

Announcement

- 1st Lab report due April 12th
- Homework (Chap 3 Torsion) due April 19th
- 2nd Lab session on April 24th and 26th

Chap. 4 Shear Forces and Bending Moments



Chap. 4 Shear Forces and Bending Moments

4.1 Introduction

4.2 Types of Beams, Loads, and Reactions

4.3 Shear Forces and Bending Moments

4.4 Relationships btw Loads, Shear Forces and Bending Moments

4.5 Shear-Force (SFD) and Bending-Moment Diagrams (BMD)

Introduction

Shear Force and Bending Moment and Deflection

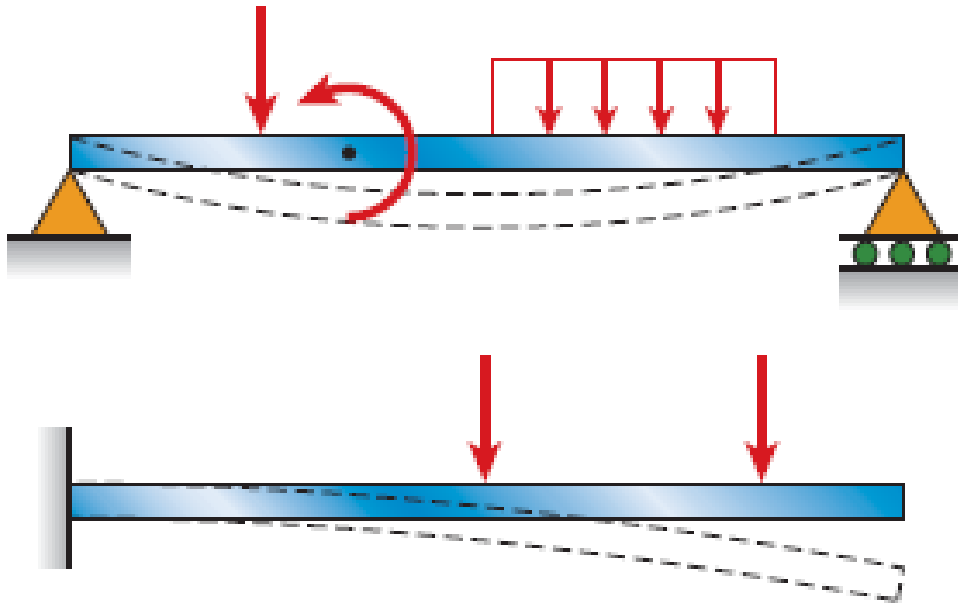
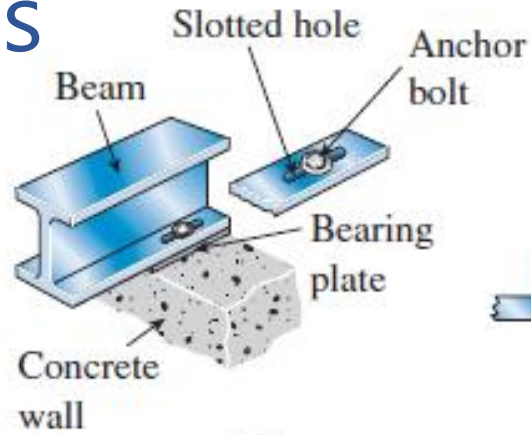
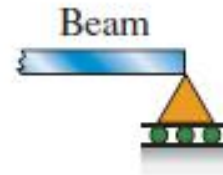


FIG. 4-1 Examples of beams subjected to lateral loads

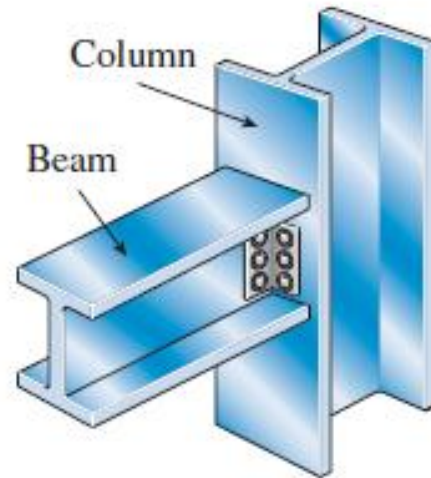
Types of Supports



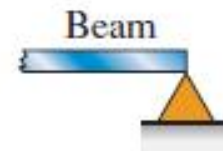
(a)



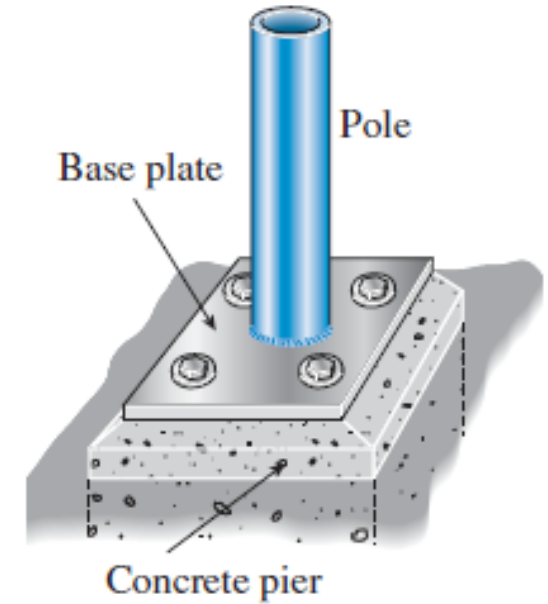
(b)



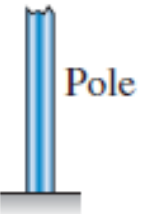
(c)



(d)

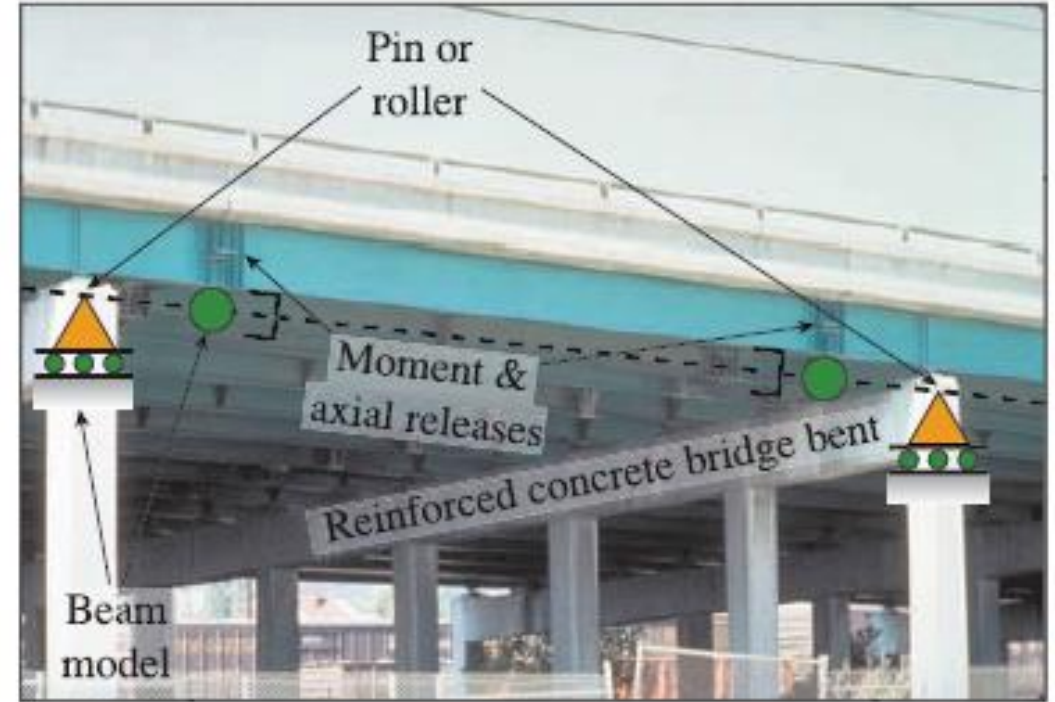


(e)

