Precision Machine Design- Flexure Hinge Design

Flexure hinge mechanism is one of high effective and precise mechanism within the limed range of motion.

An effective method for achieving a motion having small range but with most precise control is to apply a force to an elastic mechanism of known stiffness. This is a different concept from techniques using kinematic design or elastic averaging design for achieving the high precision, because the driving force is never applied directly against the stiffness in them, although the driving force needs to overcome the friction.

The pros and cons of the flexure hinge or the elastic mechanism are as follows;

Pros; (Advantage)

1) Ideal for ultra-precision motion of small stroke with fine resolution; the flexure hinge uses the elasticity for motion generation, thus very high precise motion can be obtained up to angstrom level resolution depending on actuator.
2) Friction free motion: friction is one of difficulties when very precise motion is required, even with well lubricated rolling bearings. The flexure hinge provides the elastic

motion due to the elastic deformation that is coming from the distance change between atoms, without giving any friction forces noticeable.

3) Smooth and continuous motion: this is another characteristics of friction free motion, because it does not make any stick-slip or discontinuity during the motion.

4) Wear free motion: because there is no sliding or rolling parts, this flexure hinge provides wear free, thus lubrication, replacing of worn parts are not needed, thus service free operating is possible.

Cons; (Disadvantage)

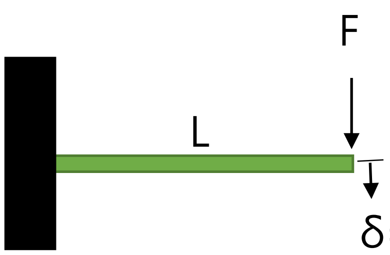
1) Limited range of motion: very small range of motion within the elastic range, typically from few tens of microns to few millimeters.

2) Limited force and stiffness in the driving direction: Because the elastic deformation of flexure is engaged, the stiffness and force in the driving direction are relatively small when compared to other actuating methods of high rigidity. Thus force and stiffness in the driving direction are quite small, and it can be quite weak to the external vibrations of disturbing.

3) Lifetime or durability can be low; because it uses the repeated elastic deformation utilizing the full elastic range, thus it is exposed to fatigue and catastrophic failure, where the allowable fatigue strength should be carefully chosen as quite low, accommodating the relatively large strain and stress concentration factor. Thus the allowable stress or strain can be reduced further.
4) Motion is much depending on material characteristics such as Young's modulus, work-hardening, and they are also varying with temperatures of operation.

<u>Leaf type linear spring</u>

The cantilever of thin plate is one of the simple flexure mechanism;



$\delta = FL^3/3EI$ and $\theta = FL^2/2EI$ at end

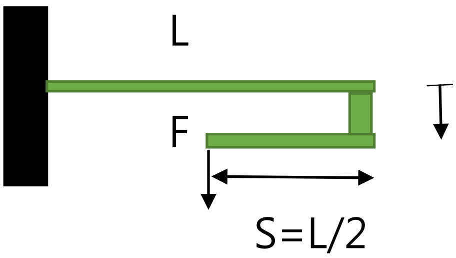
where E=Young's modulus,

I=area moment of inertial=$bt^3/12$, and b is width, t is the thickness of plate,

Stiffness, K, is

$K=F/\delta=3EI/L^3$

The arch shape of deflected cantilever is very strange to use for motion control due to the nonlinearity, and the angle θ experienced at the end is never desirable because it may generate undesirable Abbe error when a device or instrument is attached at the end. In order to minimize or cancel the angle deformation. Thus a coupled force can be applied to negate the nonlinearities at the offset of S as in the fig.



$\delta=FL^3/12EI$ and $\theta=0$ at end

Thus the rotation angle θ is cancelled by the ***superposition*** of moment, FL/2, at the end, and the stiffness, K is,

$K=F/\delta=12EI/L^3$

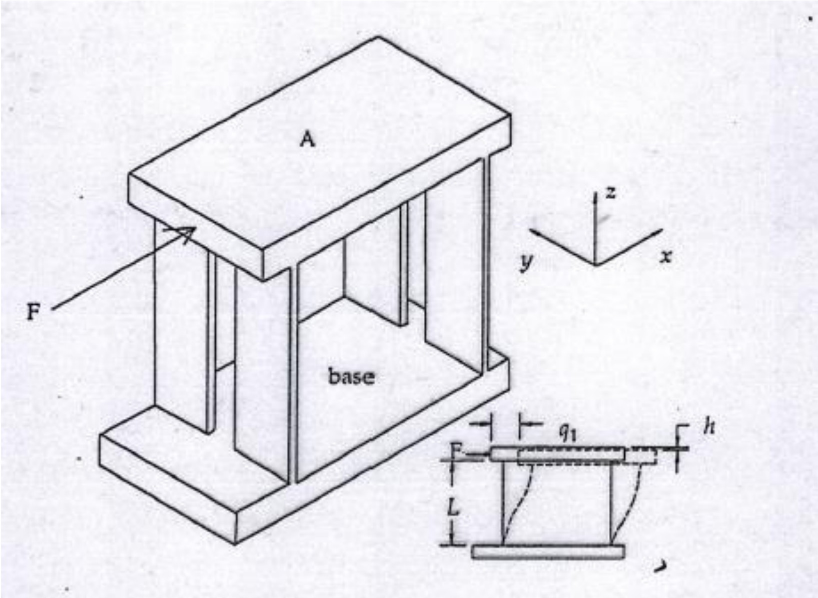
It is four times increased than the cantilever case. This mechanism is enhanced, but it may have still instability because small misalignment of force in direction or location may lead large ***parasitic deflection*** that is a kind of unwanted motion.

