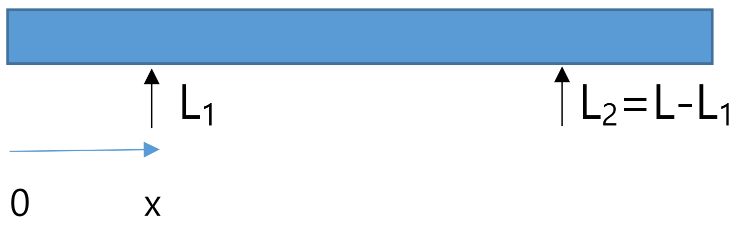
Precision Design –Airy support

<u>Support positions and deformation</u>:

Deformation varies with the supporting positions, thus it can be optimized for specific application

<u>Beam of distributed load with double support</u>

Weight per unit length=w, Length=L



Let $\langle x\rangle=x$ if $x\geq 0$, $\langle x\rangle=0$ if $x<0$

$Q(x)=-w+wL/2\langle x-L_1\rangle_{-1}+ wL/2\langle x-L_2\rangle_{-1}$

$V(x)=-\int Q(x)dx$

$=wx-wL/2\langle x-L_1\rangle^0-wL/2\langle x-L_2\rangle^0+C_1$, and $C_1=0$ ($\because$ $V(0)=0$)

$M(x)=EId^2y/dx^2=-\int V(x)dx$

$=-wx^2/2+ wL/2\langle x-L_1\rangle+wL/2\langle x-L_2\rangle+C_2$, and $C_2=0$ ($\because$ $M(0)=0$)

$EIdy/dx=\int M(x)dx$

$=-wx^3/6+ wL/4\langle x-L_1\rangle^2+wL/4\langle x-L_2\rangle^2+C_3$

$EIy(x)=-wx^4/24+ wL/12<x-L_1>^3+wL/12<x-L_2>^3+C_3x+C_4,$

Applying $y(x)=0$ at L_1 and L_2

$EIy(L_1)=-wL_1{}^4/24+C_3L_1+C_4=0$

$EIy(L_2)=-wL_2{}^4/24+C_3L_2+C_4=0$

Thus

$C_3=(wL/24)(L_1{}^2+L_2{}^2)-(wL/12)(L_2-L_1)^2$

$C_4=wL^4/24-C_3L_1=(w/24)[L_1{}^4-LL_1(L_2{}^2+L_1{}^2)+2LL_1(L_2-L_1)^2]$

Therefore,

$\delta(x)=(w/24EI)\ [-x^4+2L<x-L_1>^3+2L<x-L_2>^3+[L(L_2{}^2+L_1{}^2)-2L(L_2-L_1)^2]x+L_1{}^4$

$-L_1L\ (L_2{}^2+L_1{}^2)+2LL_1(L_2-L_1)^2]$

Thus,

$\underline{\delta(0)=(w/24EI)\ [L_1{}^4-L_1L\ (L_2{}^2+L_1{}^2)+2LL_1(L_2-L_1)^2]}$ **eq(1)**

$\delta(L/2)=(w/24EI)\quad [-L^4/16+2L(L/2-L_1)^3+[L(L_2{}^2+L_1{}^2)-2L(L_2-L_1)^2](L/2)+L_1{}^4-L_1L\ (L_2{}^2+L_1{}^2)+2LL_1(L_2-L_1)^2]$

$= (w/24EI)\ [-L^4/16+L_1{}^4+2L(L_2-L_1)^3/8+(L_1{}^2+L_2{}^2)(L^2/2-L_1L)+(L_2-L_1)^2(-L^2+2LL_1)]$

From $L/2-L_1=(L_2-L_1)/2$, $-L^2+2LL_1=2L(L_1-L/2)=-2L(L_2-L_1)/2=-L(L_2-L_1)$

Thus,

$\underline{\delta(L/2)=\ (w/24EI)\ [-L^4/16+L_1{}^4-(3/4)L(L_2-L_1)^3+(L_1{}^2+L_2{}^2)L(L_2-L_1)/2]}$ **eq(2)**

For Double support case, $L_1=0$, $L_2=L$, then from eq(1), eq(2)

$\delta(0)=0$

$\delta(L/2)= (wL^4/24EI)\ [-1/16-(3/4)+(1/2)]$

$= (wL^4/24EI)\ [(-1-12+8)/16]=-(5/384)wL^4/EI=-0.01302wL^4/EI$

Airy Points Support:

Support for standard metre-bars to remove any bending at both ends, it means to locate L_1, L_2 such that end faces of beams are vertical, that is $y'(0)=0$ and $y'(L)=0$. Thus,

