Rock Mechanics & Experiment
암석역학 및 실험

Lecture 5. Friction and Failure Criteria
Lecture 5. 암석의 마찰 및 파괴조건

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Stress
deviatoric stress/stress invariant

- Deviatoric stress

$$s_{ij}=\sigma_{ij}-\frac{\sigma_{kk}}{3}\delta_{ij},$$

$$\begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} - \begin{bmatrix} \pi & 0 & 0 \\ 0 & \pi & 0 \\ 0 & 0 & \pi \end{bmatrix}$$

$$= \begin{bmatrix} \sigma_{11}-\pi & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22}-\pi & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33}-\pi \end{bmatrix}.$$

- Stress invariant

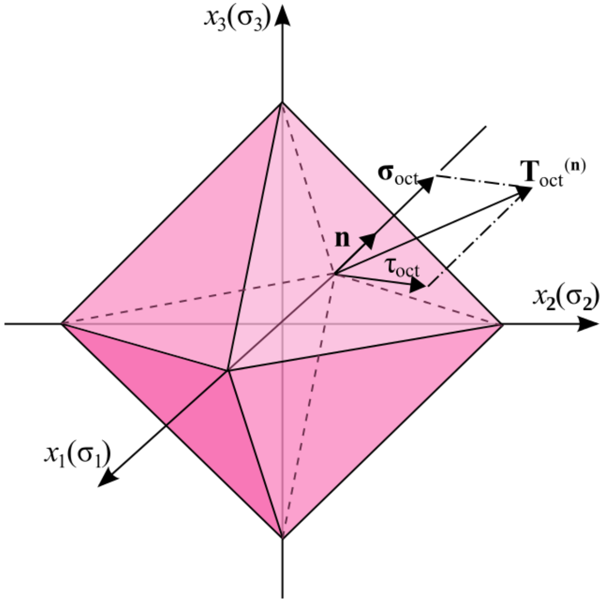
$$\begin{aligned} I_1 &= \sigma_x+\sigma_y+\sigma_z \\ I_2 &= \sigma_x\sigma_y+\sigma_y\sigma_z+\sigma_z\sigma_x-\tau_{xy}^2-\tau_{yz}^2-\tau_{zx}^2 \\ I_3 &= \sigma_x\sigma_y\sigma_z+2\tau_{xy}\tau_{yz}\tau_{zx}-\sigma_x\tau_{yz}^2-\sigma_y\tau_{zx}^2-\sigma_z\tau_{xy}^2 \end{aligned}$$

$$\begin{aligned} I_1 &= \sigma_1+\sigma_2+\sigma_3 \\ I_2 &= \sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1 \\ I_3 &= \sigma_1\sigma_2\sigma_3 \end{aligned}$$

$$\begin{aligned} J_1 &= s_{kk}=0, \\ J_2 &= \tfrac{1}{2}s_{ij}s_{ji}=\tfrac{1}{2}\mathrm{tr}(\boldsymbol{s}^2) \\ &= \tfrac{1}{2}(s_1^2+s_2^2+s_3^2) \\ &= \tfrac{1}{6}\left[(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2\right]+\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2 \\ &= \tfrac{1}{6}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \\ &= \tfrac{1}{3}I_1^2-I_2=\frac{1}{2}\left[\mathrm{tr}(\boldsymbol{\sigma}^2)-\frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})^2\right], \\ J_3 &= \det(s_{ij}) \\ &= \tfrac{1}{3}s_{ij}s_{jk}s_{ki}=\tfrac{1}{3}\mathrm{tr}(\boldsymbol{s}^3) \\ &= s_1s_2s_3 \\ &= \tfrac{2}{27}I_1^3-\tfrac{1}{3}I_1I_2+I_3=\tfrac{1}{3}\left[\mathrm{tr}(\boldsymbol{\sigma}^3)-\mathrm{tr}(\boldsymbol{\sigma}^2)\mathrm{tr}(\boldsymbol{\sigma})+\tfrac{2}{9}\mathrm{tr}(\boldsymbol{\sigma})^3\right]. \end{aligned}$$

Stress
Octahedral stress

- Octahedral stress



$$\begin{aligned}\sigma_{\text{oct}} &= T_i^{(n)} n_i \\ &= \sigma_{ij} n_i n_j \\ &= \sigma_1 n_1 n_1 + \sigma_2 n_2 n_2 + \sigma_3 n_3 n_3 \\ &= \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) = \tfrac{1}{3} I_1\end{aligned}$$

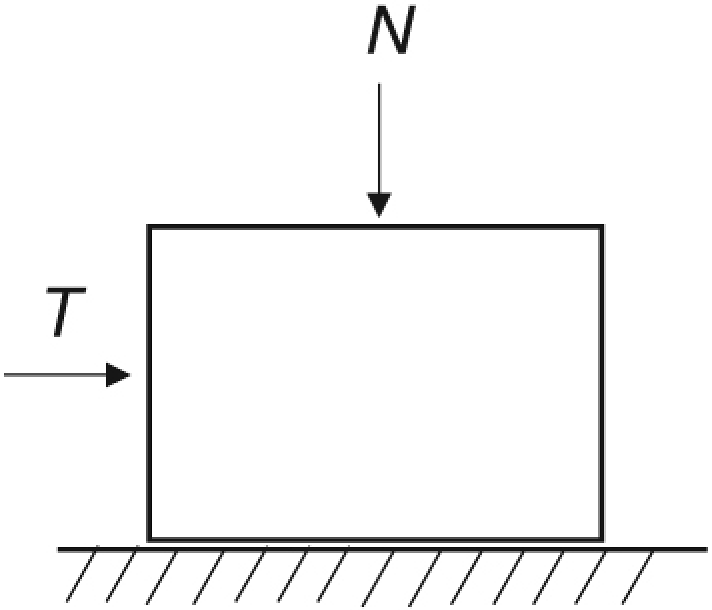
$$\begin{aligned}\tau_{\text{oct}} &= \sqrt{T_i^{(n)} T_i^{(n)} - \sigma_{\text{n}}^2} \\ &= \left[\tfrac{1}{3}(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - \tfrac{1}{9}(\sigma_1 + \sigma_2 + \sigma_3)^2\right]^{1/2} \\ &= \tfrac{1}{3}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2} = \tfrac{1}{3}\sqrt{2I_1^2 - 6I_2} = \sqrt{\tfrac{2}{3} J_2}\end{aligned}$$

Outline
Mechanical behavior of intact rock

- Friction on rock surface
- Stick-slip oscillation
- Coulomb Failure criterion
- Mohr-Coulomb Failure Criterion
- Hoek-Brown Failure Criterion
- Other Failure Criterion
- Anisotropic rock behavior

Failure Criteria
Friction on rock surface - Friction coefficient

- Friction
  - Phenomenon by which a tangential shearing force is required in order to displace two contacting surfaces along a direction parallel to their nominal contact plane
  - Importance: friction between grains, fracture and fault



$$\tau = \mu\sigma$$

τ : shear stress

σ : normal stress

μ : coefficient of friction

Also called ‘friction angle’. Why?

$$\tau = \mu_d\sigma$$

μ_d : coefficient of dynamic friction

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

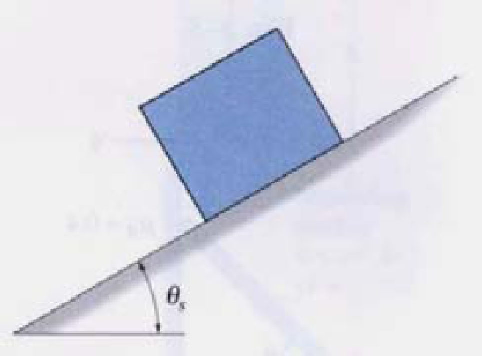
Failure Criteria
Friction on rock surfaces - Friction coefficient

- Friction angle

DETERMING μ_s EXPERIMENTALLY

$$\mu = \tan\phi$$

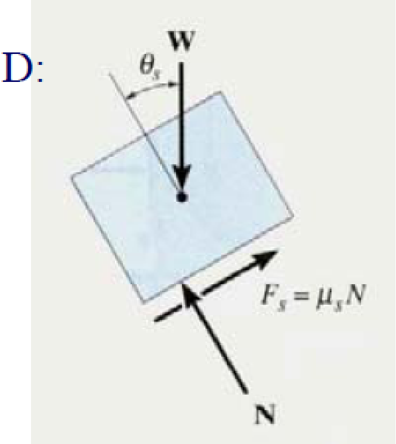
$$\phi = \text{friction angle}$$



A block with weight **W** is placed on an inclined plane. The plane is slowly tilted until the block just begins to slip.

The inclination, θ_s, is noted. Analysis of the block just before it begins to move gives (using $F_s = \mu_s N$):

FBD:



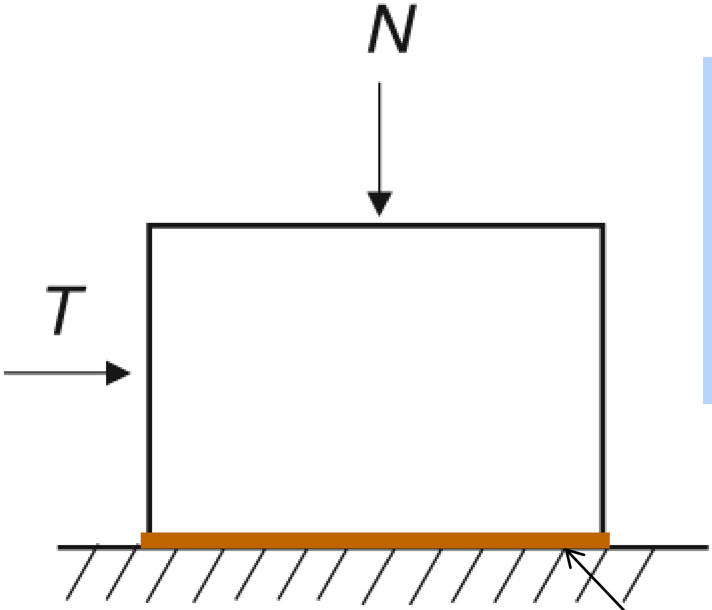
$$\nwarrow + \quad \sum F_y = N - W\cos\theta_s = 0$$

$$\nearrow + \quad \sum F_x = \mu_s N - W\sin\theta_s = 0$$

Using these two equations, we get $\mu_s = (\mathbf{W}\sin\theta_s) / (\mathbf{W}\cos\theta_s) = \tan\theta_s$
This simple experiment allows us to find the μ_s between two materials in contact.

Failure Criteria
Friction on rock surface - Friction coefficient

- Coulomb failure criterion (on fractures)



$$|\tau| = S_0 + \mu\sigma = S_0 + \sigma\tan\phi$$

S_0 : cohesion (often, c is used), or 'shear strength'

ϕ: friction angle

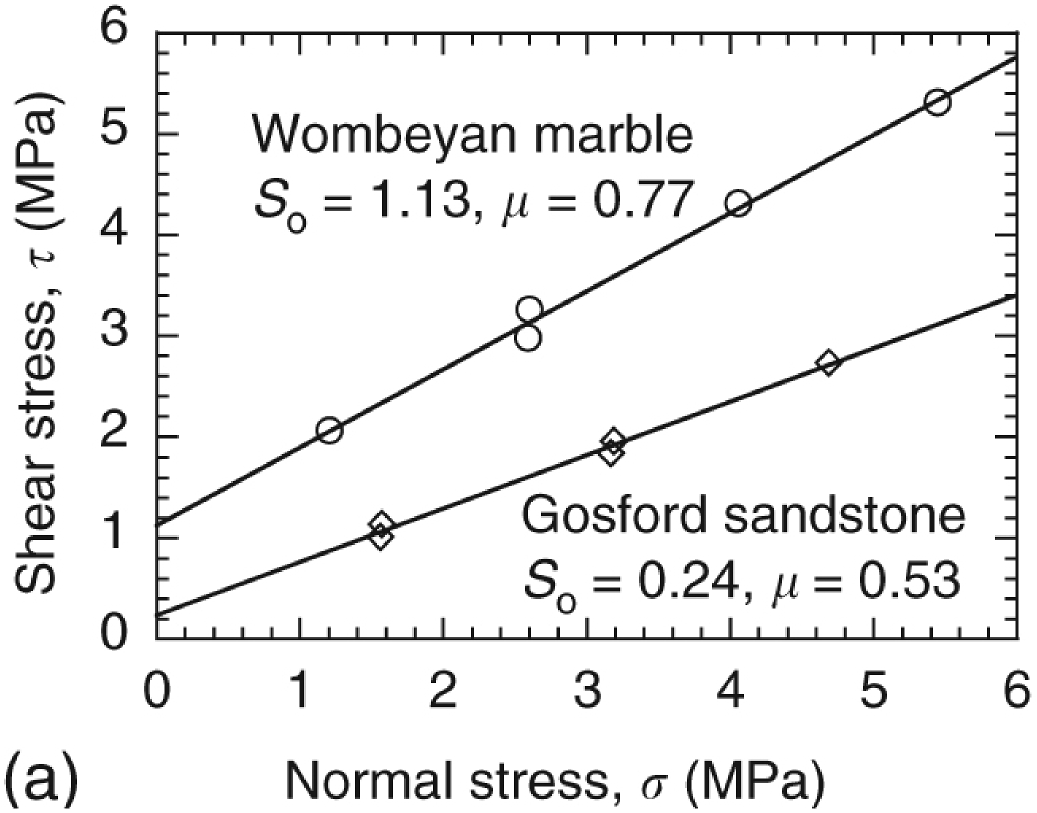
μ : coefficient of friction angle

Glue or something

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Failure Criteria
Sliding on a plane of weakness