This situation can be improved if one or more plates are **symmetrically superposed** in parallel as in fig.

Due to the symmetrically superposed plates, the y rotation angle at the end becomes zero due to the fixing of parallel plates; and the torsional stiffness of z rotational axis is greatly increased due to the increased polar moment of inertia of plates. This mechanism is a type of parallelogram motion spring, and sometimes is called as the single leaf type linear spring. Also, the stiffness in x direction becomes as twice as the previous one, assuming the split section is small.

$K= F/\delta=24EI/L^3$

Please note there exists a parasitic motion, h, in the z direction, which is a kind of unwanted motion.

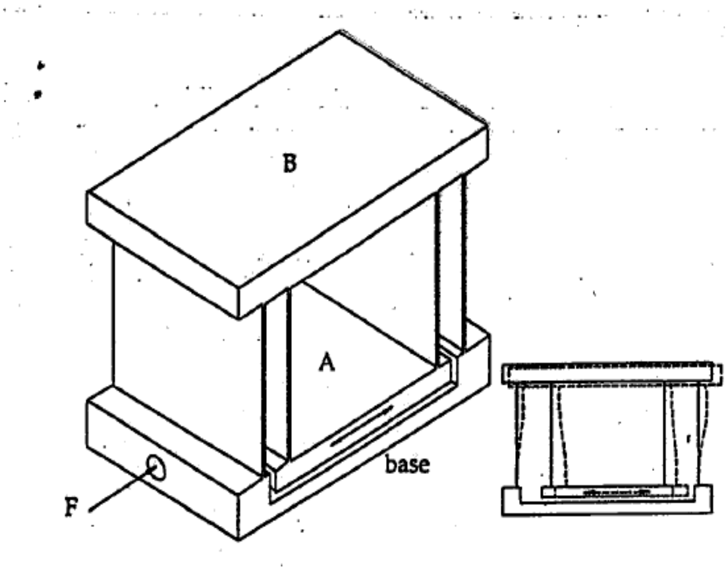


Single leaf type linear spring

(source: Smith's Ultra precision mechanism design)

The unwanted parasitic motion also can be removed by **symmetric superposition** of leaf spring in compound form as in the fig., where the parasitic motion becomes zero or very little, as the top and bottom cancel each other for the parasitic motion. The superposition leads to decrease the stiffness in the driving direction by half when compared to the single leaf type linear spring, as they are serially connected. Thus,

$K= F/\delta=12EI/L^3$



Compound leaf type linear spring

(source: Smith's ultra precision mechanism design)

This mechanism is called as compound leaf type linear spring, and are commonly used for single axis drive mechanism using the leaf type springs, as it can provide virtually no parasitic error motion.

Notch hinge structure

An alternative mechanism for flexure hinge is to use the notch spring or notch hinge, in which several notch holes are manufactured onto the solid structure to have functioning as the flexure mechanism.

Comparisons can be made between the notch hinge and the leaf spring.

For the notch hinge;

1) It is well suited to monolithic structure for higher precision due to simple manufacturing holes or notches onto the monolithic structure; while the bolting, screwing, welding, etc. are required during the assembly for the leaf spring.

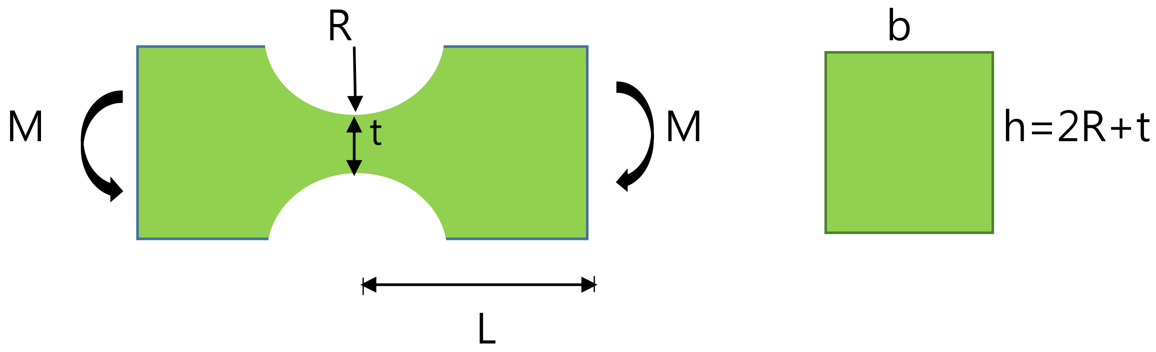
2) It has very good agreement between the theoretical analysis and experiment for the flexure characteristics; while about 30% discrepancy is typically observed between the theory and experiment for the cantilever parts.

3) Easy manufacturing via EDM, drilling, or turning; while screwing, bolting, welding, fastening for leaf spring; thus the notch hinge is of cost efficiency with compact/simple design.

4) Stronger bucking resistance due to shorter length in thin section when compared to the leaf spring

5) Notch location equals to the flexing position, while only the notch part is flexing and the rest section is quite flat for the leaf spring.Notch hinge gives more flexible design.

