# Precision Machine Design Stiffness-Enhance2

## Stiffness enhancement by reinforcing the cross sectional shape

For enhancement of bending or torsional stiffness of machine, the cross sectional shape modification is very effective and it can be implemented by adding ribs;

Bending stiffness= EI to increase

Torsional stiffness=GJ to increase

where E, G are the Young's modulus, and shear modulus of material.

$I = \int y^2 dA$ is the area moment of inertia, and y is the distance from the neutral axis that can be calculated from the $\int y dA = 0$, and $\int dA$ is the integration over the whole cross section.

Thus I becomes large as more material parts is farther away from the neutral point.

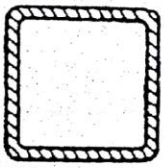
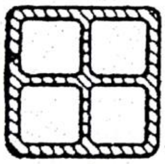
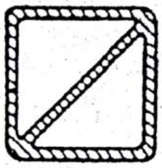
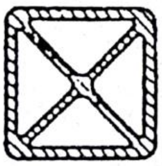
$J=\int r^2 dA=\int (x^2+y^2)dA=\int x^2 dA+\int y^2 dA=Ix+Iy$ is the polar moment of inertia, and the sum of area moment of inertia Ix and Iy.

r is the distance from the neutral point that can be calculated from the $\int rdA=0$.

Thus J becomes large as more material parts is farther away from the neutral point.

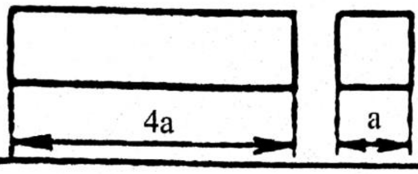
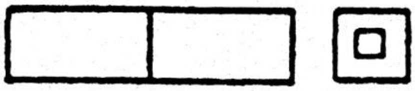
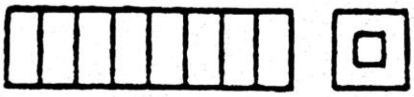
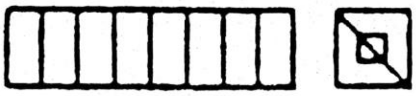
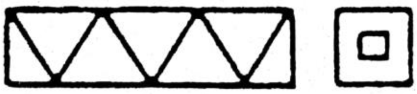
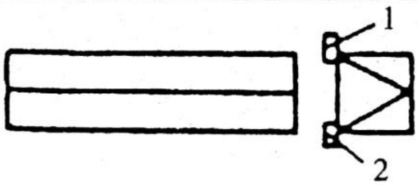
Diagonal ribs give very effective increase in stiffness typically upto 1.78 times, and torsional stiffness upto by 3.7 times as shown in the figures.

Adding ribs means adding more material for the structure, thus the ratio of Stiffness to Area is very meaningful indicator for the efficiency in stiffness enhancement.

| Profile | $I_{ben}$ | $I_{tors}$ | A | $\frac{I_{ben}}{A}$ | $\frac{I_{tors}}{A}$ |
|---|---|---|---|---|---|
|  | 1 | 1 | 1 | 1 | 1 |
|  | 1.17 | 2.16 | 1.38 | 0.85 | 1.56 |
|  | 1.55 | 3 | 1.26 | 1.23 | 2.4 |
|  | 1.78 | 3.7 | 1.5 | 1.2 | 2.45 |

Stiffness increase by reinforcing ribs on the cross section geometry

(Source: E.I.Rivin's Handbook on stiffness and damping)

| Case # | Rib location | $K_x$ % | $K_t$ | W % |
|---|---|---|---|---|
| 1 |   | 100 | 100 | 100 |
| 2 |  | 101 | 103 | 108 |
| 3 |  | 102 | 109 | 125 |
| 4 |  | 116 | 132 | 130 |
| 5 |  | 113 | 112 | 115 |
| 6 |   | 135 | – | 140 |

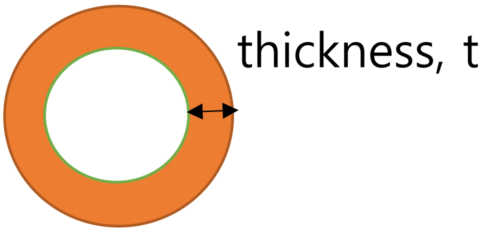
Stiffness increase by reinforcing ribs on outer/inner geometry

