Precision Design- Stiffness Enhancement

Stiffness:

-Capacity of mechanical system to sustain load;

-Stiffness, K = Force / Deformation = F/δ, or δ=F/K

-Compliance, C=Deformation per unit Load=δ/F=1/K

-Higher stiffness gives lower deformation of structure

Thus the stiffness is affecting on;

1)Static deformation

2)Dynamic performance such as high speed motion, vibration, etc

3)Accuracy/precision of machine

Factors affecting stiffness

1. Elastic moduli of material, E(Young's modulus), G(Shear Modulus,$G=E/2(1+\nu)$), K(Spring constant);
2. Structure dimension; Length(L), Width(W), Height(H), Radius(R), Thickness(t)

3. Cross sectional geometry of deforming segments

   A(cross sectional area) :

   $\int dA$, for tension/compression/shear

   Ix,Iy (Area moment of Inertia):

   $Ix=\int x^2 dA$, $Iy=\int y^2 dA$, for bending

   Jp (Polar moment of Inertia):

   $Jp=\int r^2 dA$, for torsion, $(=\int (x^2+y^2)dA=Ix+Iy)$

4. Variation of above geometric parameters along the structure

5. Loading/Supporting condition of structural components

6. Elastic stability for the slender/thin walled segments such as buckling

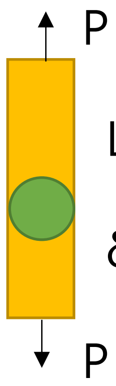
7. Joints between structures

<u>Modes of Loading on Stiffness</u>

Principal modes of structural loading:

Tension, Compression, Bending, Torsion

1. Tension/Compression mode

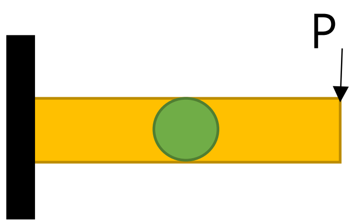


L,E,A,d

$\delta_T = PL/EA \ \therefore \text{Stiffness} = P/\delta = EA/L$

2. Bending in cantilever mode

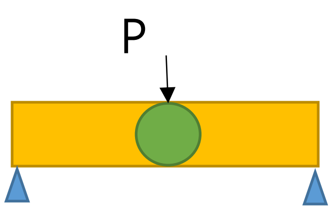
Built-In



$\delta_{BC} = PL^3/3EI$, Stiff. $= 3EI/L^3$

L,E,I,d

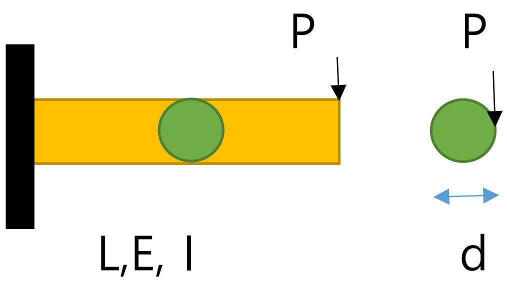
3. Bending in Double Supported mode



$\delta_{BD} = PL^3/48EI$, Stiff. $= 48EI/L^3$

L,E,I,d

4. Bending with eccentric load



$\delta_e = \delta_c + \delta_{tor}$

$\delta_{tor} = (d/2)(TL/GJp) = PLd^2/4GJp$

where $A = \pi d^2/4$, $I = \pi d^4/64$, $Jp = \pi d^4/32$

Obervation1:

$\delta_{BC}/\delta_T = \{L^2/I\}/\{3/A\} = 16L^2/3d^2 = 2133$ (if $L/d = 20$)

$\delta_{BD}/\delta_T = \{L^2/I\}/\{48/A\} = L^2/3d^2 = 133$ (if $L/d = 20$)

∴Tension/Compression is much stiffer than the bending mode by few $10^2 \sim 10^3$ times

Observation2:

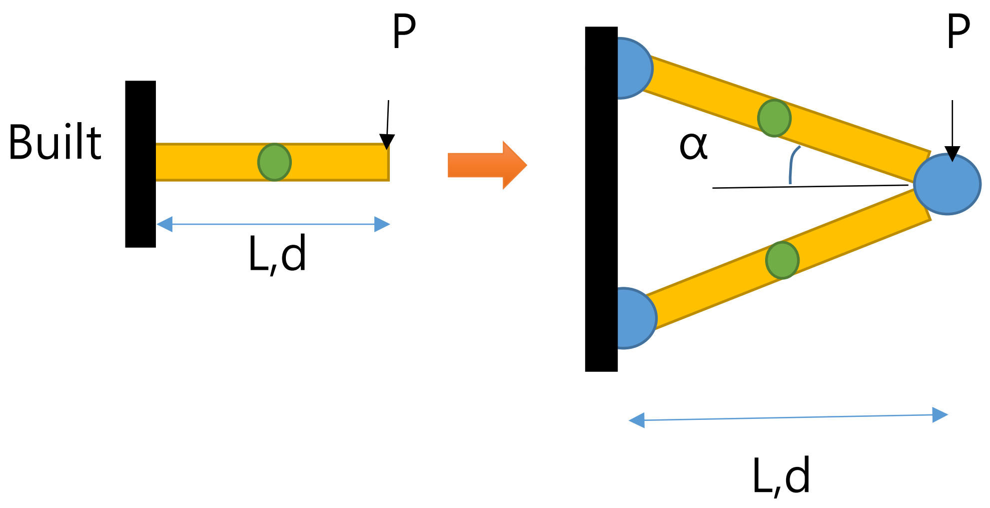
$\delta_{tor}/\delta_T = \{PLd^2\}/\{4GJp\}/[PL/\{EA\}] = 2E/G \fallingdotseq 5$

∴Tension/Compression is stiffer than Torsion by about 5 times

Strategy:

<u>To replace Bending mode with Tension/Compression mode</u>

Ex1) Cantilever->Truss



δ_B/δ_T = $(32/3)(L/d)^2\sin^2\alpha\cos\alpha$ eq(1)

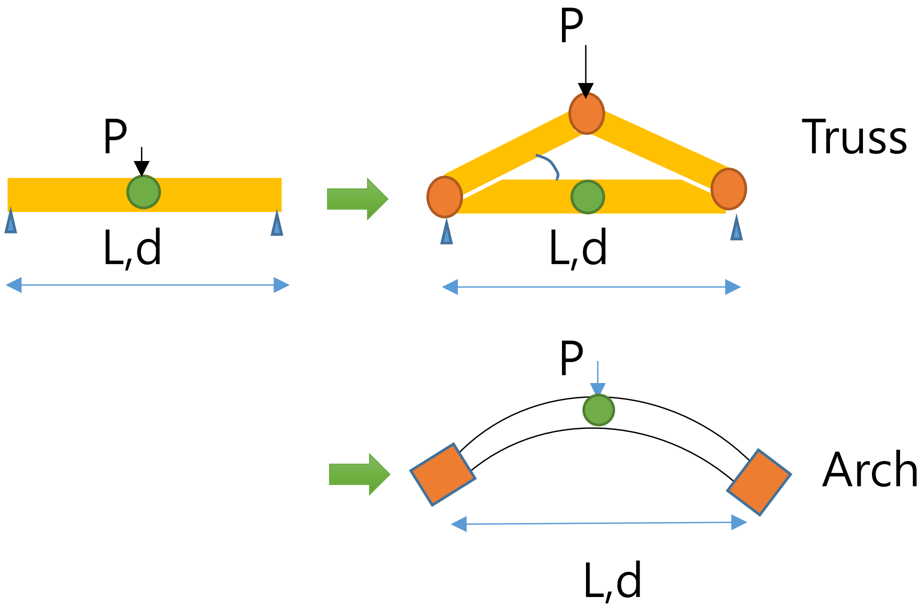
where α=half angle between trusses

When L/d=20, α=45˚,

then $\delta_B/\delta_T=(32/3)(20)^2(1/\sqrt{2})^3=1509$

∴ Truss mode has 1509 times higher stiffness than the beam mode

Ex2) Double Supported Beam-> Truss



$\delta_B/\delta_{TRUSS} \fallingdotseq 1.3(L/d)^3\sin^2\alpha\cos\alpha$ eq(2)

α= truss angle

when $L/d=20$, $\alpha=45^\circ$, then $\delta_B/\delta_{TRUSS}=3677$

Similar effect can be obtained with the Arch structure.