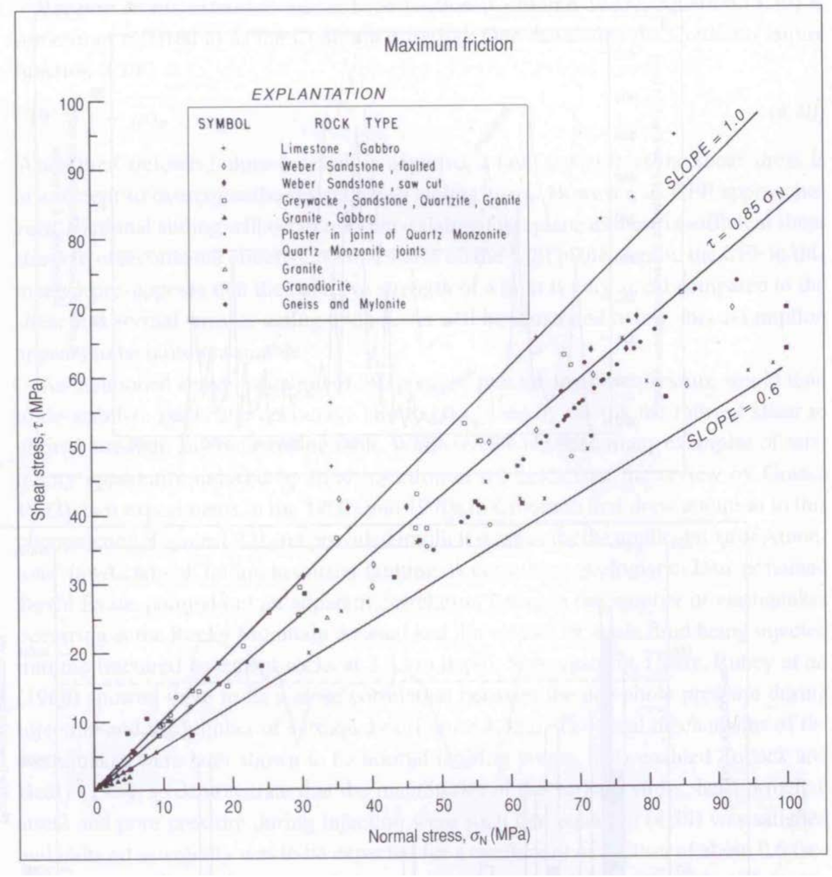
- Example



Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

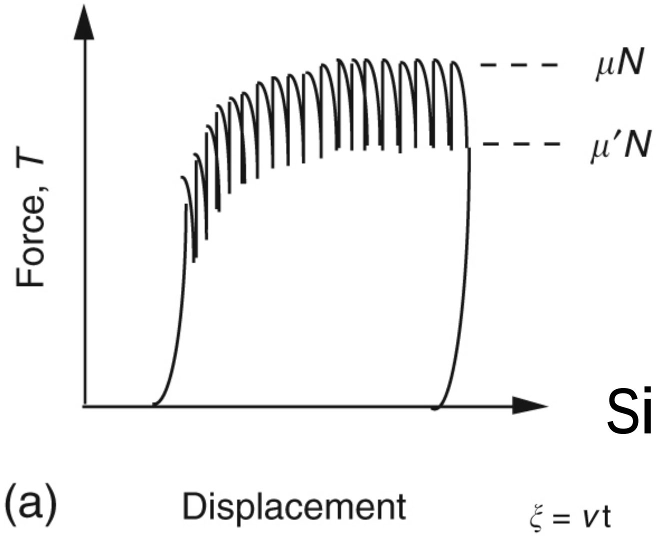
Failure Criteria
Friction on rock surfaces - Friction coefficient

- Typical Range of Friction coefficient (Byerlee, 1978)
  - 0.6 ~ 1.0
  - Wider variability in low normal stress



Failure Criteria
Friction on rock surfaces – Stick-slip oscillation

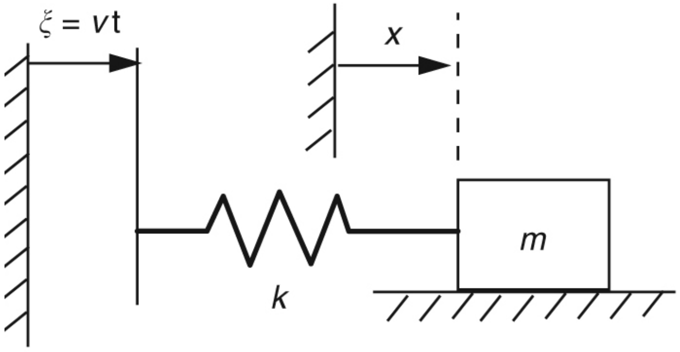
- Stick-slip oscillation
  - May provide a mechanism for earthquakes

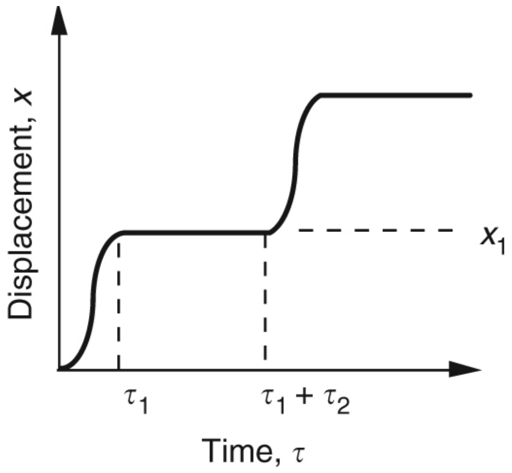


← Static friction coefficient

← Dynamic friction coefficient

Simple model





Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Failure Criteria

Sliding on a plane of weakness

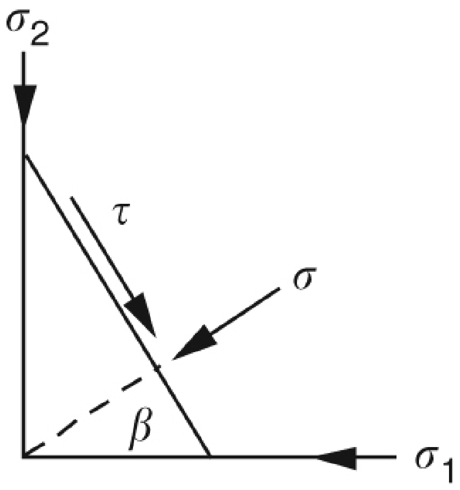
- Coulomb Failure criteria of a fracture (plane of weakness)

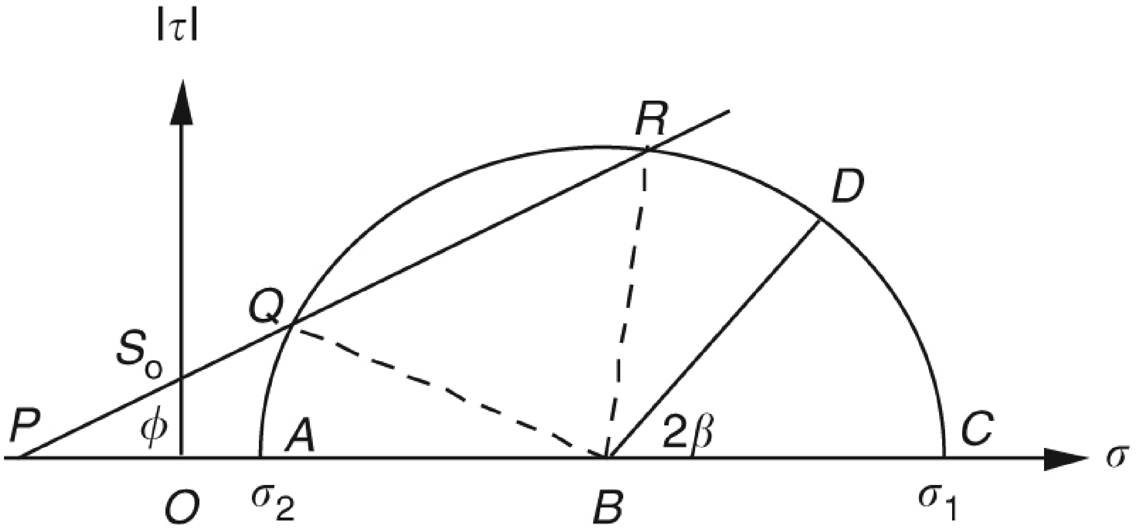
$$|\tau| = S_0 + \mu\sigma = S_0 + \sigma\tan\phi$$

S_0 : cohesion (often, c is used), or 'shear strength'

ϕ : friction angle

μ : coefficient of friction





Failure Criteria

Sliding on a plane of weakness

- The stress difference that is required to cause a slip with a given β and σ_2

$$\sigma_1 - \sigma_2 = \frac{2\left(S_0 + \mu\sigma_2\right)}{\left(1-\mu\cot\beta\right)\sin\beta}$$

$$\sigma_1 = \frac{2S_0\cos\phi}{(1-k)\sin(2\beta-\phi)-(1+k)\sin\phi}$$

$$\sigma_{1,\min} = \sigma_2 + 2\left(S_0 + \mu\sigma_2\right)\left(\sqrt{\mu^2+1}+\mu\right)$$

- Solution exists only for

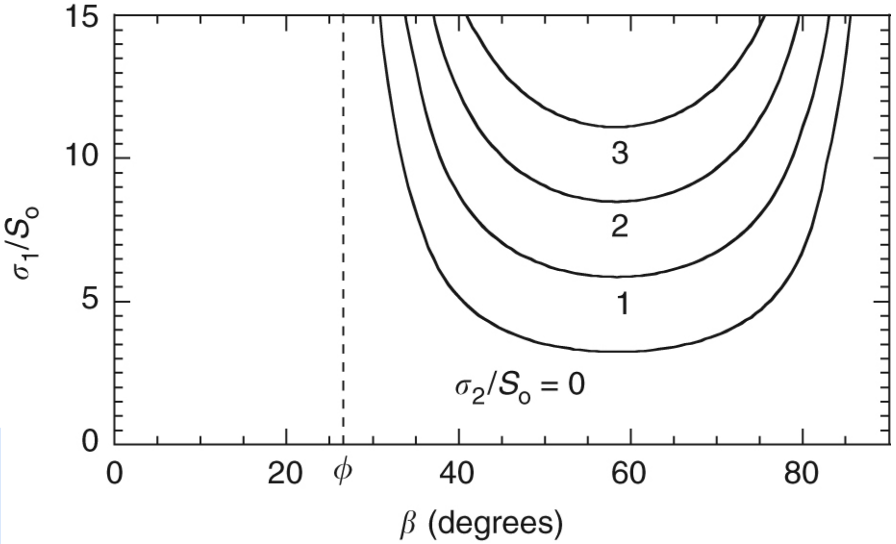
$$\phi < \beta$$

- Range of β (for a given stress state)

$$\beta_1 < \beta < \beta_2$$

$$2\beta_1 = \phi + \sin^{-1}\left[\left\{\left(\sigma_m + S_0\cot\phi\right)/\tau_m\right\}\sin\phi\right]$$

$$2\beta_2 = \pi + \phi - \sin^{-1}\left[\left\{\left(\sigma_m + S_0\cot\phi\right)/\tau_m\right\}\sin\phi\right]$$