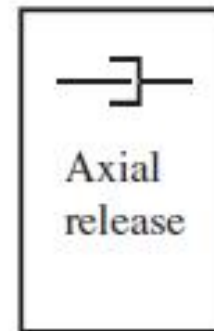


(f)

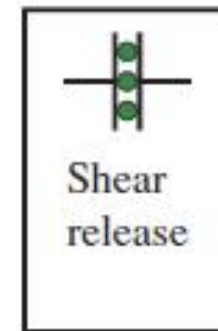
Types of Supports



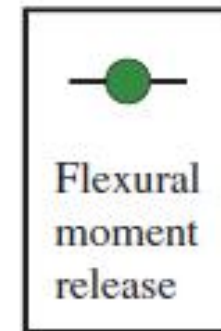
Internal releases and end supports in model of bridge beam



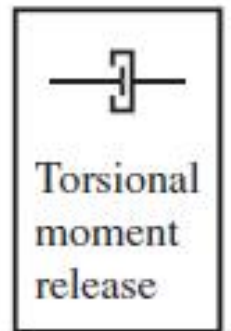
Axial
release



Shear
release



Flexural
moment
release



Torsional
moment
release

Types of beams and loads

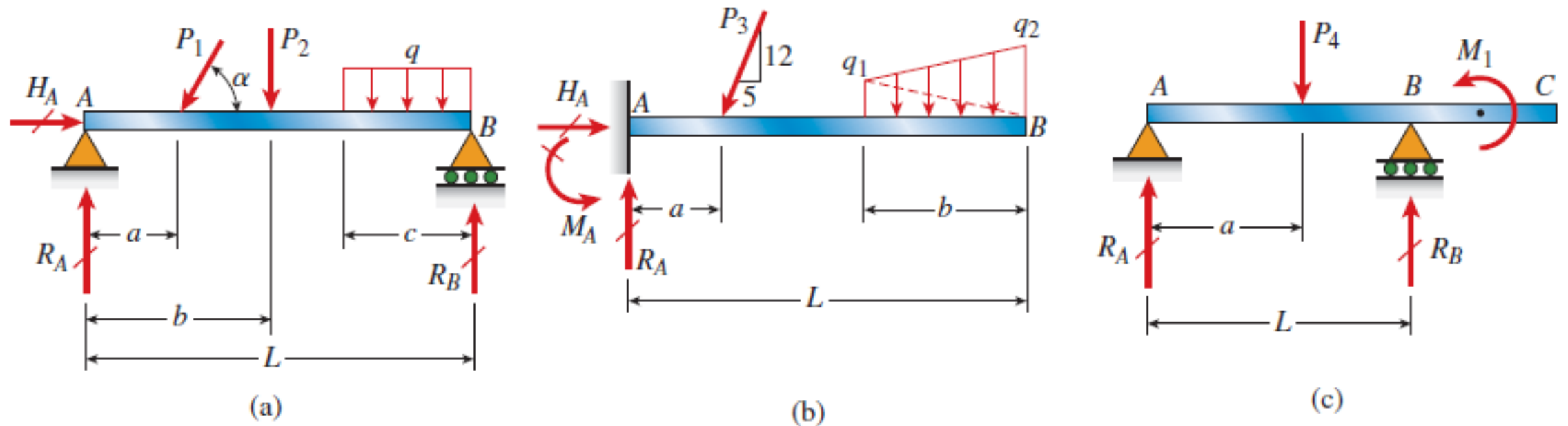
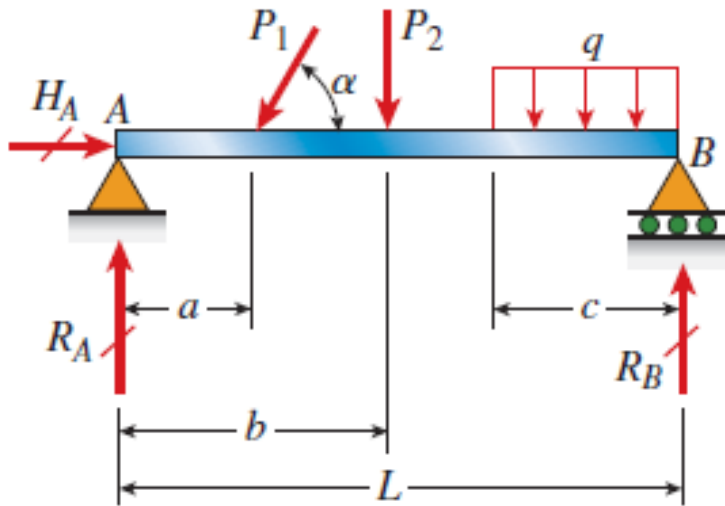


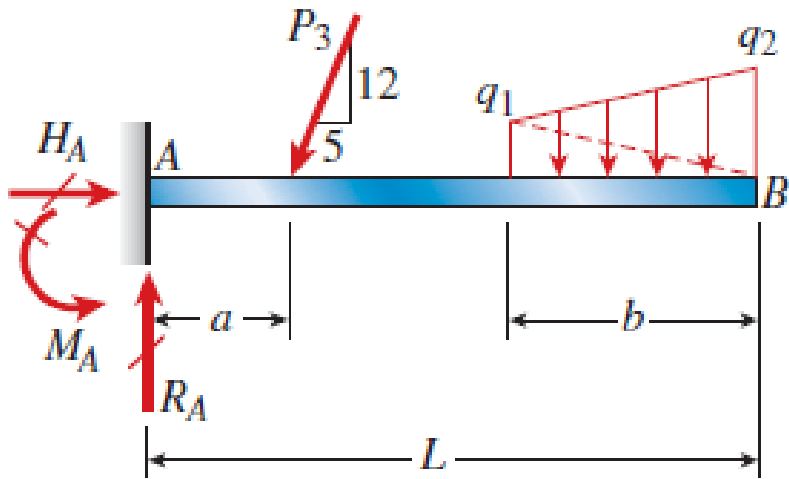
FIG. 4-2 Types of beams: (a) simple beam, (b) cantilever beam, and (c) beam with an overhang

