고성능 콘크리트 공학

High Performance Concrete Engineering

<UHPC Part II>

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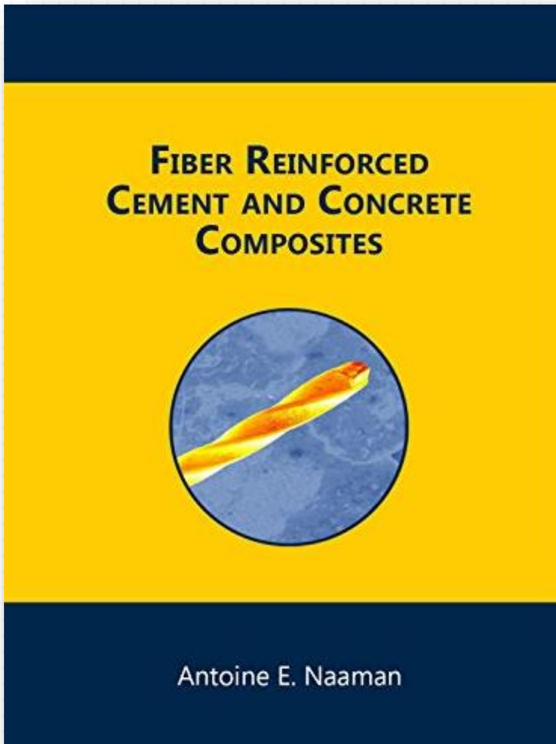
Context



(Naaman 2018)

The following notation is used:

- A_c = cross-sectional area of composite
- A_f = cross-sectional area of one fiber
- A_m = cross-sectional area of matrix
- E_f = elastic modulus of fiber; if the elastic modulus in the longitudinal direction is different from that in the transverse direction, the terms $E_{f//}$ and $E_{c\perp}$ are used.
- E_m = elastic modulus of the matrix
- d = diameter of fiber
- L = length of fiber
- N_v = average number of fibers per unit volume of composite
- N_s = average number of fibers crossing a unit area of composite, or number of fiber intersections per unit area
- V_f = volume fraction of fibers
- V_m = volume fraction of matrix
- ε_{fu} = tensile strain at failure of the fiber
- ε_{fy} = tensile strain at yield of the fiber
- ε_{mu} = tensile strain at failure of the matrix
- σ_{fy} = yield strength of the fiber
- σ_{fu} = tensile strength of the fiber
- σ_{mu} = tensile strength of the matrix
- τ = bond strength at the fiber matrix interface

Note that $V_f + V_m = 1$.

Introduction

High Performance Fiber Reinforced Cement Composites (HPFRCC)

<u>Advanced in recent years:</u>

(1) the commercial introduction of a new generation of additives (superplasticizers and viscous agents) which allow for high matrix strengths to be readily achieved with little loss in workability,

(2) the increasing use of active or inactive micro-fillers such as silica fume and fly ash and a better understanding of their effect on matrix porosity, strength, and durability,

(3) the increasing availability for use in concrete of fibers of different types and properties which can add significantly to the strength, ductility, and toughness of the resulting composite,

(4) the use of polymer addition or impregnation of concrete which adds to its strength and durability but also enhances the bond between fibers and matrix thus increasing the efficiency of fiber reinforcement,

(5) some innovations in production processes (such as self-consolidation or self-compacting) to improve uniform mixing of high volumes of fiber with reduced effects on the porosity of the matrix.

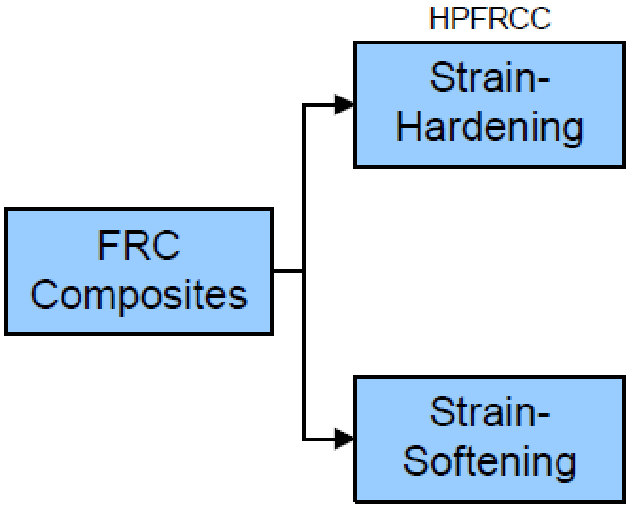
Introduction

High Performance Fiber Reinforced Cement Composites (HPFRCC)

Definition:

High performance is limited to the particular class of FRC composites that shows *strain-hardening behavior in tension after first cracking, accompanied by multiple cracking up to relatively high strain levels.*

Such an attribute applies to low and high strength, normal weight or lightweight cement composites.



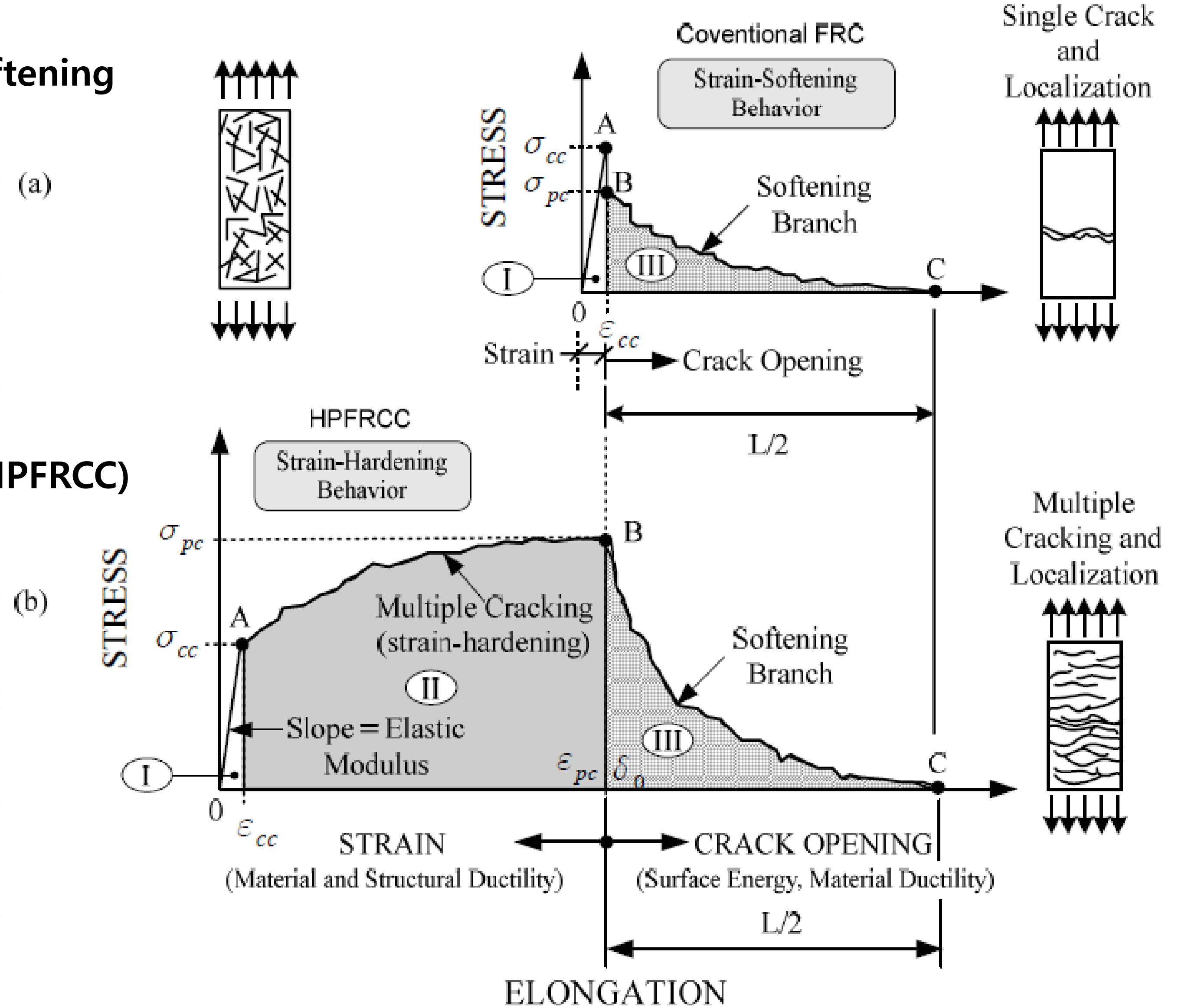
Simple classification of FRC composites based on their tensile response.

Stress-strain or stress-elongation curve in tension

Conventional strain-softening FRC composites

Strain-hardening FRC (HPFRCC)

$\sigma_{pc} \geq \sigma_{cc}$



Stress-strain or stress-elongation curve in tension

Strain-hardening FRC (HPFRCC)

Part I:

- Steep initial ascending portion up to first structural cracking

Part II:

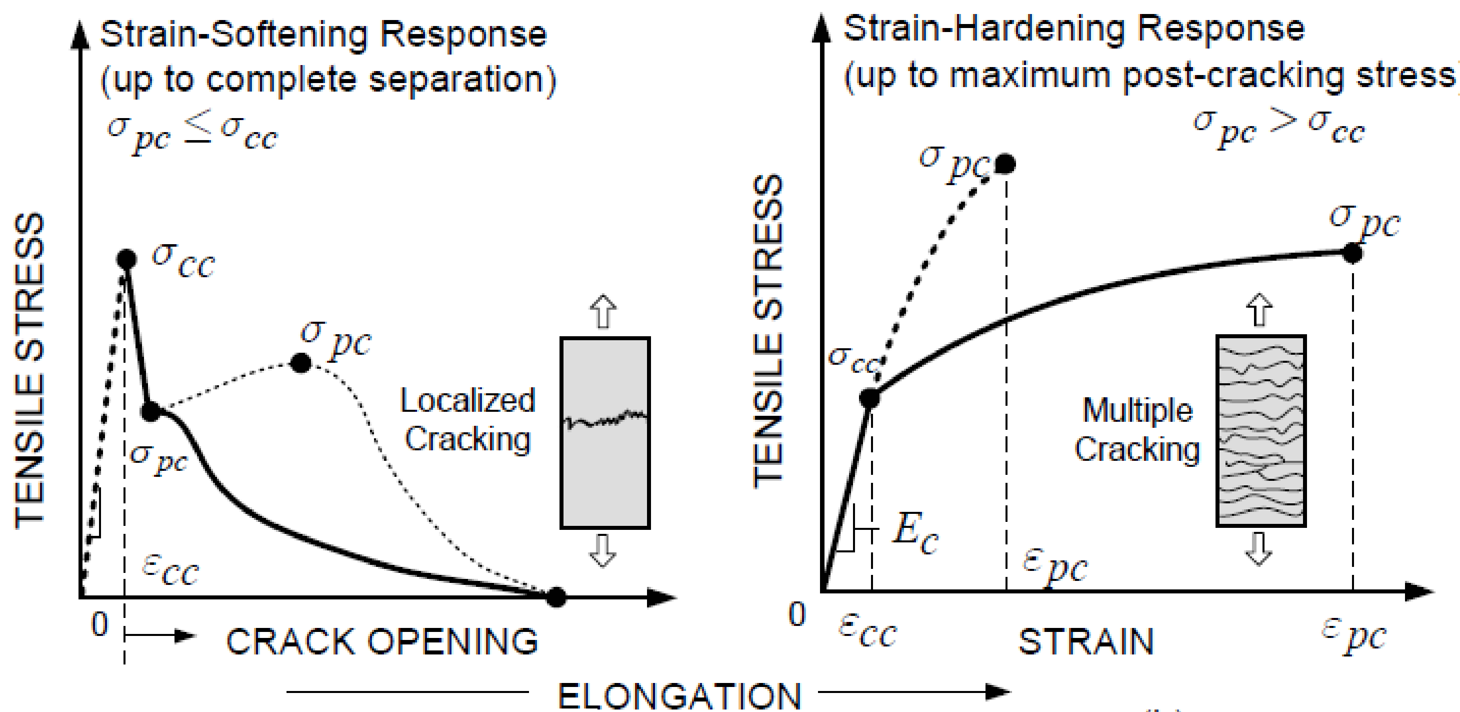
- Strain-hardening branch where multiple cracking develops.
- At the peak point, one crack becomes critical defining the onset of crack localization.
- The point where first structural cracking occurs at $(\sigma_{cc}, \varepsilon_{cc})$
- Maximum post-cracking peak point at $(\sigma_{pc}, \varepsilon_{pc})$

Part III:

- Resistance drops thereafter. No more cracks can develop, and only the critical crack will open under increased deformation. Other cracks will gradually unload or become smaller (narrower in width).
- Descending branch corresponding to the load vs. opening of the critical crack. Along that branch fibers can pull-out, fail, or a combination of these phenomena may occur.
- After this localization, the elongation is controlled by the opening of the critical cracks.