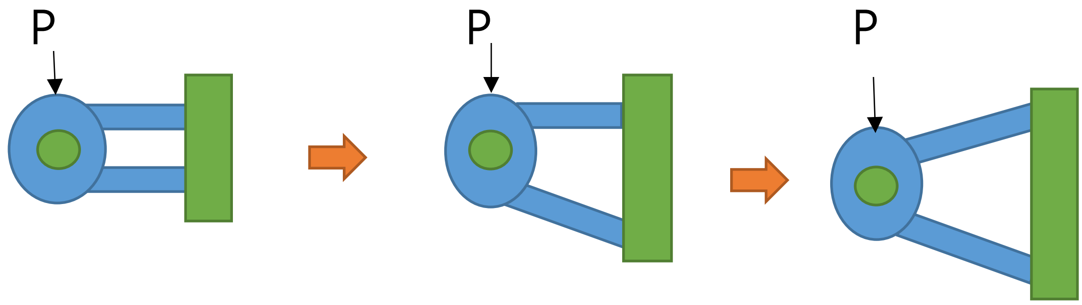
HW4)

1. Derive eq(1) and eq(2)
2. Find the ratio δ_B/δ_{ARCH} for the Arch structure of 45 deg angle.

(Hint: Use the energy method, and compare the results with the Finite Element Method)

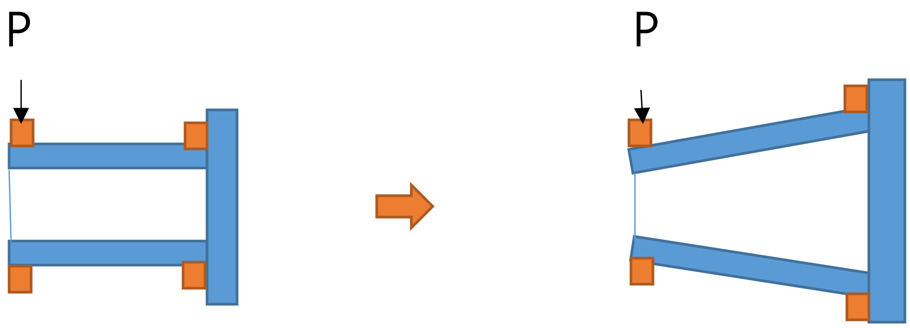
Truss concept modification

Brackets



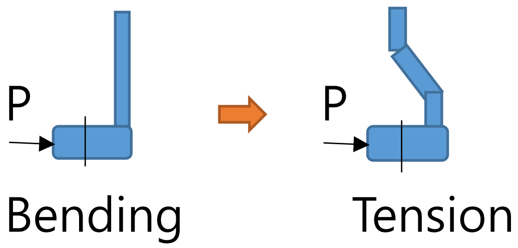
U: Bending	Bending	Tension
L: Bending	Comp.	Comp.

Thin Wall Cylinder Bracket

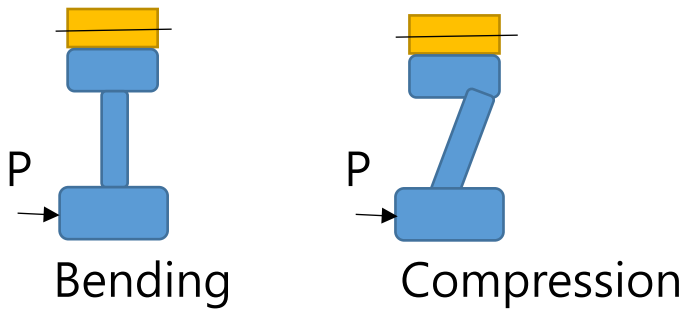


U: Bending	U: Tension
L: Bending	L: Comp.