Kx: Bending stiffness, Kt: Torsional stiffness

(Source: E.I.Rivin's Handbook on stiffness and damping)

Torsional Stiffness

Hollow Cylinder without open section



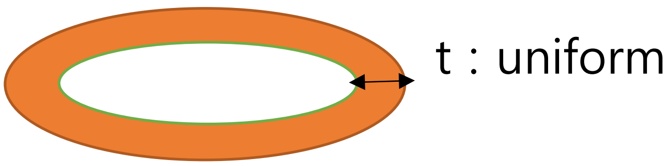

Outer Diametre=Do; Inner Diametre=Di

Torsional stiffness $K_t = T/\theta = GJp/L = G\pi(Do^4 - Di^4)/32L$;eq(1)

Where G=shear modulus, Jp=Polar moment of Inertia

L=Tube Length

Hollow cylinder of arbitrary shape with uniform thickness, t without open section




$K_t = T/\theta = 4GA^2t/LM$ ; eq(2)

and Max stress $\tau_{max}$=T/2At; eq(3)

where A=area within the outside perimeter, L=Cylinder length, M=peripheral length of the wall

From eq(1)

$(D_o^4-D_i^4)=(D^2+(D-2t)^2)(D^2-(D-2t)^2)$

$=(2D^2-4Dt+4t^2)(4Dt-4t^2)$

$=D^4(2-4t/D+4t^2/D^2)(4t/D-4t^2/D^2)$

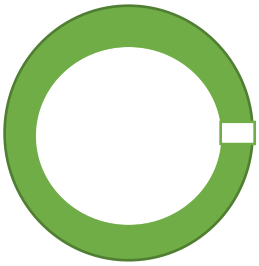
$\fallingdotseq D^4(8t/D)=8D^3t$, if $t/D \ll 1$

$K_t=G\pi(8D^3t)/32L=G\pi D^3t/4L=4G(\pi D^2/4)^2t/L(\pi D)$

$=4GA^2t/LM$ ; same as eq(2)

Torsional Stiffness with open section

Uniform thickness, t



$K_t = T/\theta = Gbt^3/3L$, where

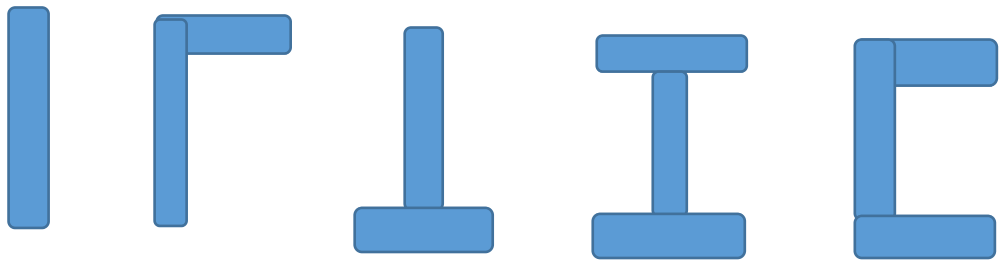
b=Peripheral length of wall with $t \ll b$

