3. Measurement of Critical Mass of AGN-201K

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Point Kinetics Equation with External Source

- With the time dependence of Λ and the $\beta_{i,eff}$ of negligible significance, the reactivity $\rho(t)$ and the effective source $Q(t)$ become the driving terms of the point kinetics equations.

$$\frac{dn(t)}{dt} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) + Q(t) \qquad \text{(1a)}$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_{i,eff}}{\Lambda} n(t) - \lambda_i C_i(t) \quad (i = 1, 2, \cdots, 6) \qquad \text{(1b)}$$

where $n(t)$ = neutron density or total neutron population,

$C_i(t)$ = i-th group delayed neutron precursor density,

λ_i = decay constant of the i-th group delayed neutron precursor,

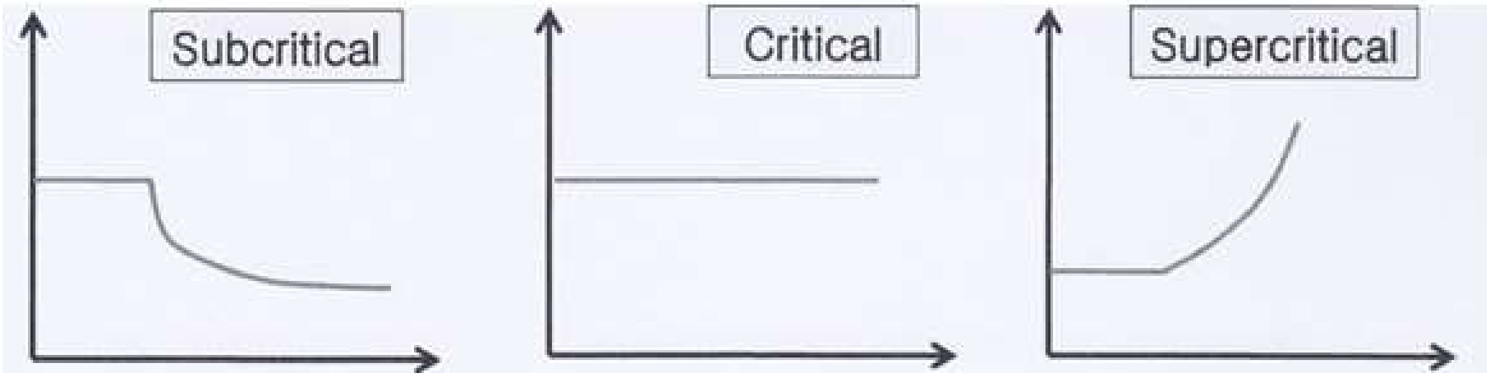
Λ = prompt neutron generation time, which is the prompt neutron lifetime l divided by k_{eff},

β_{eff} = effective delayed neutron fraction

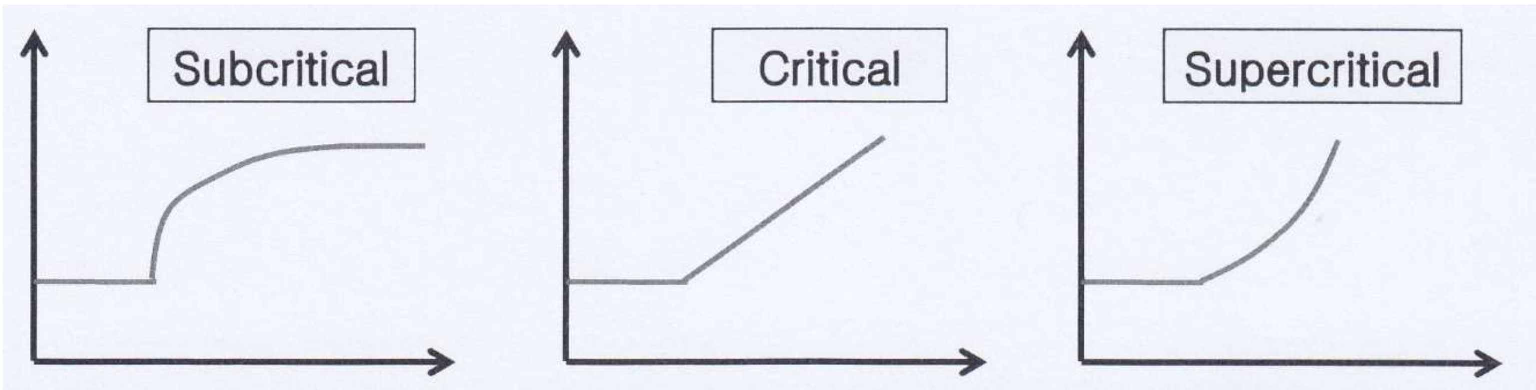
$\beta_{i,eff}$ = effective delayed neutron fraction of i-th delayed neutron precursor group

Time-Dependent Behavior at Critical Condition

- When there are no external neutron sources,



- When there is an external source,



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Steady-State Solution of a Subcritical System

- For a subcritical system without any external source, the neutron population becomes negligible after long time as

$$n(t) \cong A_1 \exp(\omega_1 t) \cong 0 \quad (t >> 0) \qquad \text{(2)}$$

- When there is an external source, $Q(t)$, its steady-state solution can be written as

$$\cancel{\frac{dn(t)}{dt}}^{=0} = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t) + Q(t) \qquad \text{(3a)}$$

$$\cancel{\frac{dC_i(t)}{dt}}^{=0} = \frac{\beta_{i,eff}}{\Lambda} n(t) - \lambda_i C_i(t) \quad (i = 1, 2, \cdots, 6) \qquad \text{(3b)}$$