Variation of σ_1 needed to cause sliding on a fracture for μ=0.5

Failure Criteria
Failure of intact rock

- Coulomb Failure Criterion (on a rock) (or Mohr-Coulomb Failure Criterion)

$$|\tau| = S_0 + \mu\sigma = S_0 + \sigma\tan\phi$$

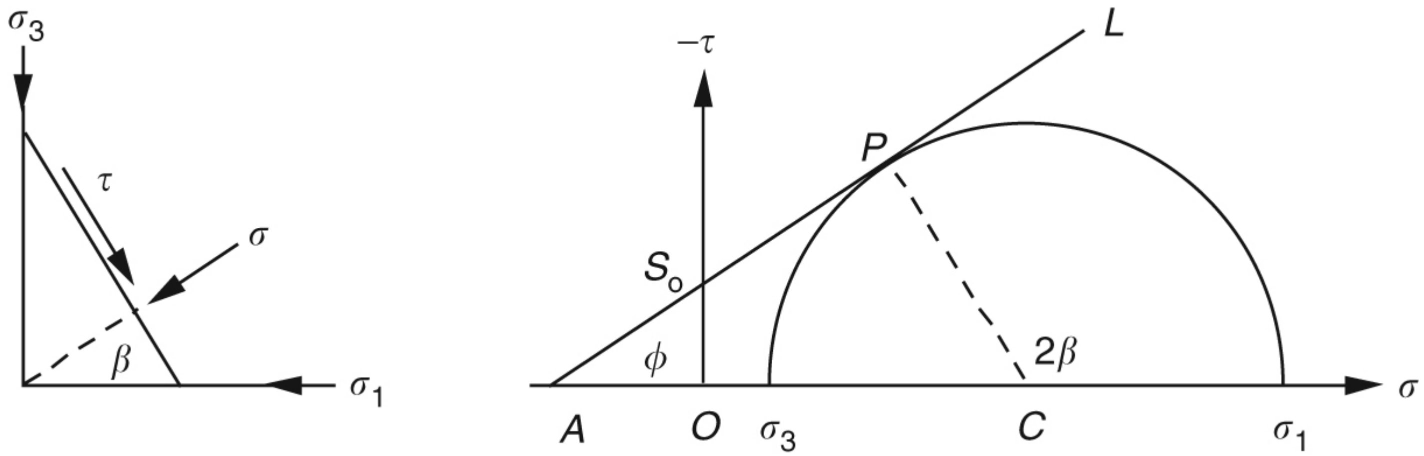
S_0 : cohesion (often, c is used), or 'shear strength'

ϕ:internal friction angle

μ : coefficient of internal friction angle

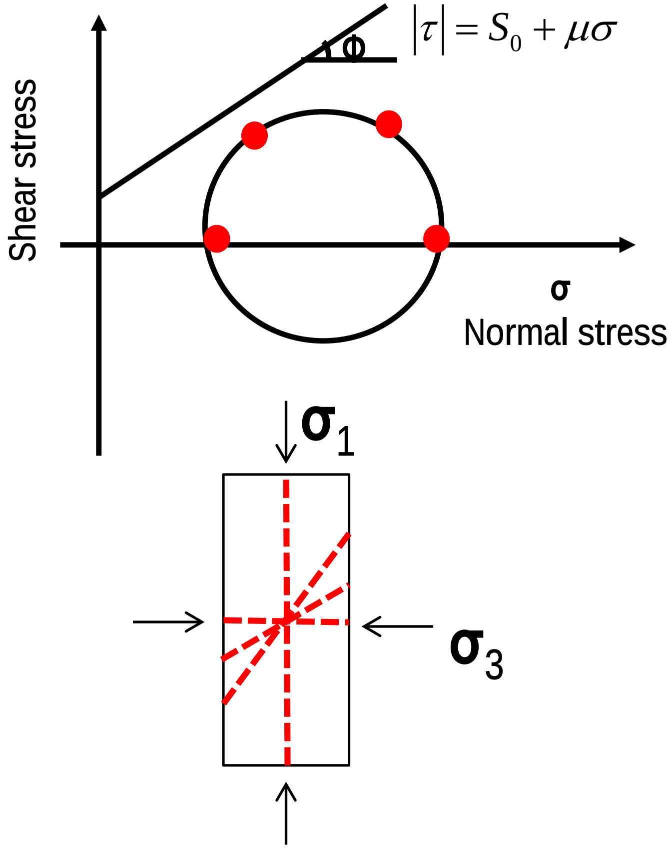
Same equation with different notation

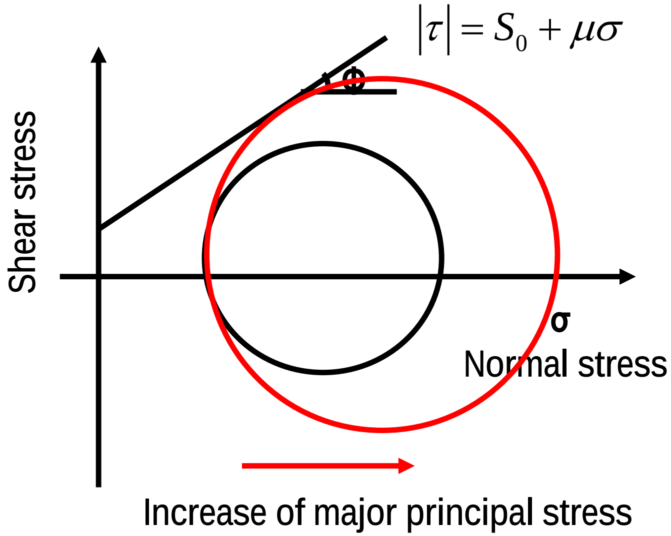
$$|\tau| = c + \mu\sigma$$



Failure Criteria
Failure of intact rock

- Conditions for failure
  - A set of normal and shear stress within a rock must satisfy failure criterion





Failure Criteria
Mohr-Coulomb Failure criteria (Example)

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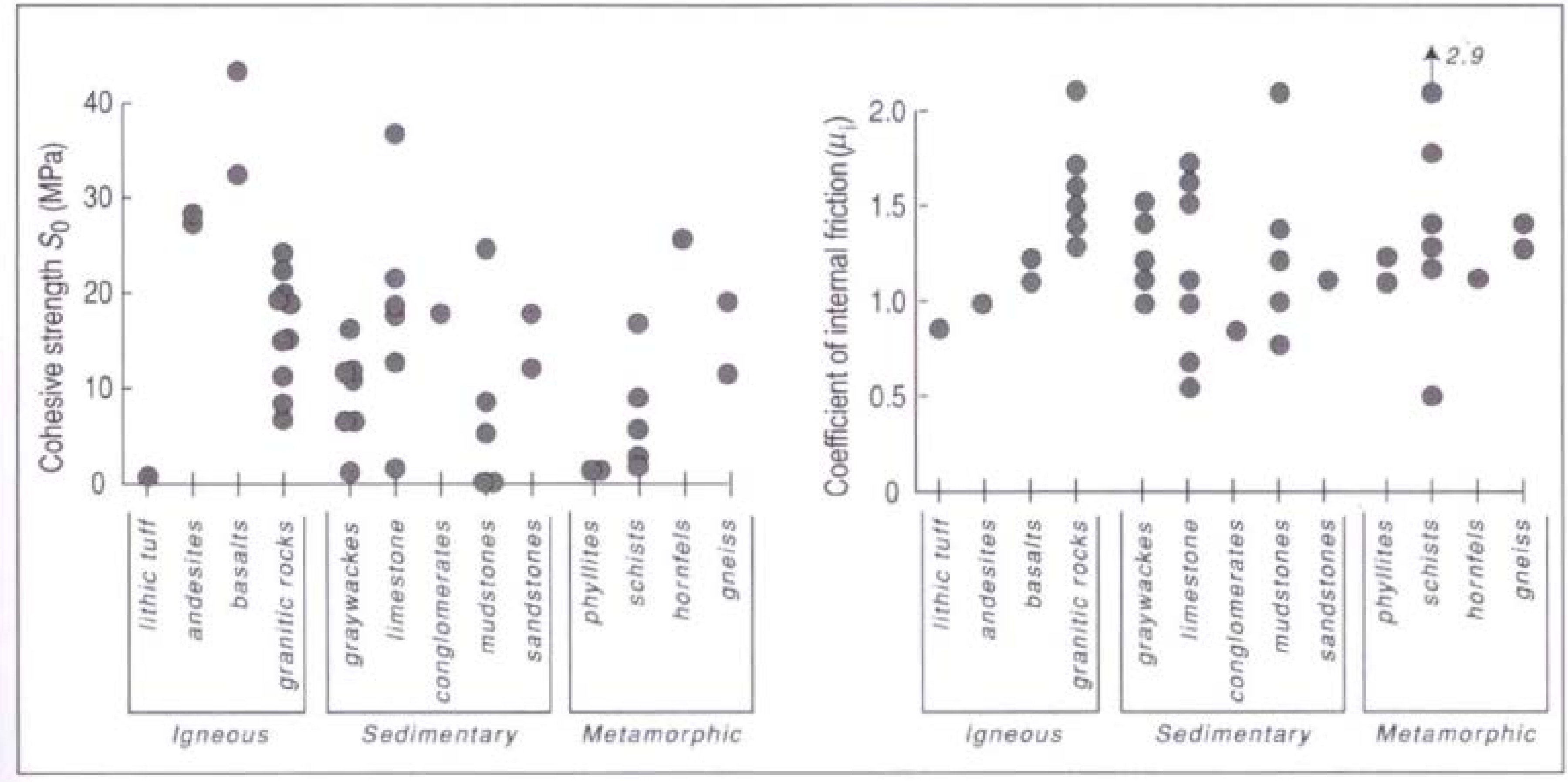
- Examples of measured cohesive strength (cohesion) and coefficient of internal friction



Zoback, 2007

Failure Criteria
Sliding on a plane of weakness

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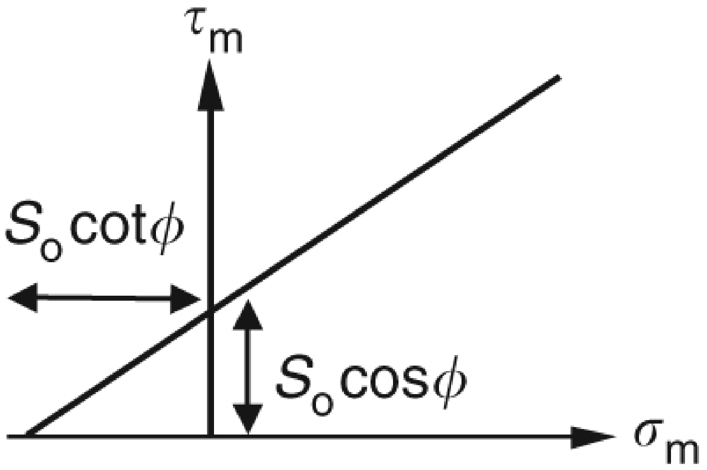
- Different expression (1)

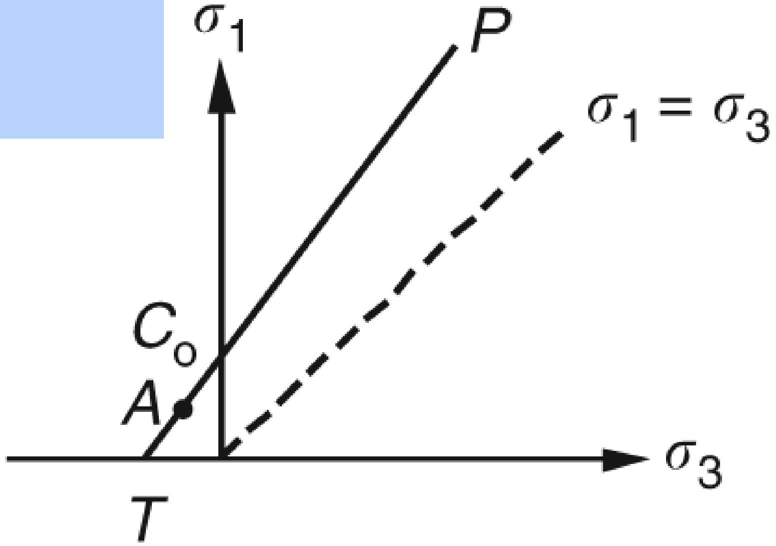
$$\sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3) \ \text{mean normal,} \ \tau_m = \frac{1}{2}(\sigma_1 - \sigma_3) \ \text{maximum shear}$$

$$\tau_m = S_0 \cos\phi + \sigma_m \sin\phi$$



- Different expression (2)

$$\sigma_1 = 2S_0 \tan\beta + \sigma_3 \tan^2\beta = C_0 + \sigma_3 \tan^2\beta = C_0 + \sigma_3 \tan^2(45 + \phi/2)$$
$$= 2S_0\left[\left(1+\mu^2\right)^{1/2} + \mu\right] + \sigma_3\left[\left(1+\mu^2\right)^{1/2} + \mu\right]^2$$



$$C_0 = 2S_0 \tan\beta = 2S_0\left[\left(1+\mu^2\right)^{1/2} + \mu\right]$$

C_0:uniaxial compressive strength

Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Failure Criteria
Effect of pore pressure

