Chapter 9. Electrical Properties

Electrical properties of materials

- Electrical conductivity of materials varied over a wide range, from values of the order of 10^{6} $(\Omega\ cm)^{-1}$ for metals to less than 10^{-16} $(\Omega\ cm)^{-1}$ for insulators.
- Conductivity of the semiconductor is strongly dependent on both the temperature and purity of the semiconductor.
- T-dependence of conductivity ~ T-dependence of free carrier density and carrier mobility

ex1) metal: no T-dependence of free carrier density

semiconductor: exponential increase of free carriers with T

ex2) carrier mobility is determined by scattering (acoustic lattice waves and charged imperfections)

Ohm's law

- In general, the current (I) flowing through a wire is proportional to the voltage (V) applied across the wire as follows. (Ohm's law)

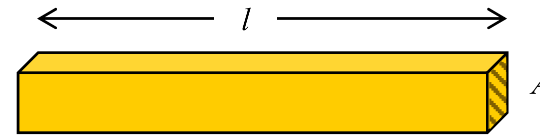
$$V = R \times I$$

where R, called the resistance, depends on the geometry of the wire.

- Electrical characteristics independent of the geometry are defined as "resistivity (ρ)" and "conductivity (σ)".

$$J = \sigma E$$

$$J \equiv \frac{I}{A}, E = \frac{V}{L}, \ \sigma = \frac{1}{\rho} = \left(\frac{1}{R}\right)\frac{l}{A}$$



$$J(\text{drift current}) = nqv_{\mathrm{d}} = qn\mu E \ (v_{\mathrm{d}} \text{ is the drift velocity})$$

$$\sigma = qn\mu$$

$$\sigma = q(n\mu_n + p\mu_p)$$ for electrons and holes

Ohm's law

- Ohm′s law requires neither n and μ be a function of E.
- However, high electric field may cause n to increase by several reasons.
  - Injection of carriers from the electrodes
  - Impact ionization of localized carriers at imperfections
  - Field emission by tunneling of localized carriers at imperfections or even electrons in the valence band to the nearest conduction band

  $n\uparrow$ as $\varepsilon\uparrow$

- Mobility may also become a function of electric field if the scattering processes are a function of carrier energy.
  - The scattering relaxation time for acoustic lattice scattering in a semiconductor decreases with increased carrier energy.

    (carrier energy ↑ → velocity ↑ → scattering probability ↑)
  - The scattering relaxation time for charged impurity scattering increases with increased carrier energy.

    (carrier energy ↑ → Coulomb force effect ↓ → scattering probability ↓)

  $\mu\downarrow$ as $E\uparrow$

Temperature-dependence of Conductivity

T − dependence on σ
$$\sigma(T) = n(T)q\mu_n(T) + p(T)q\mu_p(T)$$

In a metal,

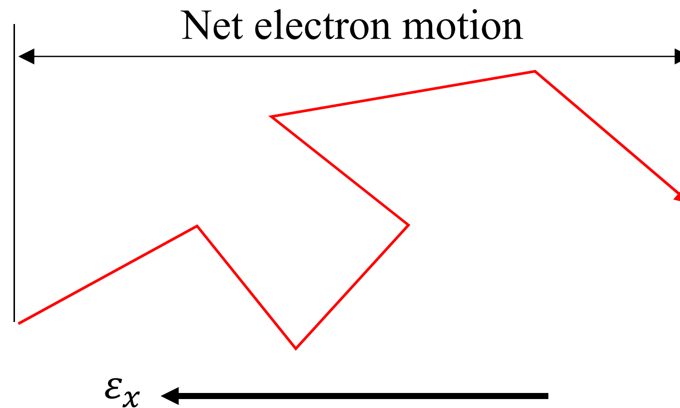
- Density of free carriers (n) is constant: $\sim 10^{22}$ cm^{-3}
- Mobility (μ): $10^2 \sim 10^3$ cm^2/Vsec
- Large conductivity: $10^5 \sim 10^6$ (Ω cm)$^{-1}$ or S cm^{-1}
- Since n is T-independent, T-dependence of conductivity comes from the T-dependence of the mobility.

