Chapter 2. General Properties of Waves

Waves

- Basic wave properties

(1) Wave motion in crystalline solids (periodic vibration of the atoms)

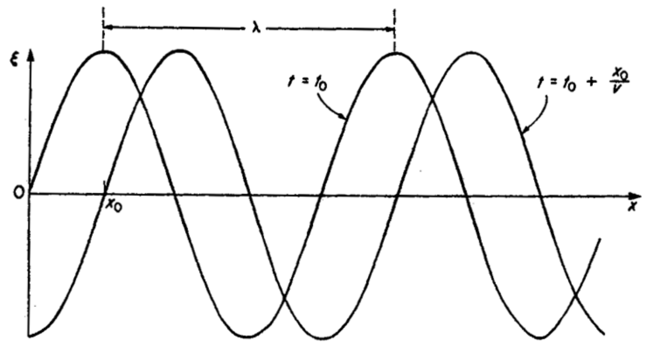
(2) Electromagnetic waves (classical view of light)

(3) Electron waves (wave-like of matter)

- A wave is any periodic displacement in time and position. $\xi(x, y, z, t)$

- The characterizing parameters are
  v: phase velocity
  λ: wavelength
  ν: frequency

- ex) sine, cosine waveforms (harmonic)



Harmonic waves moving in $+x$ direction with displacement ξ, velocity v, and wavelength λ. A point corresponding to a specific displacement ξ on the wave travels to $+x$ with the phase velocity v.

Dispersion relationship

- Relation between frequency and k, *i.e.* $\omega(k)$.

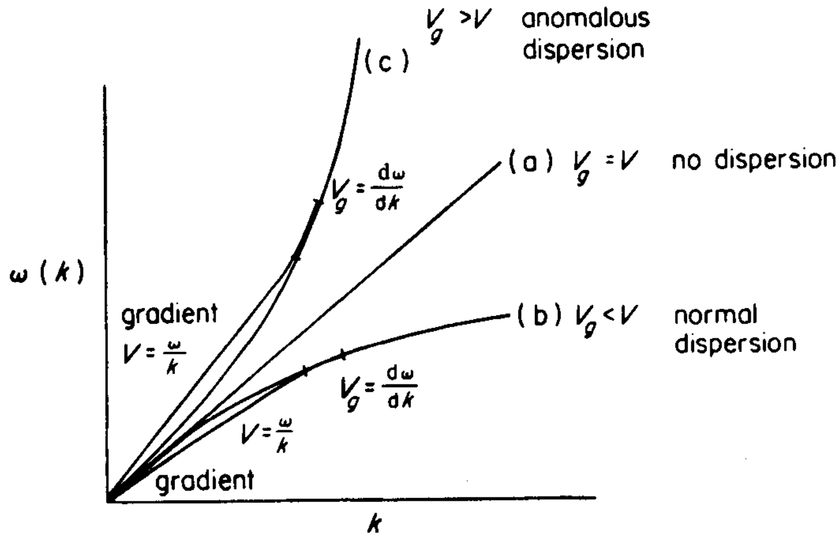


Fig. 5.12. Curves illustrating dispersion relations: (a) a straight line representing a non-dispersive medium, $v = v_g$; (b) a normal dispersion relation where the gradient $v = \omega/k > v_g = d\omega/dk$; (c) an anomalous dispersion relation where $v < v_g$

Group velocity: $v_g = \partial\omega/\partial k$
Motion of a pulse, propagation of energy

Relationship between v_g and v

$$\partial\omega/\partial k = v_g = v + k\frac{\partial v}{\partial k}$$

(1) In a nondispersive system. $v_g = v$
(2) In a dispersive system, $v_g \neq v$

Wave equation

- Wave equations are general equations of motion relating the time and space dependence of the wave displacement, which can be represented by

$$\sum_n a_n \frac{\partial^n \xi}{\partial q^n} = \sum_m b_m \frac{\partial^m \xi}{\partial t^m}$$

ξ: displacement
q: generalized coordinate (x, y, z)
a_n, b_m,; constant coefficients

- Application of boundary conditions to $\xi(q,t)$ may limit the allowed modes of vibration.
- The choice of the solution: harmonic waves

$$\left.\begin{aligned} \xi(x,t) &= A\cos(\omega t - kx) = A\cos(\omega t - \frac{2\pi}{\lambda}x) \\ \xi(x,t) &= B\sin(\omega t - kx) = B\sin(\omega t - \frac{2\pi}{\lambda}x) \end{aligned}\right]$$

The wave is moving $+x$ direction

A and B are amplitudes of the wave.
kx : phase of the wave

Wave equation

- Or more conveniently (use of complex form)

$$\xi(x,t) = A\exp\{i(kx - \omega t)\} \qquad \text{traveling to the } +x \text{ direction}$$
$$\xi(x,t) = A\exp\{-i(kx + \omega t)\} \qquad \text{traveling to the } -x \text{ direction}$$

  - At $t = 0$,

$$\xi(x,0) = A\exp(ikx)$$

  - At $t > 0$,

$$\xi(x,t) = A\exp\left[i\left\{k(x + \frac{\omega}{k}t) - \omega t\right\}\right]$$
$$= A\exp(ikx) = \xi(x,0)$$

$\therefore$ The wave is traveling with the velocity of w/k.

