Error Budget

"Design a machine of accuracy (or uncertainty) less than 10um for 300mm travel"

How and where can we start the design in terms of accuracy?

->We need to get the error budget, error allocation, or error identification, which is to identify the order of importance among the sources of errors to achieve the target performance of the machine.

Error budget is to list of all error sources and their effect on the machine accuracy (or total error). It is a powerful tool to identify a order of importance for the sources of error during the design optimization. The total error is the kinematic sum of all individual error components, where the individual error components have the *physical causes that can be identified, measured, and controlled, in order to be used for error prediction/compensation/reduction, or for design optimization.

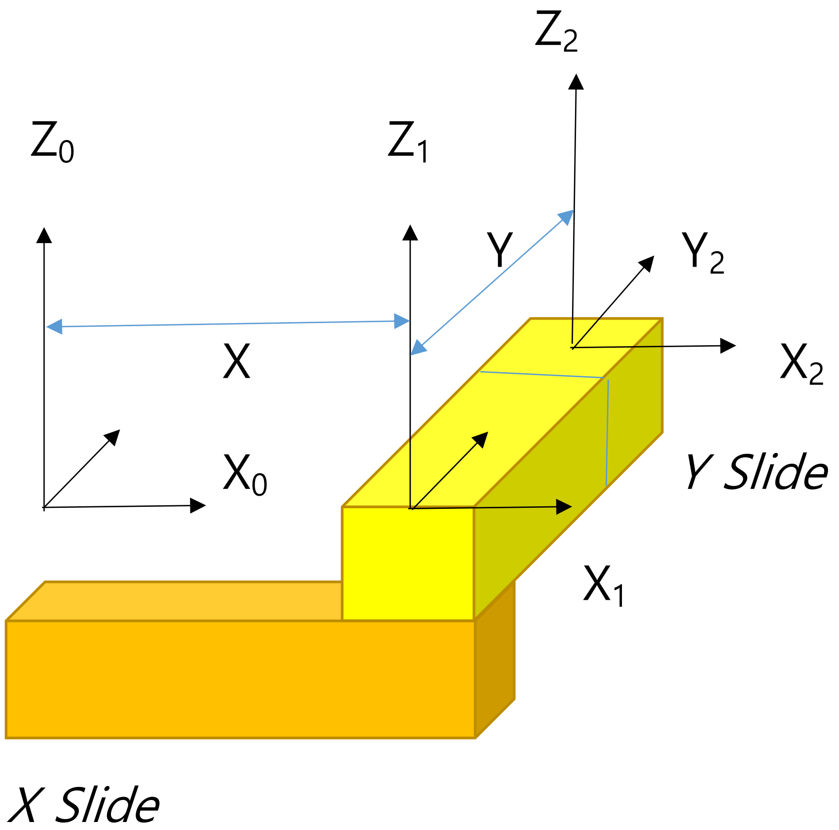
*Physical causes: Physical sources of error including all possible errors such as geometric/kinematic errors , coupling mechanism of stiffness, thermal expansion, damping, etc.

Case example: Error budgeting for XY stage

A example task is to indentify the error sources for the XY stage and to perform design optimization.

The error propagation theory provides a very useful tool for the error budget.

A commercial XY stage (Source from Dover Motion)



Kinematic Configuration XY stage;

O->X->Y (kinematic chain)

$[X_0Y_0Z_0]$: Reference Coordinate system

$[X_1Y_1Z_1]$: Coordinate system fixed on X slide

$[X_2Y_2Z_2]$: Coordinate system fixed on Y slide

The influence of geometric error components on the total positioning accuracy of XY stage is to investigate. The functional, or mathematical relationship between them is great help, say, the error propagation of each error components into the total error.

Translational Errors;

$\delta x(x)$, $\delta y(x)$, $\delta z(x)$; Translational errors of X axis

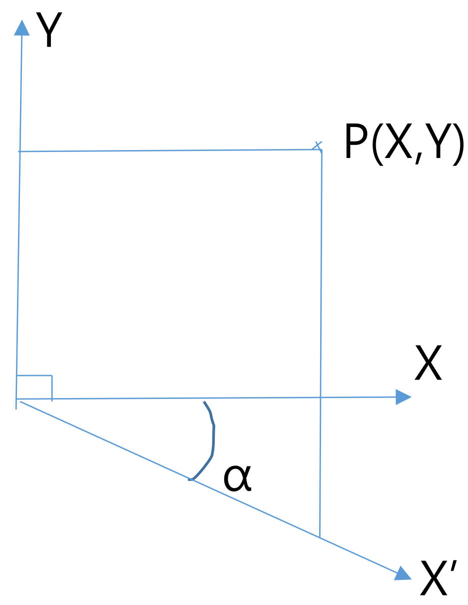
$\delta x(y)$, $\delta y(y)$, $\delta z(y)$; Translational errors of Y axis