A Notch Hinge



When E is the young's modulus of elasticity for the material,

1) $h \fallingdotseq 2R+t$; when the notch is close to half circle

Rotation angle, $\theta= 9\pi R^{1/2}M/[2Ebt^{5/2}]$

Thus Angular stiffness, λ_θ

$\lambda_\theta=M/\theta=2Ebt^{5/2}/[9\pi R^{1/2}]$ eq(1) (by Paros and Weisbord)

2) $t<R<5t$

Angular stiffness, $\lambda_\theta=M/\theta=Ebt^3/[24KR]$ eq(2)

where $K=0.565t/R=0.166$ (by Smith etal.)

Maximum stress, σ_{max}, is observed at the top part of hinge,

$\sigma_{max} = K_tM(t/2)/[bt^3/12]=K_tM/[bt^2/6]$

where K_t is the stress concentration factor for the circular notch shape, and $K_t=0.325+[2.7t+5.4R]/[t+8R]$

For the allowable maximum stress, σ_{max} , the allowable maximum moment, M_{max}, is

$M_{max}=bt^2\sigma_{max}/[6K_t]$

Thus the maximum allowable angle, θ_{max}, is

$\theta_{max}=M_{max}/\lambda_\theta$

$= 9\pi R^{1/2}M_{max}/[2Ebt^{5/2}]$ for $h \fallingdotseq 2R+t$

$= 24KRM_{max}/[Ebt^3]$ for $t<R<5t$

The right tip of the notch hinge will experience the maximum deflection q_{max} relative to the left end of the notch hinge,

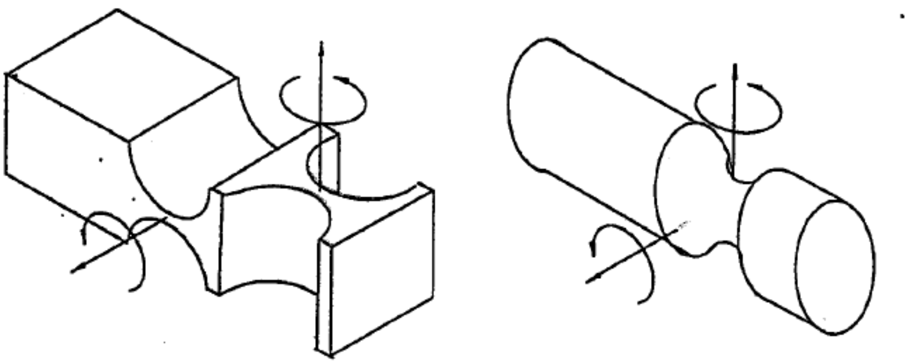
$q_{max}=L\theta_{max}$

where L is the distance from the notch centre to the right tip.

Multi directional flexure hinge

The notch flexure hinge can generate the angular motion in the perpendicular direction to the plane of force. Thus when multi directional motion is desired, the notch flexure of another axis can be superposed or added to give the desired motion as in fig.

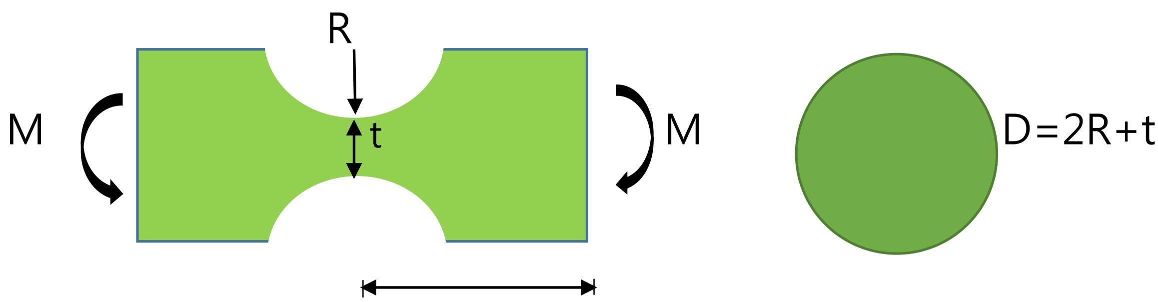
The circular flexure hinge can be used for the universal direction of motion as in the fig.



Two axis notch hinge and universal circular hinge

(source: Smith's ultraprexision mechanism design)

Circular flexure hinge, or, Flexure for universal direction



When monolithic material is cylinder instead of cuboid, the circular notch can be manufactured by turning. Then direction of bending can be any direction as it just follows the direction of force. This is called as circular notch flexure hinge or universal flexure hinge due to its all directional bending. This flexure is useful to the situation where actuation is required under which some misalignments are occurring such as in driving section by PZT actuator, thus the misalignment can be compensated by the universal flexure hinge like universal joint for the power transmission.

In this case, the rotation angle and moment relationship can be similarly derived as,

$\theta=20MR^{1/2}/[Et^{7/2}]$, and angular stiffness λ_θ is

$\lambda_\theta=Et^{7/2}/[20R^{1/2}]$

Sensitivity analysis for notch hinge

It is of interest to assess the sensitivity analysis for the notch hinge. When there are variations in the dimensions or material property for the hinge, it affects to the stiffness of hinge, and it can be derived from eq(1)

$\lambda_\theta=M/\theta=2Ebt^{5/2}/[9\pi R^{1/2}]$ eq(1)

$\delta\lambda=(\partial\lambda/\partial E)\delta E+(\partial\lambda/\partial b)\delta b+(\partial\lambda/\partial t)\delta t+(\partial\lambda/\partial R)\delta R$ eq(10)