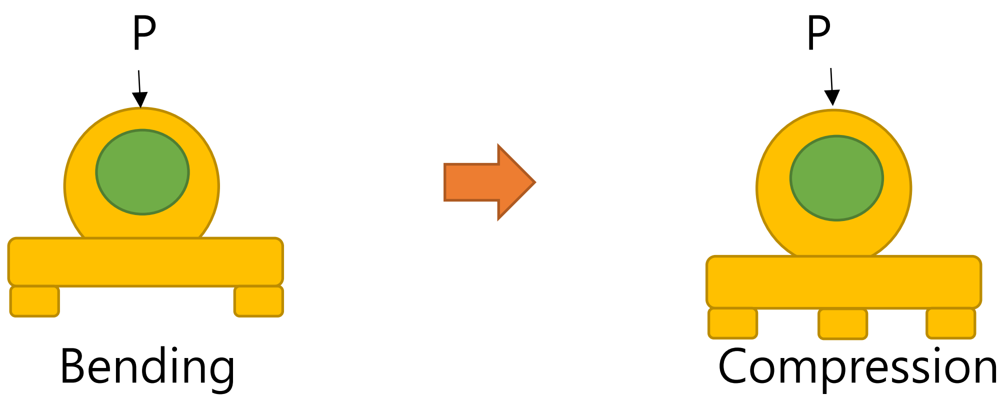
"Pocketing" for Mounting foot



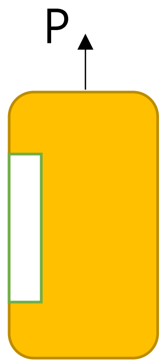
"Inclination" of Wheel or Gear-hub



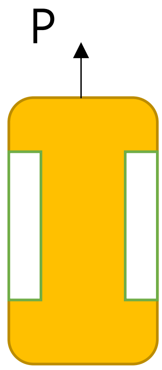
Added central support for Pillow block



Symmetric modification of structure member



Bending



Tension

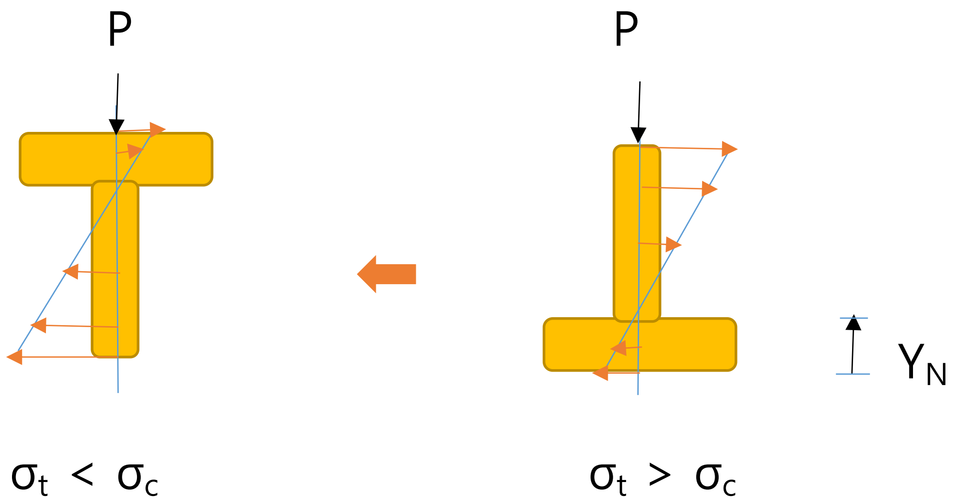
Modification of Tension to Compression

Some material such cast iron or ceramics are better suited to accommodate compressive stress rather than tensile stress. Some micro cracks in the material are quite often of weak resistance against for the tensile stress rather than compressive stress. Thus tension to compression modification is a good design strategy, or to increase the compressive stress at the expense of tensile stress, while the net force or moment are not changed.