Stress-strain or stress-elongation curve in tension

Strain-hardening FRC (HPFRCC)



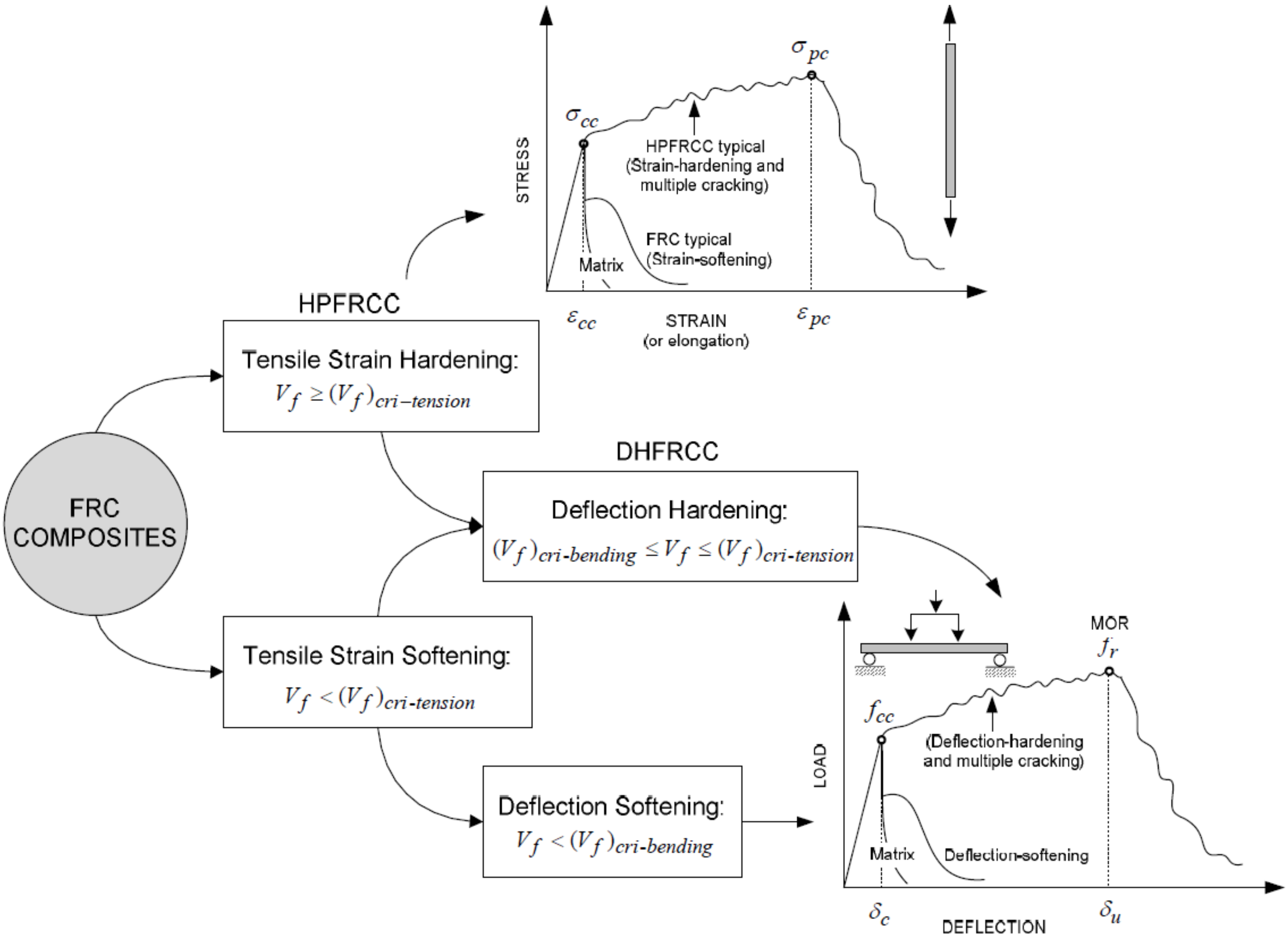
Strain-softening behavior | **Strain-hardening behavior (HPFRCC)**

$\sigma_{pc} < \sigma_{cc}$ | $\sigma_{pc} \geq \sigma_{cc}$

The strain at maximum post-cracking stress is ε_{pc}. Note that after the maximum post-cracking point, localization occurs at the critical crack and the term "strain" becomes inappropriate; instead crack opening, crack width, or member elongation become more appropriate to describe the behavior.

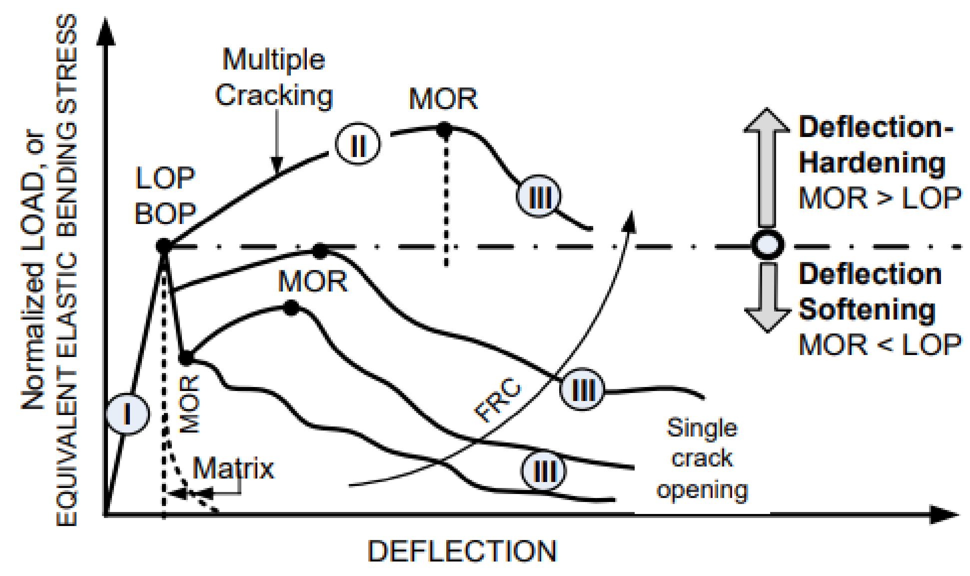
Strain-hardening and deflection-hardening

Classification of FRCC based on their tensile response and implication for bending response of structural elements.



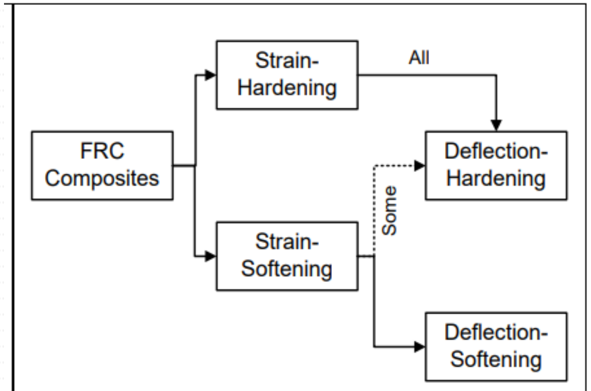
Stress-strain or stress-elongation curve in tension

(a) Schematic bending response of all FRC composites



(a)

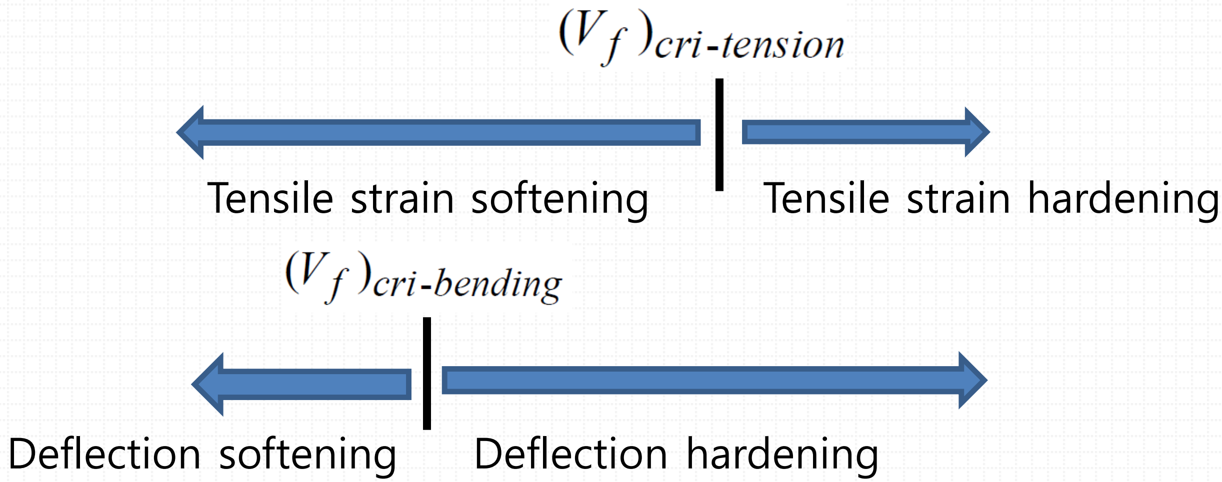
(b) Classification of all FRC composites based on their tensile response and implication for bending response of structural elements.



(b)

(Naaman and Reinhardt, Proposed classification of FRC composites based on their tensile response, 2006)

Strain-hardening and deflection-hardening



(1) all strain-hardening composites lead to deflection-hardening structural elements,

(2) a tension strain-softening composite can lead to structural elements with either deflection-hardening or deflection softening behavior

(3) a strain-hardening composite provides a better mechanical performance than a strain-softening one.

Note that, as with other materials, scale and size effects can be significant; thus the tensile response of very small specimens may not be indicative of the response of real scale structural elements in either tension or bending.

(Stang and Li, Scale effects in FRC and HPFRCC Structural Elements 2003)

Typical mix design and fibers

Table 3.1 Typical examples of mix proportions (by weight of cement) of steel fiber reinforced mortar and concrete.