Types of beams and loads



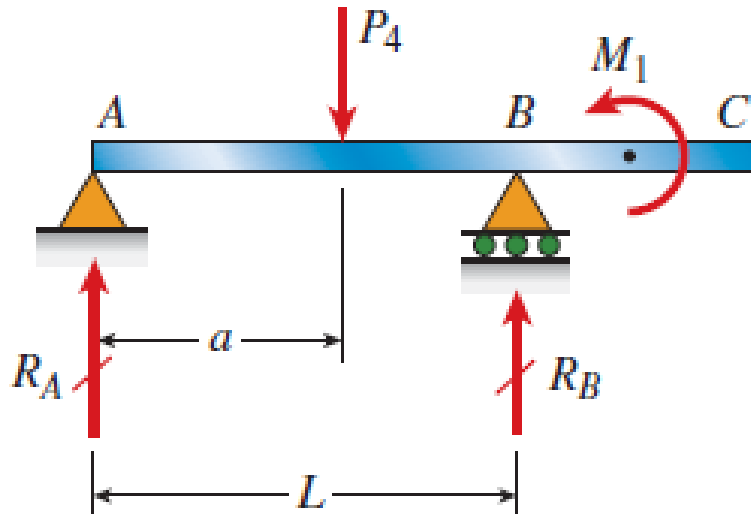
Simple beam

Types of beams and loads



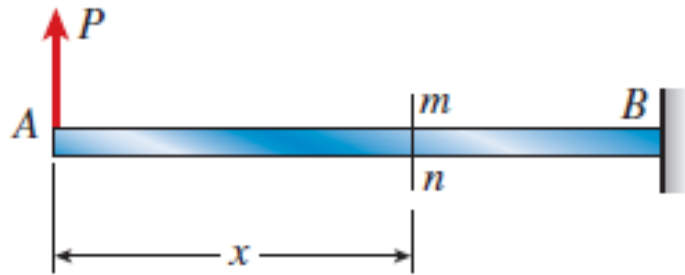
Cantilever beam

Types of beams and loads



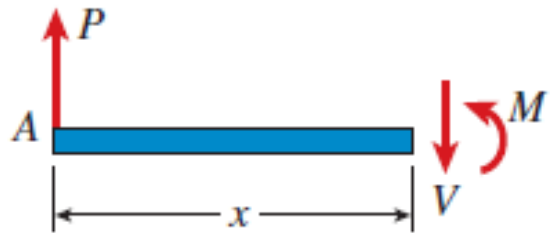
Beam with an overhang

Shear Forces and Bending Moments



(a)

$$\sum F_{\text{vert}} = 0 \quad P - V = 0 \quad \text{or} \quad V = P$$
$$\sum M = 0 \quad M - Px = 0 \quad \text{or} \quad M = Px$$



(b)



(c)

Shear Forces and Bending Moments

Notation

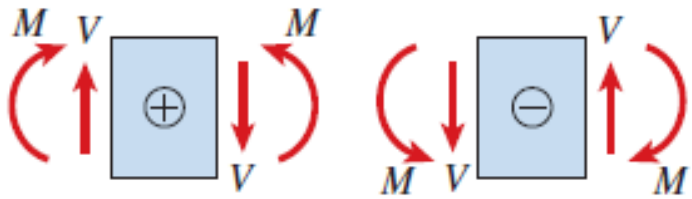
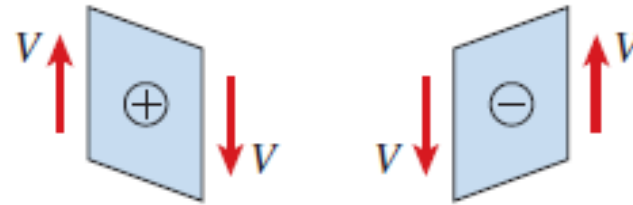
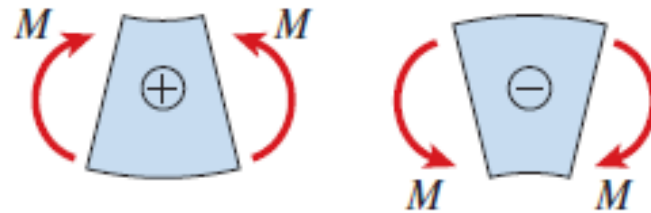


FIG. 4-9 Sign conventions for shear force V and bending moment M



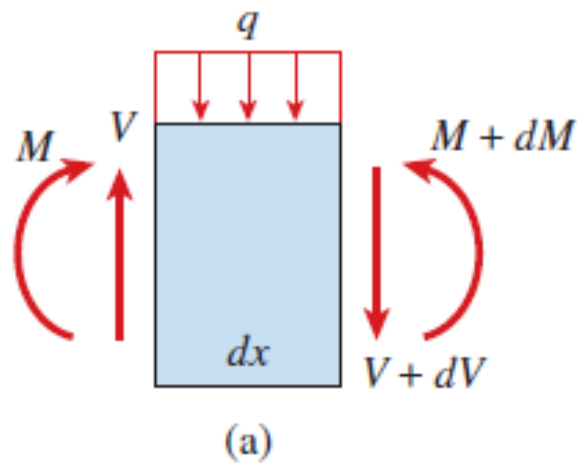
(a)



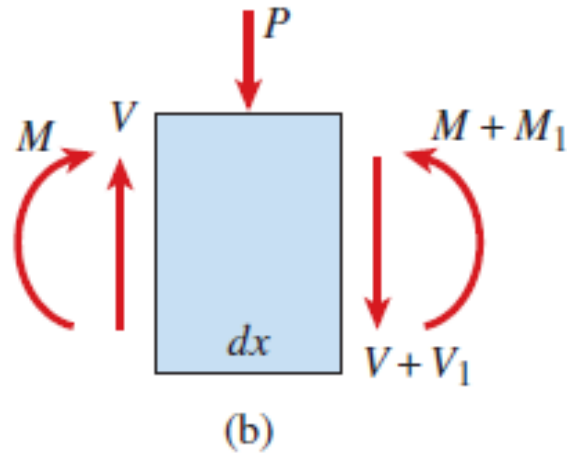
(b)

FIG. 4-10 Deformations (highly exaggerated) of a beam element caused by (a) shear forces, and (b) bending moments

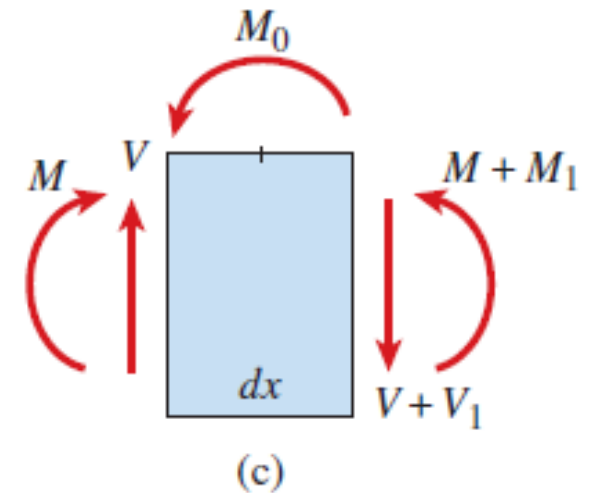
Relationships btw Loads, Shear Forces and Bending Moments



Distributed loads



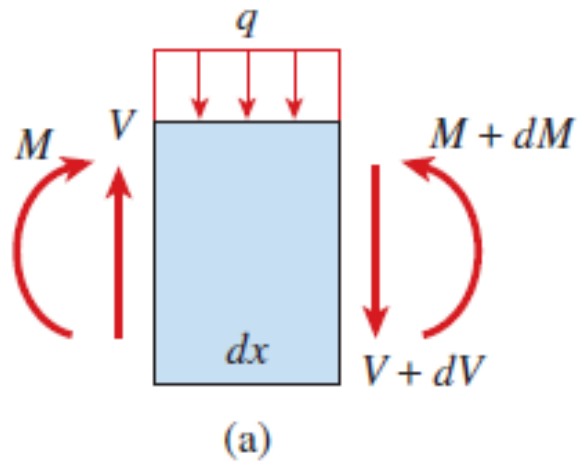
Concentrated loads



Loads of couples

Relationships btw Loads, Shear Forces and Bending Moments

Equilibrium of forces in the vertical direction



Distributed loads

$$\sum F_{\text{vert}} = 0 \quad V - q \, dx - (V + dV) = 0$$

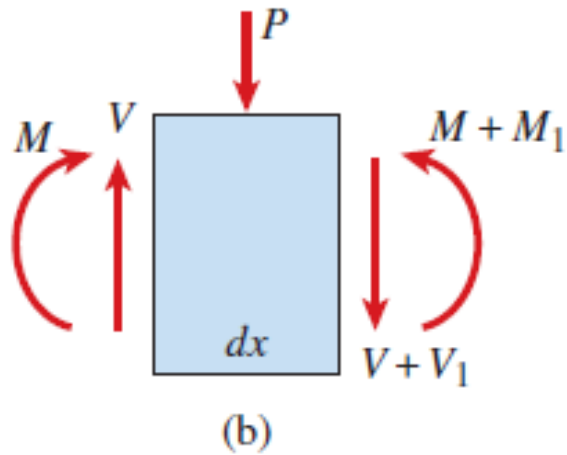
$$\rightarrow \frac{dV}{dx} = -q$$

Equilibrium of moment about an axis at the left-hand side of the element

$$\sum M = 0 \quad -M - q \, dx \left(\frac{dx}{2} \right) - (V + dV) dx + M + dM = 0$$