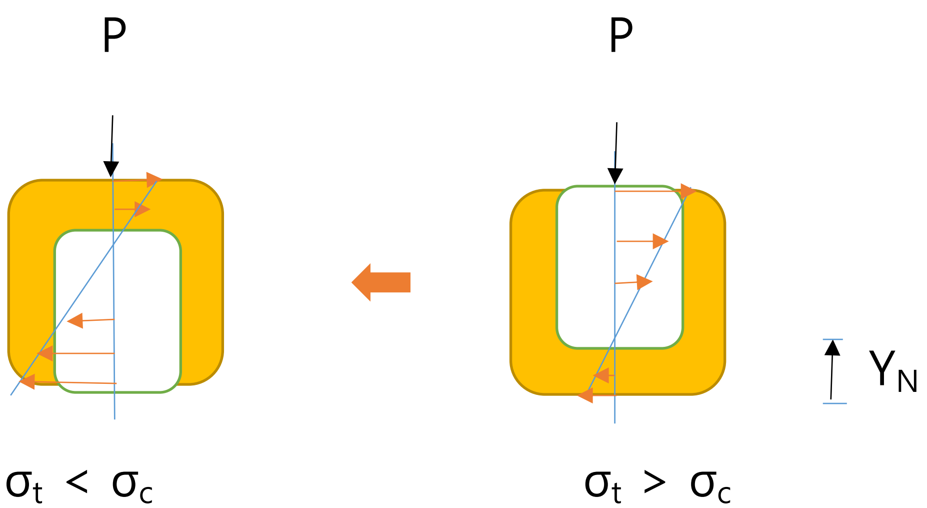
<u>Turning upside down of beam structure</u>

Neutral line, Y_N , determined from $\int YdA=0$

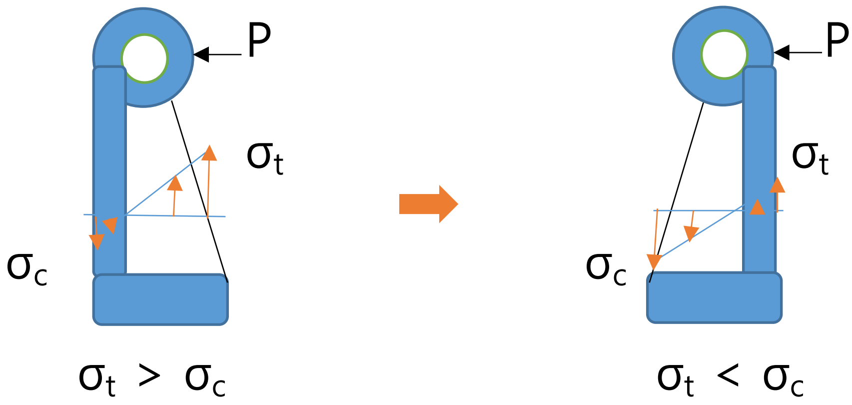
L Beam



C Beam



<u>Opposite Mounting of Bracket with stiffening wall</u>



After the design modification concept is chosen, the detail calculation is performed for the optimization using such as FEM.

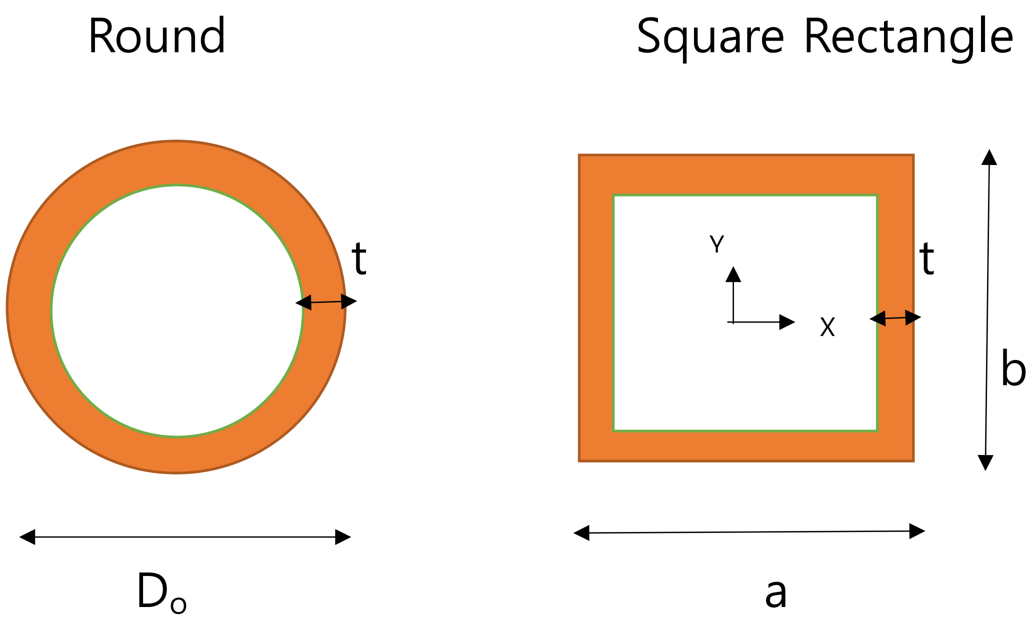
Optimization of Cross Sectional Shape

Significant gains in stiffness can be achieved depending on the cross sectional shape.