Mix ID	Cement	Sand	Coarse Agg.	Water	Target f_c' MPa
Mortar 1	1	3	----------	0.5	49
Mortar 2	1	2	----------	0.5	60
Mortar 3	1	1	----------	0.35	70
Concrete 1	1	2.5	2.1	0.6	25
Concrete 2	1	2	2.3	0.5	42
Concrete 3	1	1	1.6	0.35	67

Note: These mixtures were used for different matrix compressive strengths, and steel fiber contents ranging from 1% to 3% by volume of mortar in the mix or about 3% to 8% by weight of mortar; cement is ASTM Type I; aggregate is crushed limestone of 12 mm maximum size; fiber aspect ratio is less than 100; fiber length is less than 30 mm; superplasticizer was added as needed especially for the high fiber content.

Typical mix design and fibers

Table 3.2 Examples of mixtures by weight of cement of high performance fiber reinforced cement composites with up to 2% fibers by volume (mostly steel and Spectra fibers were used).

ID	C	FA	SF	W	S	A	SP	f'_c MPa
M1	0.3	0.7	--	0.9	5[c]	--	--	7
M2	0.3	0.7	--	0.575	3.5 [c]	--	--	13
M3	0.7	0.3	--	0.65	3.5 [a]	--	--	20
M4	0.8	0.2	--	0.45	1 [a]	--	--	44
M5	0.8	0.2	--	0.45	1 [b]	--	0.03	55
M6	0.8	0.2	--	0.27	1.1 [d] +0.38 [b]	--	0.02	63
M7	0.8	0.2	0.07	0.26	1[d]	--	0.04	76
M8	0.8	0.2	0.07	0.26	1[a]	--	0.04	84
M9	0.8	0.2		0.26	0.5[b] +0.5[d]	--	0.02	86
M10	1	--	0.24	0.27	1.1 [d] +0.38 [b]	--	0.10	90
M11	1	--	0.12	0.28	0.67[a]	1	0.05	101

Note: C = Cement; FA = Fly Ash; SF = micro-silica from Silica Fume; W = Water; S = Sand; A = coarse Aggregate, here crushed limestone with maximum size 10 mm; SP = Superplasticizer; f'_c = compressive strength from cylinders. The sand used is a silica sand with the following characteristics: (a) ASTM -50-70; (b) ASTM -270; (c) ASTM 30-70; silica sand passing ASTM sieve No. 16. These mixtures were selected from various investigations carried out by the author and his students at the University of Michigan.

Micro-fibers:
equivalent diameter of 100 μm or less

Common steel fibers:
diameter ranging from 0.4 to 0.8 mm and a length ranging from 25 to 80 mm

Typical mix design and properties of fibers in FRC

Material	Specific Gravity (1 for water)	Tensile Strength MPa	Tensile Modulus GPa	Remark
Steel	7.8	Up to 3500	200	The elastic modulus of steel is almost independent of its tensile strength. Yield strength of steel fibers varies widely depending on the fabrication process and alloying. The higher the strength the lower the strain capacity to failure.
E-Glass	2.6	Up to 3500	76	Modulus of glass can vary from 33 GPa for A-Glass to 98 GPa for S-Glass. While filament strength remains high from 3300 to 4800 MPa, the strength of a yarn or fiber bundle, which is made from a large number of filaments, will have much lower equivalent tensile strength.
Carbon	1.8	Up to 4500	100 to 300	Depending on grade and fabrication, such as pitch carbon or PAN carbon, significantly different properties can be achieved.
Kevlar (Aramid)	1.44	2800	124	Kevlar is a trade name of Dupont. The fiber material source is aramid. Aramid fibers have properties similar to Kevlar.
Spectra (HPPE)	0.97	2585	117	Spectra is a trade-name. The material is an ultra-high molecular weight polyethylene, also termed high performance polyethylene (HPPE). Other similar trade-named fiber include: Dyneema.
PVA (PolyVinyl Alcohol)	1.31	880-1600	25 to 40	A large variety of fibers exists with a wide range of tensile strengths and moduli. The higher the diameter, the lower the properties.
PP (Poly-Propylene)	0.91	Up to 800	Up to 10	Strength and modulus depend on the manufacturing process, and heat stretching leading to highly oriented long-chain molecules.

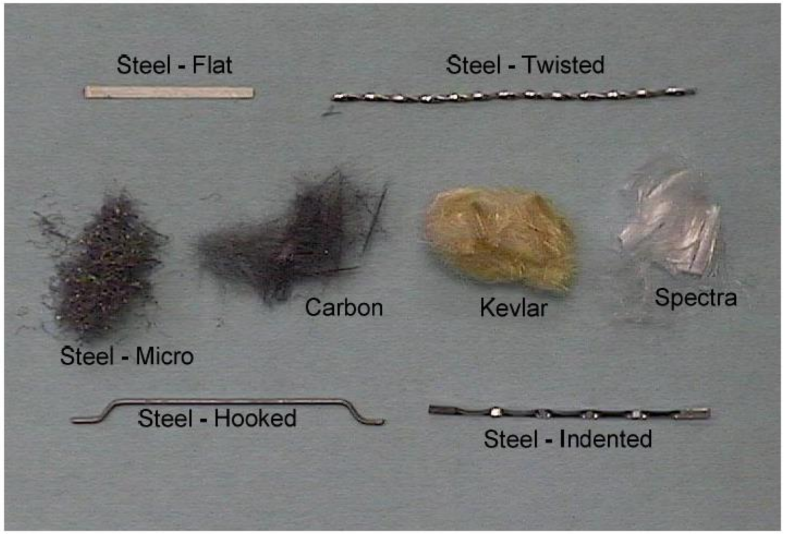


Fig. 3.8 Photograph illustrating typical fibers and micro-fibers used in cement composites.

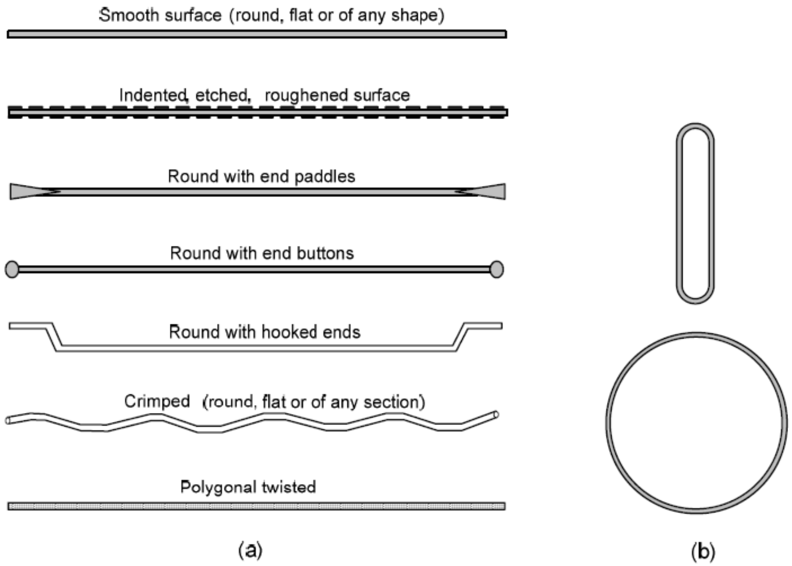


Fig. 3.7 (a) Typical profiles of steel fibers commonly used in concrete (twisted fiber is new). (b) Closed loop fibers tried in some research studies.

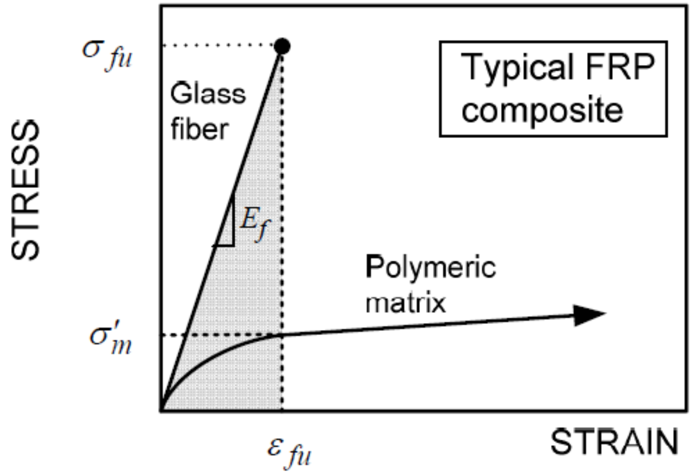
Mechanical Modelling

Key Difference Between Fiber Reinforced Cement and Fiber Reinforced Polymeric Composites for Mechanical Modeling

Fiber reinforced polymeric composite

Fiber reinforced polymeric and metallic composites use fibers with tensile strain at failure smaller than that of the matrix.

Hence, failure of the composite implies either failure of the fibers, or their complete debonding while the matrix may be in a yielding state.

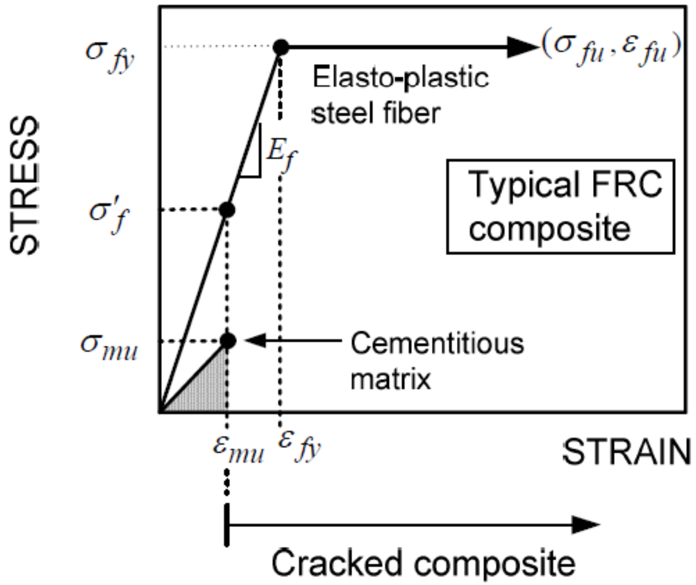


Fiber reinforced cement or concrete composite

Main characteristic of this is that the ultimate tensile strain of the fiber is significantly larger than that of the matrix.

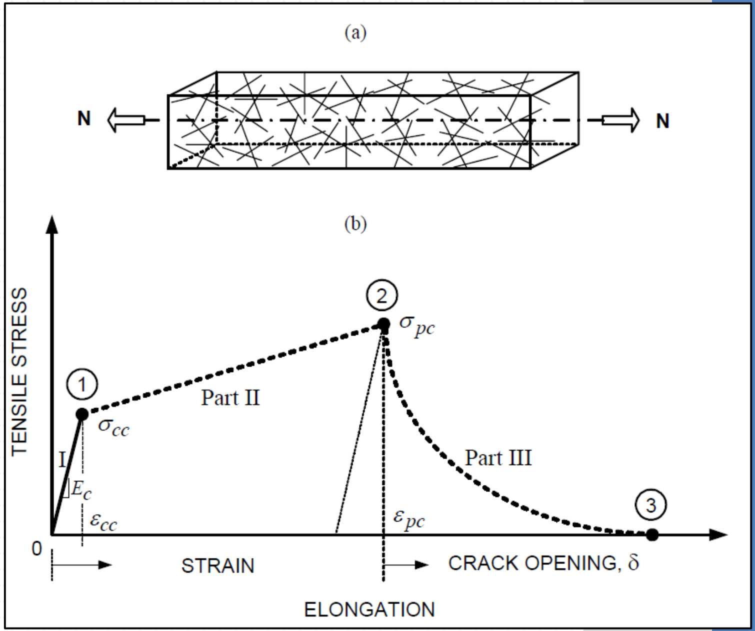
This implies that at some level of tensile loading, the matrix will crack and the resistance to full separation is entirely born by the fibers bridging the cracked surfaces.