In a semiconductor,

- Density of free carriers (n) increases rapidly with temperature due to thermal excitation of electrons, either from imperfections or across the band gap.
- T-dependence of mobility in a semiconductor is similar to that in a metal.
- However, T-dependence of conductivity in a semiconductor is dominated by the T-dependence of the carrier density.

Temperature-dependence of mobility

- An electron in a solid in an applied electric field moves in a pattern shown below. The electron progress is constantly disturbed by scattering.



- The mean free time or scattering relaxation time (τ): average time between scattering events ($\tau = \tau(E)$)
- mean free path (l): average distance between the scattering events

$\tau = l/v$ (v: thermal velocity)

Temperature-dependence of mobility

- Scattering mechanisms
  - acoustic lattice waves: scattering of free carriers by interaction with lattice atoms as they move in acoustic vibrational modes due to thermal energy.
  - optical lattice waves
  - charged imperfections: scattering by interaction with charged impurities through Coulomb force
  - neutral imperfections
  - dislocations
  - grain boundaries
  - surfaces
  - inhomogeneities

Acoustic lattice scattering

: Scattering of the free carriers by interaction with lattice atoms in acoustic vibration modes due to thermal energy

- Probability of scattering by acoustic wave

  $\propto$ Energy in lattice wave $\propto kT$
- Mean free path (l) $\propto T^{-1}$
- Relaxation time, $\tau(E) = \frac{l}{v} \propto Tv^{-1}$

For a semiconductor

$\tau \propto \mu \propto T^{-3/2}$ at all T (always elastic scattering)

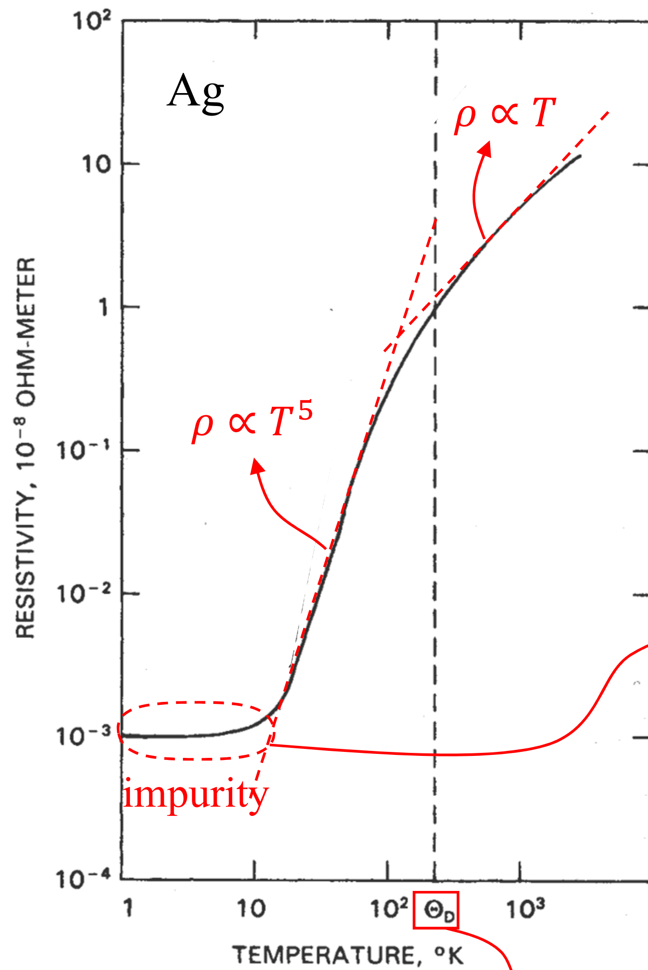
For a metal

$\tau \propto \mu \propto T^{-1}$ at high T (> kT) (elastic scattering)

Because $\tau(E)$ is $\tau(E_F)$. (Most scattering events are experienced by electrons near the Fermi energy.)

$\tau \propto \mu \propto T^{-5}$ at low T (inelastic scattering)

Temperature-dependence of Resistivity in a Metal



$T_{Debye} = 226$ K

For a metal

$\tau \propto \mu \propto T^{-1}$ at high T

$\tau \propto \mu \propto T^{-5}$ at low T

At very low T, ρ is constant due to impurity scattering.