Hollow shaft or thin walled structure have advantage for using machine parts or robotic links such as;

1. Very high bending or torsional stiffness/weight ratio, minimizing the material needed
2. Light weight to reduce inertial forces and to allow the larger payload per given sizes of motors and actuators
3. Internal hollow provides area for electrical lines for power and communication, hoses, control rods, etc

Round vs Rectangular shape for Hollow shaft



D_i:Inner Dia.

D_o:Outer Dia.(D)

t: thickness

a: Width

b: High

t: thickness

For round:

$I_r=\pi(D_o^4-D_i^4)/64=\pi\{D^4-(D-2t)^4\}/64$

$\fallingdotseq\pi D^3t\{1-3t/D+4t^2/D^2\}/8$

$A_r=\pi(D_o^2-D_i^2)/4=\pi(D^2-(D-2t)^2)/4$

$=\pi t(D-t)=\pi Dt(1-t/D)$

For square rectangle

$Ix=ab^3/12-(a-2t)(b-2t)^3/12$

$Iy=a^3b/12-(a-2t)^3(b-2t)/12$

$A_s=ab-(a-2t)(b-2t)=2t(a+b)-4t^2$

When a=b,

$Is=Ix=Iy=a^4/12-(a-2t)^4/12$

$\fallingdotseq \frac{2}{3}a^3t(1-3t/a+4t^2/a^2)$

$A_s=4ta-4t^2=4ta(1-t/a)$

Case1

D=a, t is the same for both shape

$Is/I_r=\frac{2}{3}/(\pi/8)=1.7$

$A_s/A_r=4/\pi=1.27$

Thus square hollow shaft gives 70% higher bending stiffness with only 27% increase in weight

Case2

$D=a$, $A_r=A_s$ (i.e. the same weight),

When $t_r=0.2D$; $A_r= \pi Dt(1-t/D)=As=0.16\pi D^2$

Thus $A_s=4ta(1-t/a)=0.16\pi D^2$,

$\therefore t/a=2-\sqrt{(4-4(0.16\pi))}=0.147$

Thus, $I_r=0.0405D^4$, $I_s=0.0632a^4$; Thus $I_s/I_r=1.56$

Thus for the same weight, square rectangle has 56% stiffness increase than round.

Conclusion

(1)Beam with hollow rectangle shape gives about 30-40% higher stiffness than the hollow round shape.

(2)Beam with hollow rectangle has advantage for being used as prismatic joints or roller guideways or machines.

(3)Beam with round shape gives easier fitting in telescopic links with sliding connection.

(4)Under the same weight, the beam stiffness can be significantly enhanced upto about 10 to 20 times if a solid cross section is replaced with the cross sectional shape in which the material is concentrated farther from the neutral axis

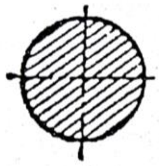
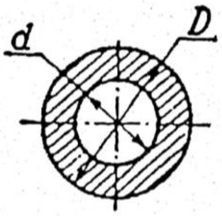
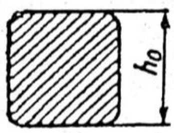
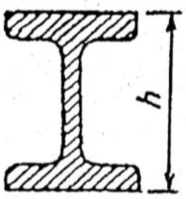
Section	Ratios d/D, h/h_0	I/I_0	W/W_0
	0	1	1
	0.6 0.8 0.9	2.1 4.5 10	1.7 2.7 4.1
	—	1	1
	1.5 2.5 3.0	4.3 11.5 21.5	2.7 4.5 7.0

Fig. 2.2.2. Relative stiffness (cross-sectional moment of inertia, *I*) and strength (section modulus, *W*) of various cross-sections having the same weight (cross-sectional area *A*).