As $\partial\lambda/\partial E=\lambda/E$, $\partial\lambda/\partial b=\lambda/b$, $\partial\lambda/\partial t=(5/2)\lambda/t$, $\partial\lambda/\partial R=(-1/2)\lambda/R$;

Thus from eq(10),

$\delta\lambda/\lambda=\delta E/E+\delta b/b+(5/2)\delta t/t-(1/2)\delta R/R$

As a maximum case, the total contribution is from the sum of absolute value of individual factors, thus

$|\delta\lambda/\lambda|=|\delta E/E|+|\delta b/b|+(5/2)|\delta t/t|+(1/2)|\delta R/R|$

Or a most probable case, the total contribution is from the square root of sum of squares of the individual factors. Thus,

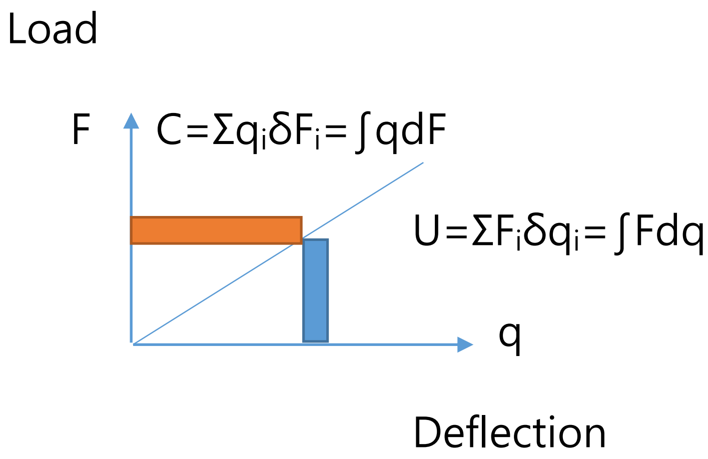
$\delta\lambda/\lambda=[(\delta E/E)^2+(\delta b/b)^2+(5/2)^2(\delta t/t)^2+(1/2)^2(\delta R/R)^2]^{1/2}$

The thickness variation, $\delta t/t$, greatly affects to the stiffness variation of the hinge, thus strict dimensional control for the thickness is needed.

Energy Method for Static Analysis

As the flexures are of purely elastic motion, the energy method can be applied to give the static analysis such as stiffness.

Let U be the total elastic energy stored in structure or mechanism, and it is a function deflection q_i such that $U=U(q_1,q_2..q_n)$, where q_i (for i=1,2..n) are the deflections(or angles) at the ith location of structure, and F_i (for i=1,2..n) are the external forces(or moments) applied to give q_i deflection.



C=Complementary energy=$\int qdF$; thus $q=\partial C/\partial F$

: by Engesser(1889), no physical meaning,

but just for mathematical convenience

U=Strain energy=$\int Fdq$; thus $F=\partial U/\partial q$

: Physical strain energy stored

For linear, elastic material; C=U

∴No difference between them,

Thus completely interchangeable such that

$q=\partial U/\partial F$ (Castigliano's theorem), or

$F=\partial U/\partial q$ (Virtual Work or Energy method)

The energy method gives the relationship between the deflection and the forces such as,

$F_i = \partial U / \partial q_i$ for i=1,2..n

Thus stiffness λ_i at the F_i location can be obtained as

$\lambda_i = F_i / q_i$ for i=1,2..n

The energy method can provide a very efficient tool for the calculation of stiffness comprising of complex structures/mechanism behaving in the elastic region.

Mobility or Kinematic analysis for DOFs

A kinematic system can consist of N elements with J joints, where the elements are not deforming thus rigid, and joints provide constraints to restrict the DOF(Degree of Freedom) of the system.

One free element can have maximum 6 DOFs in a space, and one element should be fixed for reference of motion, thus the maximum DOFs what the system can have is 6(N-1) for N elements.

The joint is for constraining the system, and the number of constraints for the joint, c, will be the 6 minus the number of freedom of that joint, f, such that $c_i = 6 - f_i$ for i=1 to J joints

The total DOFs what the kinematic system can have will be;

Total DOFs=6(N-1) - Σc_i (for i=1 to J)

=6(N-1) – Σ (6- f_i) (for i=1 to J)

The above equation provides very useful for the mobility analysis of general kinematic structures. For the plane mechanism, this equation reduces to

DOFs in 2D=3(N-1)-2J

This is called as Grubler's equation, and is because the maximum DOFs what the plane mechanisn can have will be 3 for each element, and number of constraints will be 2 for each joint such that δx, δy are constrained while θ is free for a hinge joint. For example four bar linkage system, N=number of elements=4, J=number of joints=4; the DOFs what the four bar linkage can have will be 3(4-1)-2(4)=1, and thus it is only for one DOF free. The Grubler's equation is very useful to analyze the mobility whether the system is kinematically constrained, over-constrained, or under-constrained.

Dynamic analysis

Dynamic analysis is required to give the dynamic characteristics such as natural frequency of the mechanism. There are mainly two reasons for dynamic analysis:

1) Fast servo time, or fast response of system is desirable to give the high precision motion control, because the smallest increment of motion can be run during the shortest time interval, Δt. The time interval, Δt, can be usually chosen as 1/2-1/3 of the fundamental period (or the inverse of natural frequency) of the system, in order to avoid biasing.