- Or

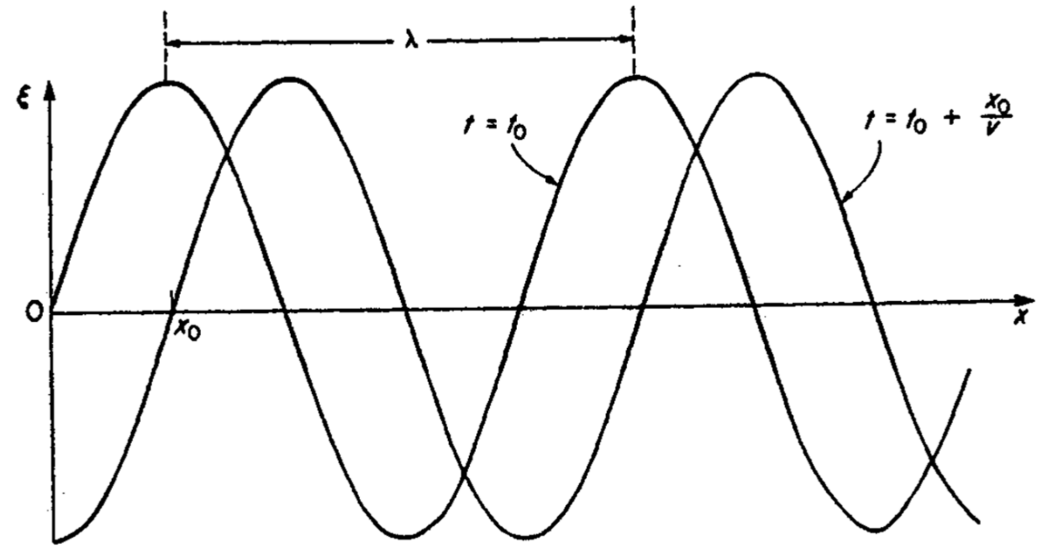
$$\xi = Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}$$

The wave traveling to $+x$ direction with $|v| = \omega/k$

The wave traveling to $-x$ direction with $|v| = \omega/k$

Traveling waves

- These waveforms correspond to traveling waves.

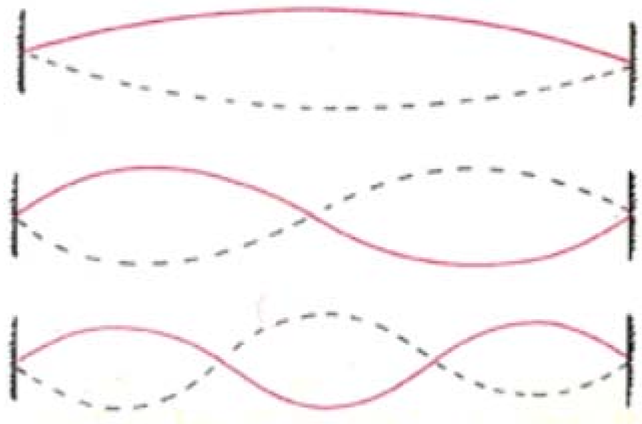


Harmonic waves moving in $+x$ direction with displacement ξ, velocity v, and wavelength λ. A point corresponding to a specific displacement ξ on the wave travels to $+x$ with the phase velocity v.

  - Moving in a particular direction
  - Unconfined by boundary conditions

Standing waves

- The dependence of the displacement on position is independent of the dependence of the displacement of time.



  - Confined boundary conditions at x=0 and x=L
  - Variation with x is $i$ndependent of variation in time

$$A\exp\{i(kx-\omega t)\}-A\exp\{-i(kx+\omega t)\}=C\sin k\,x\exp(-i\omega t)$$

+x direction

-x direction (reflected at x=L)