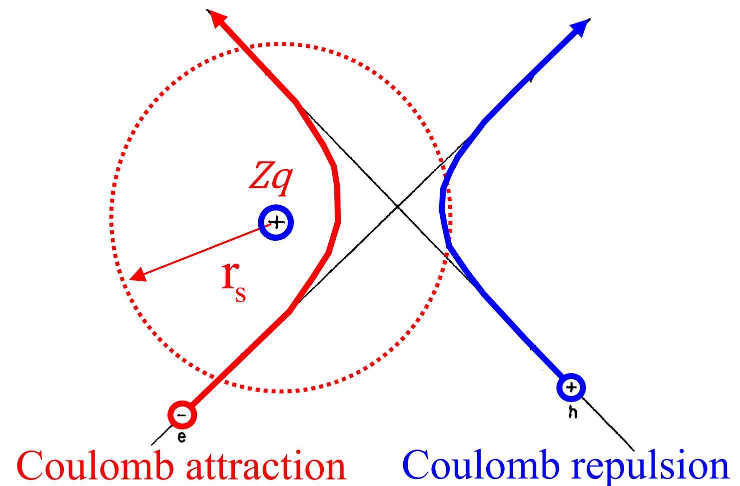
Therefore, small ρ at low T indicates that metal has small density of impurities.

Charged Impurity Scattering in a Semiconductor

- Scattering by charged impurities in semiconductors
- Coulomb attraction or repulsion
- Scattering effect is large only when the Coulomb interaction energy is comparable to the thermal energy, *i.e.*, if

$$\frac{Zq^2}{4\pi\varepsilon_r\varepsilon_0 r_s} = kT$$

where r_s radius of scattering.



Scattering cross section $(S_I) = \pi r_s^2$

$$S_I = \frac{Z^2q^4}{16\pi\varepsilon_r^2\varepsilon_0^2(kT)^2}$$

: When carriers come within this radius, scattering occurs.

If N_I is the charged impurity scattering centers, Rate of scattering $= \tau^{-1} = N_I S_I v$
(τ: relaxation time for charged impurity scattering, $v \propto T^{1/2}$)

$$\tau \propto \mu \propto T^{3/2}$$

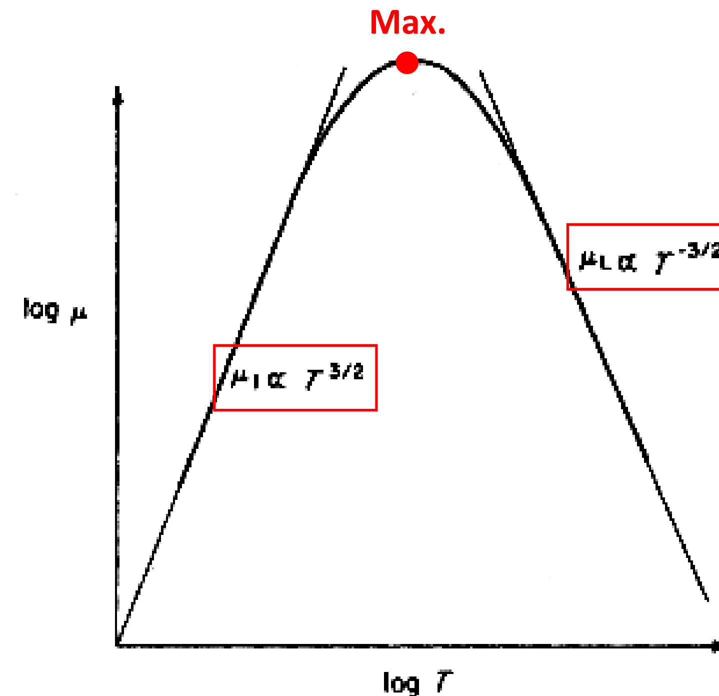
Temperature-dependence of Mobility in a Semiconductor

For a semiconductor with both acoustic lattice scattering and charged impurity scattering

$\mu^{-1} \simeq \mu_L^{-1} + \mu_I^{-1}$ Matthiessen's rule

$\mu_L = AT^{-3/2}, \qquad \mu_I = BT^{3/2}$

maximum mobility
$\mu_L = \mu_I \rightarrow T = (A/B)^{1/3}$



$\mu_I \propto (N_I)^{-1}$
: As N_I decreases (purity increases), μ_I increases.
Therefore, maximum mobility moves to lower temperature with increasing purity of the material.