Rotational Error;

Ex(x), Ey(x), Ez(x); Angular error of X axis

Ex(y), Ey(y), Ez(y); Angular error of Y axis

Squareness error, α, between X and Y axis



The total error in X direction, $\Delta X=X_R-X$ and the total error in Y direction, $\Delta Y=Y_R-Y$;

They can be proposed by kinematic consideration as follows. Also, the full mathematical formulation can be possible from the kinematics.

$$\Delta X= F= \delta x(x)+\delta x(y)-YEz(x) \quad eq(1)$$

$$\Delta Y= G= \delta y(y)+\delta y(x)-\alpha X \quad eq(2)$$

These two equation give the total error in X,Y direction respectively, and clearly shows how the geometric error components of each axis are contributing to the total errors.

Initial investigation shows that <u>positional error</u> components, <u>angular error</u> components, <u>straightness error</u> components, and <u>squareness error</u> component are the major error sources.

And, they are as follows;

<u>Positional error</u>: Possible sources are lead screw error, thermal expansion, backlash error, etc; longer axis may

give larger error, typically

Angular error: It is magnified by the Abbe offset(Y), and possible sources are guide straightness, bearing errors.

Squareness error: It is magnified by X, possible error source is non-square assembly or alignment of Y axis w.r.t. X axis

Straightness error of guide: Non-straightness of guide gives not only the straightness error, but also significant angular error of block moving along the guide as it can be magnified by the Abbe offset.

Detail error propagation of each error sources as follows;

Total error in X direction, $\Delta X(x,y)$

There are three components, $\delta x(x)$, $\delta x(y)$, $Ez(x)$; and they are all from the different error sources and physically identifiable or measurable independently, thus the systematic part and random part can be identified for every geometric error components.

Positional error, $\delta x(x)$

= Systematic part ± Random part

$= S\delta x(x) \pm R\delta x(x)$

Straightness error, $\delta x(y)$

= Systematic part ± Random part

$= S\delta x(y) \pm R\delta x(y)$

Yaw angular error, $Ez(X)$

= Systematic part ± Random part

$= SEz(x) \pm REz(x)$

As the geometric error components are physically identifiable and measureable, the systematic part and random part can be evaluated.

The error propagation rules can be applied to the ΔX, total error in X direction, as follows, assuming the first order approximation for systematic part and most probable case for the random part.

ΔX=Systematic part±Random part

$=S\Delta X \pm R\Delta X$

Systematic part:

$S\Delta X$

=Algebraic sum of systematic part of components from eq(1)

$= S\delta x(x)\ \partial F/\partial \delta x(x) + S\delta x(y)\ \partial F/\partial \delta x(x) + SEz(x)\ \partial F/\partial Ez(x)$

$= S\delta x(x) + S\delta x(y) - Y \bullet SEz(x)$ eq(3)

Thus the absolute sum of the systematic part is,

$A\Delta X = |S\delta x(x)| + |S\delta x(y)| + |Y \bullet SEz(x)|$ eq(4)

For the random parts,

$R\Delta X$=RMS of random part of components from eq(1)

$= [\partial F/\partial x(x)\ R\delta x(x)^2 + \partial F/\partial x(y)\ R\delta x(y)^2 + \partial F/\partial Ez(x)\ REz(x)^2]^{1/2}$

$= [R\delta x(x)^2 + R\delta x(y)^2 + Y^2 REz(x)^2]^{1/2}$ eq(5)

These equations indicate that the random part as well as the systematic part are also the function of x,y; contributing to the total error in X direction, $\Delta X(x,y)$.