$$\rightarrow \frac{dM}{dx} = V$$

Relationships btw Loads, Shear Forces and Bending Moments



Concentrated loads

Equilibrium of forces in the vertical direction

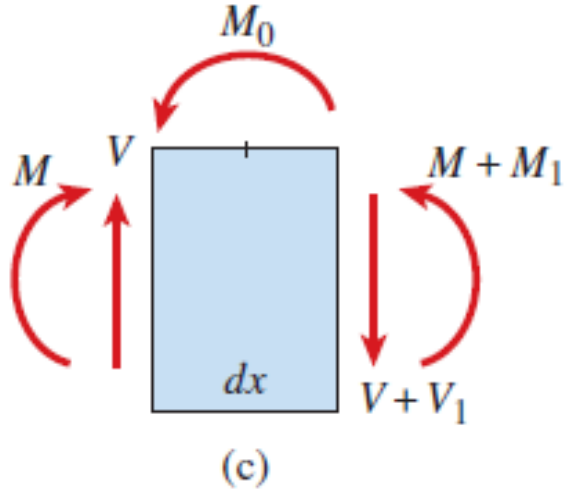
$$V - P - (V + V_1) = 0 \text{ or } V_1 = -P$$

Equilibrium of moments about the left-hand face of the element

$$-M - P\left(\frac{dx}{2}\right) - (V + V_1)dx + M + M_1 = 0$$

$$\Rightarrow M_1 = P\left(\frac{dx}{2}\right) + V dx + V_1 dx = 0$$

Relationships btw Loads, Shear Forces and Bending Moments



Loads of couples

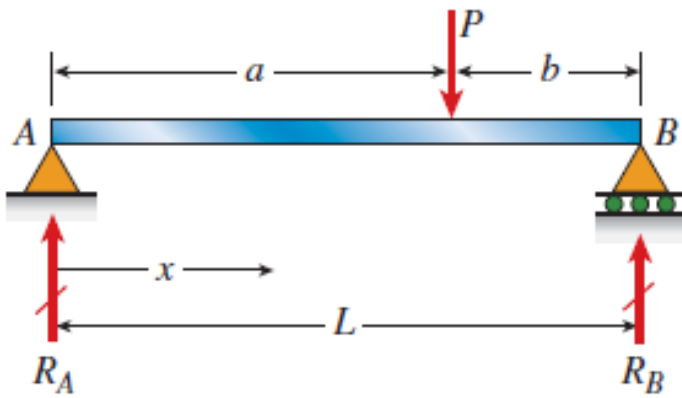
Equilibrium of moments about the left-hand side of the element:

$$-M + M_0 - (V + V_1)dx + M + M_1 = 0$$

➔ $M_1 = -M_0$

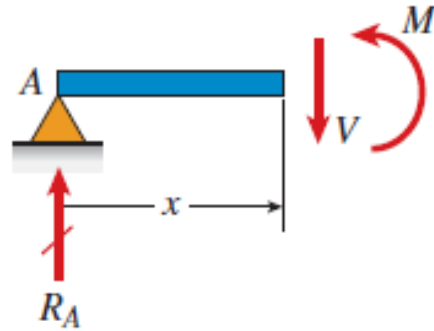
Shear-Force (SFD) and Bending Moment Diagrams (BMD)

Point loading



$$R_A = \frac{Pb}{L} \quad R_B = \frac{Pa}{L}$$

$$(0 < x < a)$$

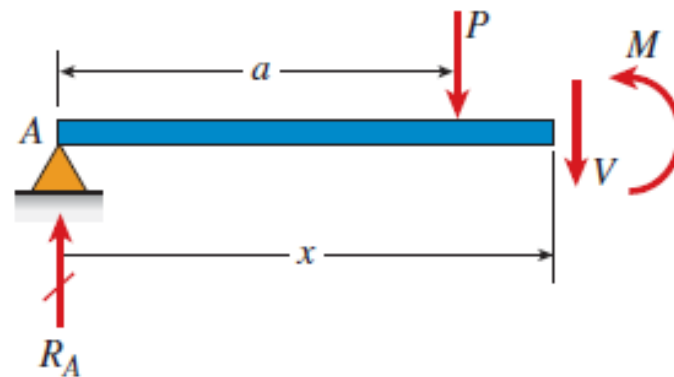


(b)

$$V = R_A = \frac{Pb}{L}$$

$$M = R_A x = \frac{Pbx}{L}$$

$$(a < x < L)$$



(c)

$$V = R_A - P = \frac{Pb}{L} - P = -\frac{Pa}{L}$$

$$M = R_A x - P(x - a) = \frac{Pbx}{L} - P(x - a)$$

$$= \frac{Pa}{L}(L - x)$$

Shear-Force (SFD) and Bending Moment Diagrams (BMD)

Point loading

$$V = \frac{Pb}{L}$$

$(0 < x < a)$

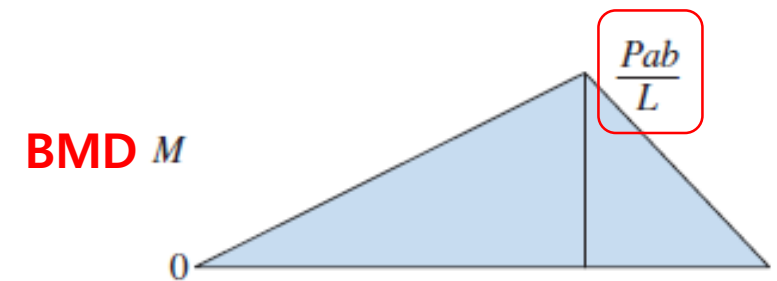
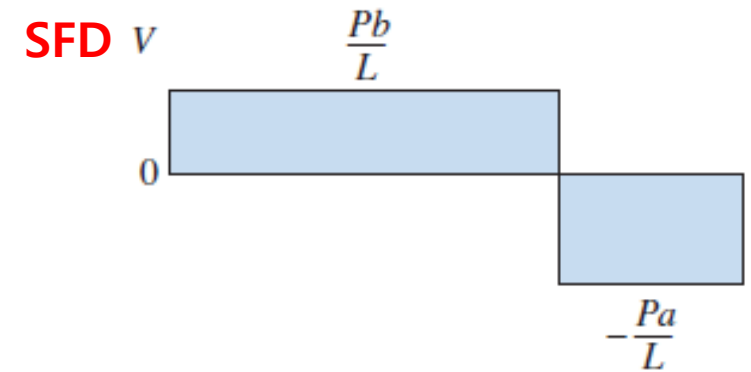
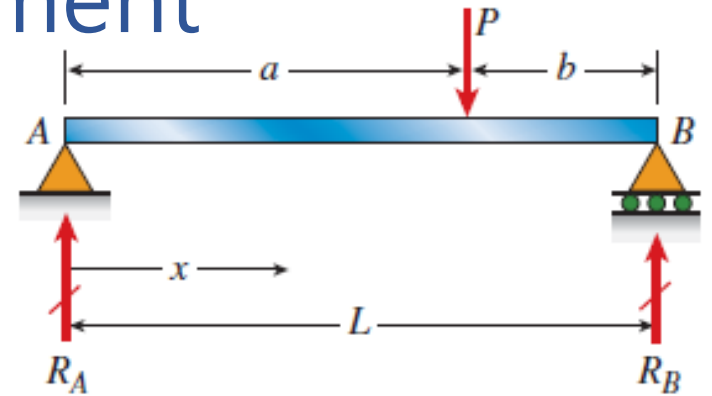
$$M = \frac{Pbx}{L}$$