<u>This also generally implies that the contribution of the fibers can be fully utilized only after matrix cracking.</u>



Mechanical Modelling: Contents

1. Assumptions for simplified model
2. Number of fibers per unit volume and per unit area
3. **Stress and strain at first cracking** of matrix in tension
    1) Stress; σ_{cc}
    2) Strain; ε_{cc}
    3) Upper bound stress
    4) Non-circular case
4. Elastic modulus
5. **Maximum post-cracking stress** in tension; σ_{pc}
    1) All fibers fail simultaneously
    2) All fibers pull-out simultaneously
    3) Non-circular case
6. **Maximum post-cracking strain** in tension; ε_{pc}
7. General pull-out response (Part III)
8. Critical volume fraction of fiber for **strain-hardening** in tension
9. Critical volume fraction of fiber for **deflection-hardening** in bending
10. Application

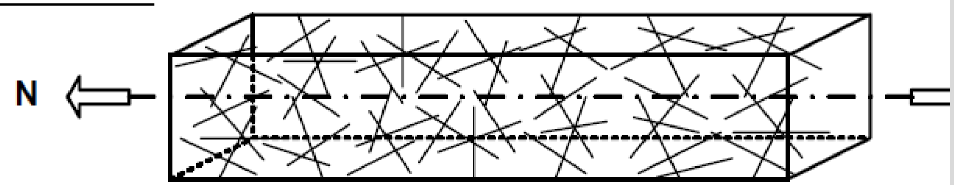


Mechanical Modelling

1. Assumptions for simplified model

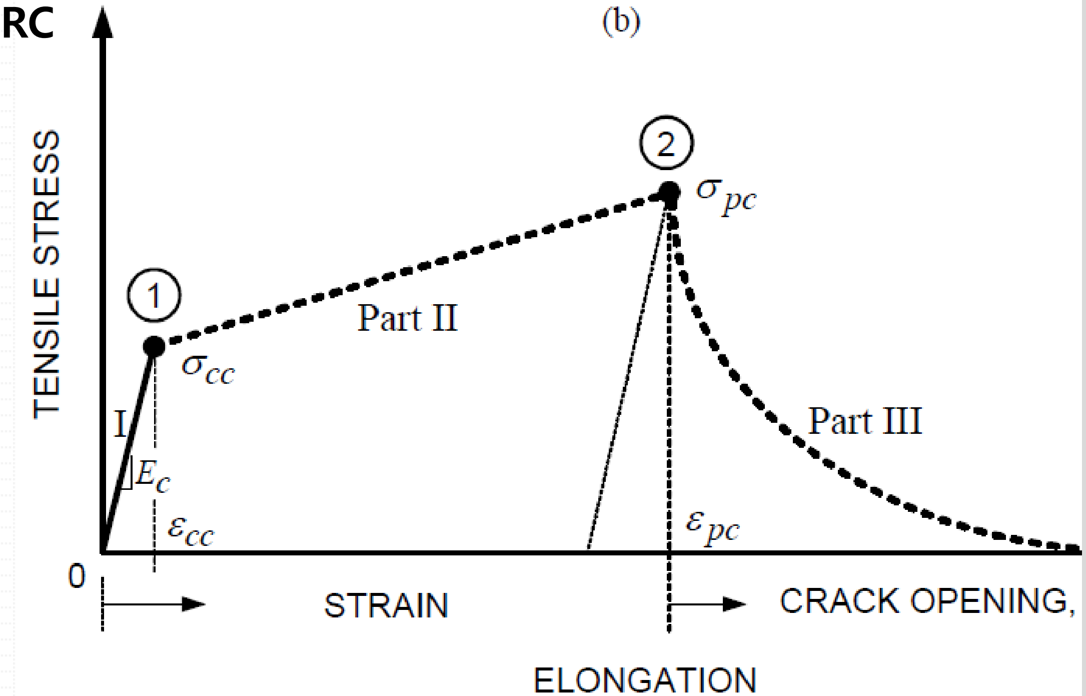
(a)

Model of tensile element considered.

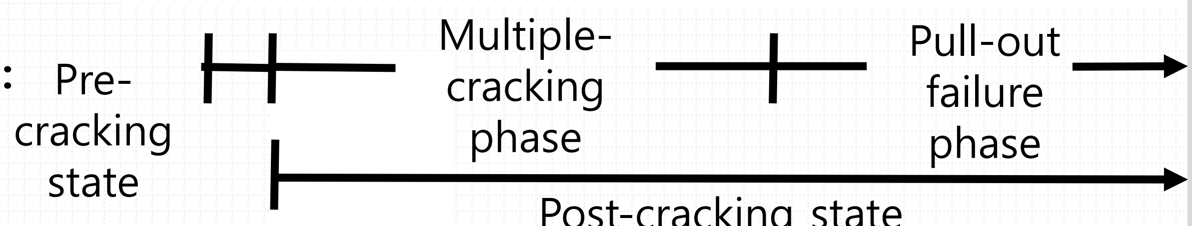


Idealized stress-elongation response in tension of a strain-hardening FRC

(b)

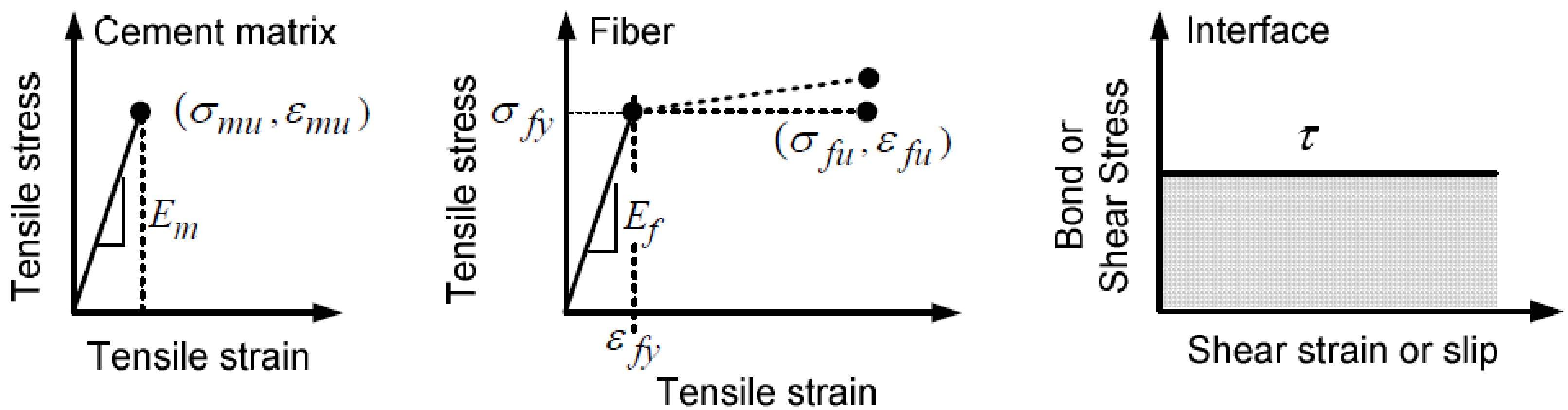


Part I, II, III :



Mechanical Modelling

1. Assumptions for simplified model

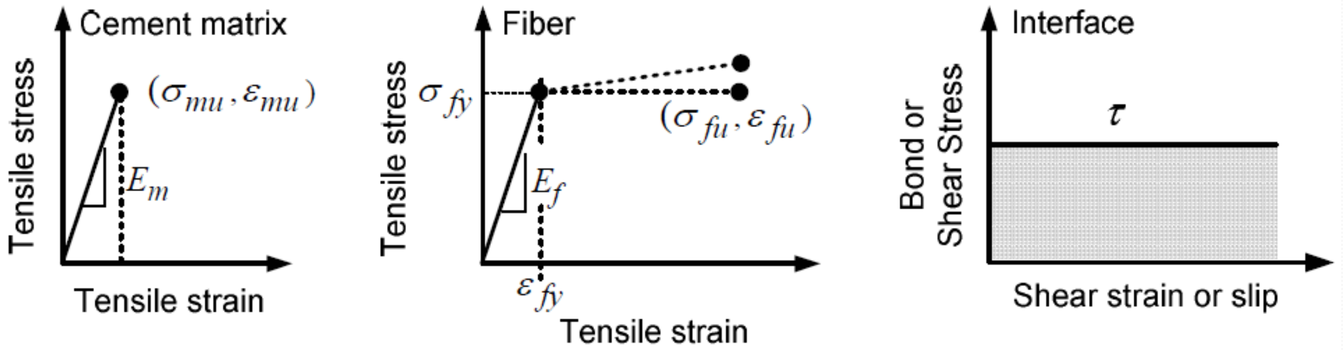


(a) **Cement matrix** (b) **Fiber** (c) **Bond stress**

a. The matrix is brittle and best characterized by a linear stress-strain curve in tension; thus two of three parameters are sufficient to characterize the matrix: E_m, σ_{mu} , and ε_{mu}.

b. The fiber is either brittle (such as glass) or ductile with an initial elastic response (such as steel or polypropylene). For brittle fibers at least three parameters are needed: E_f , σ_{fu} , and ε_{fu} . For ductile fibers with well defined yielding behavior, the yield stress, σ_{fy} and the corresponding strain ε_{fy} are also needed.

1. Assumptions for simplified model



(a) **Cement matrix** (b) **Fiber** (c) **Bond stress**

c. The tensile strain at failure of the matrix is smaller than that of the fiber, that is, for brittle fibers: $\varepsilon_{mu} < \varepsilon_{fu}$ (or for ductile fibers $\varepsilon_{mu} < \varepsilon_{fy}$); this implies that cracking will occur in the matrix prior to failure of the fibers and the composite.

d. Although a weak, non-flowing, non-plastic bond generally exists at the fiber matrix interface, the properties of the interface are assumed characterized by an equivalent elastic perfectly plastic bond stress versus slip response, at small slips. The value of bond strength selected is assumed to represent an average value over a reasonable range of slip between the fiber and the matrix.

e. While it is conceivable that the bond at the interface between fiber and matrix could fail prior to failure of the matrix in tension (thus nullifying composite action), this case is most unlikely and is not considered here. In fact, there is evidence that the bond is mobilized only partly prior to matrix cracking, and develops fully only after cracking.

Mechanical Modelling

2. Number of fibers per unit volume and per unit area

N_v = average number of fibers per unit volume of composite assuming fibers of circular cross section

$$N_v = \frac{4V_f}{\pi d^2 L} \quad \text{(Eq. 1)}$$

N_s = average number of fiber crossing a unit area or unit surface of composite. If a plane is cut through the composite, N_s is the average number of fiber intersections per unit area.

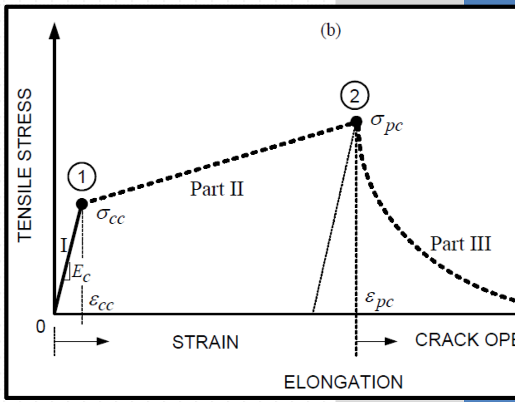
$$N_s = \frac{4V_f}{\pi d^2}\alpha_2 \quad \text{(Eq. 2)}$$

$\alpha_2 = 1$	for unidirectional fibers (1 Dimensional)
$\alpha_2 = \frac{2}{\pi}$	for fibers randomly oriented in planes (2 Dimensional)
$\alpha_2 = \frac{1}{2}$	for fibers randomly oriented in space (3 Dimensional)

Note that, while the total number of fibers put in a given composite can be considered a deterministic value (V_f is an input parameter), the actual number of fibers per unit volume or number crossing a unit area are random variables with a probability distribution function of the Poisson's type. For this chapter, the average values, N_v and N_s, are considered deterministic variables.