- From Eq. (3b), the following relationship is obtained

$$C_i(t) = \frac{\beta_{i,eff}}{\Lambda \lambda_i} n(t) \ (i = 1, 2, \cdots, 6) \qquad \text{(4)}$$

- By inserting Eq. (4) into Eq. (3a), one can obtain

$$0 = \frac{\rho - \beta_{eff}}{\Lambda} n(t) + \underbrace{\boxed{\sum_{i=1}^{6} \cancel{\lambda_i} \frac{\beta_{i,eff}}{\Lambda \cancel{\lambda_i}} n(t)}}_{= \frac{\beta_{eff}}{\Lambda} n(t)} + Q(t) \Rightarrow \boxed{n(t) = -\frac{\Lambda}{\rho} Q(t)} \qquad \text{(5)}$$

Steady-State Solution of Sub. Sys. (Contd.)

- For a fixed strength of the external source, Q_0, one can find that the neutron population at steady-state, denoted by $n(\infty)$, becomes inversely proportional to reactivity as

$$n(\infty) = -\frac{\Lambda}{\rho} Q_0 \qquad \text{(6a)}$$

- Because every detector signal located at the critical assembly is proportional to $n(t)$, Eq. (6a) can be written for its detector signal or count rate A as

$$\boxed{A(\infty) \propto -\frac{\Lambda}{\rho} Q_0} \qquad \text{(6b)}$$

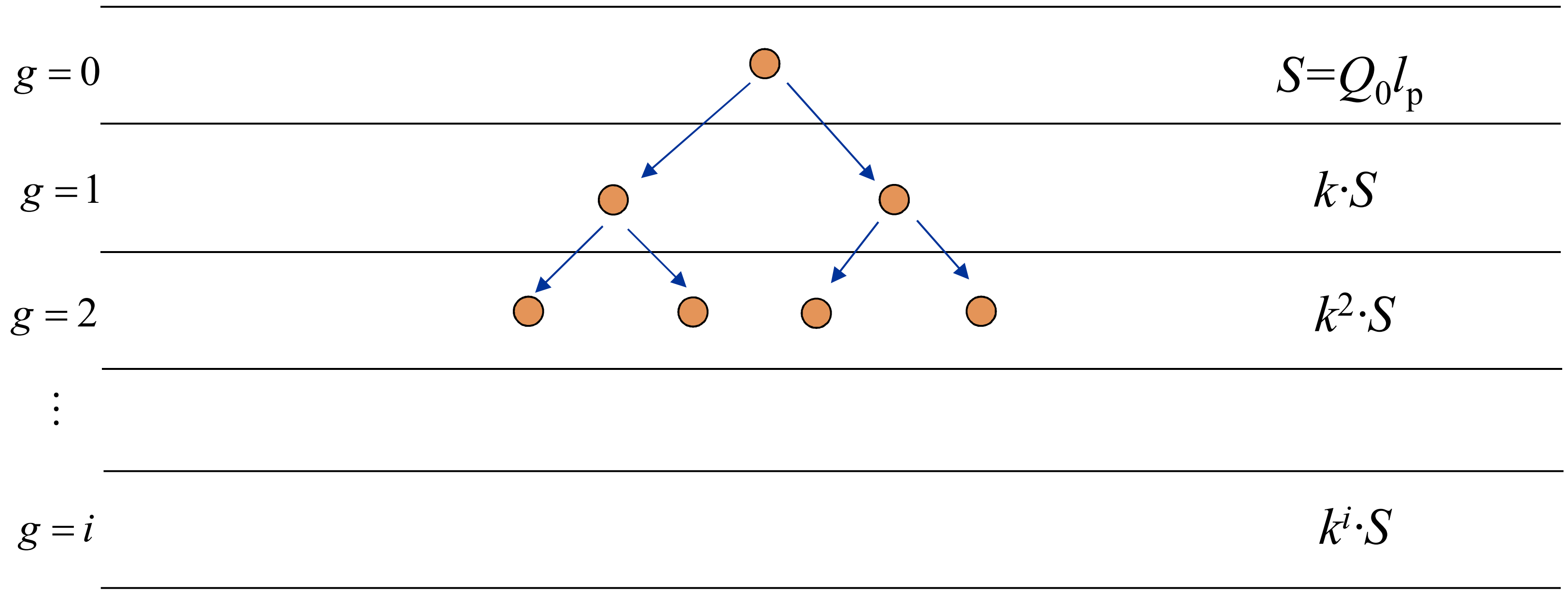
Physical Interpretation of the Critical Approach Equation

$$n(\infty) = -\frac{\Lambda}{\rho} Q_0 \quad \cdots\cdots \quad (6a)$$

$$A(\infty) \propto -\frac{\Lambda}{\rho} Q_0 \quad \cdots\cdots \quad (6b)$$

Number of Neutrons Existing in Subcritical System

- When the external source strength is given as Q_0 [#/sec], it means that $Q_0 l_p$ neutrons are produced during this prompt neutron lifetime, l_p.
- Then, how many numbers of neutrons exist in this subcritical system after a considerable time?



$$\Rightarrow \quad n(\infty) = S + kS + k^2S + k^3S + \cdots = \boxed{\frac{1}{1-k}} S = \frac{Q_0 l_p}{1-k} = \frac{Q_0 (l_p/k)}{(1-k)/k} = \frac{Q_0 \Lambda}{-\rho}$$

