

$$\vec{c} = (u, v, w)$$

of collisions on A

$$= n_0 A w dt$$

rate of collisions on A

$$= \frac{n_0 A w dt}{dt} = n_0 A w [\#/s]$$

$$P = (\dot{p}_+ + \dot{p}_-) / A = \frac{1}{3} n m \bar{c}^2$$

$$PV = \frac{1}{3} n V m \bar{c}^2 = \frac{1}{3} N m \bar{c}^2$$

$$\boxed{\frac{1}{2} m \bar{c}^2} = \bar{E}_k = \frac{3}{2N} \frac{1}{3} N m \bar{c}^2 = \frac{3}{2N} RT$$

$$= \frac{3}{2N} \frac{N}{N_A} RT = \frac{3}{2} \frac{R}{N_A} T = \boxed{\frac{3}{2} k T}$$

$$\left(R = 8.31 \text{ J/mole}, N_A = 6.02 \times 10^{23} / \text{mole} \right)$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

* molecule partial current

$$J_+ = (n_1 w_1 + n_2 w_2 + \dots) A = n_+ \bar{w}_+ A$$

$$\bar{w}_+ = \frac{n_1 w_1 + n_2 w_2 + \dots}{n_1 + n_2 + \dots} = \frac{n_1 w_1 + n_2 w_2 + \dots}{n_+}$$

$$\bar{n}_+ = \bar{n}_- = \frac{1}{2} n, \quad \bar{w}_+ = \frac{1}{2} \bar{c}$$

$$\therefore J_+ = \frac{1}{4} n \bar{c} A [\#/s]$$

* momentum partial current $[(kg \cdot m/s)/s]$

$$n_w A w m w = n_w A m w^2$$

$$\dot{p}_+ = (n_1 w_1^2 + n_2 w_2^2 + \dots) A m = n_+ \bar{w}_+^2 A m = \frac{1}{6} n m \bar{c}^2 A$$

$$\bar{w}_+^2 = \frac{n_1 w_1^2 + n_2 w_2^2 + \dots}{n_1 + n_2 + \dots} = \frac{n_1 w_1^2 + n_2 w_2^2 + \dots}{n_+}$$

$$c^2 = u^2 + v^2 + w^2, \quad \bar{u}^2 = \bar{v}^2 = \bar{w}^2 = \frac{1}{3} \bar{c}^2$$

$$\bar{w}^2 = \bar{w}_+^2 = \bar{w}_-^2$$

$$\therefore \dot{p}_+ = \frac{1}{6} n m \bar{c}^2 A [kg \cdot m/s^2]$$

$$PV = \frac{1}{3} N m \bar{c}^2 = \frac{1}{3} N 3 k T = N k T$$

$$PV = N k T \rightarrow P = \frac{N}{V} k T = n k T$$

$$\text{Box 1} + \text{Box 2} = P_1 + P_2 = n_1 k T + n_2 k T = (n_1 + n_2) k T$$

$$\text{Box 3} = P = \frac{N_1 + N_2}{V} k T = (n_1 + n_2) k T = P_1 + P_2$$

$$\therefore \frac{PV}{T} = N k$$

* number density

$$P = n k T \quad \text{space } 10^4 \# / m^3$$

$$n = \frac{P}{k T} \quad \text{KSTAR } 10^{20} \# / m^3$$

$$n [\# / m^3] = 2.45 \times 10^{25} P [\text{atm}]$$

at 300K

* molecular speed

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} k T$$

$$H_2: 1.93 \times 10^3 \text{ m/s (at 300K)}$$

Maxwell's speed distribution

$$dN = A \exp\left(-\frac{\frac{1}{2} m c^2 + P.E.}{k T}\right) dv dw du$$

$$dN = A \exp\left(-\frac{\frac{1}{2} m c^2 + P.E.}{k T}\right) 4\pi c^2 dc$$

$$= n B c^2 \exp\left(-\frac{\frac{1}{2} m c^2}{k T}\right) dc$$

$$= n f(c) dc$$

$$\int_0^\infty f(c) dc = 1$$

$$B = \sqrt{\frac{2}{\pi}} \left(\frac{m}{k T}\right)^{\frac{3}{2}}$$

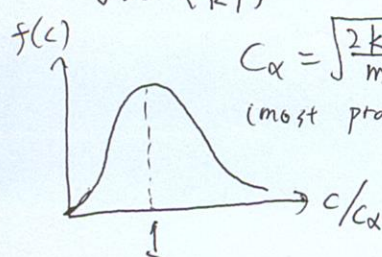
$$f(c) = C_\alpha \exp\left(-\frac{m c^2}{2 k T}\right)$$

(most probable speed)

$$\bar{c} = \int_0^\infty c f(c) dc = \sqrt{\frac{8 k T}{\pi m}}$$

$$\sqrt{\bar{c}^2} = \left\{ \int_0^\infty c^2 f(c) dc \right\}^{\frac{1}{2}} = \sqrt{\frac{3 k T}{m}} \Rightarrow \frac{1}{2} m \bar{c}^2 = \frac{3}{2} k T$$

$$\left(\int_0^\infty c^{2n} e^{-ac^2} dc = \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^{n+1} a^n} \sqrt{\frac{\pi}{a}}, \quad \int_0^\infty c^{2n+1} e^{-ac^2} dc = \frac{n!}{2 a^{n+1}} (a > 0) \right)$$