(Naaman PhD thesis MIT, 1972)

Mechanical Modelling



3. Stress and strain at first cracking of matrix in tension

1) Stress σ_{cc}

Consider a fiber reinforced cement composite tensile prism with randomly oriented and distributed fibers.

Assume that if a crack develops in the matrix, it will be normal to the direction of loading; also assume that the fibers are "circular" in cross section. It can be shown that the tensile stress in the composite at onset of first percolation cracking (that is, just before first cracking) of the matrix can be expressed as follows:

$$\sigma_{cc} = \sigma_{mu}(1 - V_f) + \alpha \tau V_f \frac{L}{d} \quad \text{(Eq. 3)}$$

where $\alpha = \alpha_1 \alpha_2 \alpha_3$

$$\frac{\sigma_{cc}}{\sigma_{mu}} = (1 - V_f) + \alpha \frac{\tau}{\sigma_{mu}} V_f \frac{L}{d} \quad \text{(Eq. 4)}$$

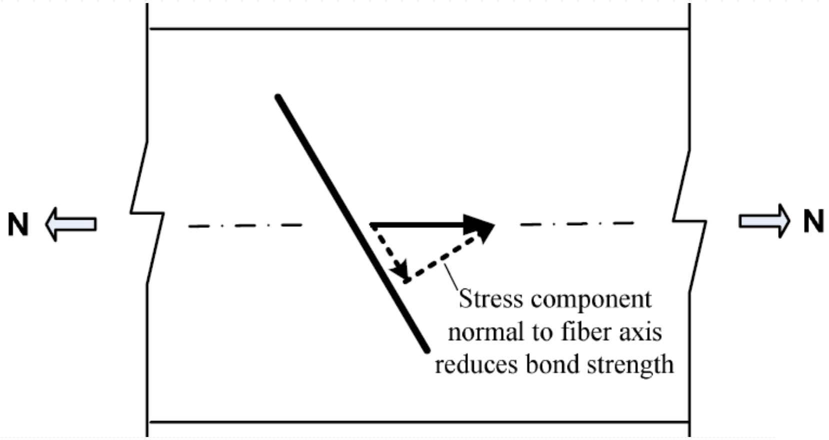
3. Stress and strain at first cracking of matrix in tension

1) Stress σ_{cc}

$$\sigma_{cc} = \sigma_{mu}(1 - V_f) + \alpha \tau V_f \frac{L}{d} \qquad \text{(Eq. 3)}$$

$$\frac{\sigma_{cc}}{\sigma_{mu}} = (1 - V_f) + \alpha \frac{\tau}{\sigma_{mu}} V_f \frac{L}{d} \qquad \text{(Eq. 4)}$$

where $\alpha = \alpha_1 \alpha_2 \alpha_3$



Bond reduction coefficient α3 in the uncracked state

τ = average or equivalent bond strength at the fiber matrix interface

α_1 = coefficient describing the average contribution of bond at onset of matrix cracking.

α_2 = efficiency factor of fiber orientation in the uncracked state of the composite; it is equal 1 for unidirectional fibers; 2/π = 0.636 for fibers randomly oriented in planes; and 0.5 for fibers randomly oriented in space. (Eq. 2)

α_3 = coefficient describing the reduction of bond strength at the fiber matrix interface due to an applied external stress radial or normal to the interface; it is equal 1 for aligned fibers.

3. Stress and strain at first cracking of matrix in tension

2) Strain ε_{cc}

Assuming linear elastic response up to first cracking, the strain at first cracking of the composite can be estimated as:

$$\varepsilon_{cc} = \frac{\sigma_{cc}}{E_c} \qquad \text{(Eq. 5)}$$

Ec is the modulus of elasticity of the composite in the uncracked state. It will be described in Section 4.

3. Stress and strain at first cracking of matrix in tension

3) Upper bound stress

The stress in the composite is limited to the value obtaining assuming the composite is made of continuous fibers with equal strain distributions in the fiber and matrix. Thus:

$$\sigma_{cc} \leq (\sigma_{cc})_{\text{upper-bound}} = \sigma_{mu}[1 + (n-1)V_f] \qquad \text{(Eq. 6)}$$

where $n = \dfrac{E_f}{E_m}$

$$\frac{\sigma_{cc}}{\sigma_{mu}} \leq 1 + (n-1)V_f \qquad \text{(Eq. 7)}$$

3. Stress and strain at first cracking of matrix in tension

4) Non-circular case

For non-circular fibers, Eqs. 3 and 4 which are derived for circular fibers can be used as a first approximation provided an equivalent diameter of fiber based on its cross sectional area is used, that is $d = \sqrt{4A_f / \pi}$

$$\sigma_{cc} = \sigma_{mu}(1 - V_f) + \alpha \times \tau \times V_f \times \frac{\psi \times L}{4A_f} \qquad \text{(Eq. 8)}$$

Where A_f is the cross-sectional area of the fiber

ψ is the perimeter of one fiber

τ is the average bond strength at the fiber-matrix interface

4. Elastic modulus

Similarly to the case of other elastic composites, the modulus of elasticity, *Ec* , of fiber reinforced cement composites assuming randomly oriented and distributed fibers can be estimated from a linear combination of its upper and lower bound values as follows

$$E_c = mE_{c//} + (1-m)E_{c\perp} \qquad \text{(Eq. 9)}$$

$m =$ coefficient between zero and 1

$E_{c//} =$ upper bound modulus

$E_{c\perp} =$ lower bound modulus

m = 3/8 for fibers randomly oriented in planes
m = 0.5 from analytical modeling
either value seems acceptable for practical applications since the fiber content is generally small

(Halpin and Tsai, Effect of environmental factors on composite materials, 1969)
(Agarwal and Broutman, Analysis and performance of composites, 1981)

4. Elastic modulus

Derivation of upper bound modulus (based on Voigt's model or equal strain model) assumes that all fibers are aligned in the direction of loading and equal strain exists in the fiber and the matrix

$$
\begin{aligned}
E_{c//} &= E_m V_m + E_{f//} V_f \\
&= E_m V_m + E_{f//}(1 - V_m) \\
&= E_{f//} V_f + E_m (1 - V_f)
\end{aligned}
\qquad \text{(Eq. 10)}
$$

where $E_{f//}$ = modulus of elasticity of the fiber along its axis

4. Elastic modulus

The derivation of lower mound modulus (based on Reuss's model or equal stress model) assumes that al fibers have their axis normal to the direction of loading and the stress in the matrix and the stress normal to the fiber axis are equal.

$$\frac{1}{E_{c\perp}} = \frac{V_f}{E_{f\perp}} + \frac{V_m}{E_m} \quad \text{(Eq. 11)}$$

$$E_{c\perp} = \frac{E_{f\perp}E_m}{E_{f\perp}V_m + E_mV_f}$$

$$= \frac{E_{f\perp}E_m}{E_{f\perp}(1-V_f) + E_m(1-V_m)}$$

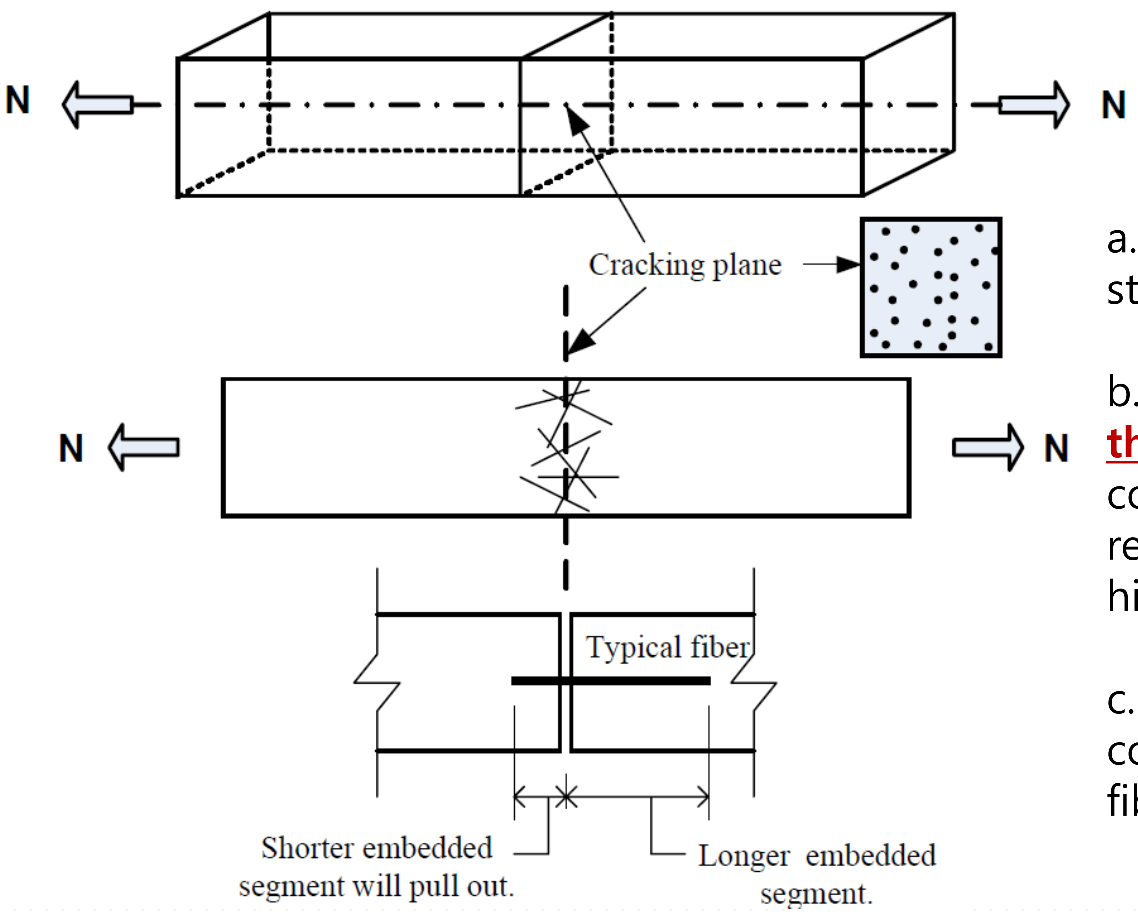
$$= \frac{E_{f\perp}E_m}{E_{f\perp}(1-V_f) + E_mV_f} \quad \text{(Eq. 12)}$$

where $E_{f\perp}$ = modulus of elasticity of the fiber normal to its axis

Mechanical Modelling

5. Maximum post-cracking stress in tension; σ_{pc}

While the cracking strength of the composite, σ_{cc}, is primarily influenced by the strength of the matrix, the post-cracking strength, σ_{pc}, is mainly dependent on the fiber reinforcing parameters and the bond at the fiber-matrix interface. Thus, improving the post-cracking strength is key to the success of the composite.



a. Upon increased elongation or straining, **all the fibers fail**;

b. Upon increased elongation, **all the fibers pull-out** such as most commonly observed in fiber reinforced cement composites with high strength steel fibers;

c. Upon increased elongation, a combined case occurs where some fibers fail and some fibers pull-out.