$=M$

=**source multiplication factor**

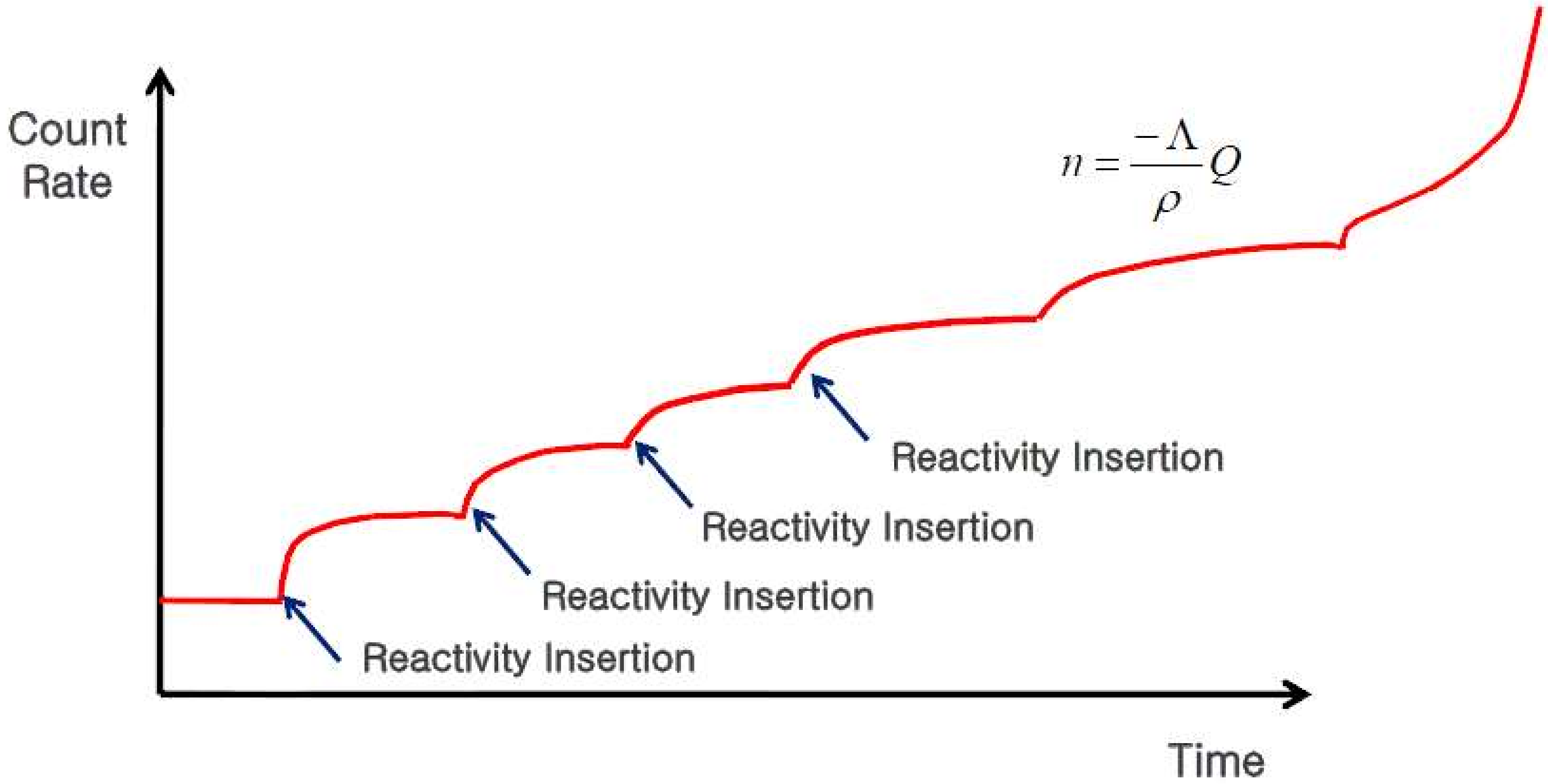
Detector Signal vs. Reactivity

Approaching to the criticality,
ramp rate becomes low and takes more time to equilibrium.

$$n(\infty) = \frac{Q_0 \Lambda}{-\rho}$$

Question #1

- Why does it take longer time when the reactor approaches to the criticality compared with an initial deep subcritical state?



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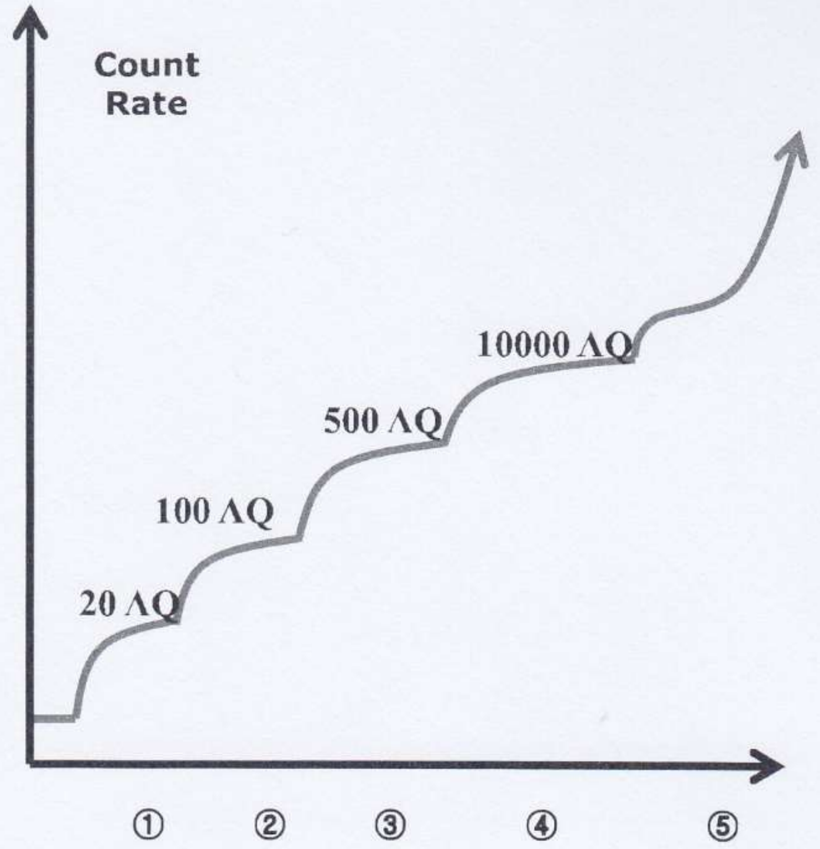
Example of Detector Signal vs. Reactivity