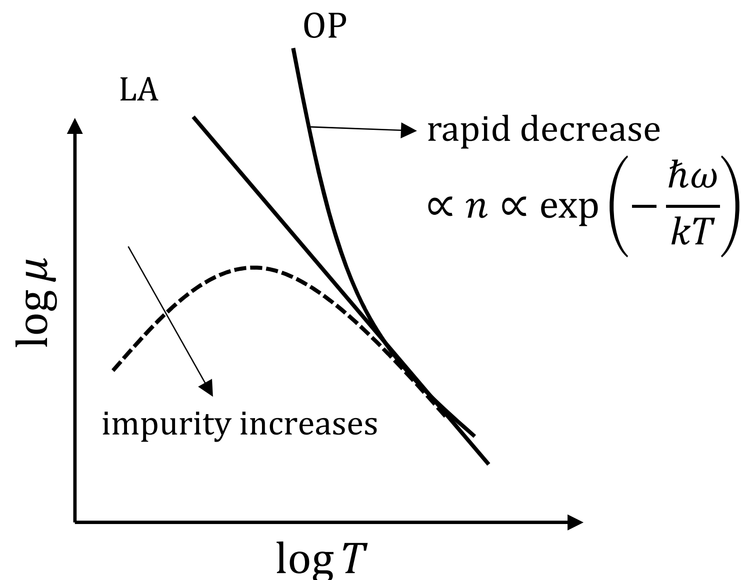
Other scattering processes

- Optical lattice waves: inelastic due to large optical phonon energy

  $\hbar\omega_{pn} \geq kT$

  Probability of scattering $\propto n \propto \frac{1}{e^{\hbar\omega/kT}-1} \propto e^{-\hbar\omega/kT}$ at low T

  (Bose-Einstein distribution)

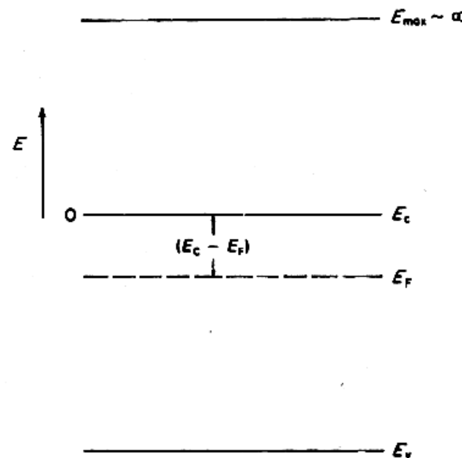


- Neutral imperfections
- Dislocations: single crystal
- Grain boundaries: poly crystal
- Surfaces: Thin films
- Inhomogeneity

Fermi Level in Semiconductors

$$\sigma(\mathrm{T}) = n(T)\mu(\mathrm{T})\, q$$

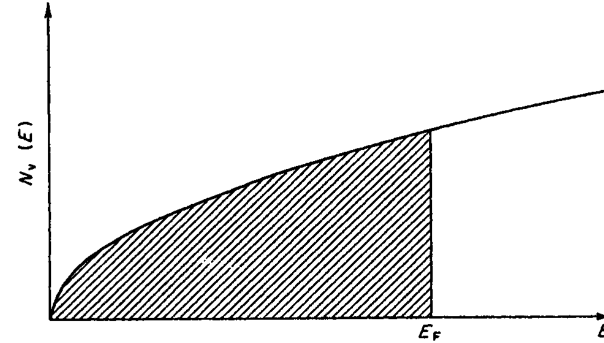
- In metal, $n(\mathrm{T})$ is nearly constant.
- In non degenerate semiconductors, E_F lies several kT away from either band so that occupancy of states in the band can be described in terms of the Boltzmann tail of the Fermi distribution.
- For calculation of the total density of free electrons, we need to consider the Fermi level in materials.