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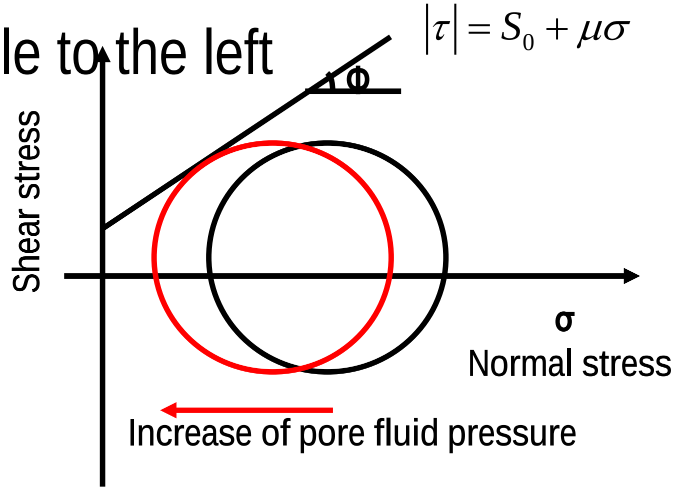
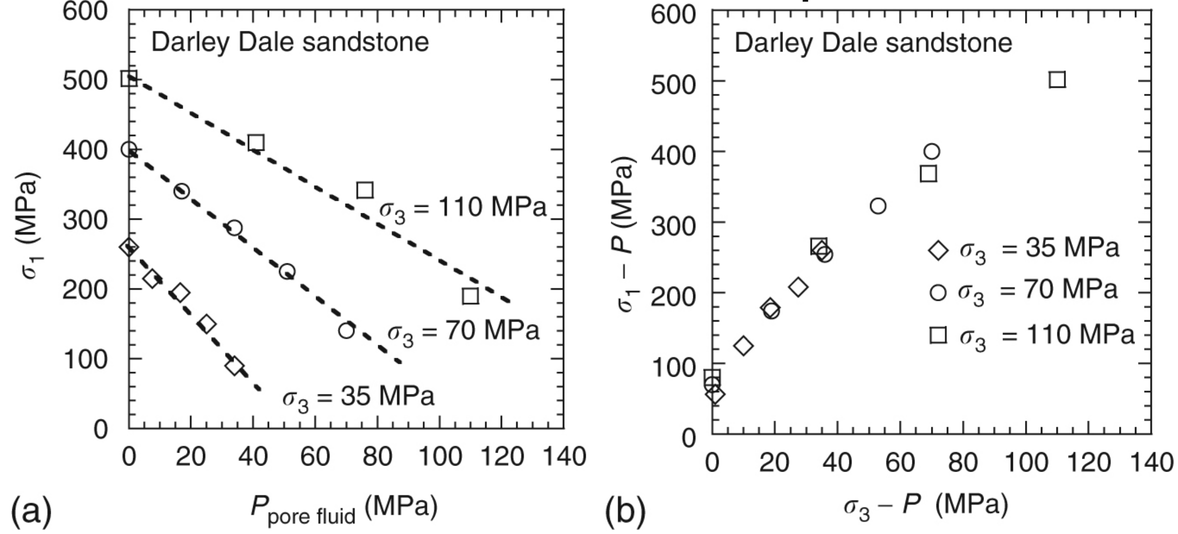
- Mechanical effect
  - Pore pressure translate the Mohr's circle to the left



- Chemical interactions
  - between rock and the fluid

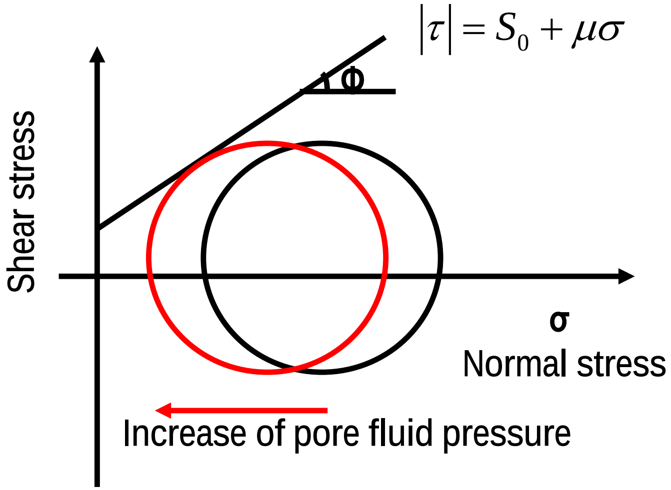


Jaeger, Cook and Zimmerman, 2007, Fundamentals of Rock Mechanics, 4th ed., Blackwell Publishing

Failure Criteria
Effect of pore pressure

- Required pore pressure to induce fracture with a given stress condition;

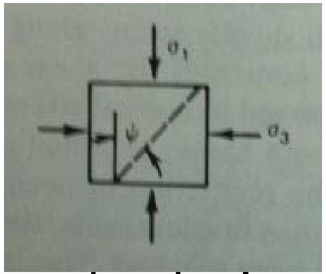
$$p_w = \sigma_3 - \frac{(\sigma_1 - \sigma_3) - C_0}{\tan^2(45 + \frac{\phi}{2}) - 1}$$

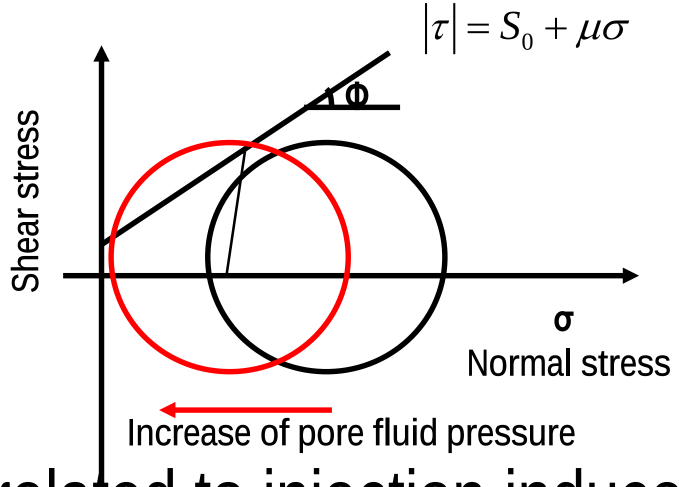


Failure Criteria
Effect of pore pressure

- Required pore pressure to induce sliding of a given fracture with a specific orientation under a specific stress condition;

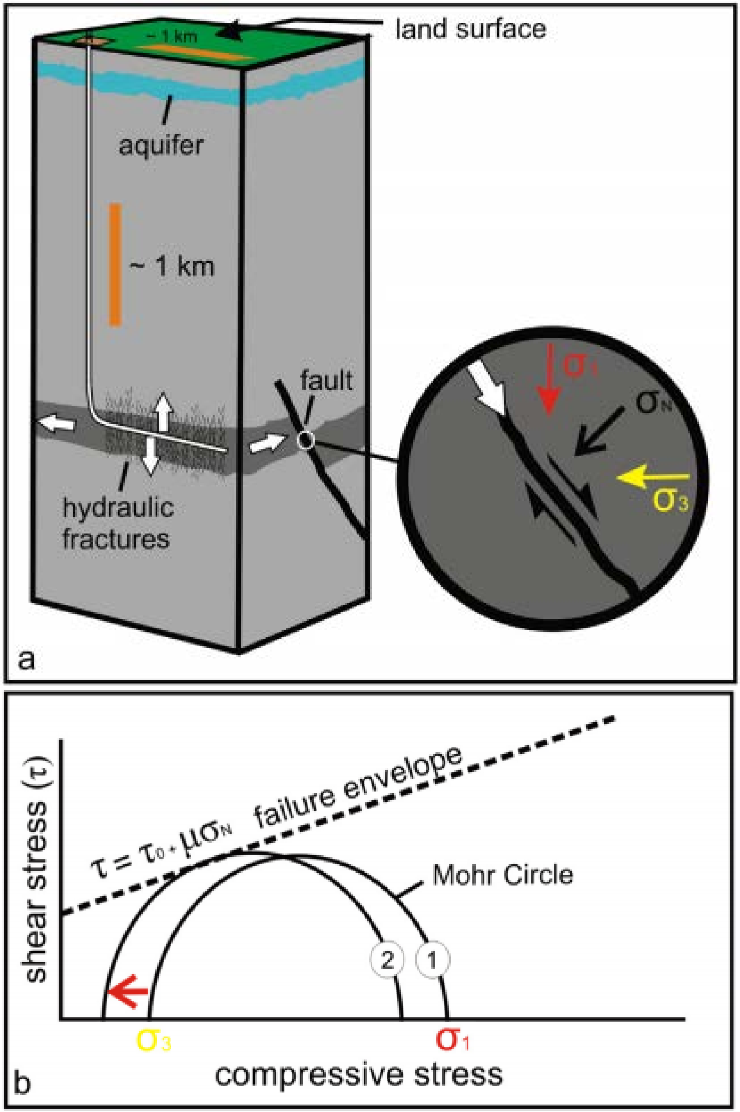
$$p_w = \frac{S_0}{\tan\phi} + \sigma_3 + (\sigma_1 - \sigma_3)\left(\sin^2\theta - \frac{\sin\theta\cos\theta}{\tan\phi}\right)$$





- Extremely important phenomenon related to injection induced microearthquake

Microseismic event 미소진동이란?
메커니즘



(Richard Davies, 2013)