$$A(\infty) \propto -\frac{\Lambda}{\rho} Q_0 \qquad \text{(6b)}$$

state	Reactivity	Count Rate
5	0	∞
4	−0.0001	10000ΛQ
3	−0.002	500ΛQ
2	−0.01	100ΛQ
1	−0.05	20ΛQ



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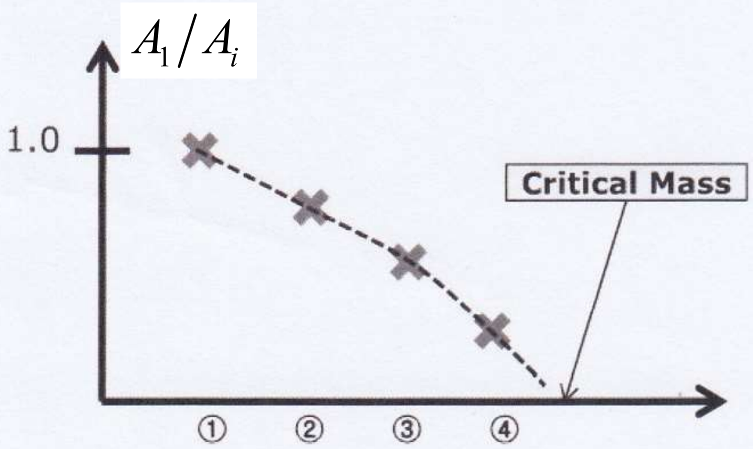
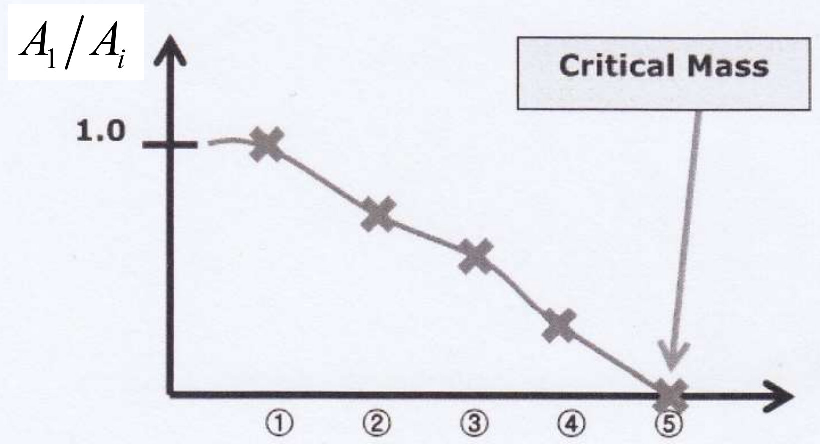
Inverse Count Rate vs. Reactivity

state	Reactivity	Count Rate	A_1/A_i
1	−0.05	20ΛQ	1
2	−0.01	100ΛQ	0.2
3	−0.002	500ΛQ	0.04
4	−0.0001	10000ΛQ	0.002
5	0	∞	0

state	Reactivity	Count Rate	A_1/A_i
1	−0.005	200ΛQ	1.0
2	−0.00325	307.7ΛQ	0.65
3	−0.0012	833.3ΛQ	0.24
4	−0.00025	4000ΛQ	0.05
5	0	∞	0





$$\boxed{A(\infty) \propto -\frac{\Lambda}{\rho} Q_0} \quad \text{............ (6b)}$$