Fermi Level in Semiconductors

In a metal, $n = \int_0^{E_F} N_{\mathrm{v}}(E)dE = \frac{1}{3\pi^2}\left(\frac{2m}{\hbar^2}\right)^{\frac{3}{2}} E_F{}^{\frac{3}{2}}$

$E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{\frac{2}{3}}$ (chapter 6)



In a semiconductor,

$$
\begin{aligned}
n &= \int_{E_C}^{E_{\max}} N_C(E) f(E) dE \\
&= \int_{E_C}^{E_{\max}} \frac{1}{2\pi^2}\left(\frac{2m_e^*}{\hbar^2}\right)^{3/2} (E - E_C)^{1/2} \frac{1}{\mathrm{e}^{(E-E_F)/kT} + 1} dE \\
&= 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \mathrm{e}^{-(E_C - E_F)/kT} \\
&= N_C exp(-(E_C - E_F)/kT)
\end{aligned}
$$

$N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$: effective density of states in conduction band

Fermi Level in Semiconductors

Density of holes in the valence band

$$p = N_V exp(-(E_F - E_V)/kT)$$

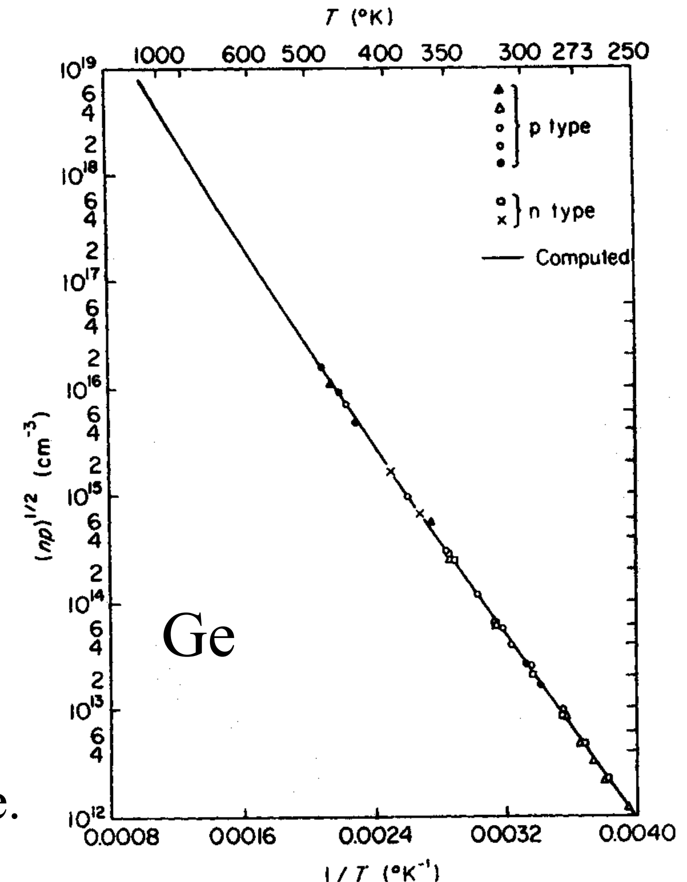
$$N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

: effective density of state in the valence band

Let $E_C - E_V = E_G$, then

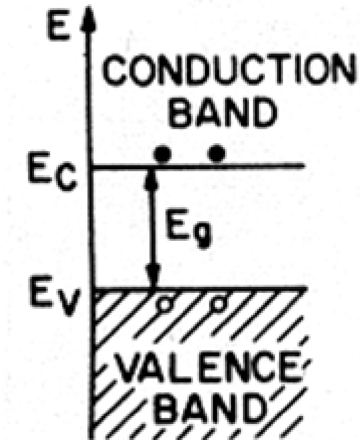
$$np = N_C N_V exp(-E_G/kT)$$

: constant for a given material at a given temperature. When n increases, p decreases to maintain constant np.



Intrinsic Semiconductor

: No role of imperfections in controlling the density of free carriers



Intrinsic carrier density

$$n_i^2 = np = N_C N_V exp(-E_G/kT)$$

mistake

Charge neutrality condition

$n = p$ (thermal excitation from the valence band)

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

$$E_C - E_V = E_G \quad N_C = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2} \quad N_V = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

$$E_C - E_F = \frac{E_G}{2} + \frac{kT}{2}\ln\left(\frac{N_C}{N_V}\right) = \frac{E_G}{2} + \frac{3kT}{4}\ln\left(\frac{m_e^*}{m_h^*}\right)$$

: The Fermi level is located at the midpoint of the energy gap at T = 0 K and departs from this point at finite temperature if $m_e^* \neq m_h^*$.