L=cylinder length

When having different wall thicknesses, $t_i$ with $b_i$ peripheral length,

$K_t = G/3L \Sigma b_i t_i^3$

## Typical open cross sections for Torsional stiffness



## Torsional stiffness with and without opening @t=0.1D

Hollow round without opening, $K_{t1}$

$K_{t1}=GJp/L=G\pi D^4(1-0.9^4)/32L=0.558GD^4/L$

Hollow round with opening, $K_{t2}$

$K_{t2}=Gbt^3/3L=G(\pi D)t^3/3L=G\pi(0.1)^3D^4/3L=1.05E\text{-}3GD^4/L$

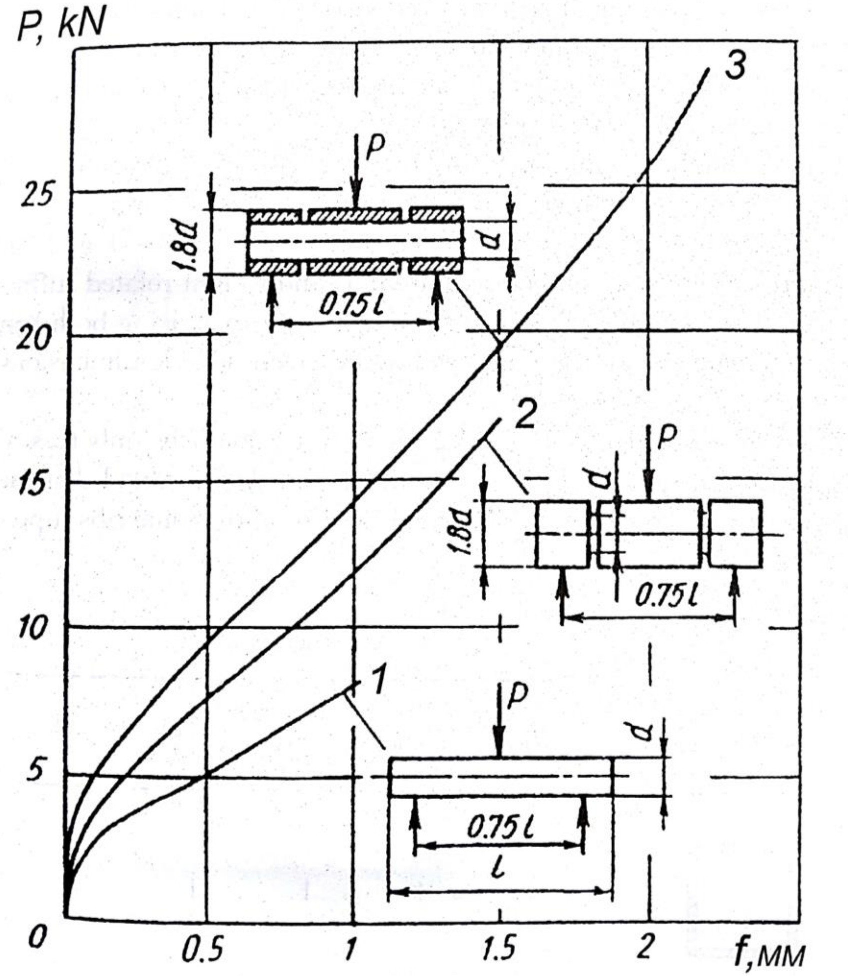
$K_{t1}/K_{t2} ≒ 531$ ; Thus the hollow round without open section gives 531 times higher torsional stiffness than the one with the open section.

## Influence of Stress Concentration on Stiffness

Stress concentration caused by sharp changes in the cross sectional area gives significant influence on strength, especially on fatigue strength.

Local stress concentration induce local deformation such as micro-displacement, and it gives increase in damping (desirable effect), and fretting corrosion (highly undesirable effect). Reinforcing stress concentration gives decrease in micro-displacement, damping, fretting; but increase stiffness, lifetime of structures.

Fig. compares three round bars under bending. Case 1 is a bar of d=10mm, L=80mm; Case 2 is a bar of 1.8d diametre with two circular grooves. The solid bar of 1.8d diametre would be almost 10.5 ($\fallingdotseq 1.8^4$) times higher bending stiffness than the case 1 when with no grooves. But the groove gives only 1.5 times increase in bending stiffness due to the stress concentration. The stress concentration can be significantly reduced by using the bar of d with reinforcement of tightly fit bushings (case 3). The result is 50% increase bending stiffness, and increase in fatigue strength from P1=8KN, P2=2.1P1, P3=3.6P1.