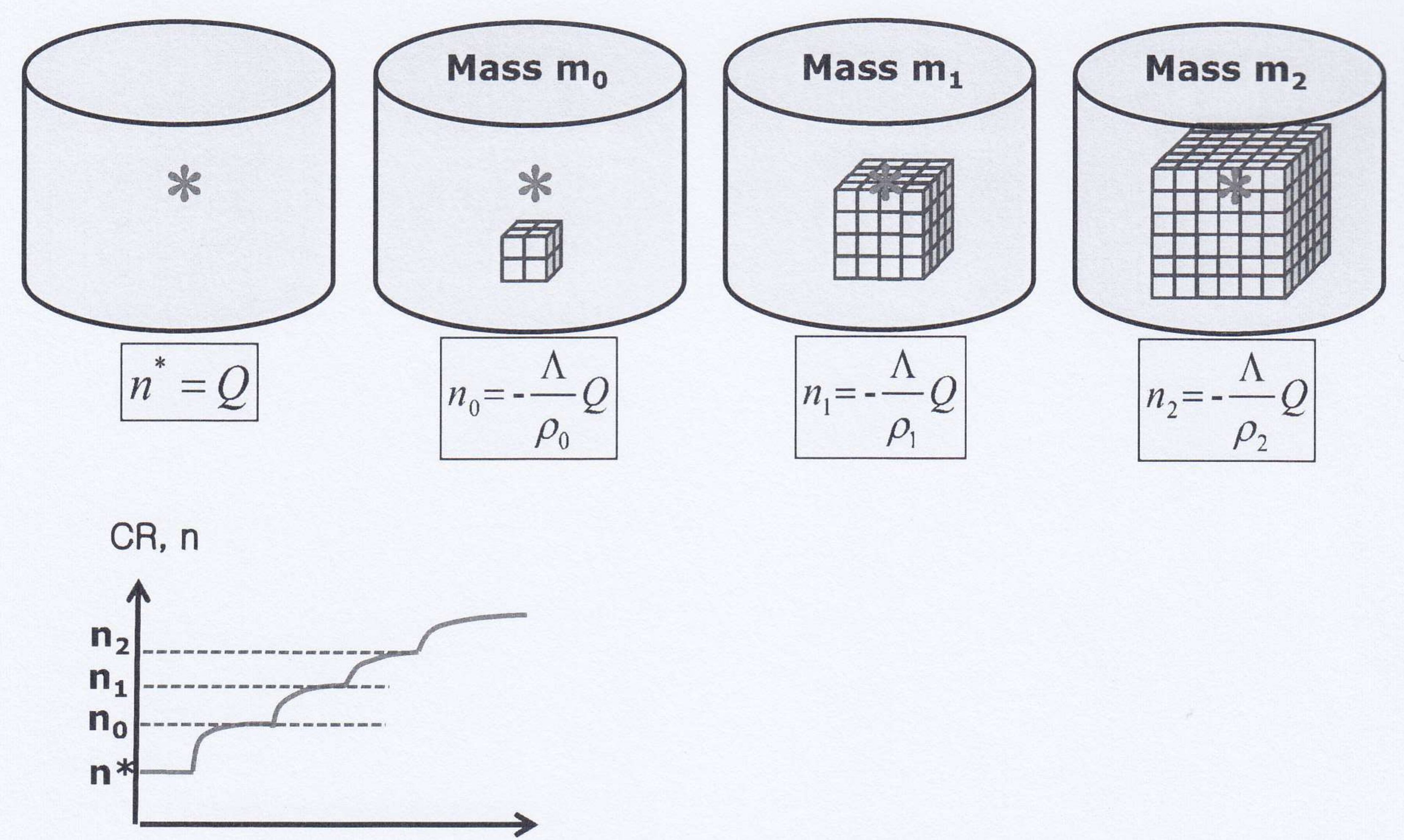
➡

$$\boxed{\frac{A_0}{A_i} \propto \frac{\rho_i}{\rho_0} \propto \frac{M_0}{M_i} \propto \frac{1}{M_i}} \quad \text{and} \quad \boxed{\rho_i = \rho_0 \frac{A_0}{A_i}}$$

Called **1/*M* Experiment**

Critical Mass Measurement (1/M Experiment)

Critical Mass Approach



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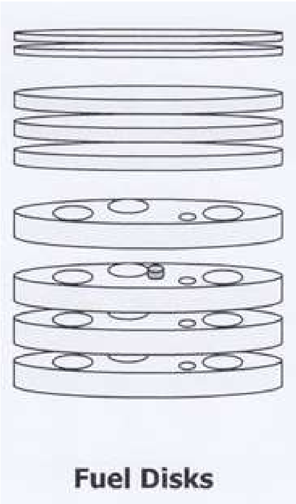
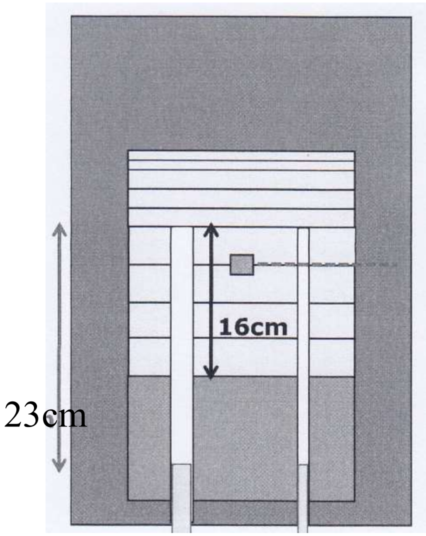
Fuel Disks Making Core