$$V = -\frac{Pa}{L}$$

$(a < x < L)$

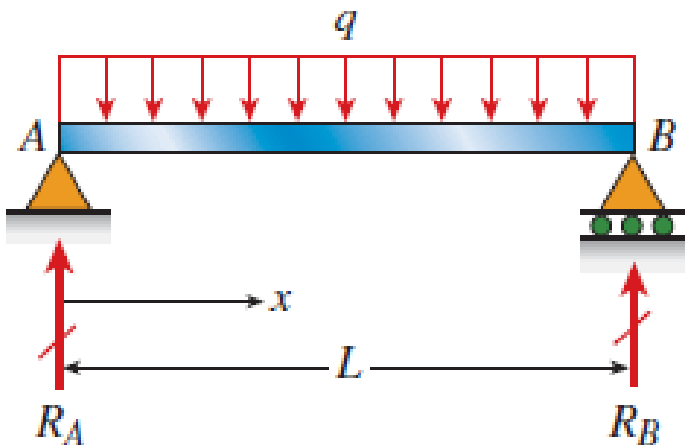
or

$$M = \frac{Pa}{L}(L - x)$$



Shear-Force (SFD) and Bending Moment Diagrams (BMD)

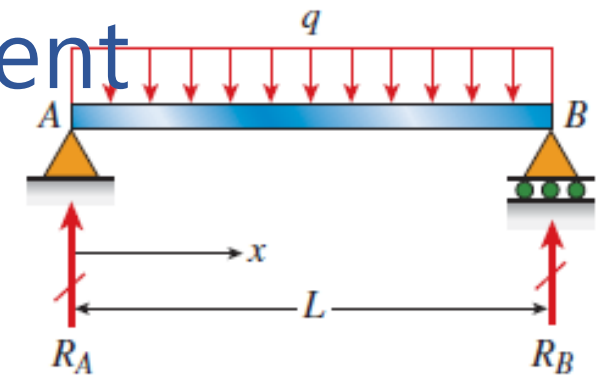
Uniform loading



$$R_A = R_B = \frac{qL}{2}$$

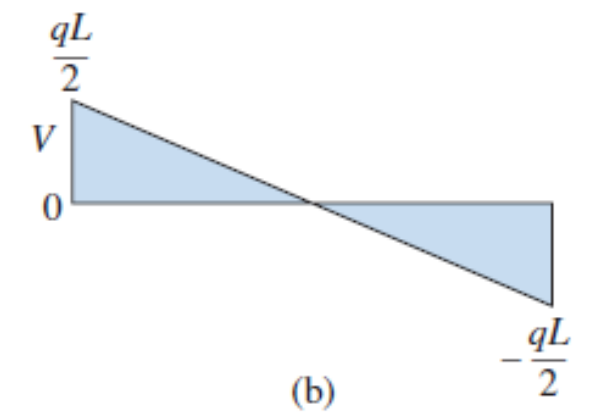
$$V = R_A - qx = \frac{qL}{2} - qx$$

$$M = R_A x - qx \left(\frac{x}{2} \right) = \frac{qLx}{2} - \frac{qx^2}{2}$$



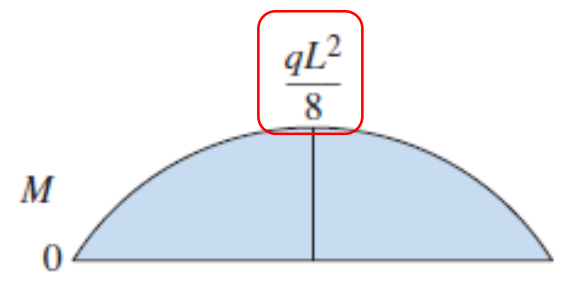
(a)

SFD



(b)

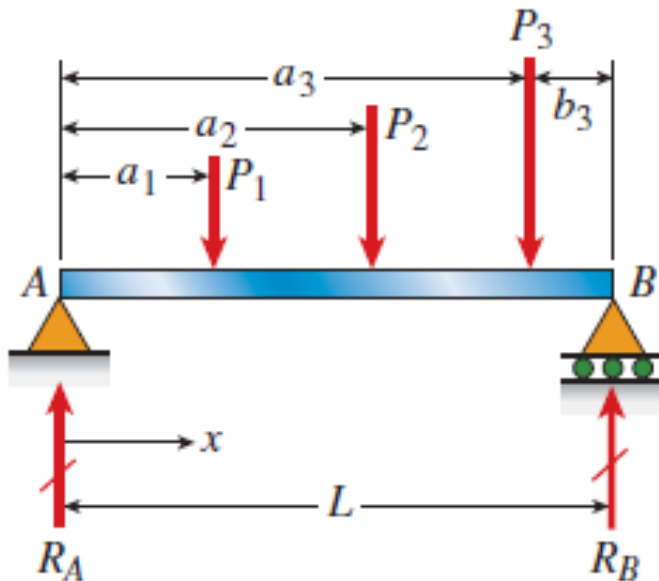
BMD



(c)

Shear-Force (SFD) and Bending Moment Diagrams (BMD)

Several concentrated loads



$$(0 < x < a_1)$$



$$V = R_A \quad M = R_A x$$

$$(a_1 < x < a_2)$$



$$V = R_A - P_1 \quad M = R_A x - P_1(x - a_1)$$

$$(a_2 < x < a_3)$$



$$V = -R_B + P_3$$

$$M = R_B(L - x) - P_3(L - b_3 - x)$$

$$(a_3 < x < L)$$



$$V = -R_B \quad M = R_B(L - x)$$

Shear-Force (SFD) and Bending Moment Diagrams (BMD)