5. Maximum post-cracking stress in tension; σ_{pc}

1) All fibers fail simultaneously

$$\sigma_{cu} \times (A_c = 1) = \sigma_{fu} N_s A_f \quad \text{(Eq. 13)}$$

σ_{cu} = Ultimate strength of composite

σ_{fu} = Ultimate tensile strength of fiber

N_s = Number of fibers crossing a unit area of composite (or of crack surface) -> (Eq. 2)

For a circular fiber:

$$\sigma_{cu} = \sigma_{fu} \alpha_2 \frac{4V_f}{\pi d^2} \frac{\pi d^2}{4} \quad \text{(Eq. 14)}$$

$$\sigma_{cu} = \alpha_2 V_f \sigma_{fu} \quad \text{(Eq. 15)}$$

Mechanical Modelling

5. Maximum post-cracking stress in tension; σ_{pc}

2) All fibers pull-out simultaneously

For the derivations below, the following assumptions are made:

(1) a critical planar crack exists across the entire section of the tensile member

(2) the crack is normal to the tensile stress field,

(3) the contribution of the matrix along the crack is negligible,

(4) the fibers crossing the crack are in a general state of pull-out. This is typically the case when steel fibers are used in concrete.

For a circular fiber:

$$\sigma_{pc} = \lambda \tau \frac{L}{d} V_f \qquad \text{(Eq. 16)}$$

$$\lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_5$$

$$\lambda_2 = 4\alpha_2 \lambda_4$$

(Reinhardt and Naaman, High performance fiber reinforced cement composites, 1992)

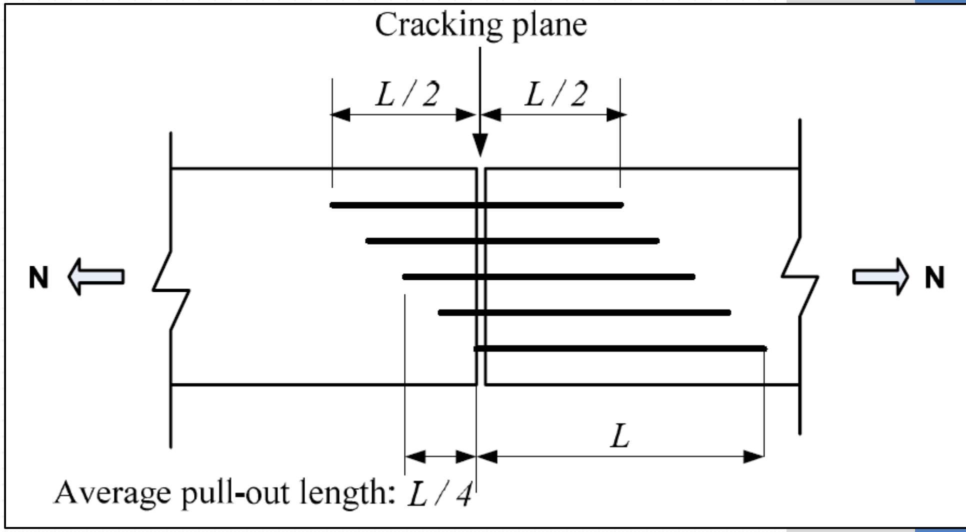
Mechanical Modelling

5. Maximum post-cracking stress in tension; σ_{pc}

2) All fibers pull-out simultaneously

$$\sigma_{pc} = \lambda \tau \frac{L}{d} V_f \quad \text{where} \quad \begin{aligned} \lambda &= \lambda_1 \lambda_2 \lambda_3 \lambda_5 \\ \lambda_2 &= 4\alpha_2 \lambda_4 \end{aligned} \qquad \text{(Eq. 16)}$$

λ_1 = average or expected value of the ratio of fiber shorter embedded distance from a forming crack to the length of the fiber. Its value is ¼ as derived from probability theory considerations.



$\lambda_2 = 4\lambda_4\alpha_2$ = coefficient that takes into consideration orientation effect on pull-out resistance. It can be considered the efficiency factor of orientation in the cracked state

Mechanical Modelling

5. Maximum post-cracking stress in tension; σ_{pc}

2) All fibers pull-out simultaneously

$$\sigma_{pc} = \lambda \tau \frac{L}{d} V_f \quad \text{where} \quad \lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_5, \quad \lambda_2 = 4\alpha_2 \lambda_4 \qquad \text{(Eq. 16)}$$

λ_3 = group reduction coefficient for bond, to simulate the fact that the bond strength resistance per fiber decreases when the number of fibers pulling out from the same area increases

λ_4 = expected value of ratio of maximum pull-out load for a fiber oriented at angle θ to maximum pull-out load of same fiber aligned with the direction of pull-out. The angle θ is the angle between the longitudinal axis of the fiber and the pull-out direction and varies between 0 and 90 degrees;

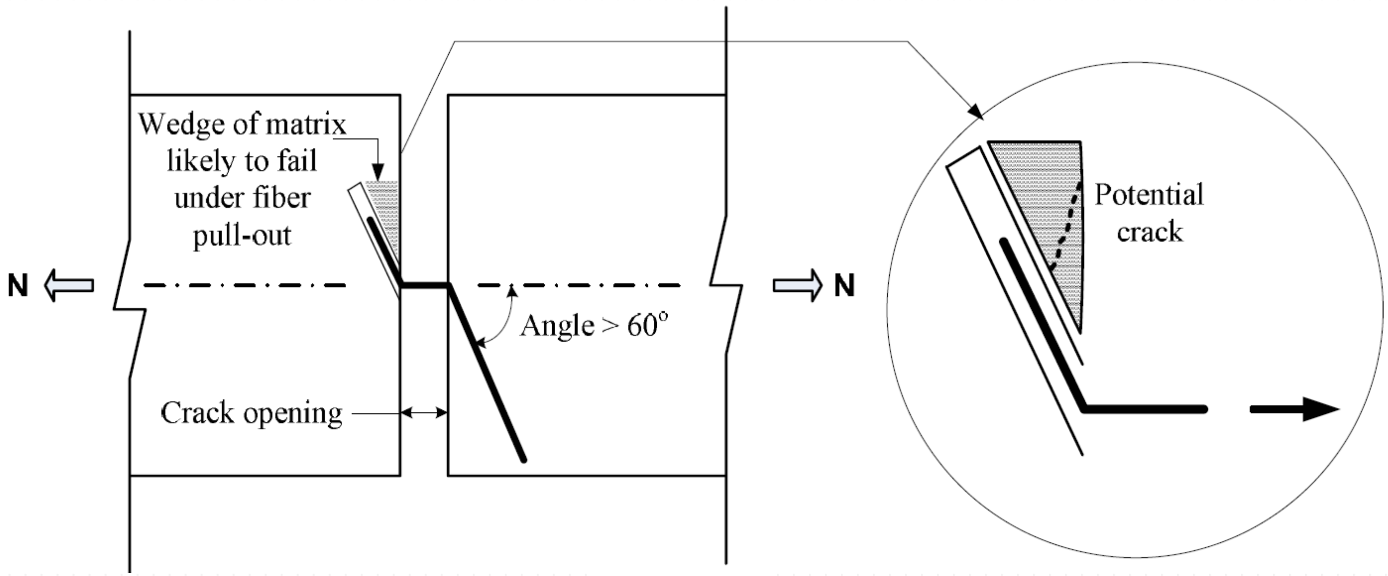
thus ($\pi / 2 - \theta$) represents the angle between the longitudinal axis of the fiber and the cracking plane. $\lambda_4 = 1$ for fibers with longitudinal axis oriented in the direction of pull-out loading.

5. Maximum post-cracking stress in tension; σ_{pc}

2) All fibers pull-out simultaneously

$$\sigma_{pc} = \lambda \tau \frac{L}{d} V_f \quad \text{where} \quad \lambda = \lambda_1 \lambda_2 \lambda_3 \lambda_5, \quad \lambda_2 = 4\alpha_2 \lambda_4 \qquad \text{(Eq. 16)}$$

λ_5 = reduction coefficient to account for the fact that fibers inclined at an angle of more that about 60 degrees with the pull-out load direction contribute very little, due to spalling of the wedge of matrix at the cracking plane. This is particularly true for stiff fibers such as steel. For aligned fibers, λ5 =1.



5. Maximum post-cracking stress in tension; σ_{pc}

3) Non-circular case

$$\sigma_{pc} = \lambda \tau V_f \frac{\psi L}{4 A_f} \qquad \text{(Eq. 17)}$$

Where A_f is the cross-sectional area of the fiber

ψ is the perimeter of one fiber

τ is the average bond strength at the fiber-matrix interface

6. Maximum post-cracking strain in tension; ε_{pc}

No simple model exists to predict the strain at maximum post-cracking stress in strain-hardening FRC composites.

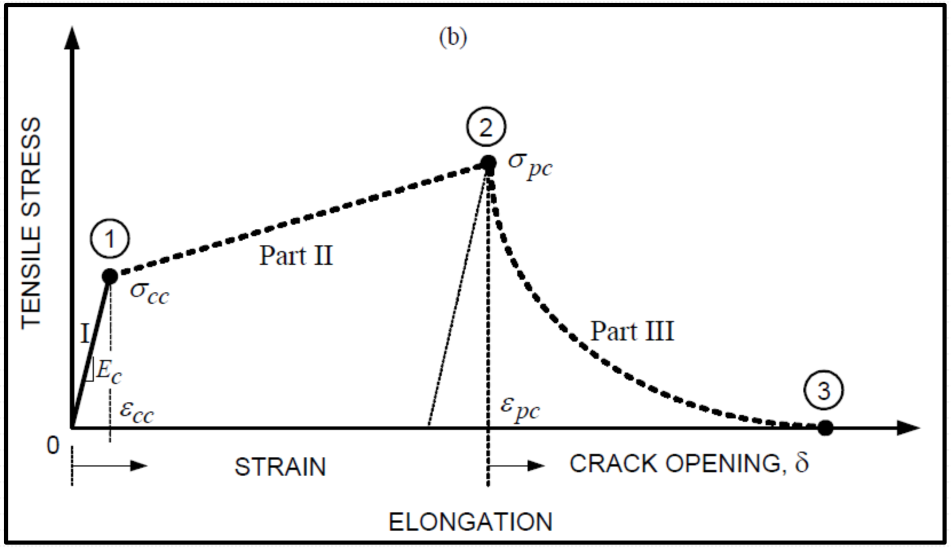
Numerical values of ε_{pc} depend on numerous parameters including not only the fiber reinforcing parameters, the matrix properties and the interface bond between fiber and matrix, but also the specimen size, the testing method, and the technique of measurement.

Experimental values observed range from about 0.1% to 5%, with a large proportion being below 2%.

For practical modeling purposes, a value of ε_{pc} = **0.5%** is recommended for strain-hardening FRC composites with **steel fibers**, and **1%** for strain-hardening FRC composites with **spectra fibers**.

(Naaman and Reinhardt, Proposed classification of FRC composites based on their tensile response, 2006)

7. General pull-out response (Part III)



$$\sigma_c(\delta) = \sigma_{pc}\left[1 - \frac{\delta}{k \times L/2}\right]^2 \quad \text{for} \quad \delta \le k \times L/2 \;\; \text{and} \; k \le 1$$ (Eq. 18)

$\sigma_c(\delta) =$ Stress in the composite at crack opening (or face displacement) δ

k = Damage coefficient

σ_{pc} = (Eq. 16)

Note that Eq. 18 applies to the general pull-out curve after crack localization for both a strain-hardening or a strain-softening composite.