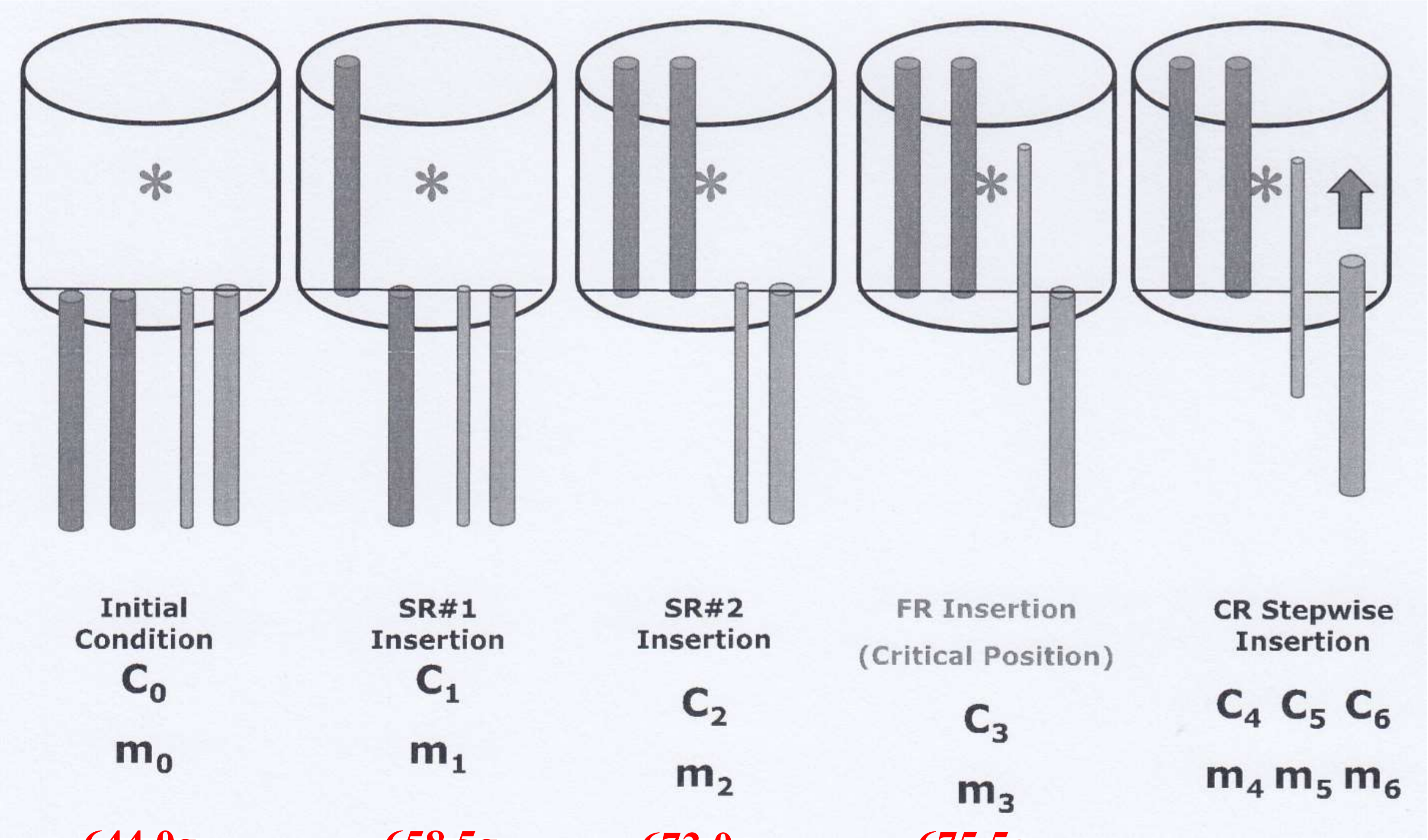
644g

Part	Height	Weight of ^{235}U	Function
Fuel disk 1, 2, 3, 4	4 × 4[cm/disk]	4 × 102[g/disk]	Core
Fuel disk 5, 6, 7	3 × 2[cm/disk]	3 × 58[g/disk]	
Fuel disk 8, 9	2 × 1[cm/disk]	2 × 29[g/disk]	
Thermal fuse*	2.2D × 0.9H [cm]	4 g (double density fuel)	Safe shutdown
Safety rod 1 & 2	4 fuel pieces (16 cm)	2 × 14.5[g/rod]	Shutdown
Coarse control rod	4 fuel pieces (16 cm)	14.5 g	Control
Fine control rod	3 fuel pieces (12 cm)	2.5 g	Control
Totol ^{235}U mass		690 g	

* Melting point of U+polystyrene = 100ºC

cf. Melting point of U+polyethylene = 200ºC

Experimental Procedure



644.0g 658.5g 673.0g 675.5g

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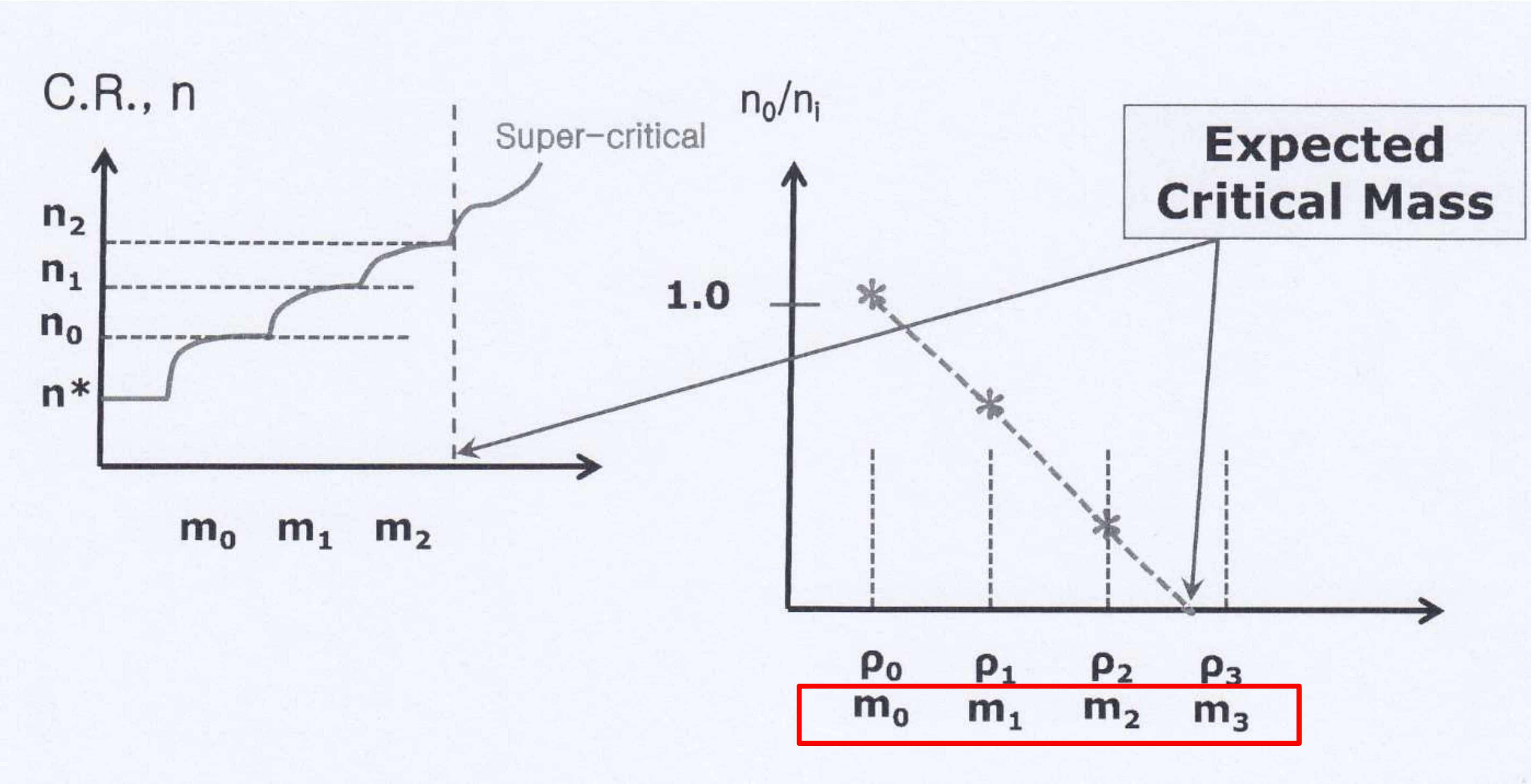
Drawing of Count Ratio and Extrapolation



