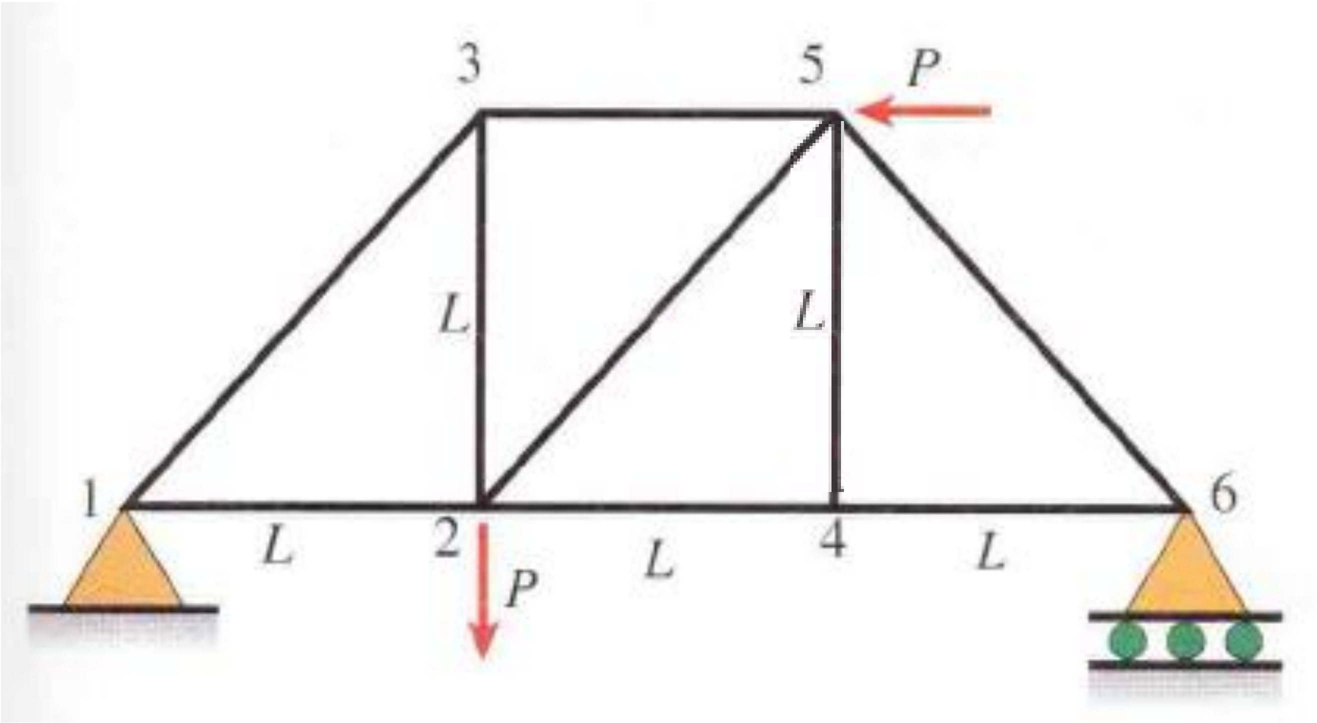
Static Review

Other supports

		(b) Rotational spring (k_r) 
(6) Pinned connection (from Figs. 1-1 and 1-3)  Pin connection on old bridge (© Barry Goodno)	 Pinned connection at *D* between members *EDC* and *DF* in plane frame (Fig. 1-1)	
(7) Slotted connection *(modified connection from that shown in Figs. 1-1 and 1-3)*	 Alternate slotted connection at *D* on plane frame (Note that the plane frame in Fig. 1-1 is *unstable* if this slotted connection is used instead of a pin at *D*.)	
(8) Rigid connection *(internal forces and moment in members joined at C of plane frame in Fig. 1-1)*	 Rigid connection at *C* on plane frame	