2) The system capability of isolation from the vibration disturbance is very important to the ultra-precision motion control. When the natural frequency is quite high, and the disturbing vibration frequency is lower than the natural frequency, the system will not experience the resonance. When the disturbing frequency is high and is close to the natural frequency of the system, the energy of disturbing frequency is very low, thus the resonance cannot happen easily. Thus it is very good practice to have the system's natural frequency higher. Thus the higher the natural frequency, the higher the capability of vibration isolation. It is the golden rule for the precision mechanism of higher performance, being isolated from external vibration disturbance.

For the system having potential energy and kinetic energy, the ***Lagrangean*** principle can be applied to give the efficient dynamic analysis.

Let F_i , q_i (i=1,2..n) be the force and displacement experienced at the sub-system i, and m_i is the mass of the sub-system in the mechanism.

The Kinetic energy, T, is the sum of kinetic energy of sub-syetem of mass, m_i

$T=\Sigma m_i(dq_i/dt)^2/2$, for i=1,2..n eq(20)

The potential energy or elastic energy, U, is the sum of the elastic energy of sub-system, whose stiffness is K_i in the q_i direction.

$U=\Sigma K_i q_i^2/2$, for i=1,2..n eq(21)

The ***Lagrangean***, L, is derived as T-U such as

$L=T-U$ eq(22)

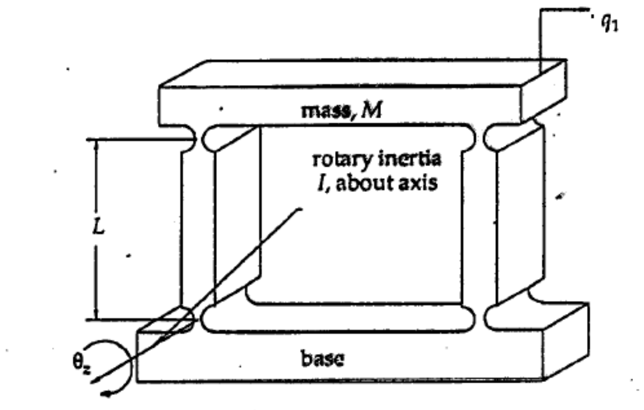
Then the motion of equation of the each sub-system is given by the ***Lagrangean*** equation;

$d[\partial L/\partial(dq_i/dt)]/dt - \partial L/\partial q_i=F_i$ for i=1,2..n eq(23)

Eq(20) and (23) lead to the motion of equations for every sub-system in the mechanism.

Simple notch type spring

When the notches are symmetrically superposed like the simple leaf type linear spring as in fig, it becomes a simple type notch spring, providing smooth motion into q_i direction under the force applied to the direction. Please also note that it can give some parasitic motion in the vertical direction due to the rotations of four hinges.



Simple notch type linear spring

(source: Smith's Ultraprecision mechanism design)

Mobility analysis

Considering the number of elements=4, and the number of joints equals to the number of notches=4. Thus allowable DOFs=3(4-1)-2(4)=1. Therefore this mechanism provides 1 DOF, and it is kinematically constrained for the 1 DOF motion into the q_1 direction.

Static analysis

Assume F is applied to the mechanism along the q_1 direction, then the moment of FL is input to the mechanism, and the moment is assumed as equally allocated to each hinge (or notch) due to the symmetry. Thus the moment of FL/4 is applied to each notch, giving the angular deflection, $\theta=M/\lambda_\theta$, giving the elastic energy storage of $0.5\lambda_\theta\theta^2$. Thus the total elastic energy, U, stored in the mechanism will be four times of this, thus

$U=4(0.5)\lambda_\theta\theta^2=2\lambda_\theta\theta^2=2\lambda_\theta(q_1/L)^2$

Applying the energy method,

$F=\partial U/\partial q_1=4\lambda_\theta q_1/L^2$

Thus stiffness in the q_1 direction, K_1, is

$K_1=F/q_1=4\lambda_\theta/L^2$ [N/m]

where λ_θ is the angular stiffness of one notch given by eq(1),eq(2).

This simple notch spring generate the parasitic motion in the vertical direction, and it would be $\delta = L(1-\cos\theta)$ downward, where θ is the angle of rotation experienced at a notch.

Dynamic analysis

Kinetic energy:

When m is the mass of the notched element of length, L , the kinetic energy of the mechanism can be considered as sum of kinetic energy of top part of mass, M and two hinge parts of mass, m, respectively, while the base is stationary.

Thus kinetic energy, T, is

$T=M(dq_1/dt)^2/2+ I(dq_1/dt/L)^2/2 + I(dq_1/dt/L)^2/2$

Where I=mass moment of inertia of hinge part about the axis of bottom hinge=$mL^2/12+m(L/2)^2 = mL^2/3$

Thus $T= M(dq_1/dt)^2/2+I(dq_1/dt/L)^2/2+ I(dq_1/dt/L)^2/2$

$=[M+2m/3](dq_1/dt)^2/2$

The potential or strain energy, U

U=sum of elastic energy equally stored in the four hinges

$=\lambda_\theta\theta^2/2+ \lambda_\theta\theta^2/2+ \lambda_\theta\theta^2/2+ \lambda_\theta\theta^2/2$

$=2\lambda_\theta(q_1/L)^2$ where λ_θ is defined as above.