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Experimental Procedure

0. Student operator should read all reactor conditions and write down at the worksheet first. They are temperatures at various locations, source position, neutron and gamma radiation dose in the reactor hall. All students should confirm that neutron source is inserted inside of the reactor and all control rods are out. At this moment, there are no significant counts from all channels except two; channel #1 and #5.

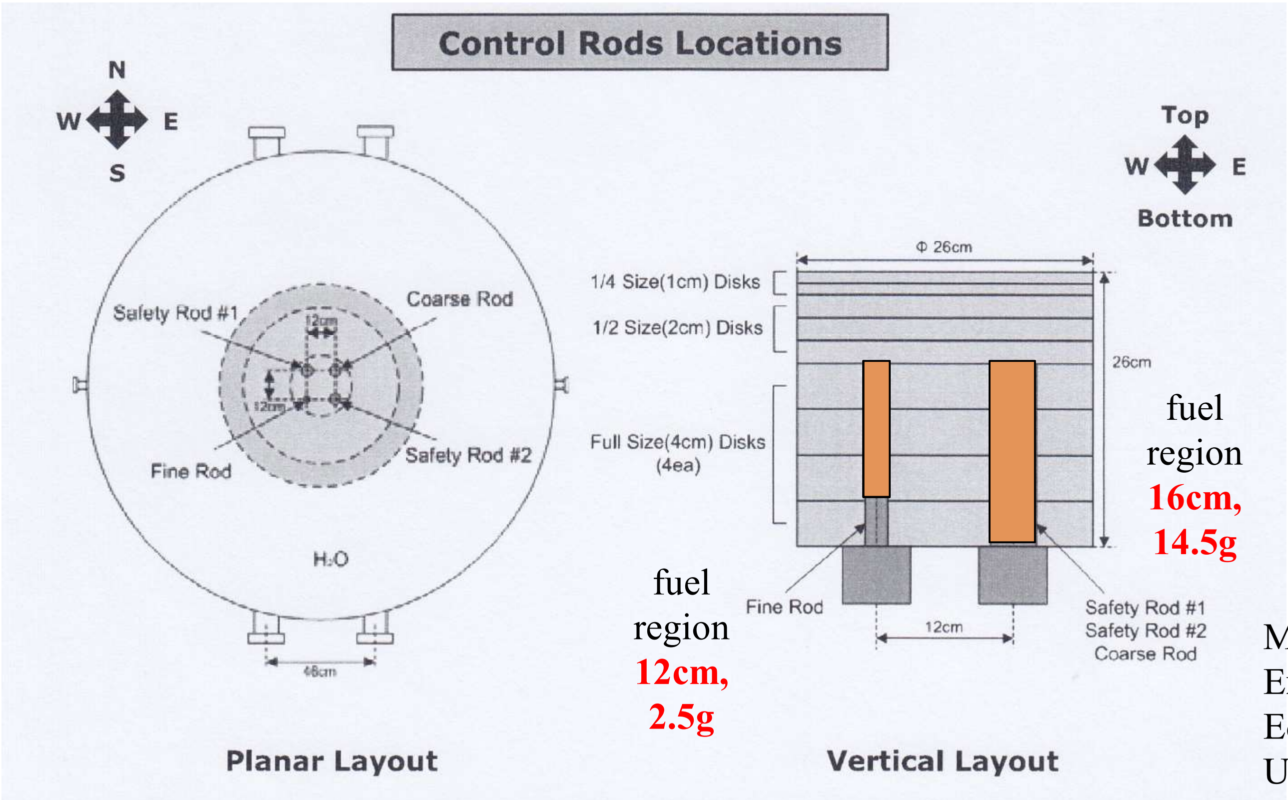
Exp. Condition			Temp	Analog Console	
SOURCE Position				Digital Reactor	
Gamma	In/Out	/		Digital Water	
Neutron	In/Out	/		Digital Room	

Core Mass = 644 g
Mass of SR = 14.5 g
Mass of CR = 14.5 g
Mass of FR = 2.5 g
Length of Rods = 23 cm

Case	Rod Position				Mass of U-235	Channel #1 - **Analog**			Channel #5 - **Digital**		
	SR#1	SR#2	CR	FR		Count Rate		C_0/C_i	Count Rate		C_0/C_i
0					m_0=		C_0=			C_0=	
1					Δm= m_1=		C_1=			C_1=	

Experimental Procedure (Contd.)