Several concentrated loads

$$(0 < x < a_1)$$



$$V = R_A \quad M = R_A x$$

$$(a_1 < x < a_2)$$



$$V = R_A - P_1 \quad M = R_A x - P_1(x - a_1)$$

$$(a_2 < x < a_3)$$



$$V = -R_B + P_3$$

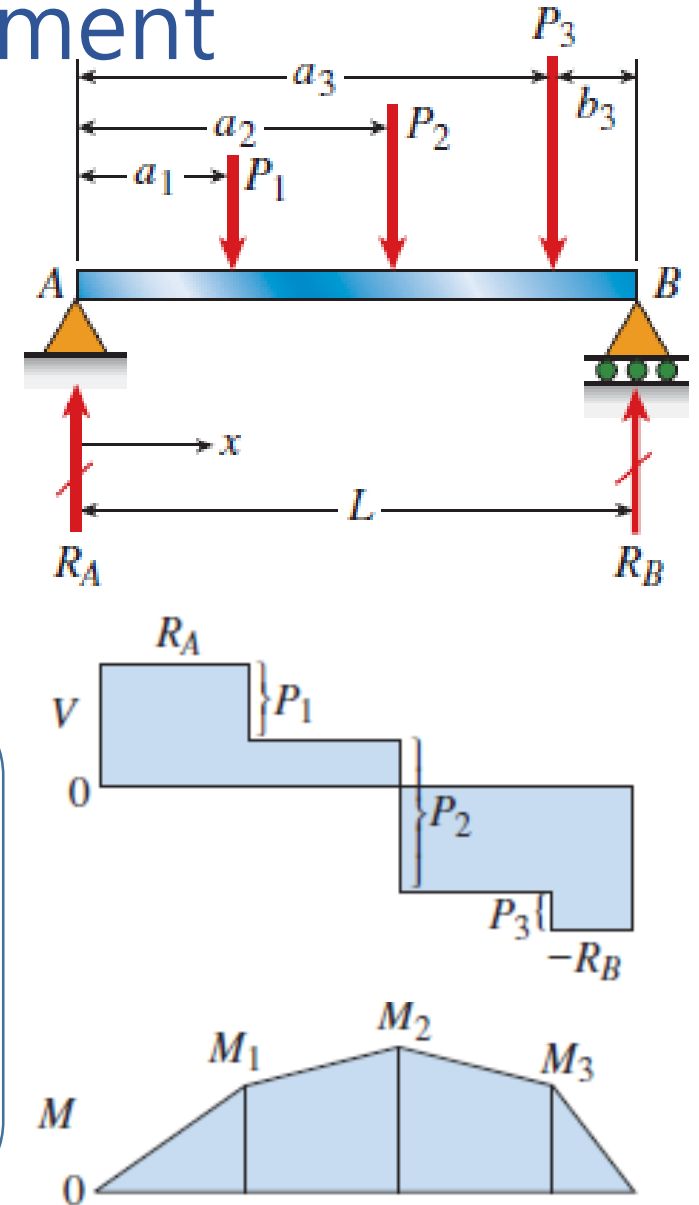
$$M = R_B(L - x) - P_3(L - b_3 - x)$$

or $(a_2 < x < a_3)$



$$V =$$

$$M =$$



Shear-Force (SFD) and Bending Moment Diagrams (BMD)

Several concentrated loads

$$(0 < x < a_1)$$



$$V = R_A \quad M = R_A x$$

$$(a_1 < x < a_2)$$



$$V = R_A - P_1 \quad M = R_A x - P_1(x - a_1)$$

$$(a_2 < x < a_3)$$



$$V = -R_B + P_3$$

$$M = R_B(L - x) - P_3(L - b_3 - x)$$

$$(a_3 < x < L)$$



$$V = -R_B \quad M = R_B(L - x)$$

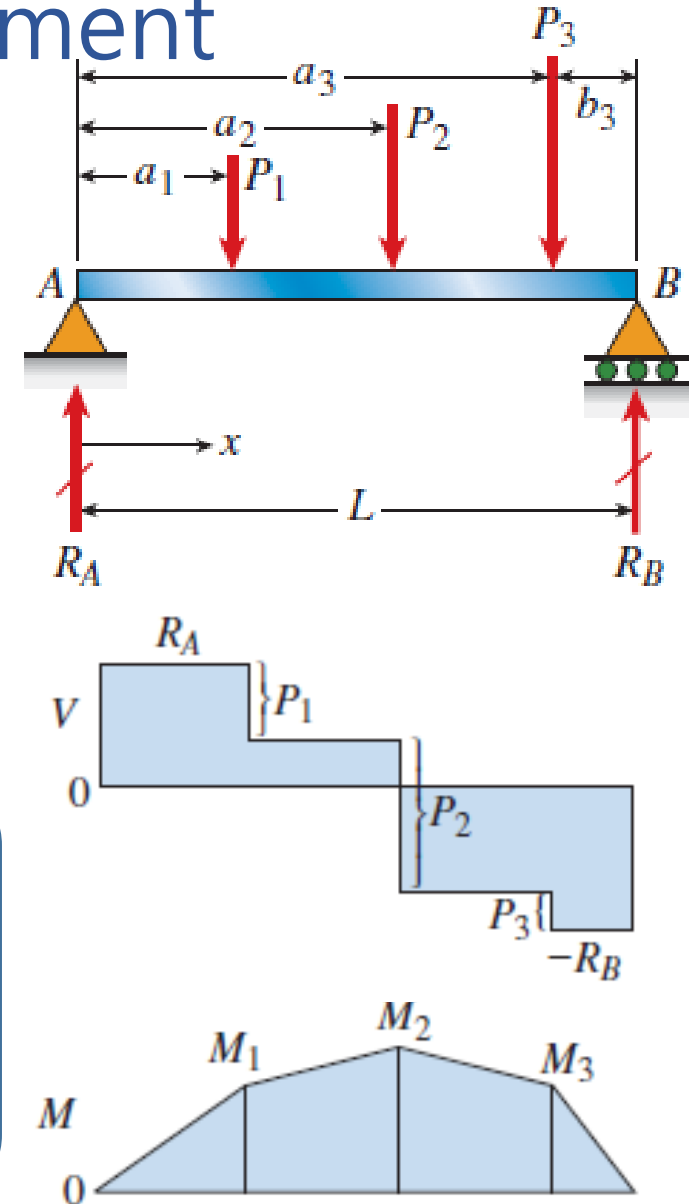
or

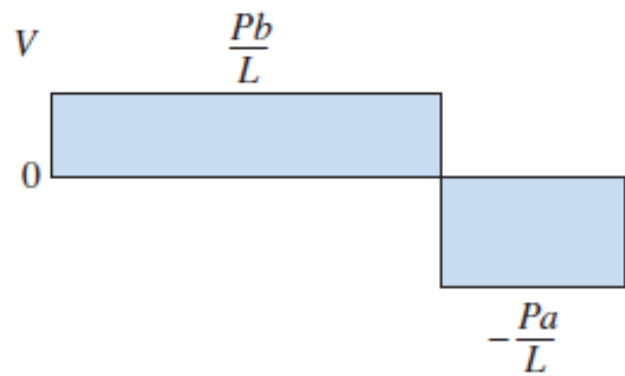
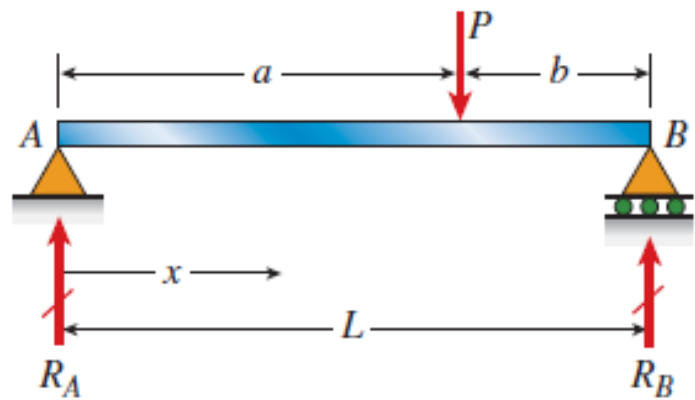
$$(a_3 < x < L)$$