8. Critical volume fraction of fiber for strain-hardening in tension

By setting that the maximum post-cracking stress must be larger than or equal to the stress at first cracking, critical volume fraction of fiber can be suggested.

$$\sigma_{pc} \geq \sigma_{cc} \qquad \text{(Eq. 19)}$$

Substituting (Eq. 3) & (Eq. 16)

$$\lambda \tau \frac{L}{d} V_f \geq \sigma_{mu}(1 - V_f) + \alpha \tau V_f \frac{L}{d} \qquad \text{(Eq. 20)}$$

Solving for V_f leads to:

$$V_f \geq (V_f)_{cri-tension} = \frac{1}{1 + \frac{\tau}{\sigma_{mu}} \frac{L}{d} (\lambda - \alpha)} \qquad \text{(Eq. 21)}$$

8. Critical volume fraction of fiber for strain-hardening in tension

$$V_f \geq (V_f)_{cri-tension} = \frac{1}{1+\frac{\tau}{\sigma_{mu}}\frac{L}{d}(\lambda-\alpha)}$$ (Eq. 21)

Assuming the value of V_f is relatively small such as in the case for FRCC, (Eq. 21) can be written in the following form which illustrates not only the influence of the fiber volume fraction but also that of the other main casual variables:

$$V_f \frac{\tau}{\sigma_{mu}}\frac{L}{d} \geq \frac{1-V_f}{(\lambda-\alpha)} \approx \frac{1}{\lambda-\alpha}$$ (Eq. 22)

For relatively small values of V_f,

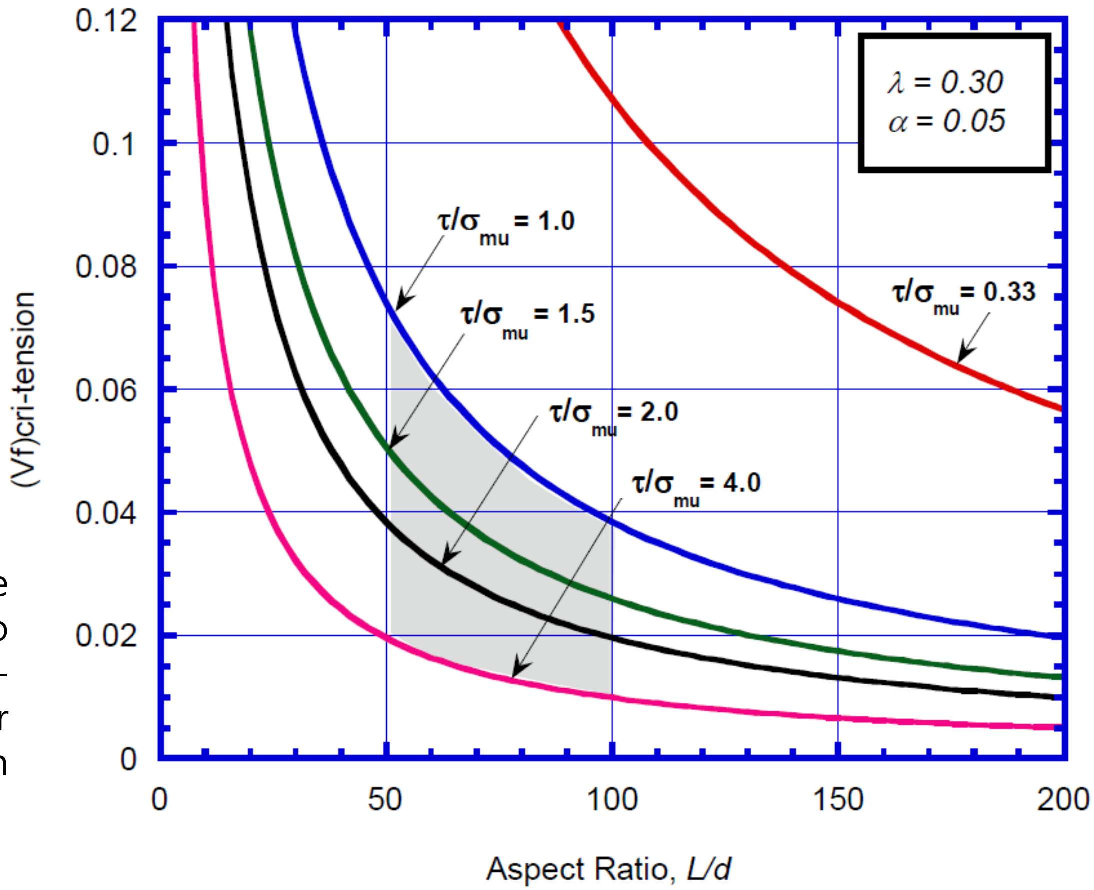
$$V_f \frac{\tau}{\sigma_{mu}}\frac{L}{d} \geq \frac{1}{\Omega}$$ (Eq. 23)

where: $\Omega = \lambda - \alpha = \lambda_1\lambda_2\lambda_3\lambda_5 - \alpha_1\alpha_2\alpha_3$ (Eq. 24)

8. Critical volume fraction of fiber for strain-hardening in tension

$$V_f \frac{\tau}{\sigma_{mu}} \frac{L}{d} \geq \frac{1}{\Omega} \qquad \text{(Eq. 23)}$$

Assuming the right-hand side to be a constant for given conditions, it shows that the aspect ratio of the fiber and the ratio of bond strength to matrix tensile strength are as influential as the volume fraction of fibers.



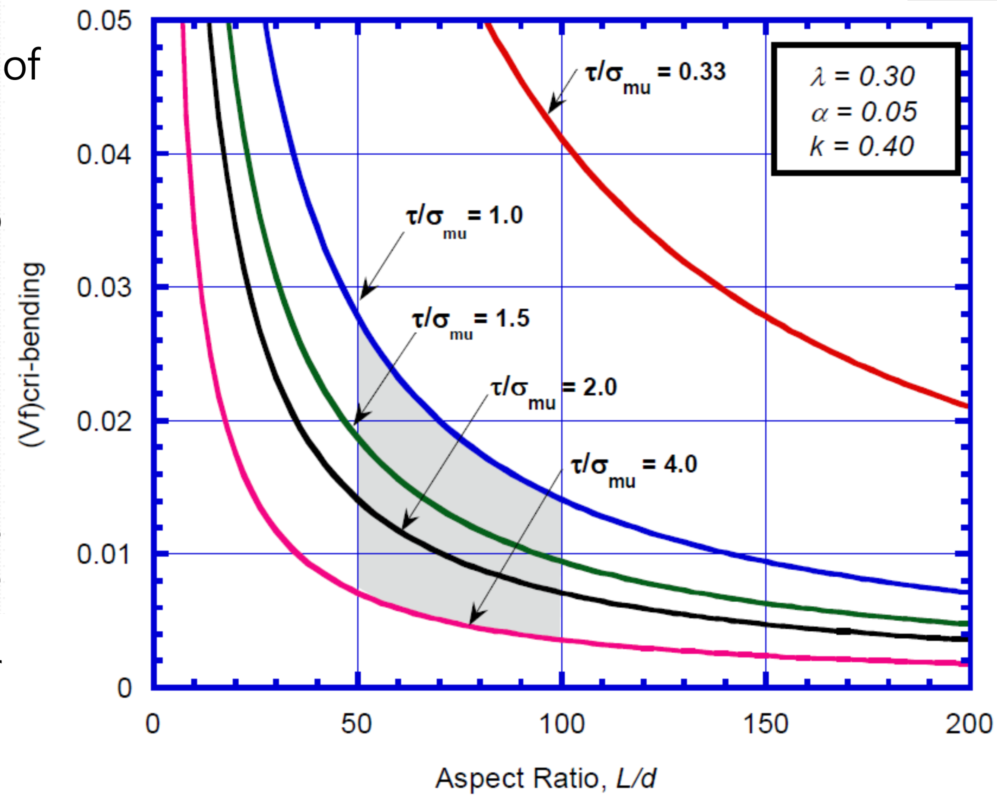
Critical volume fraction of fibers to achieve strain-hardening behavior in tension

9. Critical volume fraction of fiber for deflection-hardening in bending

Deflection-hardening implies that the maximum equivalent elastic bending stress (or modulus of rupture) after first cracking is larger than the stress at first cracking in bending, and that multiple cracking would generally occur after first cracking.

$$V_f \geq (V_f)_{cri-bending} = \frac{k}{k + \frac{\tau}{\sigma_{mu}}\frac{L}{d}(\lambda - k\alpha)}$$ (Eq. 24)

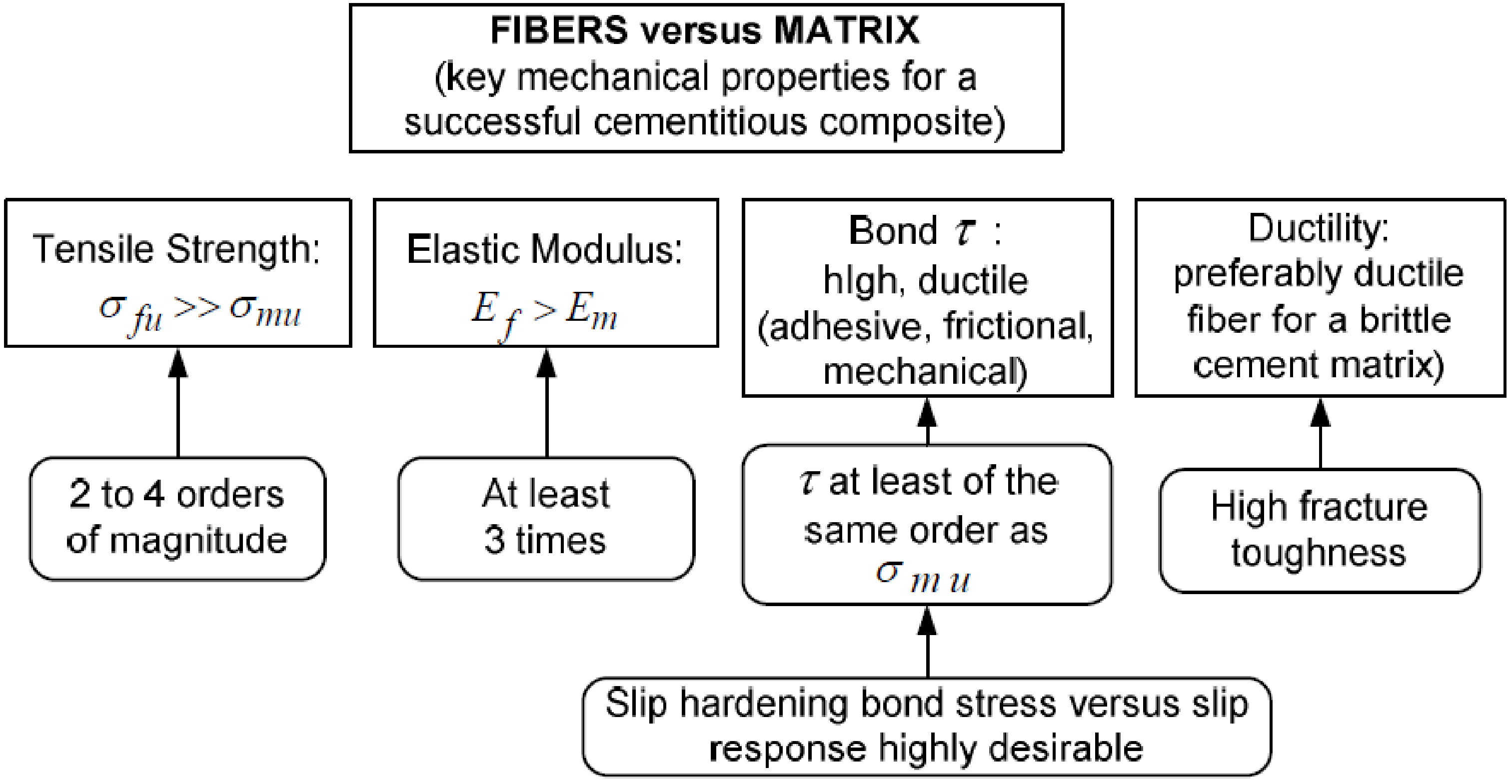
k is a coefficient less than 1. A value of *k*=0.4 is recommended for practical applications of steel fiber reinforced concrete. For k=1, (Eq. 24) reverts to (Eq. 21).