미소 진동 발생 원인

(a) 공극압 증가: 유효 응력 감소

(b) 지하 암반의 온도 변화

(c) 유체 생산/주입

(d) 화학 물질 첨가: 마찰계수 변화

⬇

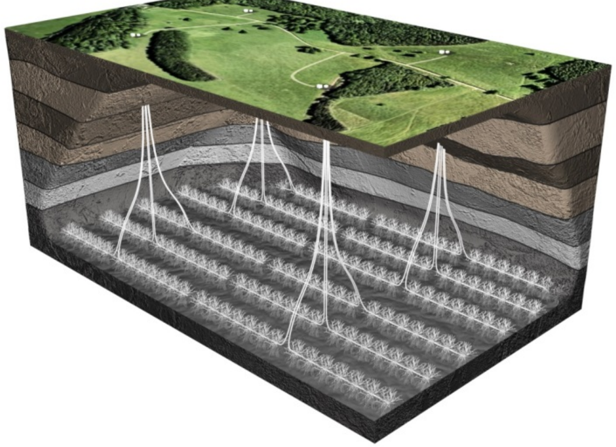
지하 암반 내 유효 응력의 감소/증가

다양한 지중 에너지 관련 응용분야에서 미소진동 발생 가능

스웨덴 Aitik 광산, 민기복, 2012

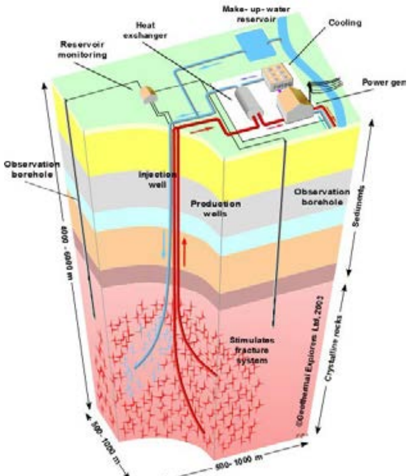
광물자원개발

http://www.statoil.com/en/NewsAndMedia/News/2010/Pages/26MarMarcellus.aspx

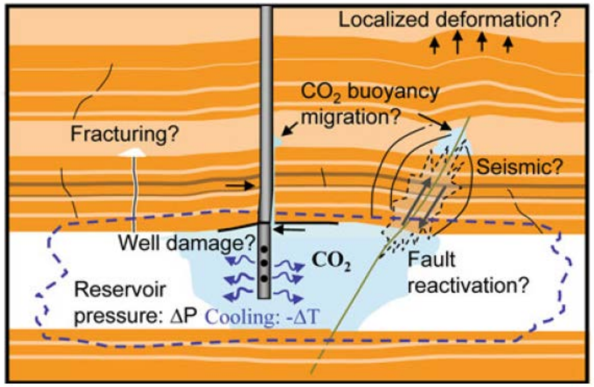


셰일가스 등 석유가스자원개발

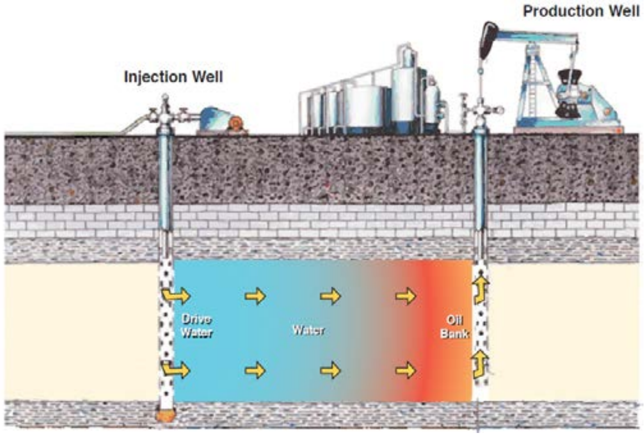
EGS 지열발전



Geothermal Explorer, 2010

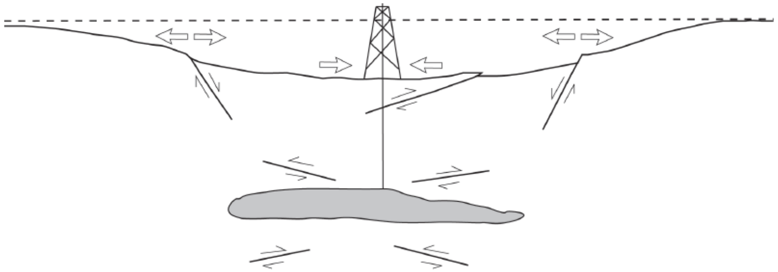


CO_2 지중저장



NRC, 2013

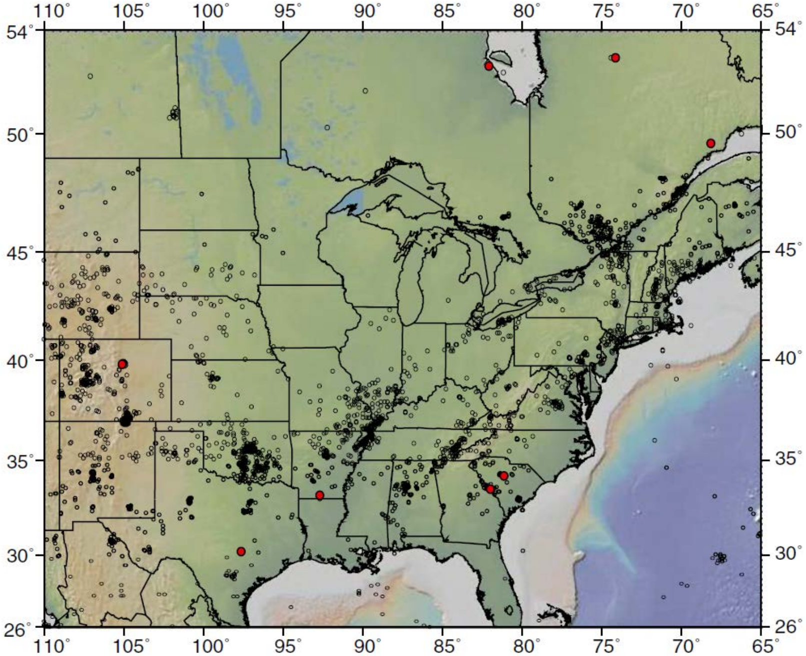
Enhanced Oil Recovery



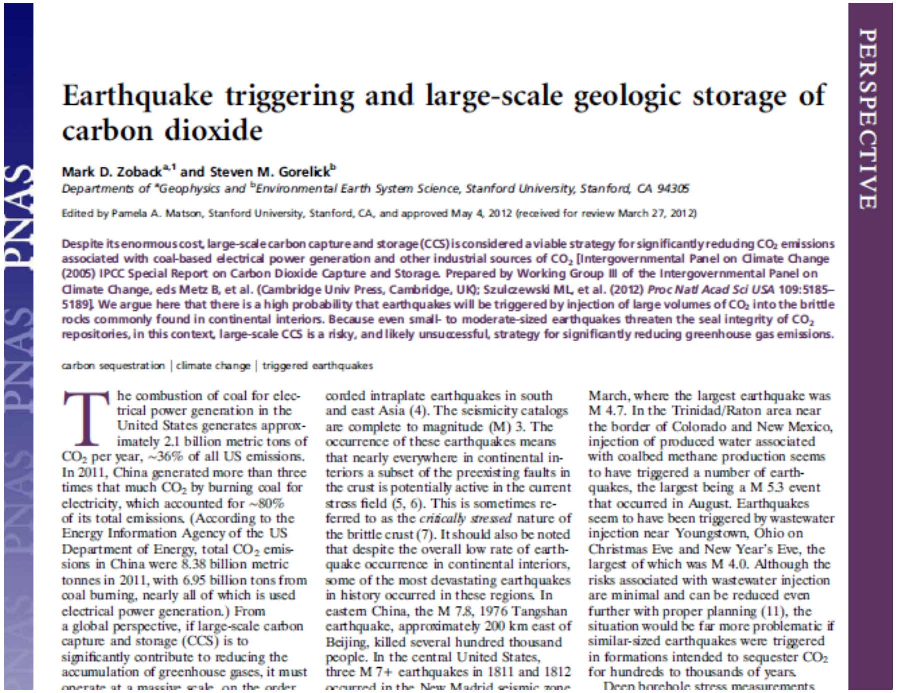
석유 가스 생산 (Segall, 1989)

주입에 의한 미소진동
이산화탄소 지중저장

- 이산화탄소 주입으로 인한 미소진동 및 이로 인한 누출로 인해 대규모 CCS는 성공할 가능성이 낮다* (스탠포드대학 Zoback교수의견)



규모 3 이상의 지진현황. • 는 인공지진



PNAS

PERSPECTIVE

Earthquake triggering and large-scale geologic storage of carbon dioxide

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Edited by Pamela A. Matson, Stanford University, Stanford, CA, and approved May 4, 2012 (received for review March 27, 2012)

Despite its enormous cost, large-scale carbon capture and storage (CCS) is considered a viable strategy for significantly reducing CO_2 emissions associated with coal-based electrical power generation and other industrial sources of CO_2 [Intergovernmental Panel on Climate Change (2005) IPCC Special Report on Carbon Dioxide Capture and Storage. Prepared by Working Group III of the Intergovernmental Panel on Climate Change, eds Metz B, et al. (Cambridge Univ Press, Cambridge, UK); Szulczewski ML, et al. (2012) *Proc Natl Acad Sci USA* 109:5185–5189]. We argue here that there is a high probability that earthquakes will be triggered by injection of large volumes of CO_2 into the brittle rocks commonly found in continental interiors. Because even small- to moderate-sized earthquakes threaten the seal integrity of CO_2 repositories, in this context, large-scale CCS is a risky, and likely unsuccessful, strategy for significantly reducing greenhouse gas emissions.

carbon sequestration | climate change | triggered earthquakes

The combustion of coal for electrical power generation in the United States generates approximately 2.1 billion metric tons of CO_2 per year, ~36% of all US emissions. In 2011, China generated more than three times that much CO_2 by burning coal for electricity, which accounted for ~80% of its total emissions. (According to the Energy Information Agency of the US Department of Energy, total CO_2 emissions in China were 8.38 billion metric tonnes in 2011, with 6.95 billion tons from coal burning, nearly all of which is used electrical power generation.) From a global perspective, if large-scale carbon capture and storage (CCS) is to significantly contribute to reducing the accumulation of greenhouse gases, it must operate at a massive scale, on the order

corded intraplate earthquakes in south and east Asia (4). The seismicity catalogs are complete to magnitude (M) 3. The occurrence of these earthquakes means that nearly everywhere in continental interiors a subset of the preexisting faults in the crust is potentially active in the current stress field (5, 6). This is sometimes referred to as the *critically stressed* nature of the brittle crust (7). It should also be noted that despite the overall low rate of earthquake occurrence in continental interiors, some of the most devastating earthquakes in history occurred in these regions. In eastern China, the M 7.8, 1976 Tangshan earthquake, approximately 200 km east of Beijing, killed several hundred thousand people. In the central United States, three M 7+ earthquakes in 1811 and 1812 occurred in the New Madrid seismic zone

March, where the largest earthquake was M 4.7. In the Trinidad/Raton area near the border of Colorado and New Mexico, injection of produced water associated with coalbed methane production seems to have triggered a number of earthquakes, the largest being a M 5.3 event that occurred in August. Earthquakes seem to have been triggered by wastewater injection near Youngstown, Ohio on Christmas Eve and New Year's Eve, the largest of which was M 4.0. Although the risks associated with wastewater injection are minimal and can be reduced even further with proper planning (11), the situation would be far more problematic if similar-sized earthquakes were triggered in formations intended to sequester CO_2 for hundreds to thousands of years.

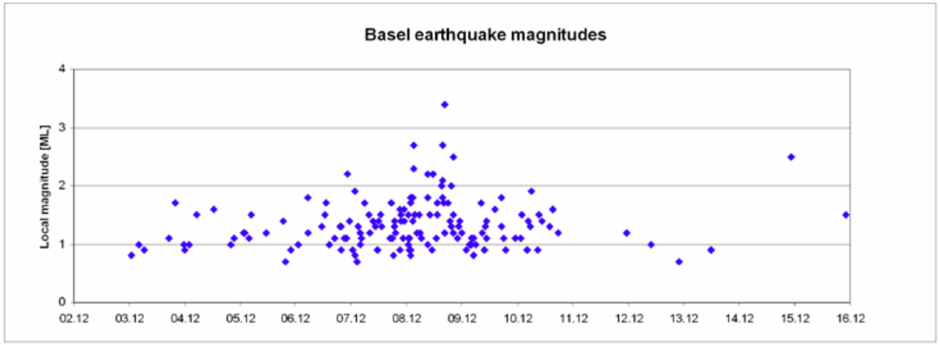
Deep borehole stress measurements

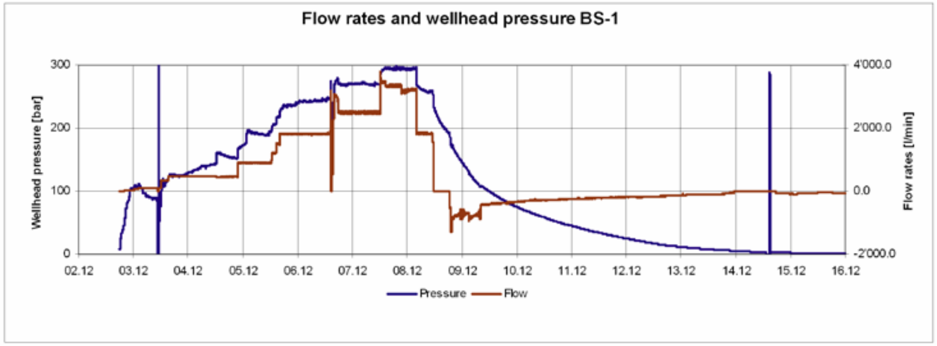
Zoback MD & Gorelick SM, Earthquake triggering and large-scale geologic storage of carbon dioxide, Proc National Academy of Science of the USA (PNAS), June 2012

주입에 의한 미소진동
스위스 Basel 프로젝트 (2006년)

3 MW 발전 + 20 MW 난방
5 km 시추, 수리자극 @4.6 km
6일간 11,500 m^3 주입
유량 ~50 liter/s, 유압 ~30 MPa
최대 M_{Lmax}: 3.4
(1356년 규모 6.6 지진 기록)
→ 프로젝트 중단