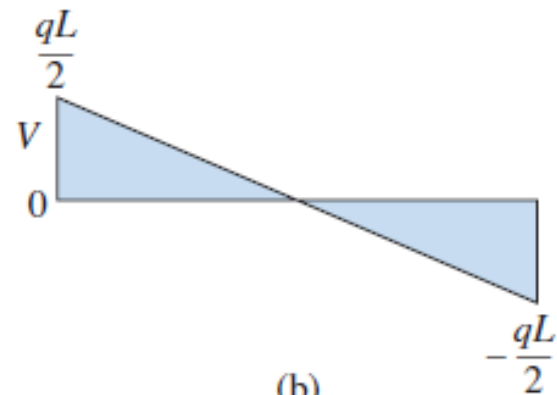
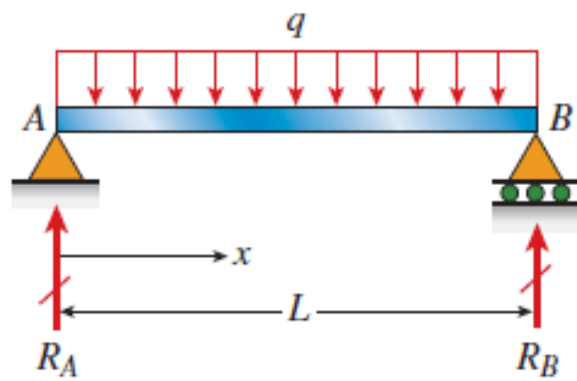
$$V =$$

$$M =$$

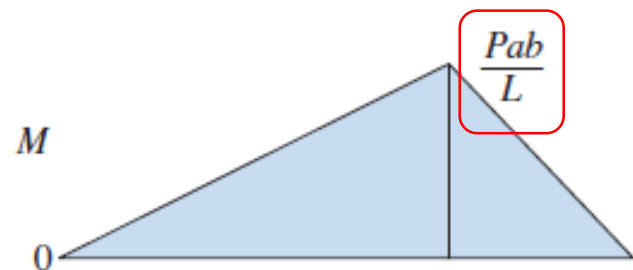




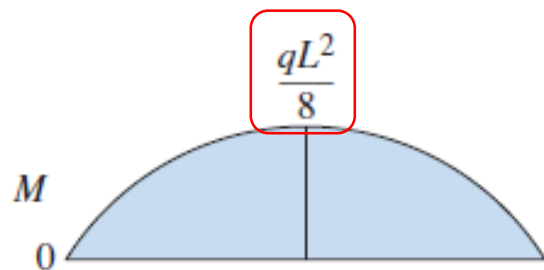
(d)



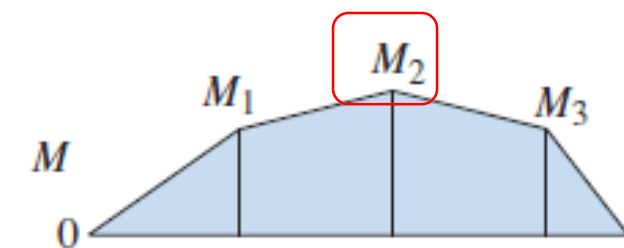
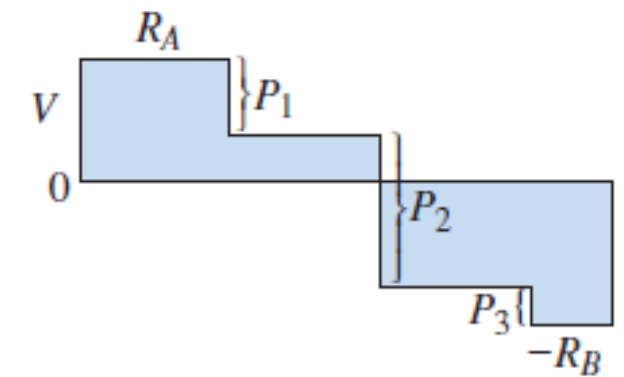
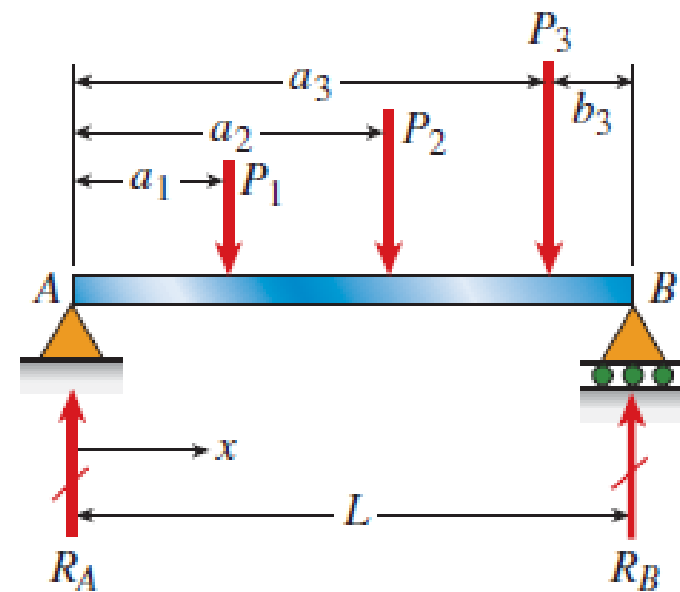
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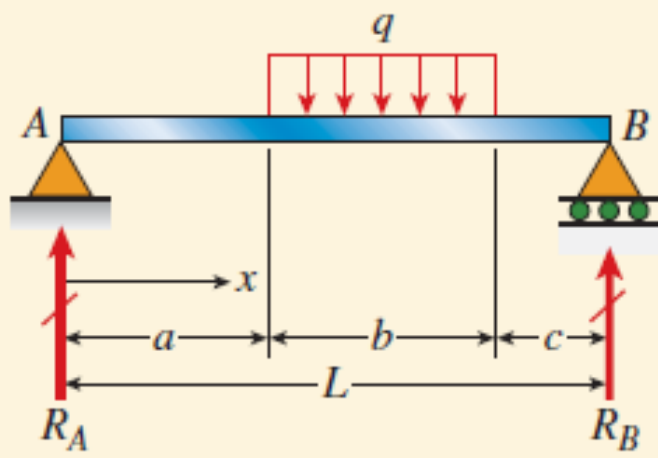


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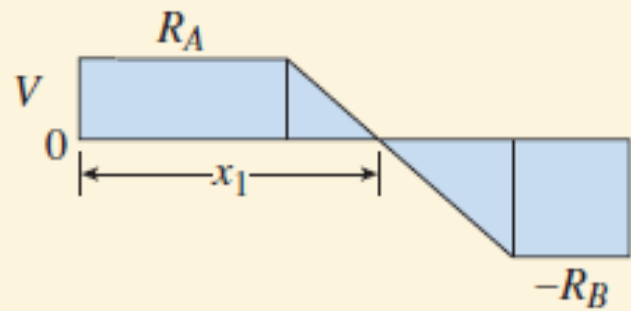


(c)

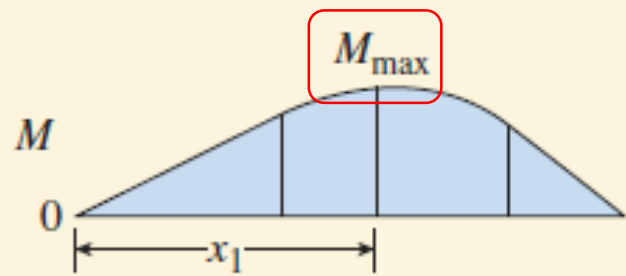




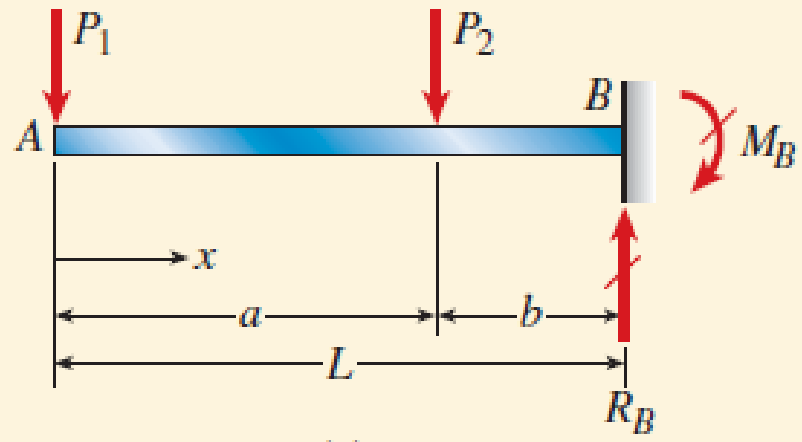
(a)