Total error in Y axis, $\Delta Y(x,y)$

Similarly, the total error $\Delta Y(x,y)$ in the Y direction also can be considered as follows;

Geometric error components contributing to Y direction;

Positional error, $\delta y(y)$

= Systematic part ± Random part

$=S\delta y(y) \pm R\delta y(y)$

Straightness error, $\delta y(x)$

= Systematic part ± Random part

$=S\delta y(x) \pm R\delta y(x)$

Sqaureness error, α

= Systematic part ± Random part

$=S\alpha \pm R\alpha$

Therefore

ΔY=Systematic part±Random part

$=S\Delta Y \pm R\Delta Y$

$S\Delta Y$

$=$Algebraic sum of systematic part of components

$= S\delta y(y)\ \partial G/\partial \delta y(y) + S\delta y(x)\ \partial G/\partial \delta y(x) + S\alpha\ \partial G/\partial \alpha$

$= S\delta y(y) + S\delta y(x) - X \bullet S\alpha$ eq(7)

$A\Delta Y$

$=$ Absolute sum of systematic part of components

$A\Delta Y = |S\delta y(y)| + |S\delta y(x)| + |X \bullet S\alpha|$ eq(8)

$R\Delta Y$=RMS of random part of components from eq(7)

$= [R\delta y(y)^2\ \partial G/\partial \delta y(y) + R\delta y(x)^2\ \partial G/\partial \delta y(x) + X^2 R\alpha^2\ \partial G/\partial \alpha]^{1/2}$

$= [R\delta y(y)^2 + R\delta y(x)^2 + X^2 R\alpha^2]^{1/2}$ eq(9)

Thus the full error propagation is,

$\Delta A \pm \Delta R$; where

$\Delta A = [A\Delta X^2 + A\Delta Y^2]^{1/2}$ eq(10)

$\Delta R = [R\Delta X^2 + R\Delta Y^2]^{1/2}$ eq(11)

Therefore the full error propagation is considered for the total errors in the X,Y direction, eq(10),(11) indicate the error budget for the XY stage.

<u>Example case of error calculation</u>

An example Case of XY stage at 300mm (or 12 inch) stroke;

Initial error allocation can be assigned as follows;

Positional error; $\delta x(x)=10\pm1$ [um], $\delta y(y)=10\pm1$ [um]

Straightness error; $\delta x(y)=5\pm1$ [um], $\delta y(x)=5\pm1$ [um]

Angular error; $Ez(x)=5\pm1$ [arcsec]

Squareness error; $\alpha=5\pm0.1$ [arcsec]

Therefore, from eq(4), (5);

$A\Delta X=10+5+0.3(4.8)(5)=22.2$ [um]

$R\Delta X=[1^2+1^2+\{(0.3)(4.8)(0.1)\}^2]^{1/2}=1.42$ [um], and

$\therefore \Delta X=22.2\pm1.42$ [um]

From eq(8), (9);

$A\Delta Y=10+5+(0.3)(4.8)(5)=22.2$ [um]

$R\Delta Y=[1^2+1^2+\{(0.3)(4.8)\ (1)\}^2]^{1/2}=2.02$ [um]

$\therefore \Delta Y=22.2\pm 2.02$ [um]

Therefore the total error, $\Delta A\pm\Delta R$, becomes,

$\Delta A=[A\Delta X^2+A\Delta Y^2]^{1/2}=[22.2^2+22.2^2]^{1/2}=31.4$

$\Delta R=[R\Delta X^2+R\Delta Y^2]^{1/2}=[1.42^2+2.02^2]^{1/2}=2.469$

->Total error=31.4±2.469

The calculation of error budgeting shows the 10um accuracy cannot be achieved with the initial error allocation. Thus the total error should be further reduced. When the systematic errors are compensated via numerical calculation by eq(1) and (2), the random parts $R\Delta X$=1.42[um], $R\Delta Y$=2.02[um], thus ΔR=2.469[um] is the only remaining part, and total error=±2.469.

Thus 10um accuracy can be achieved!

But this is a very ideal case, and further reduction on the error allocation has to be made.

The squareness error and angular errors should be treated more significantly than the rest of errors. It is also to note that the positional errors are also much contributing.

This is a practical procedure for error budgeting, and each causes should be identified, adjusted, and optimized, in such as; precision grade of screw, preloaded screw mechanism, adequate servo-drive, friction, guideway bearing types, preloaded bearing structure; indicator/talyvel/edge/square assisted assembly for guideway with checking of straightness errors, angular errors, etc.

The repeat procedures can be followed until the performance is met with the requirement.

Other error sources such as vibration, machining, or temperature influence can be identified and added to the error budget in terms of error accumulation as discussed earlier.

HW3) Perform the error budget analysis for XY stage of 300mmX300mm stroke, where the Abbe offset is 50mm (X direction) from the centre of the Y slide.