Critical volume fraction of fibers to achieve deflection-hardening behavior in bending (*k*=0.4)

10. Application

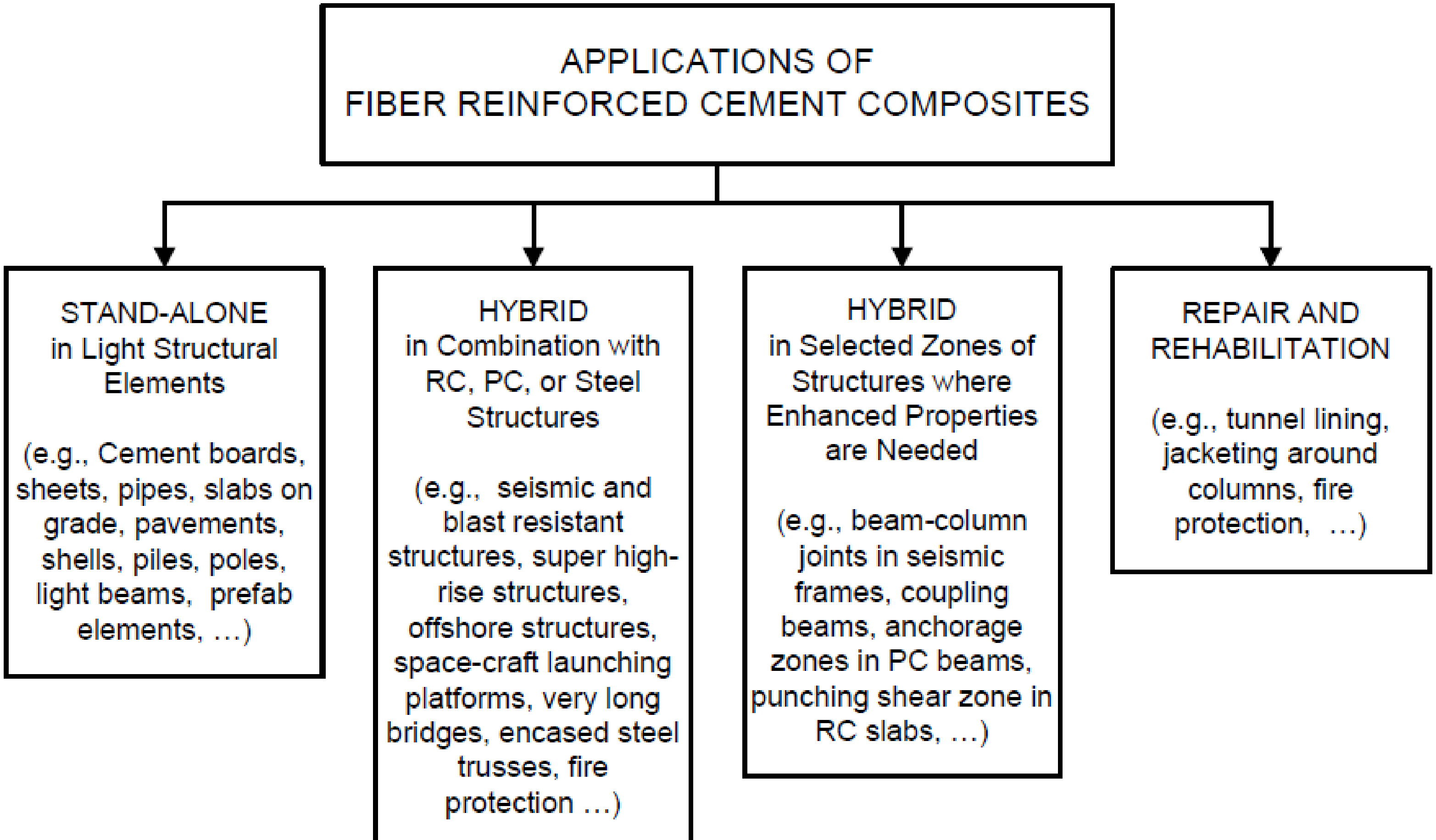
Desirable fiber versus matrix properties for a successful cementitious composite



(Hannant, "Fiber cements and fiber concretes, 1978)

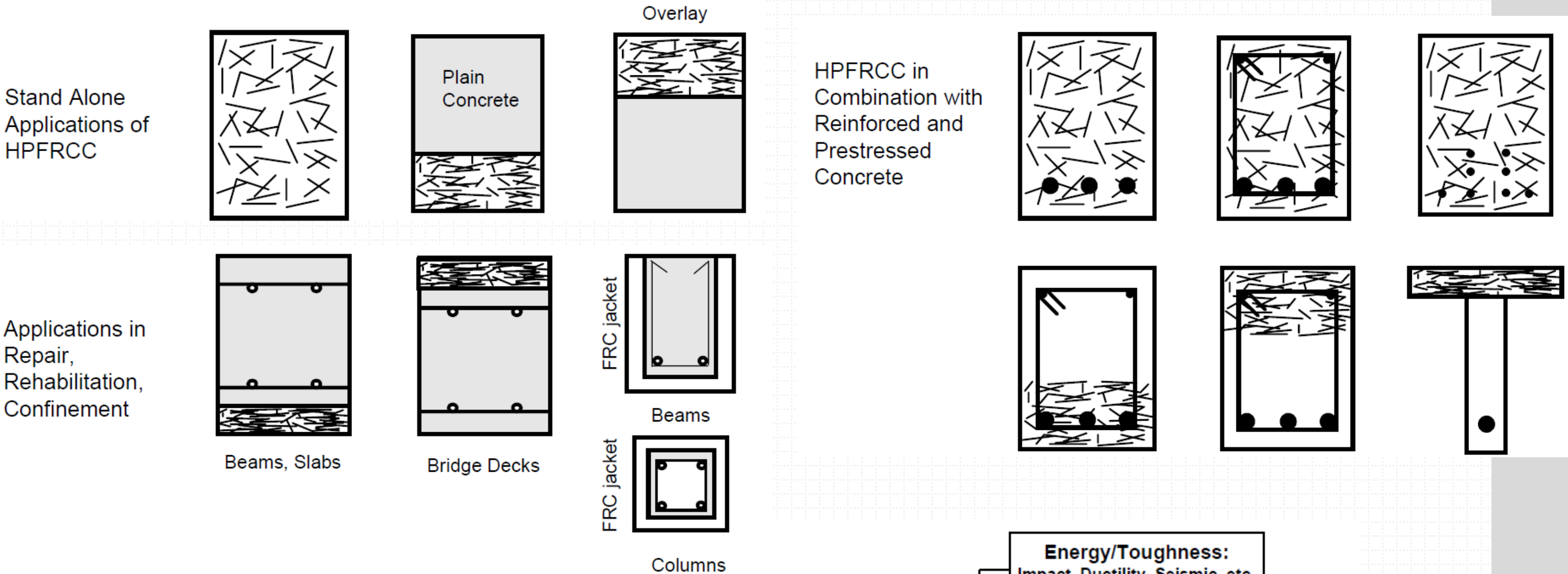
10. Application

Classes of applications of fiber reinforced cement composites

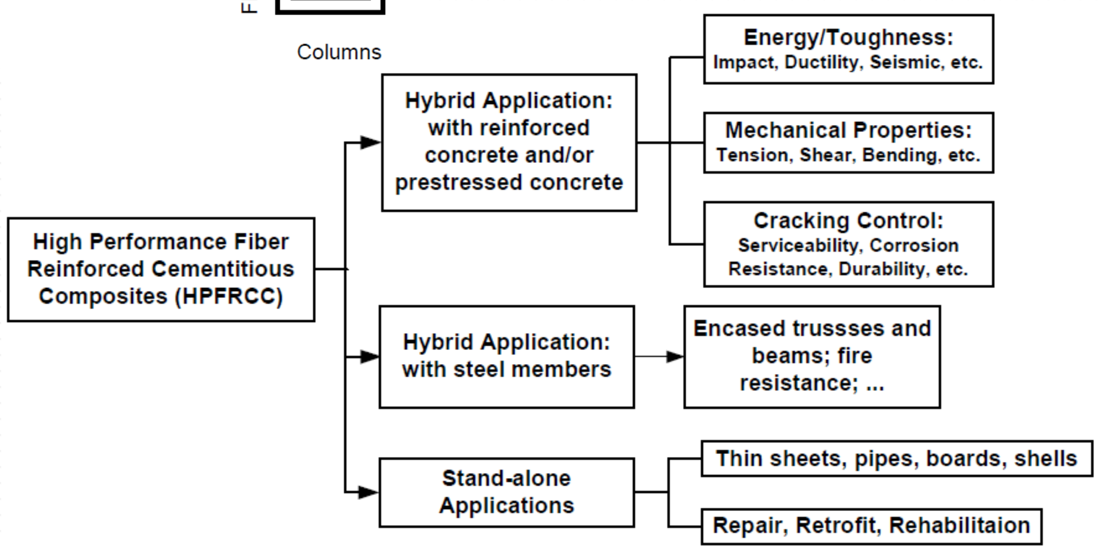


10. Application

Illustration of the applications of FRC composites in various concrete structures



Advantages of using HPFRC composites in structural applications



10. Application

The use of FRC composites, when considered as an alternative in design, is generally not necessary throughout the structure. Commonly, only a small part (selected zone) of the structure may be in need of strengthening or toughening. In such a case their use is often competitive and economically justifiable.

Applications in selected zones of structures include:

- punching shear zone around columns in two-ways slab systems [1];
- end blocks and anchorage zones in prestressed concrete beams;
- beam-to-column connections in seismic resistant frames;
- beam to shear wall connections; coupling beams for seismic cyclic resistance [2];
- out-rigger beams; in-fill damping structural elements [3];
- lower end of shear walls;
- tension zone of RC and PC beams to reduce crack widths and improve durability;
- compression zone of beams and columns to improve ductility;
- compression zone of RC and PC beams using fiber reinforced polymeric (FRP) reinforcements to improve ductility and take advantage of the strength of FRP reinforcements [4, 5].

ACI code to replace part or all the shear reinforcement in concrete members [2]

[1] Parra-Montesinos, High Performance Fiber Reinforced Cement Composites: an Alternative for Seismic Design of Structures,. ACI Structural Journal, 2005
[2] Parra-Montesinos, Proposed addition to ACI Code 318-05 on shear design provisions for fiber reinforced concrete members,. 2006.
[3] Xia and Naaman, Behavior and Modeling of Infill FRC Damper Element for Steel-Concrete Hybrid Shear Wall,. *ACI Structural Journal*, 2002.
[4] Naaman and Jeong, Structural Ductility of Beams Prestressed with FRP Tendons.. RILEM Proceedings 29
[5] Park and Naaman, Shear Behavior of Concrete Beams Prestressed with FRP Tendons,. PCI Journal, 1999