The Lagrangean, L

$L=T-U= [M+2m/3](dq_1/dt)^2/2 - 2\lambda_\theta(q_1/L)^2$

$\partial L/\partial(dq_1/dt)=[M+2m/3](dq_1/dt)$

$d[\partial L/\partial(dq_1/dt)]/dt=(M+2m/3)d^2q_1/dt^2$

$\partial L/\partial q_1=-4\lambda_\theta q_1/L^2$

Thus the motion of equation for q_1 is

$(M+2m/3)d^2q_1/dt^2 +4\lambda_\theta q_1/L^2 = F$

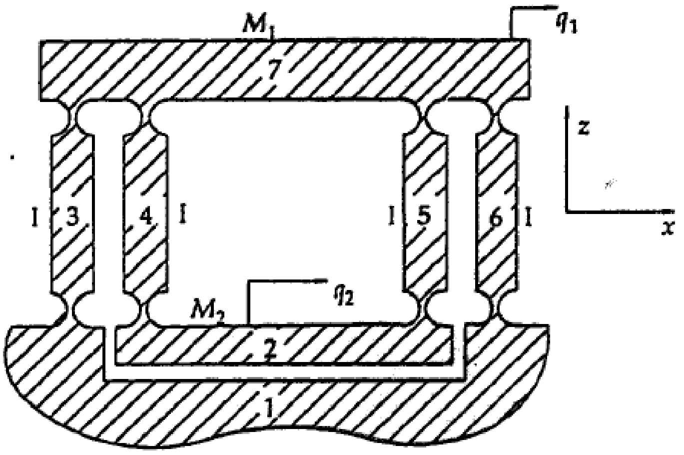
When F=0, it gives the fundamental response of q_1 of the mechanism. Thus

$(M+2m/3)d^2q_1/dt^2 +4\lambda_\theta q_1/L^2=0$

Therefore the natural frequency, ω_n, of the mechanism can be obtained as follows;

$\omega_n=[4\lambda_\theta/L^2/(M+2m/3)]^{1/2}$ [rad/sec]

Compound notch type spring



Compound notch type linear spring (source: Smith's Ultraprecison mechanism design)

The notched hinge structure can be added (or symmetrically superposed) as shown in fig, then it can give improved flexure hinge structure that is much similar to the compound leaf type linear spring, giving zero parasitic motion, but with the less stiffness in the driving direction when compared to the simple notch type linear spring.

Mobility analysis

There are 7 elements including base fixed, and 8 joints (hinges); thus N=7, J=8

From Grubler's equation for plane mechanism,

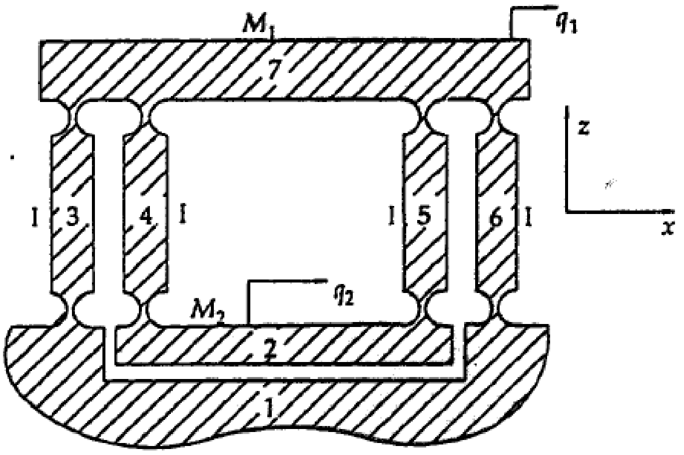
DOF=3(N-1)-2J=3(7-1)-2(8)=18-16=2

This mechanism provides 2 DOFs such as q_1 and q_2;

Although there exists 2 DOFs along the driving direction, they can be related by $q_1=q_2/2$ if the applied force F_1 becomes zero due to the symmetric structure, as it is explained in the static analysis

Static Analysis

Stiffness Calculation



Let F_1 , F_2 be the forces applied to the q_1, q_2 displacement, respectively.

For the hinges connected between the top parts and base, the angle of rotation, θ_1, due to the q_1 displacement become

$\theta_1 = q_1/L$, where L is the length of hinged part.

For the hinges connected between the top parts and bottom parts, the angle of rotation, θ_2, become

$\theta_2 = (q_2 - q_1)/L$

The total elastic energy, U, stored the in the mechanism is the sum of the elastic energy stored in the 8 hinges.

$U = \lambda_\theta \theta_1^2/2 \times 4 + \lambda_\theta \theta_2^2/2 \times 4$

$= 2\lambda_\theta q_1^2/L^2 + 2\lambda_\theta (q_2 - q_1)^2/L^2$

Applying the energy method to obtain the force F_1 and F_2;

$F_1 = \partial U/\partial q_1 = 4\lambda_\theta q_1/L^2 + 4\lambda_\theta (q_2 - q_1)(-1)/L^2$

$= 4\lambda_\theta (2q_1 - q_2)/L^2 = 0$ if there is no force applied to the q_1

Thus $q_1 = q_2/2$, and it is the same result as we expect, due to the symmetry of structure.