(Source: Eugene I. Rivin, 'Handbook on Stiffness and Damping in Mechanical Design' ASME Press,2010)

Stiffness and Damping of Helically Patterned Tube

Helically patterned tubes are frequently used in heat exchangers and some structural applications such as vibration-assistant smokestacks.

Width(w) and wall thickness(t) gives very small influence on the stiffness and damping characteristics.

Depth(h) and pitch(s) of the groove pattern are important, and test results of brass tube approximately show under D=16-24mm, h=0.4-1.5mm, s=8.3-24mm.

$(EI)_{hp}=(EI)_s \exp(-4.8(h/s))$

Where $(EI)_s$=Bending stiffness of solid tube

$(EI)_{hp}$= Bending stiffness of helical patterned tube

The damping performance is also enhanced up to 7 times higher damping as shown in fig., probably due to the stress concentration introduced by the helical grooves, where δ indicates the logarithmetic decrement, that is,

$\delta = nT^* = 2\pi c/\sqrt{(4mk - c^2)} \doteqdot \pi c/\sqrt{mk}$,

where $n = c/2m$ for $md^2x/dt^2 + cdx/dt + kx = 0$, or simply,

$\delta = Ln(X_0/X_1)$, and damping factor $\xi \doteqdot \delta/2\pi$

and $d^2x/dt^2 + 2\xi\omega_n dx/dt + \omega_n^2 x = 0$

where $2\xi\omega_n = c/m$, $\omega_n^2 = k/m$

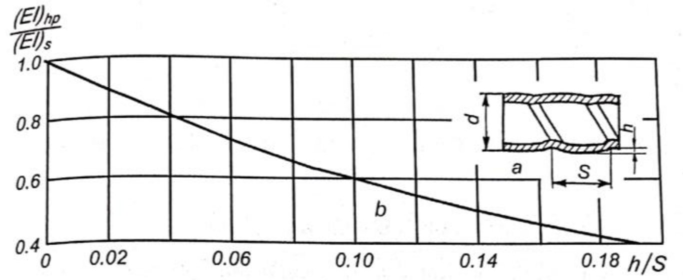


Fig. 2.2.3. Helically grooved tube (a) and its bending stiffness as function of the groove dimensions (b).

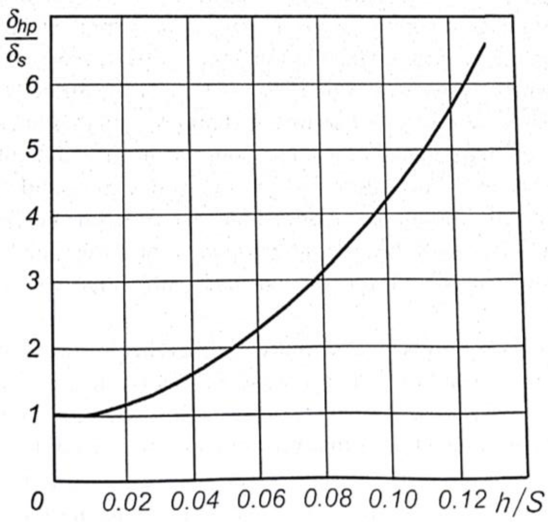


Fig. 2.2.4. Damping as function of helical groove dimensions.

(Source: Eugene I. Rivin, 'Handbook on Stiffness and Damping in Mechanical Design' ASME Press,2010)

Stiffness of composites beam

When beams are of laminated multilayer composites, then the bending stiffness becomes the sum of stiffness each layers 1 to n; with high damping materials for the middle layer, optionally.

Composite Layers 1 to n

Beam with clamped ends

Total bending stiffness$=\Sigma E_i I_i = E_1 I_1 + E_2 I_2 + .. + E_n I_n$

where $E_i I_i$ is the bending stiffness of the ith layer.

Solid adhesion between layers, or split beam with clamped ends give the beam to behave like one beam with no slippage at the interfaced surface.

Soft adhesion between layers, or split beam with non-clamped ends give the beam to behave like spit beams with slippage at the interfaced surface.