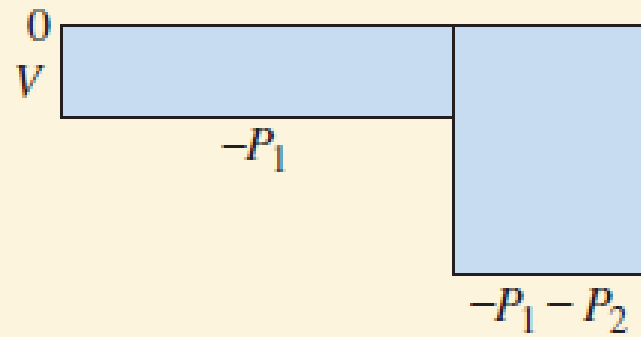
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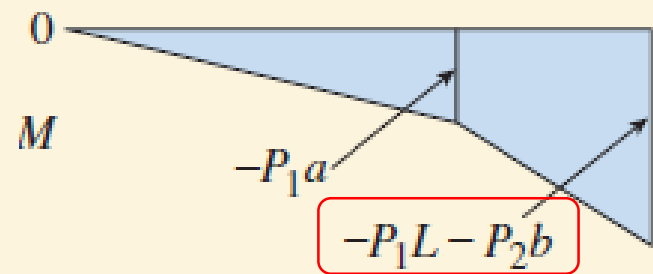
(c)



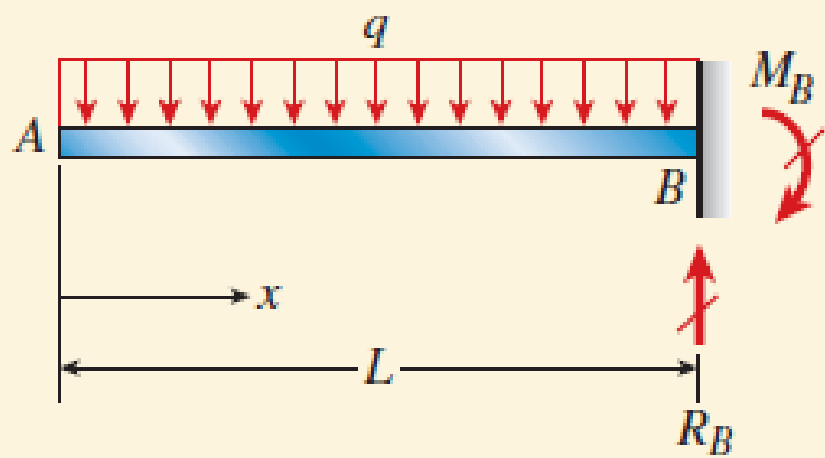
(a)



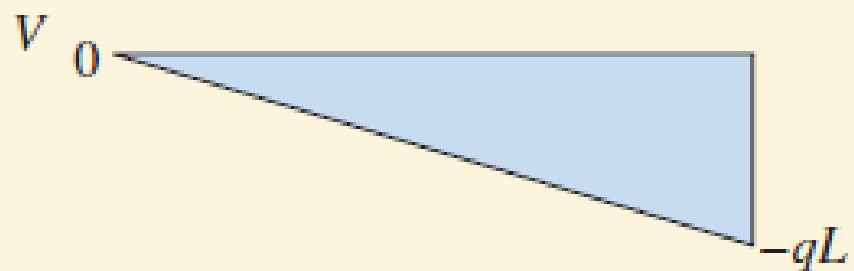
(b)



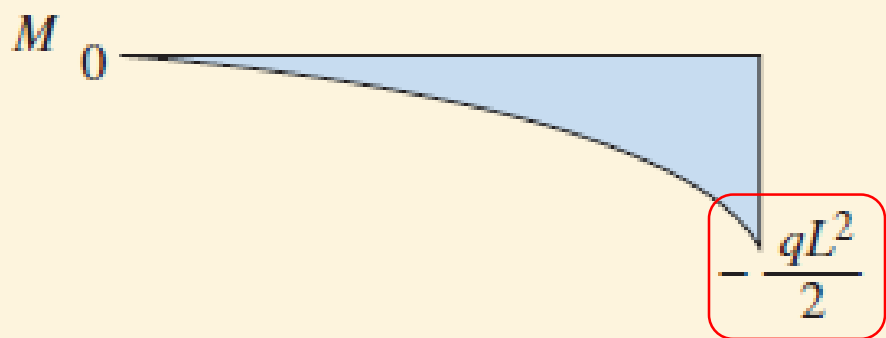
(c)



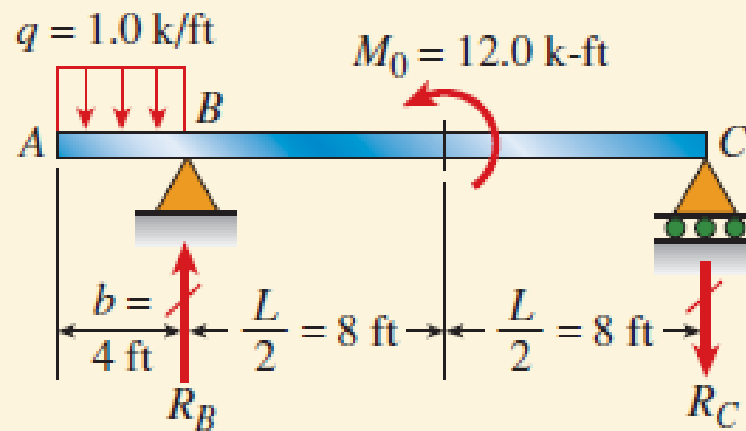
(a)



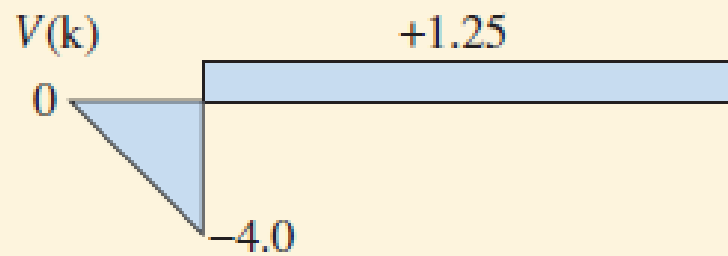
(b)



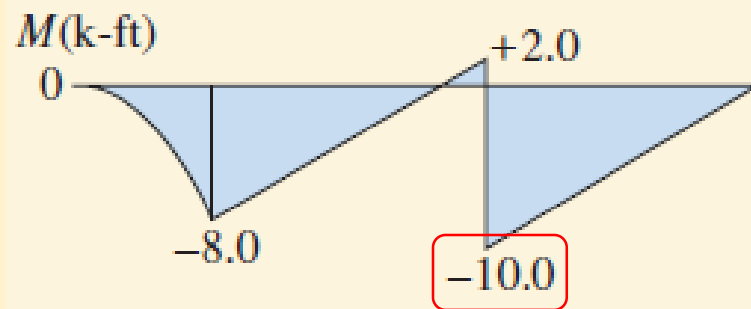
(c)



(a)



(b)



(c)