Sharp change of cross section on stiffness

(Source: Eugene.I.Rivin's 'Handbook of Stiffness and Damping in Mechanical Design)

Holes/Openings/Windows on stiffness

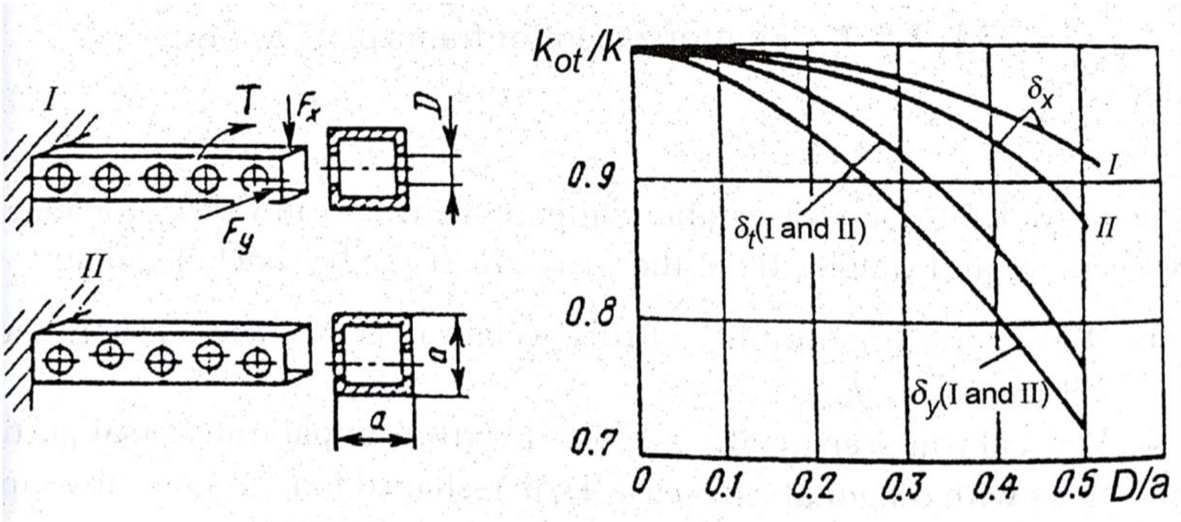
Machine frame components have numerous openings or holes or windows for accesing mechanisms, and other units located inside. Theses openings can significantly reduce the stiffness, depending on their dimension and positioning. Guidelins are as follows;

(1) Holes (windows) significantly reduce the torsional stiffness, althogh may not for the bending stiffness

(2) Holes should be located close to the neutral plane

(3) Holes of parallel direction to the applied force are better to be avoided; Stiffness(Ky) in the parallel direction is significantly decreased when compared with the orthogonal direction (Kx)

(4) Holes exceeding 1/2 of cross sectional dimension (D/a>0.5) should be avoided




Influence of holes in frame parts on stiffness

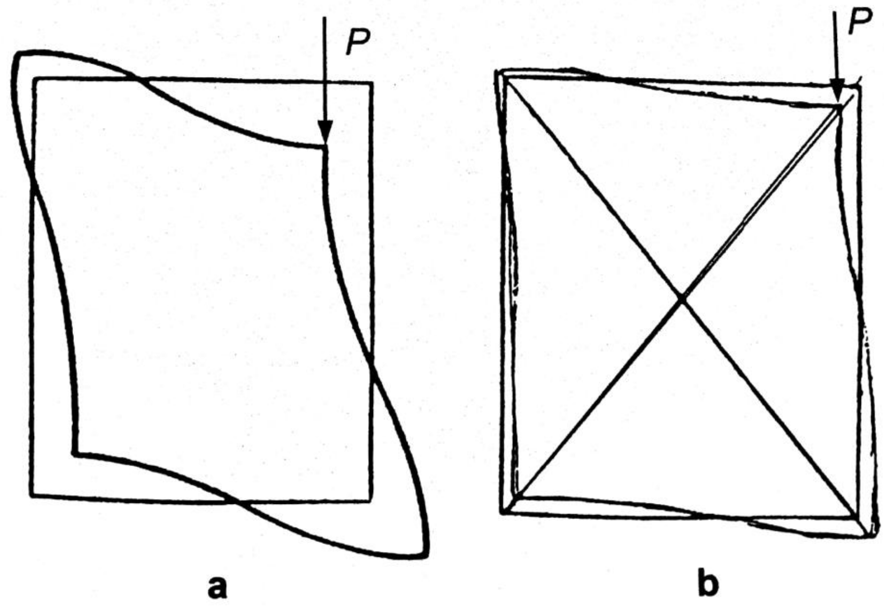
(Source: E.I.Rivin's Handbook on stiffness and damping)

## Ribs for reduction of local deformation

Local elastic deformation occurs due to local bending and torsional loading.