$EIy'(0)=C_3=0$

$EIy'(L)=-wL^3/6+wL(L-L_1)^2/4+ wL(L-L_2)^2/4=0$

Divide by wL; $-L^2/6+(L-L_1)^2/4+(L-L_2)^2/4=0$,

$*12/L^2$, and remembering $L_1+L_2=L$, and $L_1/L=x$, $L_2/L=1-x$

$-2+3(1-x)^2+3x^2=0$, and $6x^2-6x+1=0$, thus $x=(3\pm\sqrt{3})/6$

Thus $L_1/L=(3-\sqrt{3})/6=0.211$, $L_2/L=(3+\sqrt{3})/6=0.788$, and

$L_2-L_1=2\sqrt{3}L/6=0.577L$: Airy Points Support

At Airy points support,

$\delta(0)= (w/24EI)[L_1^4-L_1L\ (L_2^2+L_1^2)+2LL_1(L_2-L_1)^2]$

$=(wL^4/24EI)[\ 0.211^4-0.211(0.788^2+0.211^2)+2(0.211)(0.577)^2]$

$=(wL^4/24EI)[0.00206]$

$=0.000086(wL^4/EI)$

$\underline{\delta(L/2)=\ (w/24EI)\ [-L^4/16+L_1^4-(3/4)L(L_2-L_1)^3+(L_1^2+L_2^2)L(L_2-L_1)/2]}$

$=\ wL^4/24EI[-1/16+0.211^4-0.75(0.577)^3+(0.211^2+0.788^2)(0.577)/2]$

$=(wL^4/24)[-0.0126]$

$\fallingdotseq -0.000525wL^4/EI$

Thus deflection, $\delta=\delta(0)-\delta(L/2)=0.000611\ wL^4/EI$

For double support condition from eq(1), eq(2): $L_1=0$, $L_2=L$

$\delta(0)=0$

$\delta(L/2)=\ (w/24EI)(-5L^4/16)=-5wL^4/384EI=-0.01302\ wL^4/EI$

∴ About 4.7% (≒0.000611/0.01302) deflection when compared to the Double Support condition.

*Minimum straightness support points

The straightness due to bending deflection=$\delta_{max}-\delta_{min}$

To find the L_1, L_2 such that $\delta_{max}-\delta_{min}$ be minimum

This condition is to find L_1 such that $\delta(0)=\delta(L/2)$;

From eq(1), eq(2);

$Left=24EI\delta(0)/w=L_1^4-L_1L\ (L_2^2+L_1^2)+2LL_1(L_2-L_1)^2$

$Right=24EI\delta(L/2)/w=-L^4/16+L_1^4-(3/4)L(L_2-L_1)^3+(L_1^2+L_2^2)L(L_2-L_1)/2$

$/L^4$, and let $x=L_1/L$, then $L_2/L=1-x$, $(L_2-L_1)/L=1-2x$

$x^4-x[(1-x)^2+x^2]+2x(1-2x)^2=-1/16+x^4-0.75(1-2x)^3+[x^2+(1-x)^2](1-2x)/2$

$Left=x^4-x(1-2x+2x^2)+2x(1-4x+4x^2)=x^4+6x^3-6x^2+x$

$Right=-1/16+x^4-0.75[1-3(2x)+3(2x)^2-(2x)^3]-(2x^2-2x+1)(x-1/2)$

$=-1/16+x^4-(0.75-4.5x+9x^2-6x^3)-(2x^3-3x^2+2x-1/2)$

$=x^4+4x^3-6x^2+2.5x+(-1-12+8)/16$

$Left-Right=2x^3-1.5x+5/16=0$

By solving the cubic equation by numerical method,

$L_1=0.223L$, $L_2=0.777L$, and $L_2-L_1=0.554L$.

Then $\delta(0)\ =(w/24EI)\ [L_1^4-L_1L\ (L_2^2+L_1^2)+2LL_1(L_2-L_1)^2]$

$=(wL^4/24EI)\ [0.223^4-(0.223)(0.777^2+0.223^2)+2(0.223)(0.554)^2]$

$=wL^4/24EI(-0.00636)=(-0.000265)wL^4/EI$

The straightness error $=\delta(L_1)-\delta(0)=-\delta(0)=0.000265wL^4/EI$

$\therefore$ About 2% ($\fallingdotseq$0.000265/0.01302) of Double support case

Therefore, the supporting locations for beam elements give very strong influence on the beam bending or deformation, and thus better to be located at the optimum position.