주입압/유량 과 관측된 미소진동


Basel earthquake magnitudes


Flow rates and wellhead pressure BS-1

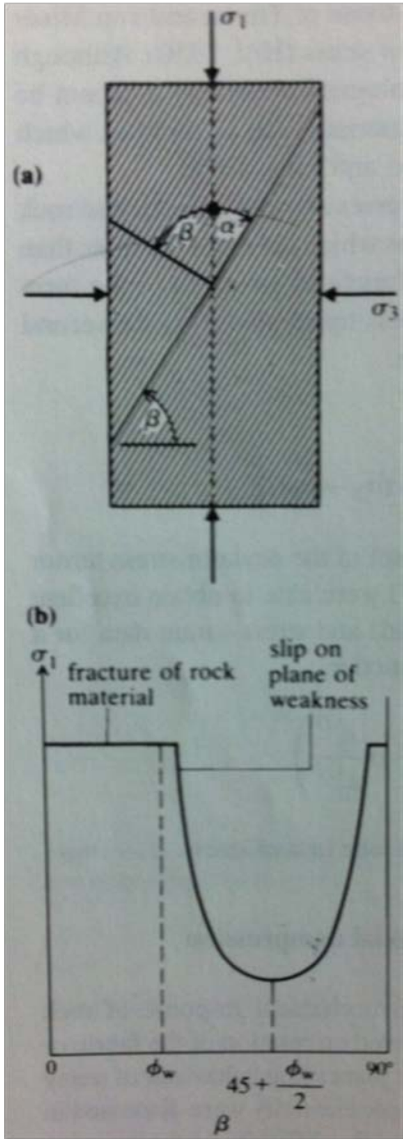
Failure Criteria
Effect of anisotropy

- Strength of anisotropic rock
  - can be estimated assuming a failure through predominant layers (which could be assumed to behave similar to fractures)

$$\sigma_1 = \sigma_3 + \frac{2\left(S_w + \mu_w \sigma_3\right)}{\left(1-\mu_w \cot\beta\right)\sin 2\beta}$$

  - Minimum strength when $\tan 2\beta_w = -\frac{1}{\mu_w}$ in other words, $\beta_w = 45 + \frac{\phi}{2}$

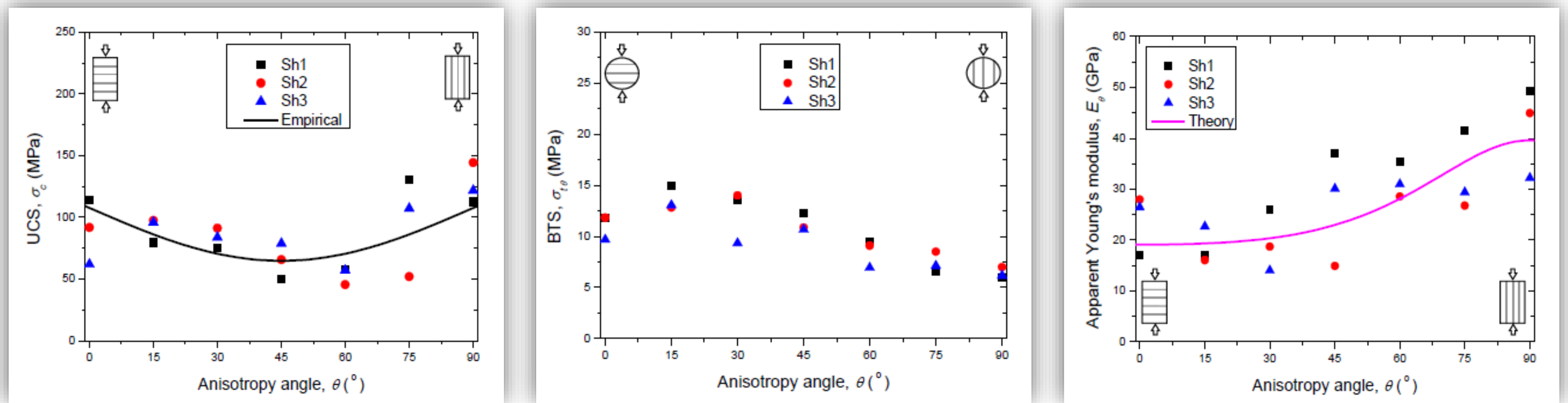
$$\sigma_1^{\min} = \sigma_3 + 2\left(S_w + \mu_w \sigma_3\right)\left[\sqrt{\mu_w^2 + 1} + \mu_w\right]$$



Failure Criteria

Effect of anisotropy

- 각도에 따른 강도 및 탄성계수의 변화 (보령셰일)



(a) 일축압축강도

(b) 인장강도

(c) 탄성계수

(Cho et al., 2012)

보령셰일의 역학적, 탄성파적, 열전도도적 이방성비				
압축강도 이방성비 (UCS/UCS′)	**인장강도 이방성비 (BTS/BTS′)**	**탄성 계수 이방성비 (E/E′)**	**P파 속도 이방성비 $(V_{P(90^\circ)}/V_{P(0^\circ)}{}')$**	**열전도도 이방성비 $(K_{(90^\circ)}/K_{(0^\circ)}{}')$**
2.6	2.2	2.1	1.5	2.1

(Cho, Kim, Min and Jeon, 2012)

Failure Criteria
Coulomb Failure Criteria (Mohr-Coulomb)

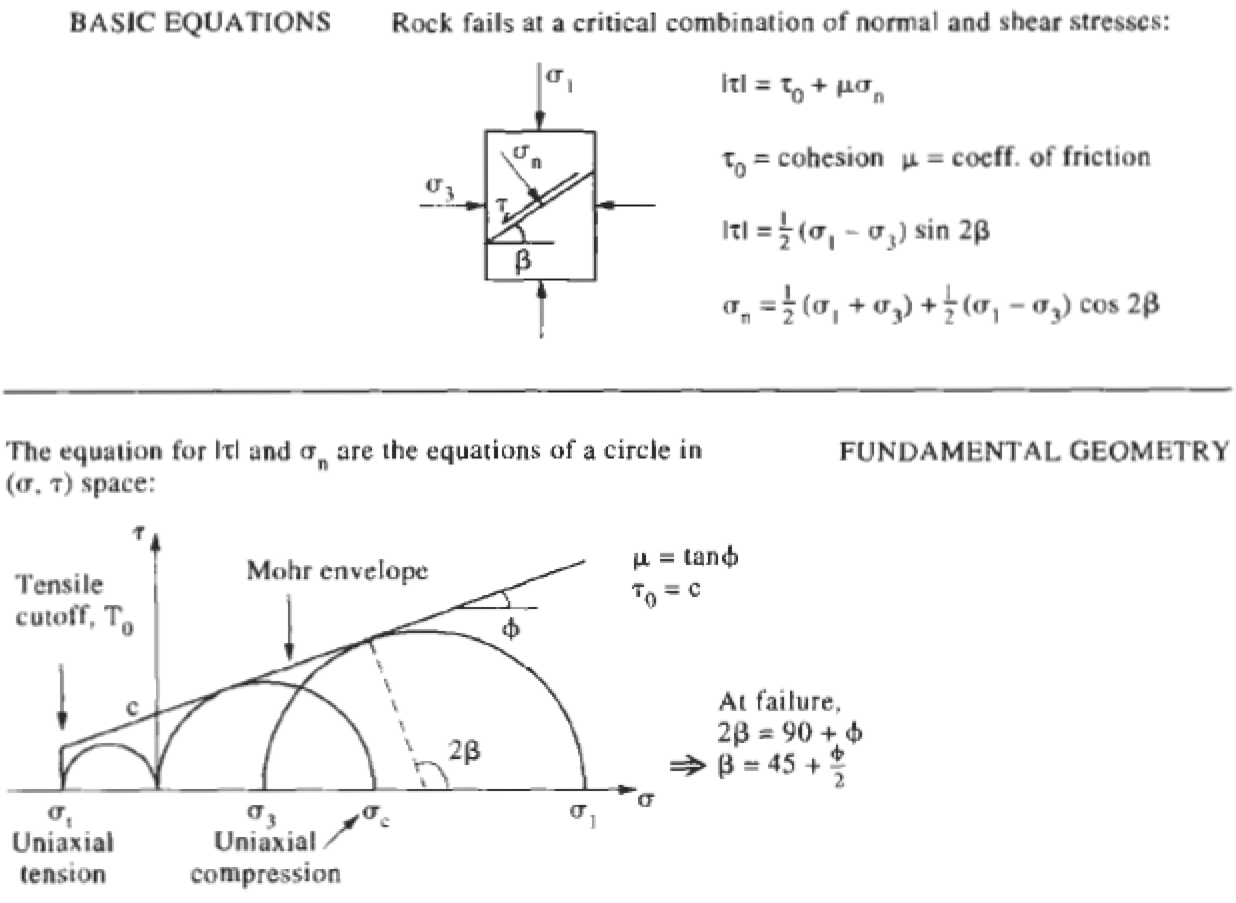
- Limitations
  - Prediction of too high tensile strength
    - Tension cut-off needed

$$\frac{\sigma_c}{\sigma_t} = \tan^2\beta = \left[\left(1+\mu^2\right)^{1/2}+\mu\right]^2$$

  - Actual σ-τ is not linear
    - Angle β decreases with higher confining pressure
  - Does not consider intermediate principal stress
    - Additional consideration is needed

Failure Criteria
Coulomb Failure Criteria (Mohr-Coulomb)

- Coulomb Failure Criterion for intact rock



BASIC EQUATIONS

Rock fails at a critical combination of normal and shear stresses:

$$|\tau| = \tau_0 + \mu\sigma_n$$

τ_0 = cohesion μ = coeff. of friction

$$|\tau| = \tfrac{1}{2}(\sigma_1 - \sigma_3)\sin 2\beta$$

$$\sigma_n = \tfrac{1}{2}(\sigma_1 + \sigma_3) + \tfrac{1}{2}(\sigma_1 - \sigma_3)\cos 2\beta$$

The equation for $|\tau|$ and σ_n are the equations of a circle in (σ, τ) space:

FUNDAMENTAL GEOMETRY

$\mu = \tan\phi$

$\tau_0 = c$

At failure,

$2\beta = 90 + \phi$

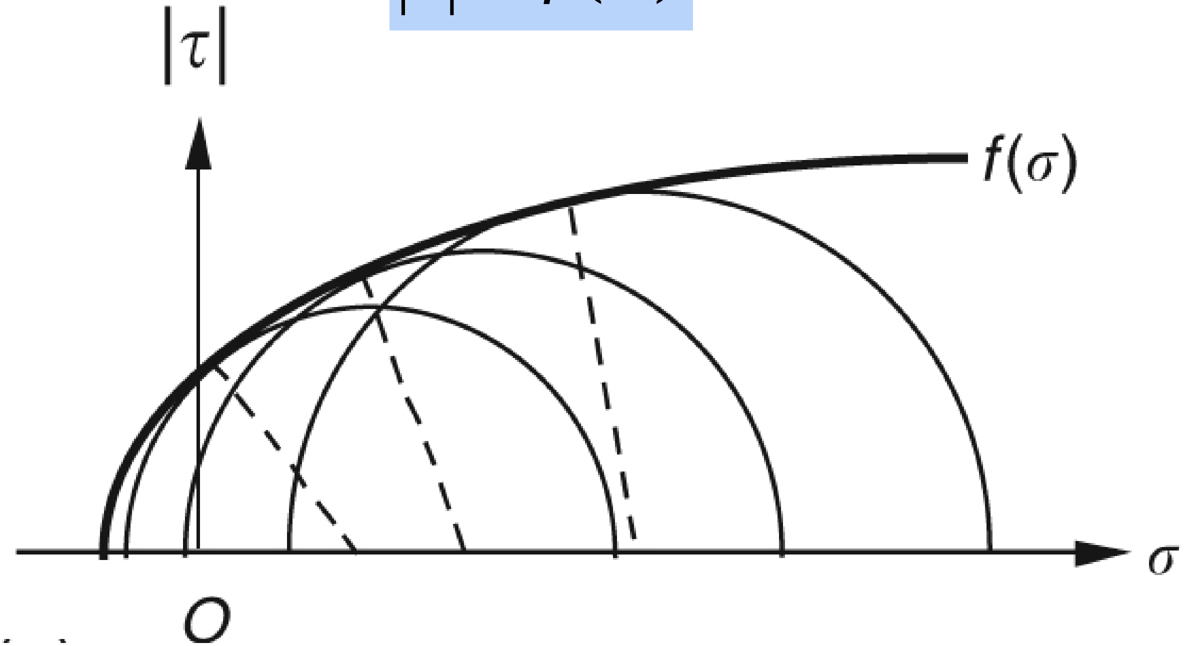
$\Rightarrow \beta = 45 + \frac{\phi}{2}$

Figure 6.18 The Mohr–Coulomb failure criterion.