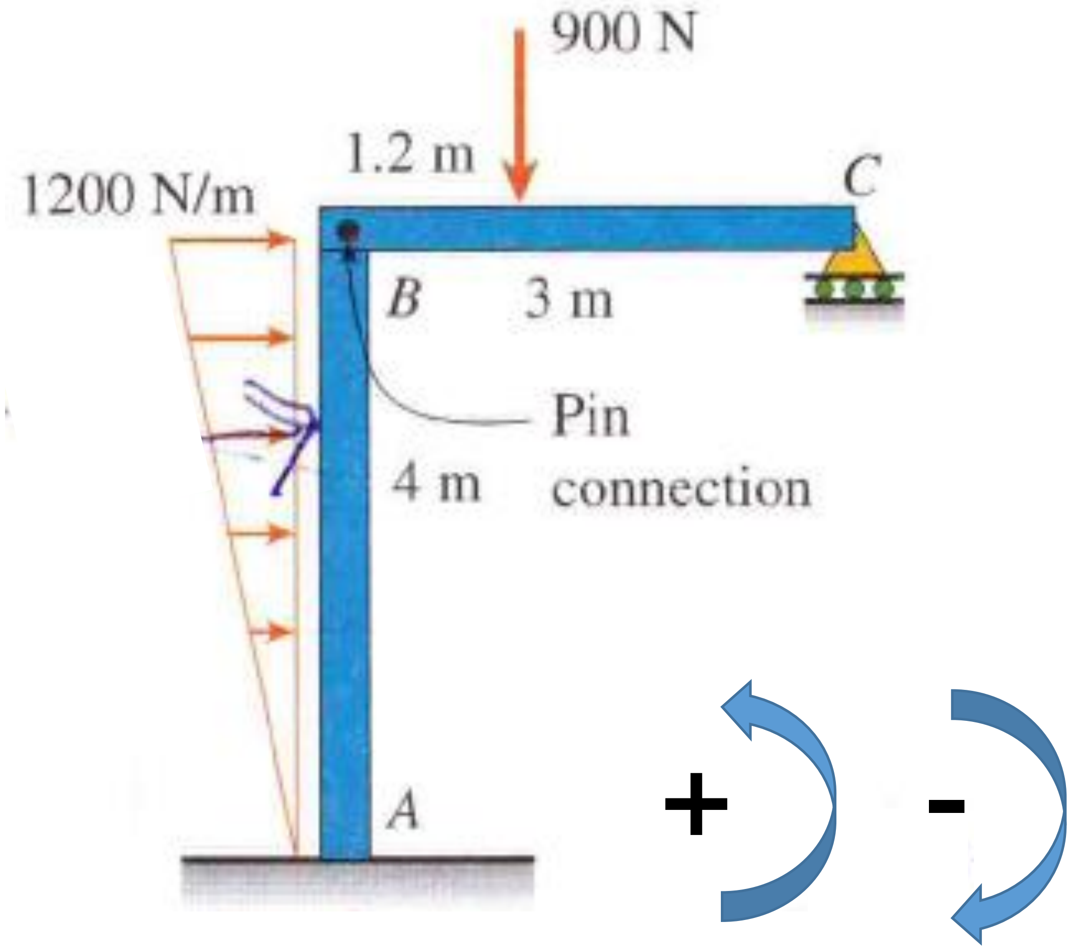
Static Review

A plane truss has downward applied load P at joint 2 and another load P applied leftward at joint 5. The force in member 4-5 is:



Static Review

The moment reaction at A in the plane frame below is approximately:



Static Review

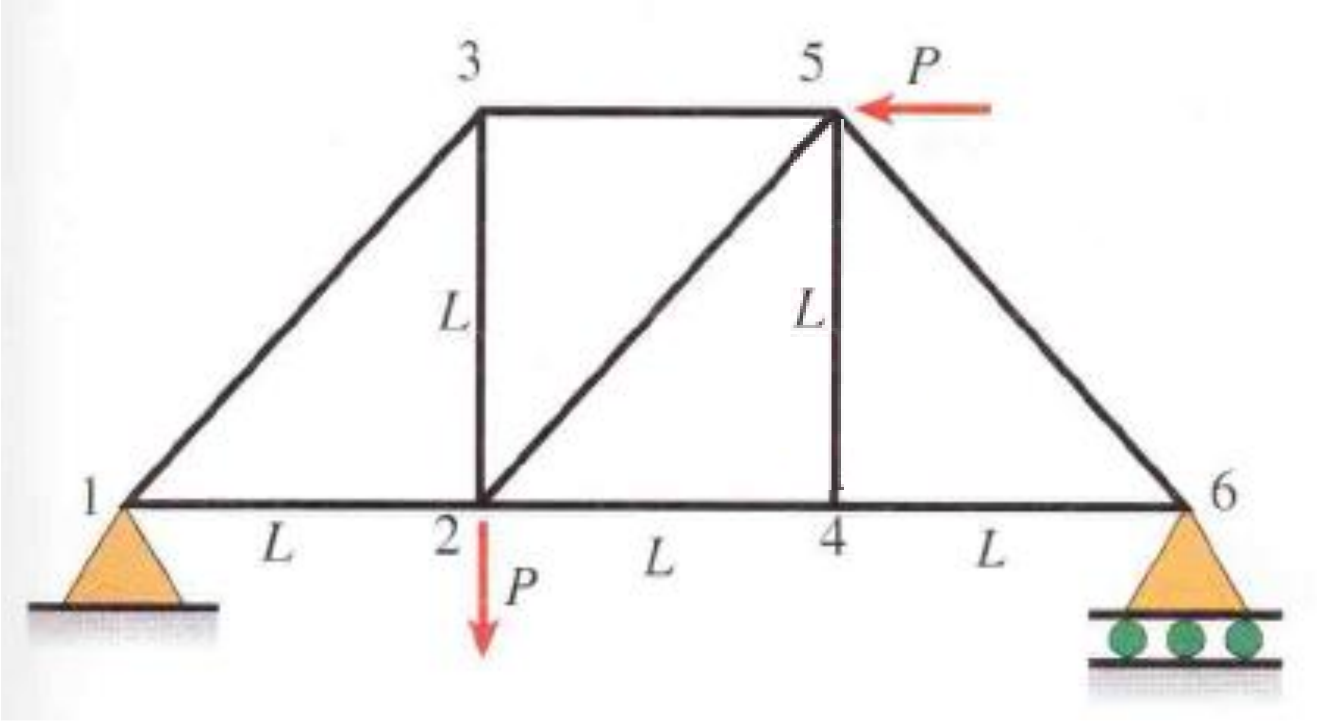
A plane truss has downward applied load P at joint 2 and another load P applied leftward at joint 5. The force in member 4-5 is:

(A) 0

(B) –P/2

(C) -P

(D) + 1.5 P



Static Review

The moment reaction at A in the plane frame below is approximately:

(A) +1400 N.m

(B) -2280 N.m

(C) -3600 N.m

(D) +6400 N.m