The effective way of reduction is to introduce the tension/compression memebers at the area of peak local deformation.

A thin-walled rectangular beam may have local deformation due to the eccentrically load induced torsion. This can be dramatically reduced by the introduction of tension/compression rib as in the fig.




Distortion in a loaded thin-walled part with and without reinforcing ribs

(Source: E.I.Rivin's Handbook on stiffness and damping)

## Diagonal reinforcement for shear loading

:Shear stiffness of thin-walled structure is very low, and the structure is prone to the deformation, as the planar frame does.

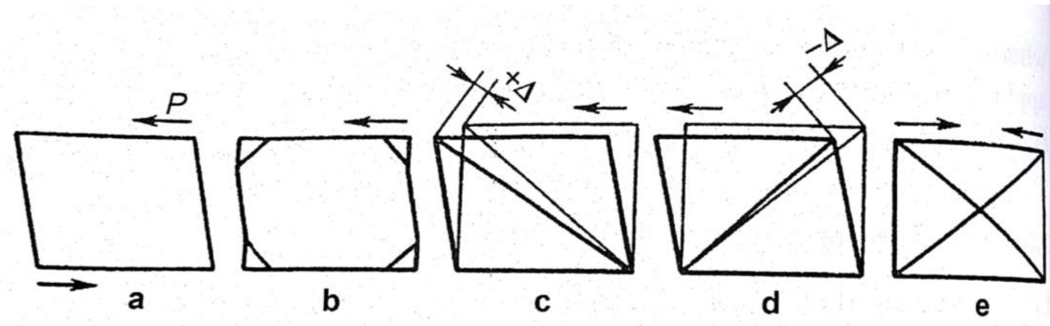
The corners, or joints, can be reinforced by introducing the corner gussets holding the shape of the corners.

More effective method is to introduce the diagonal ribs in tension/compression. As the stiffness of the diagonal rib is much higher than the bending stiffness of the wall, the overall shear stiffness is greatly increased. Diagonal rib of tension is preferable, as the compressive rib is prone to buckling.

Buckling Load, or Critical Load, due to unstable equilibrium such that slightest lateral force gives jump onto the buckling

$P_{cr}=\pi^2EI/(KL)^2$, K=column effective length factor

When the force direction is alternating, crossed diagonal ribs can be used.

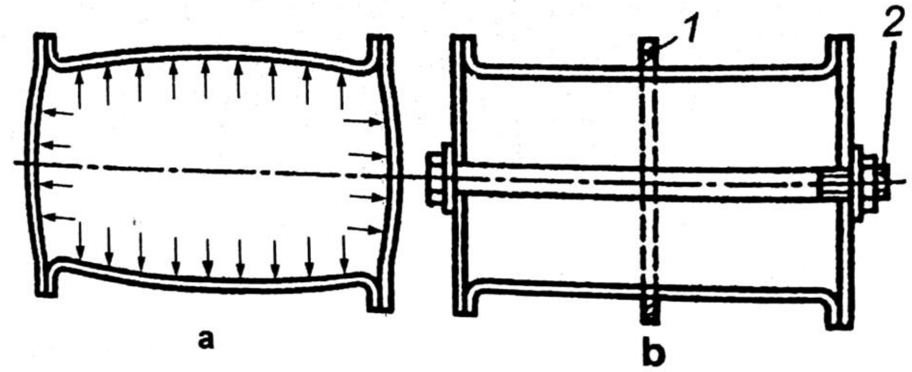



Diagonal ribs reinforcement for shear loading

(Source: E.I.Rivin's Handbook on stiffness and damping in mechanical design)

Reinforcing for Local deformation due to internal pressure:

Local deformation of the wall can be caused by the internal/external pressure. The tensile reinforcing member such as lug bolt increases the stiffness in the axial direction, and the reinforcing ring increase the stiffness in the circumferential direction to prevent bulging of the side wall. These reinforcing members not only reduce local deformation, but also reduce vibration and ringing of walls as diaphragms.