Mohr-Coulomb Failure criteria

- Mohr's nonlinear failure criterion
  - Experiment shows that σ_1 increase at a rate less than linear rate with σ_3
  - Failure angle (β) decrease with increasing confining stress.

$$\left|\tau\right| = f(\sigma)$$



Failure Criteria
Hoek-Brown Failure Criterion

- Advantage
  - Non-linear form fits better with experimental data over a range of confining pressure
  - Developed through extensive lab tests on a wide range of rock type
  - Straightforwardly used

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c{}^2}$$

σ_1 : maximum principal stress at failure

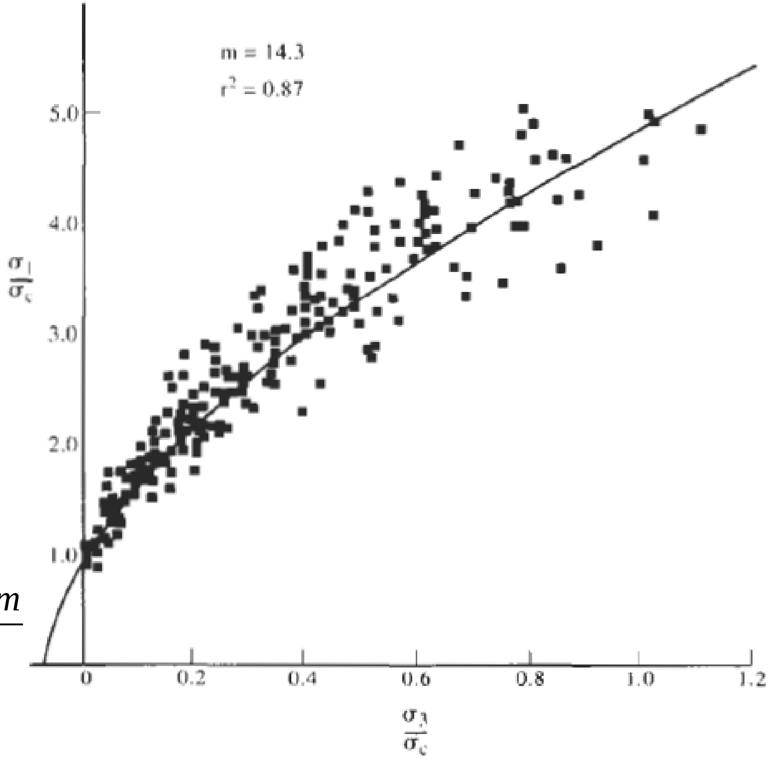
σ_3 : minimum principal stress at failure

σ_c : uniaxial compressive strength

m: Hoek-Brown material constants ($0 \le$ m)

s: Hoek-Brown material constants ($0 \le$ s ≤ 1)

- More realistic tensile strength $\dfrac{\sigma_c}{\sigma_t} = \dfrac{\sqrt{m^2 + 4s} + m}{2s}$



More general form: $\sigma_1 = \sigma_3 + \left(m\sigma_c\sigma_3 + s\sigma_c{}^2\right)^a$

Figure 6.20 The Hoek–Brown empirical failure criterion.

Failure Criteria

Hoek-Brown Failure Criterion

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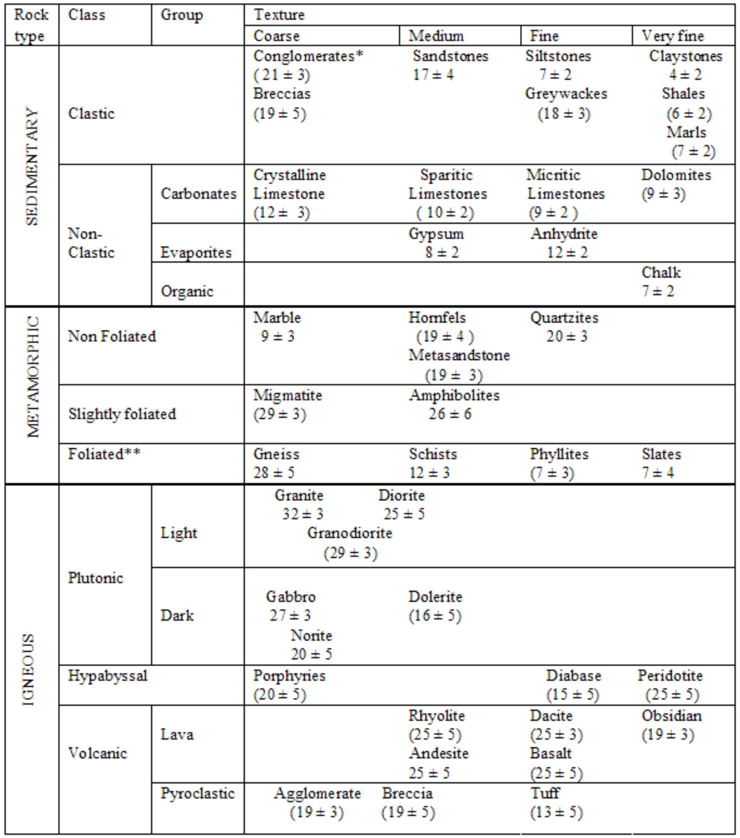
- Values of the constant *m* for intact rock, by rock group (Note that values in parenthesis are estimates)



Rock type	Class	Group	Texture			
			Coarse	Medium	Fine	Very fine
SEDIMENTARY	Clastic		Conglomerates* (21 ± 3) Breccias (19 ± 5)	Sandstones 17 ± 4	Siltstones 7 ± 2 Greywackes (18 ± 3)	Claystones 4 ± 2 Shales (6 ± 2) Marls (7 ± 2)
	Non-Clastic	Carbonates	Crystalline Limestone (12 ± 3)	Sparitic Limestones (10 ± 2)	Micritic Limestones (9 ± 2)	Dolomites (9 ± 3)
		Evaporites		Gypsum 8 ± 2	Anhydrite 12 ± 2	
		Organic				Chalk 7 ± 2
METAMORPHIC	Non Foliated		Marble 9 ± 3	Hornfels (19 ± 4) Metasandstone (19 ± 3)	Quartzites 20 ± 3	
	Slightly foliated		Migmatite (29 ± 3)	Amphibolites 26 ± 6		
	Foliated**		Gneiss 28 ± 5	Schists 12 ± 3	Phyllites (7 ± 3)	Slates 7 ± 4
IGNEOUS	Plutonic	Light	Granite 32 ± 3 Diorite 25 ± 5 Granodiorite (29 ± 3)			
		Dark	Gabbro 27 ± 3 Norite 20 ± 5	Dolerite (16 ± 5)		
	Hypabyssal		Porphyries (20 ± 5)		Diabase (15 ± 5)	Peridotite (25 ± 5)
	Volcanic	Lava		Rhyolite (25 ± 5) Andesite 25 ± 5	Dacite (25 ± 3) Basalt (25 ± 5)	Obsidian (19 ± 3)
		Pyroclastic	Agglomerate (19 ± 3)	Breccia (19 ± 5)	Tuff (13 ± 5)	

* Conglomerates and breccias may present a wide range of m_i values depending on the nature of the cementing material and the degree of cementation, so they may range from values similar to sandstone to values used for fine grained sediments.

* *These values are for intact rock specimens tested normal to bedding or foliation. The value of m_i will be significantly different if failure occurs along a weakness plane.

Failure Criteria

Failure under true triaxial stress conditions

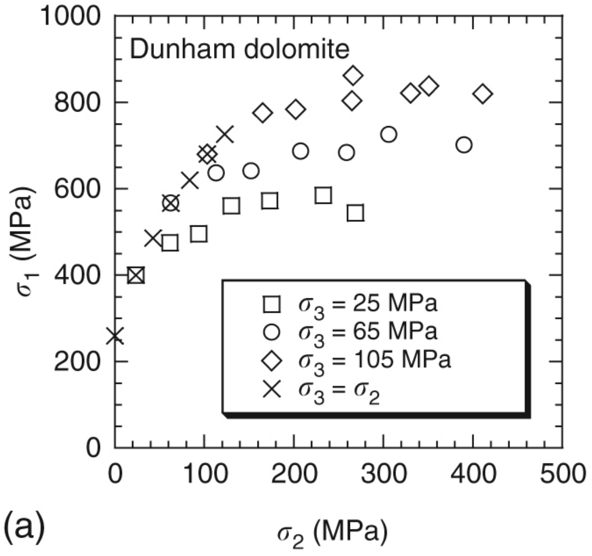
- It is (generally) known that intermediate principal stress also affect the failure.
  - Failure criterion under true triaxial stress conditions is of the form;

$$\tau_{oct} = f(\tau_{m2})$$

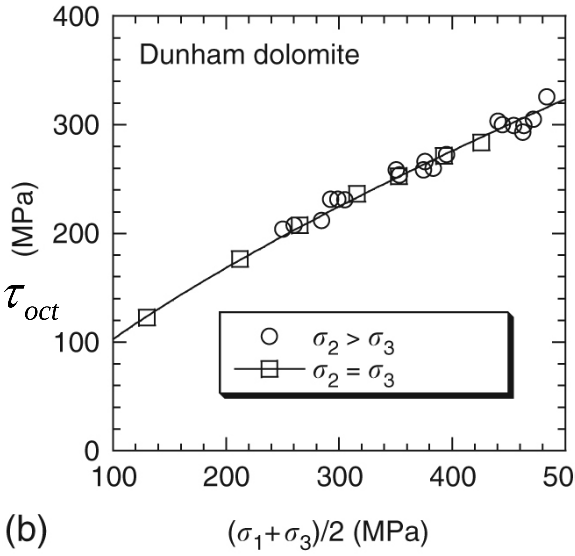
$$\left|\tau_{oct}\right| = \frac{1}{3}\left\{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right\}^{1/2} = \frac{\sqrt{2}}{3}\left\{I_1^2+3I_2\right\}^{1/2} = \sqrt{\frac{2}{3}J_2}$$

$$\tau_{oct} = a + b\tau_{m2}$$

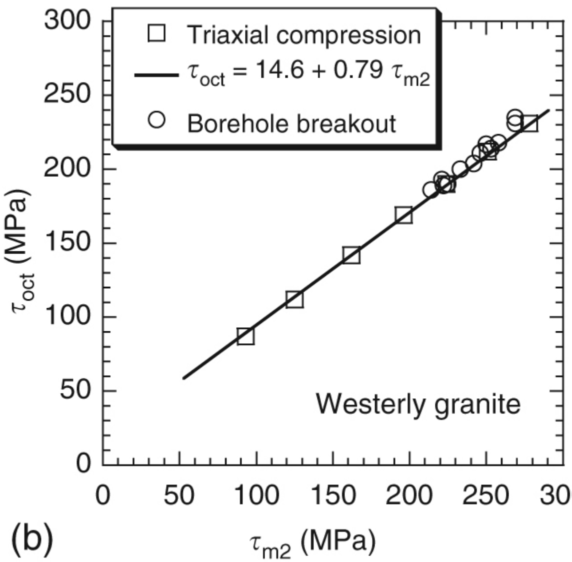
$$\tau_{m2} = \frac{(\sigma_1+\sigma_3)}{2}$$



(a)



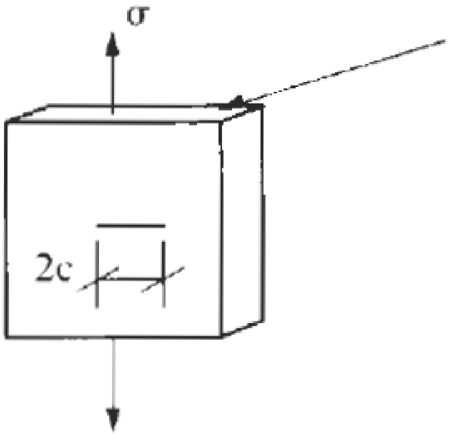
(b)



(b)

Failure Criteria
Griffith Failure Criterion

Material fractures when sufficient strain energy is released to enable cracks to propagate



$$\sigma_t = \sqrt{\frac{k\,\alpha\,E}{c}}$$

$k = \frac{2}{\pi}$ for plane stress

$= \frac{2}{\pi}(1 - \nu^2)$ for plane strain

α = unit surface energy of the crack

In compression:

$(\sigma_1 - \sigma_3)^2 = 8T_0(\sigma_1 + \sigma_3)$ when $\sigma_1 + 3\sigma_3 > 0$

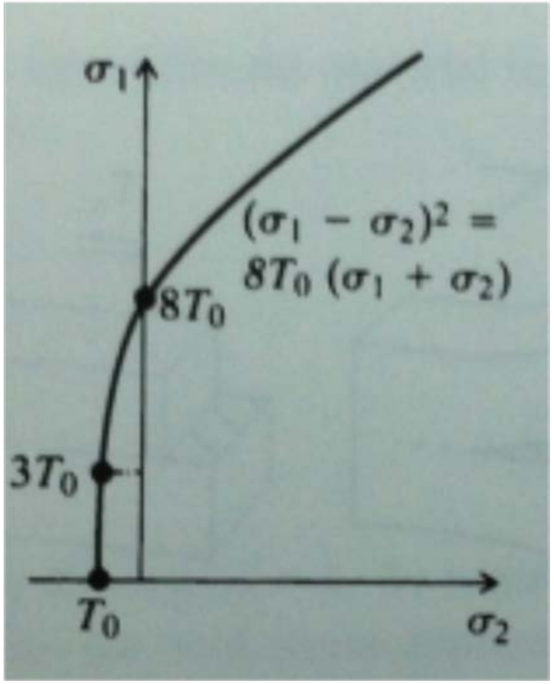
$\sigma_3 = -T_0$ when $\sigma_1 + 3\sigma_3 < 0$

Note: compression positive, T_0 positive ($-T_0 = \sigma_t$)

Figure 6.19 The plane Griffith failure criterion.