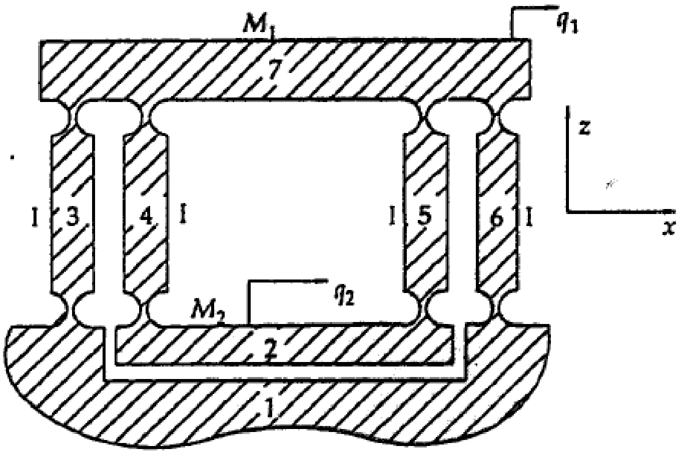
$F_2 = \partial U/\partial q_2 = 4\lambda_\theta (q_2 - q_1)/L^2 = 4\lambda_\theta (q_2/2)/L^2 = 2\lambda_\theta q_2/L^2$

Thus the Stiffness, K_2, in the driving direction of q_2 , becomes

$K_2 = F_2/q_2 = 2\lambda_\theta/L^2$

Thus the stiffness in the driving direction becomes half of the simple notch spring, but the vertical parasitic motions are cancelled for the bottom moving part, when compared to the simple notch spring case.

Dynamic Analysis



Let M_1, M_2 be the mass of top part and bottom part, respectively, and m is the mass of hinged part.

Kinetic Energy

Top part: $M_1(dq_1/dt)^2/2$

Bottom part: $M_2[dq_2/dt]^2/2$

For the one hinged part connected between base and top:

$m[dq_1/dt/2]^2/2 + I[dq_1/dt/L]^2/2$ eq(30)

where I=mass moment of inertial of one hinged part about axis of rotation centered=$mL^2/12$

Eq(30) becomes

$=(m/4+m/12)(dq_1/dt)^2/2=(m/3)(dq_1/dt)^2/2$

and the kinetic energy for the two hinged parts

$=(2m/3)(dq_1/dt)^2/2=(m/3)(dq_1/dt)^2$

One hinged part connected between the top moving part and bottom moving part:

$m[d\{q_1+(q_2-q_1)/2\}/dt]^2/2 + I[d(q_2-q_1)/dt/L]^2/2$

$=m[d(q_1+q_2)/dt]^2/8 + m[d(q_2-q_1)/dt]^2/24$

Thus for two hinged parts

$=m[d(q_1+q_2)/dt]^2/4 + m[d(q_2-q_1)/dt]^2/12$

Thus the total kinetic energy, T

$= M_1(dq_1/dt)^2/2+M_2[dq_2/dt]^2/2$

$+(m/3)(dq_1/dt)^2+m[d(q_1+q_2)/dt]^2/4 + m[d(q_2-q_1)/dt]^2/12$

Potential or strain energy, U

Potential energy, U, is the same as the static analysis, and

$U=2\lambda_\theta q_1^2/L^2 + 2\lambda_\theta(q_2-q_1)^2/L^2$

Lagrangean, L=T-U

For q_1 displacement,

$\partial L/\partial(dq_1/dt)=M_1(dq_1/dt)$

$+(2m/3)(dq_1/dt)+(m/2)[d(q_1+q_2)/dt]+(m/6)[d(q_2-q_1)/dt(-1)]$

$=M_1dq_1/dt+4m/3(dq_1/dt)+(m/3)(dq_2/dt)$

$\partial L/\partial q_1=-[4\lambda_\theta q_1+4\lambda_\theta(q_2-q_1)(-1)]/L^2=-4\lambda_\theta(2q_1-q_2)/L^2$

Thus

$(M_1+4m/3)d^2q_1/dt^2+(m/3)d^2q_2/dt^2+4\lambda_\theta(2q_1-q_2)/L^2$

$=F_1=0$ if homogeneous solution eq(31)

For q_2 displacement;

$\partial L/\partial(dq_2/dt)=M_2dq_2/dt+(m/2)[d(q_1+q_2)/dt]$

$+(m/6)[d(q_2-q_1)/dt]=(M_2+2m/3)dq_2/dt+m/3dq_1/dt$

$\partial L/\partial q_2=-4\lambda_\theta(q_2-q_1)/L^2$

Thus

$(m/3)d^2q_1/dt^2+(M_2+2m/3)d^2q_2/dt^2-4\lambda_\theta(q_2-q_1)/L^2$

$=F_2=0$ if homogeneous solution; eq(32)

Eq(31),(32) give the motion of equations, and they are the second order differential equations.

Let $q_1=c_1\exp(j\omega t)$, $q_2=c_2\exp(j\omega t)$;

then $d^2q_1/dt^2=-\omega^2q_1$ and $d^2q_2/dt^2=-\omega^2q_2$

From eq(31)

$(M_1+4m/3)d^2q_1/dt^2+(m/3)d^2q_2/dt^2+4\lambda_\theta(2q_1-q_2)/L^2$

$=[-(M_1+4m/3)\omega^2+8\lambda_\theta/L^2]q_1+[-(m/3)\omega^2-4\lambda_\theta/L^2]q_2=0$ eq(33)

From eq(32)

$[-(m/3)\omega^2-4\lambda_\theta/L^2]q_1+[-(M_2+2m/3)\omega^2+4\lambda_\theta/L^2]q_2=0$ eq(34)

In order to have nontrivial solution for q_1 and q_2, the determinant of eq(33),(34) are zero;

The characteristics equations are;

$[-(M_1+4m/3)\omega^2+8\lambda_\theta/L^2][-(M_2+2m/3)\omega^2+4\lambda_\theta/L^2]$