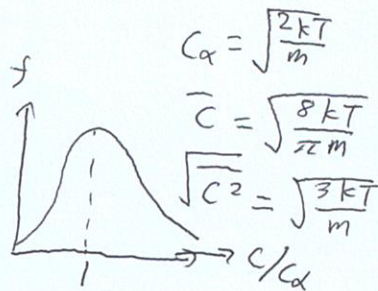
$$P = \frac{1}{3} n m \bar{c}^2$$

$$PV = \frac{1}{3} N m \bar{c}^2 = \frac{2}{3} RT$$

$$\frac{1}{2} m \bar{c}^2 = \frac{3}{2} kT$$

$$dn = A \exp\left(-\frac{\frac{1}{2} m c^2 + P.E.}{kT}\right) 4\pi c^2 dc$$

$$= n B c^2 \exp\left(-\frac{\frac{1}{2} m c^2}{kT}\right) dc = n f(c) dc$$

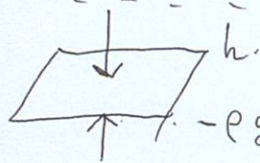
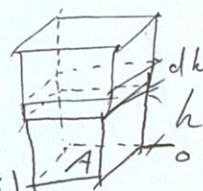


* Effect of potential energy - Isothermal atmosphere

$$4\pi A \exp\left(-\frac{P.E.}{kT}\right) c^2 \exp\left(-\frac{\frac{1}{2} m c^2}{kT}\right) dc$$

$$= n B c^2 \exp\left(-\frac{\frac{1}{2} m c^2}{kT}\right) dc$$

$$\Rightarrow n = \frac{4\pi A}{B} \exp\left(-\frac{P.E.}{kT}\right)$$



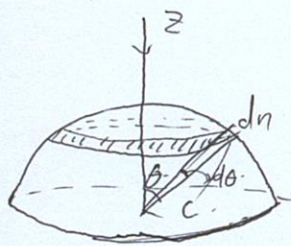
$$-p g dh A = A dp$$

$$-n m g dh A = A k T dn$$

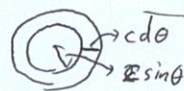
$$\frac{dn}{n} = -\frac{m g}{k T} dh$$

$$\int_{n_0}^n \frac{dn}{n} = -\int_0^h \frac{m g}{k T} dh \Rightarrow \ln \frac{n}{n_0} = -\frac{m g h}{k T}$$

$$= \frac{4\pi A}{B} \exp\left(-\frac{m g h}{k T}\right) = n_0 \exp\left(-\frac{m g h}{k T}\right)$$



$$dn = n f(c) dc \frac{d\Omega}{4\pi}$$



$$dS = 2\pi c \sin\theta c d\theta = c^2 d\Omega$$

$$\therefore d\Omega = 2\pi \sin\theta d\theta$$

$$\therefore n = n_0 \exp\left(-\frac{m g h}{k T}\right)$$

$$dJ_z = dn c \cos\theta A$$

$$= n f(c) dc \frac{2\pi}{4\pi} \sin\theta d\theta c \cos\theta A$$

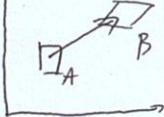
$$= \frac{1}{2} \sin\theta \cos\theta dn c n f(c) dc$$

$$\therefore J_z = \int_0^{\pi/2} \frac{A}{2} \sin\theta \cos\theta \int_0^\infty c n f(c) dc d\theta = \frac{A}{2} n \bar{c} \int_0^{\pi/2} \sin\theta \cos\theta d\theta = \frac{A}{4} n \bar{c} \left[-\frac{1}{2} \cos 2\theta\right]_0^{\pi/2}$$

$$= \frac{1}{4} n \bar{c} A$$

* Boltzmann equation

v_x



$$0 = \frac{dN}{dt} = \frac{d}{dt} \int f(x, v_x, t) dx dv_x = \int \frac{df}{dt} dx dv_x$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial v_x} \frac{dv_x}{dt}$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{a} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$= \frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \frac{8(\vec{E} + \vec{v} \times \vec{B})}{m} \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

$$\epsilon_0 \nabla \cdot \vec{E} = \rho \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{B} = \mu_0 \vec{J} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

Perrin의 실험



if there is collision

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \nabla f + \vec{a} \cdot \frac{\partial f}{\partial \vec{v}} = \left(\frac{\partial f}{\partial t}\right)_c$$

$$\left(\frac{\partial f}{\partial t}\right)_c = 0; \text{ Vlasov equation}$$

$$\left(\frac{\partial f}{\partial t}\right)_c = \frac{f}{\tau}; \text{ Krook collision term}$$

$$\sigma = \sum_j n_j \sigma_j = \sum_j \sigma_j \int f_j dv_j$$

$$\vec{j} = \sum_j n_j \sigma_j \vec{u}_j = \sum_j \sigma_j \int \vec{v}_j f_j dv_j$$

$$(\vec{v} = \vec{u} + \vec{w})$$