Failure Criteria
Griffith Failure Criterion

$$
\begin{aligned}
(\sigma_1-\sigma_2)^2 &= 8T_0(\sigma_1+\sigma_2), & \sigma_1+3\sigma_2 &> 0\\
\sigma_2 &= -T_0, & \sigma_1+3\sigma_2 &< 0
\end{aligned}
$$



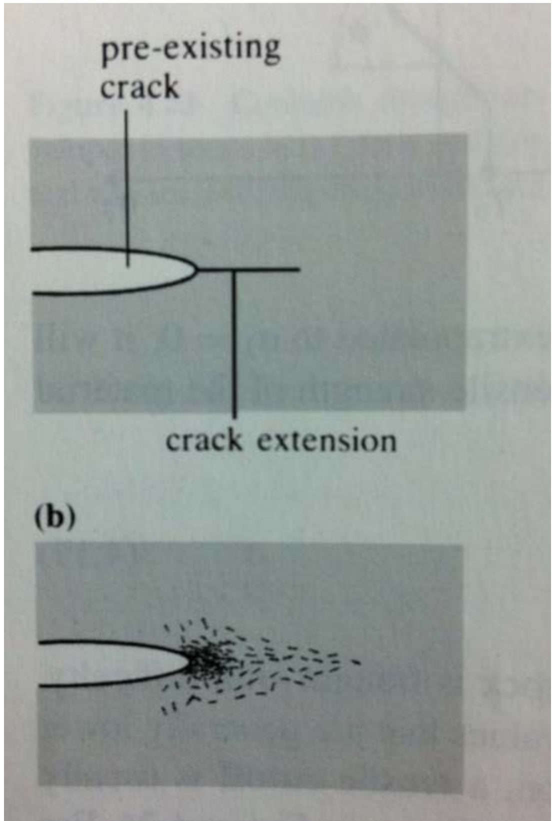


Figure 4.25 Extension of a preexisting crack, (a) Griffith's hypothesis, (b) the actual case for rock.

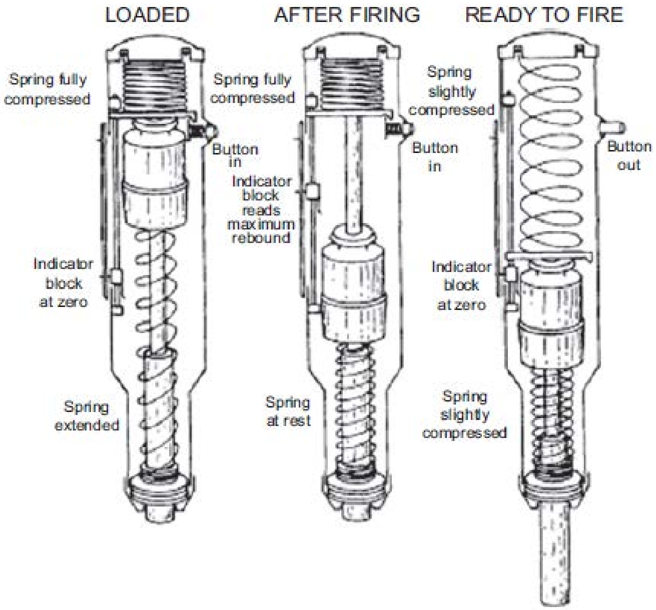
Hypothesis (model)

reality

Other Strength Test
Schmidt Hammer Rebound Hardness Test

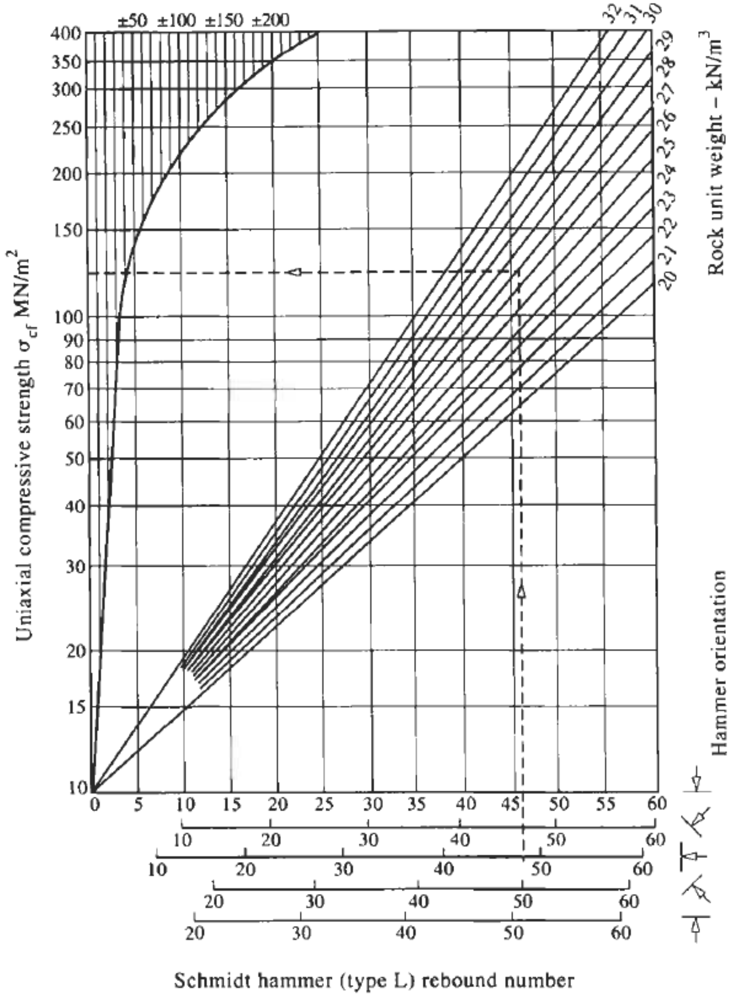
- Spring-driven cylindrical hammer rebounds off the rock surface
- The rebound distance is a measure of rock quality (e.g., strength)
- Often used on rock fracture surface
- Condition of rock surface has significant effect on the results

http://rammedearth.blogspot.kr/2006/06/hammer-time.html



Other Strength Test
Schmidt Hammer Rebound Hardness Test

- Use chart relating the rebound number and UCS



Other Strength Test
Point Load Test

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- Index test used mainly to predict the uniaxial compressive strength of rock
- Measures the 'Point Load Strength Index' $I_{s(50)}$
- Rock specimens in the form of either core, cut blocks, or irregular lumps are broken by application of concentrated load through a pair of spherically truncated, conical platens.
- Little or no specimen preparation is needed.

$$I_{s(50)} = \frac{P}{D^2}$$

$I_{s(50)}$: Point Load Strength Index (50 mm)

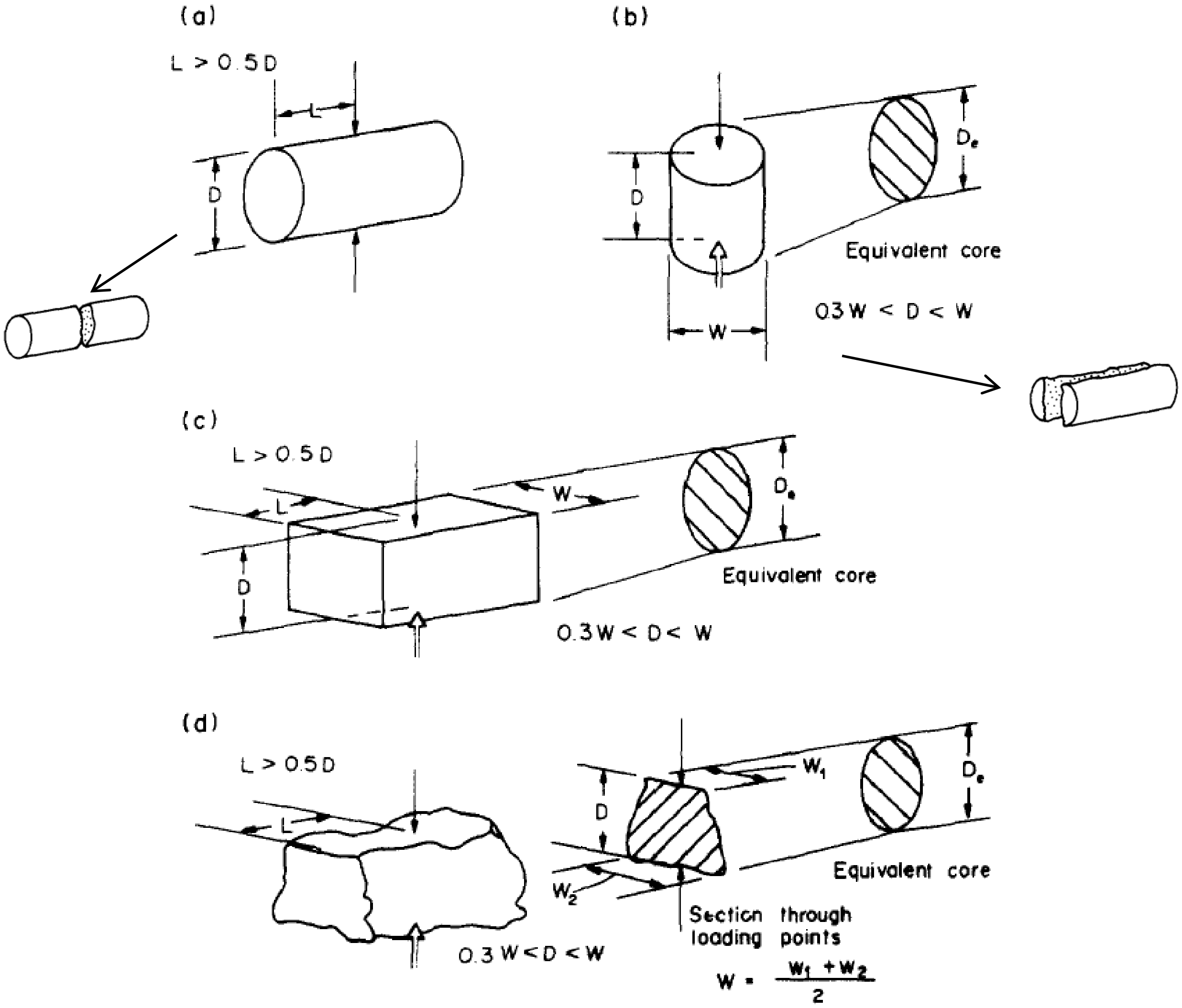
P: Peak load

D: Distance between the two platen contacts

$$UCS = (20 \sim 25) * I_{s(50)}$$

http://www.controls-group.com/eng/rock-mechanics-testing-equipment/rock-strength-index-apparatus.php

Other Strength Test
Point Load Test



. Specimen shape requirements for (a) the diametral test, (b) the axial test, (c) the block test, and (d) the irregular lump test.

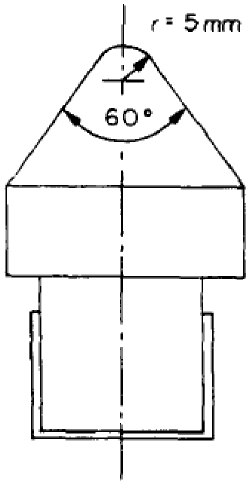


Fig. 2. Platen shape and tip radius.

Other Strength Test

Point Load Test