$-[-(m/3)\omega^2-4\lambda_\theta/L^2]^2=0$

Thus

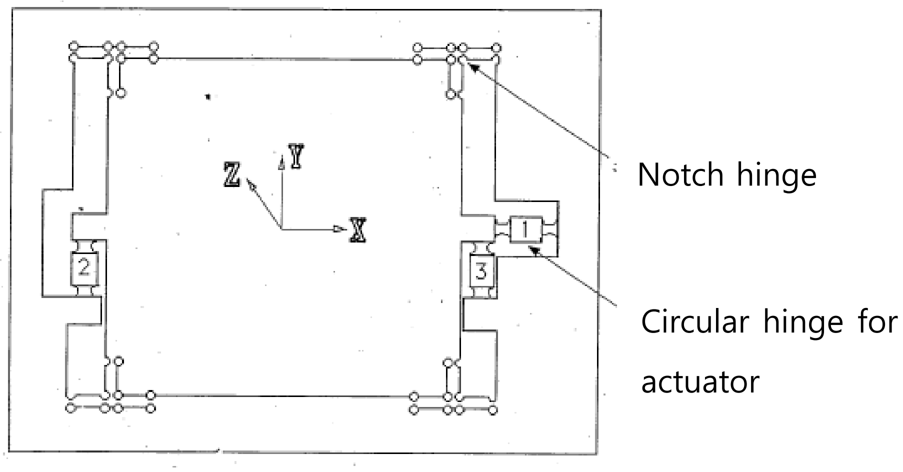
$$\omega^4[(M_1+4m/3)(M_2+2m/3)-(m/3)^2]$$

$$-\omega^2[4M_1+8M_2+40m/3]\ \lambda_\theta/L^2+16(\lambda_\theta/L^2)^2=0$$

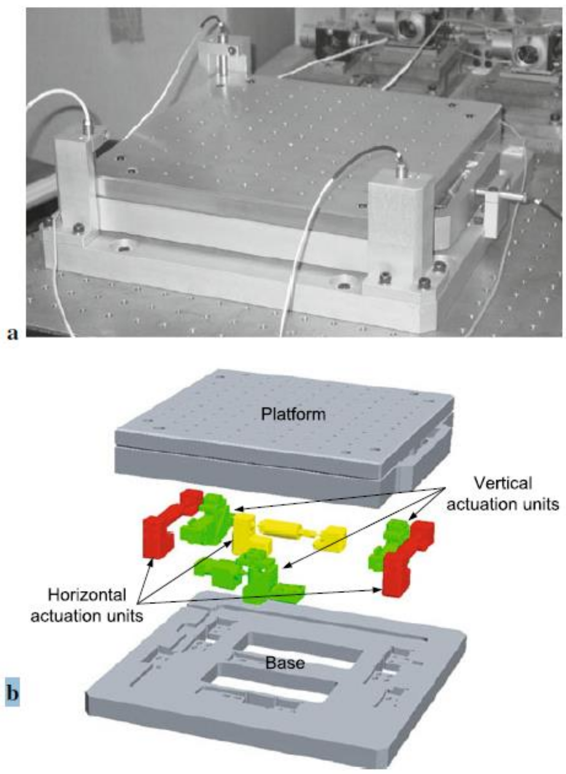
Therefore the natural frequency of the mechanism, ω, can be obtained. Also, the mode shape, or eigen vector can be obtained for the q_1, q_2 displacement.

Multi DOF mechanism

The multi DOF mechanism can be designed by fully utilizing the symmetry and superposition of hinge elements. Fig shows the 3DOF stage, and 6 DOF stage, respectively. The stress analysis and design analysis are very much essential for the stiffness, vibration isolation capability, allowable range and lifetime. These analysis is quite complex thus, it is much efficient to use the FEM during the design stage.



3 DOFs stage designed (source: SNU Metrology Lab)

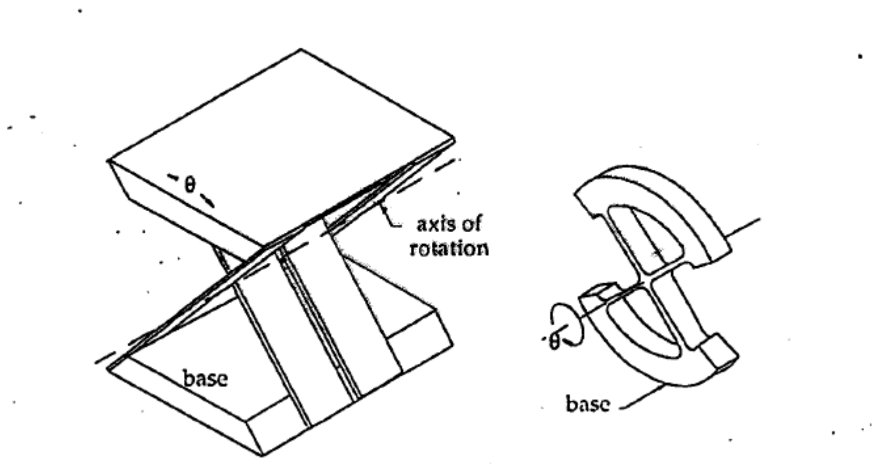


6 DOFs Stage Designed

(source: Mun and Pahk, Int.J.Advanced Manuf. Tech)

Angular motion flexure hinge:

In order to generate the angular motion, there are also some angular flexure hinges such as crossed strip hinge, monolithic torsional hinge, etc., as in fig, where the bending modes or the torsion modes of the thin sections or notched parts are fully utilized.



Cross strip hinge and Torsional hinge

(source: Smith's Ultraprecision Mechanism Design)