Reinforcing ring for wall under internal pressure

(Source: E.I.Rivin's Handbook on stiffness and damping)

## General Guidelines for Stiffness Enhancement

1. Repacement bending by tension/compression
2. Optimization of load distribution and support conditions when bending is inevitable

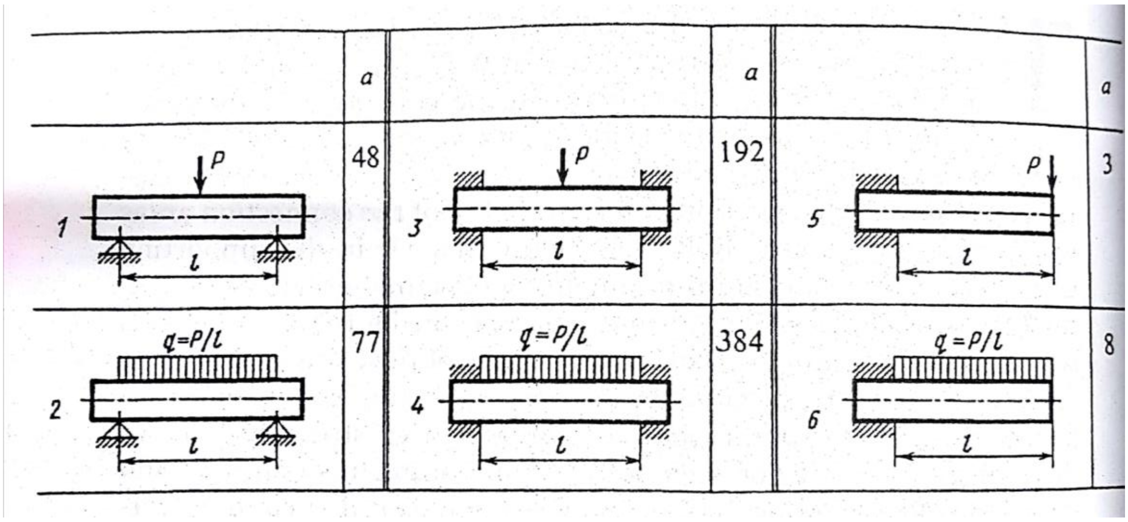
   Bending deflection, $\delta = PL^3/aEI$

   Bending stiffness, $K = P/\delta = aEI/L^3$

   where E is the Young's modulus,

   I is the area moment of inertia

   and a is the constant shown in fig.




Influence of load pattern and support conditions on beam stiffness

(Source: E.I.Rivin's Handbook on stiffness and damping in Mechanical Design

Stiffness Increase Ratio

Distributed / Concentrated=1.67~2.67

Double Support / Cantilever=16~9.62

Double Built / Double Support=4~4.98

Double Built / Cantilever=64~48

Maximum / Minimum = over 100 times!!

3. Optimum design of cross sectional area to have largest I ($=\int y^2 dA$), J ($=\int r^2 dA$) with consideration of Area for material increase
4. Reduce stress concentration by reinforcing such as to avoid sharp change of cross sectional area or to use tightly fit bushing
5. Stiffness reinforcing ribs, preferably in tension
6. Reinforcing ribs for shear loading
7. Solid or cross sections without opening for torsional stiffness
8. Cross sectional shape modification can enhance

stiffness

9. Holes/windows give significant decrease in torsional stiffness, and partly decrease in bending stiffness (direction and size of holes)

10. Composite laminates for combined stiffness, and possible anisotropic stiffness.

11. Shear deformation consideration

1) For long beam with $h/L \le 0.1$, shear deformation is 1-3% for solid cross section, 6-9% for I-beams. Thus it is negligible.

2) For short beams (e.g, spindle, gear shaft) with $h/L > 0.1$, where h=height of beam, L=length of beam, shear deformation may exist up to 30% of total deformations. (Stiffness reduction)

$\delta = [1 + 3h^2/L^2] PL^3/48EI$ for rectangle cross section with double supported beam

3) For laminated beam, shear deformation increase up to 50%