Transverse and longitudinal waves

- Transverse wave
  : Displacement is vertical to the direction of wave

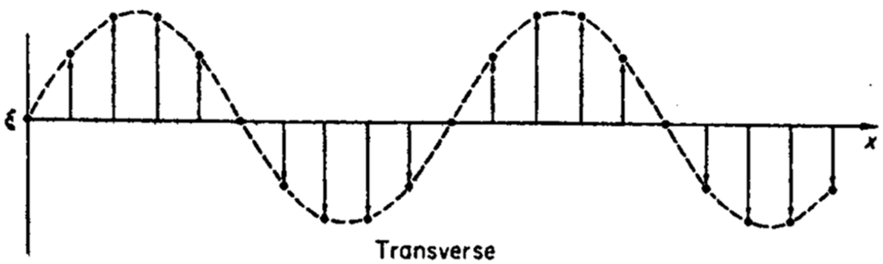
  $$\vec{\xi} \perp \vec{k}$$

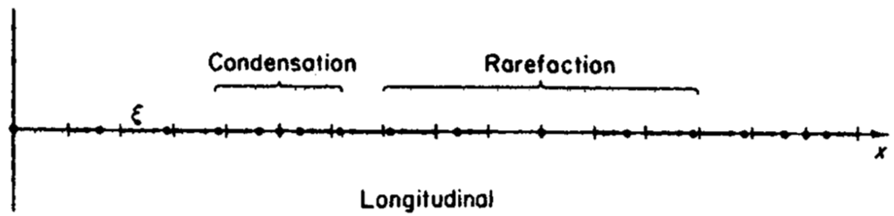
  – e.g. string, lattice, light

- Longitudinal wave
  : Displacement is parallel to the propagation of wave

  $$\vec{\xi} // \vec{k}$$

  – e.g. lattice (sound), rod

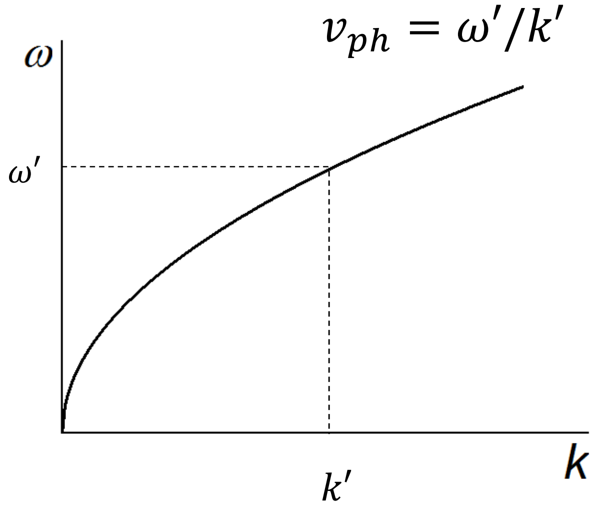




Velocity

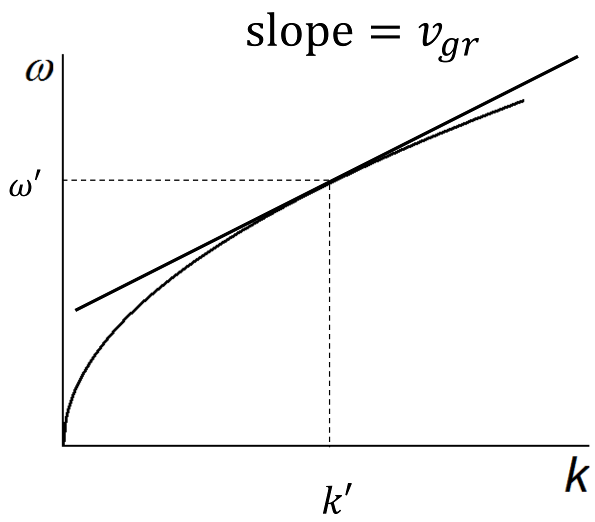
- Phase velocity

$$v_{ph} = \omega/k$$



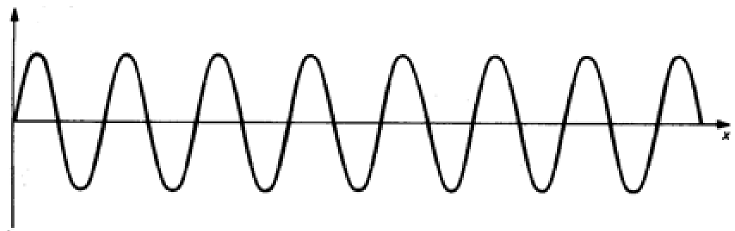
- Group velocity

$$v_{gr} = d\omega/dk$$

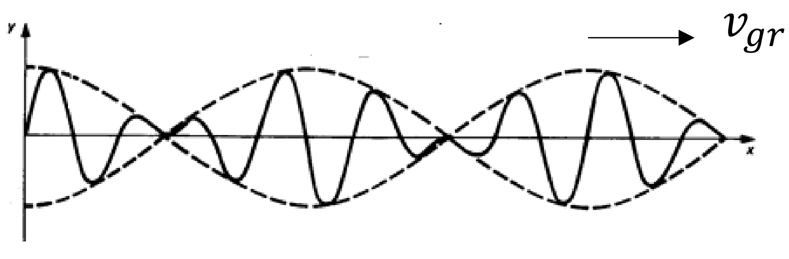


Velocity

- Phase velocity
  - Velocity of individual wave



- Group velocity
  - *v* of which energy is transported
  - *v* of a pulse
  - Sum of individual waves



Red: phase velocity
Green: group velocity