1. As the first step, two students should read cpm from channel #1 and #5 repeatedly to
644.0g have a saturated values.
2. As the second step, one safety rod SR#1 is inserted as an addition of U-235 by
658.5g 14.5g. Channel #1 & 5 is monitored to read saturated cpm. These records are sent to the working table to log on the spread sheet and make a graph.



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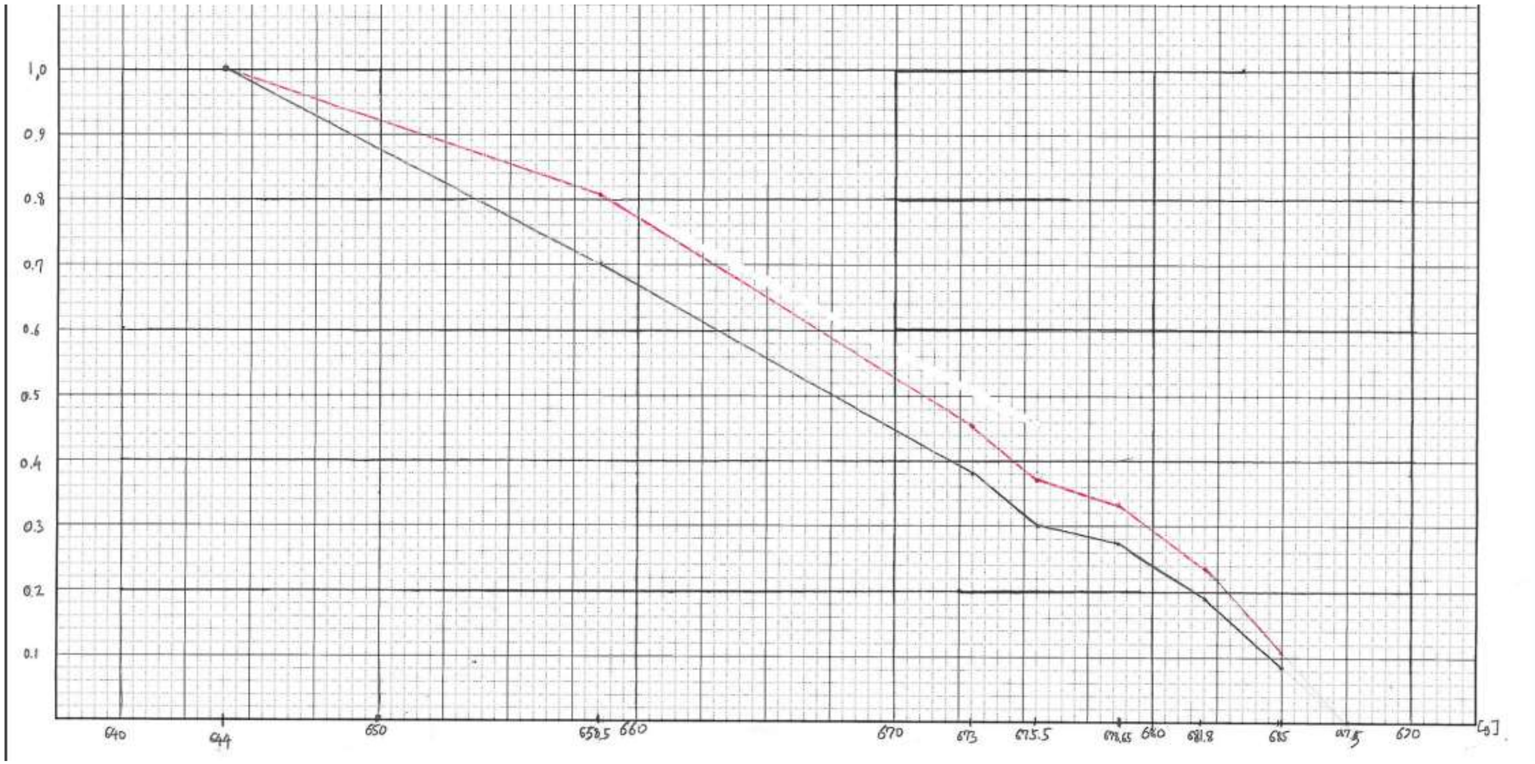
Experimental Procedure (Contd.)

3. As the third step, another safety rod SR#2 is inserted as an addition of U-235 by
673.0g 14.5g. Channel #1 & 5 is monitored again to read saturated cpm. These records are sent to the working table to log on the spread sheet and make two dots on the graph.
4. As the 4th step, a fine control rod FR is inserted as an addition of U-235 by 2.5g.
675.5g Channel #1 & 5 is monitored again to read saturated cpm. These records are sent to the working table to log on the spread sheet and make two dots on the graph.
5. As the 5th step, a coarse control rod CR is inserted by 4cm as an addition of U-235 by 2.52g (14.5g × **4/23**=2.52g). Channel #1 & 5 is monitored again to read saturated cpm. These records are sent to the working table to log on the spread sheet and make two dots on the graph. *Standard Procedure*
6. As the 6th step, a coarse control rod CR is inserted more by 4cm as an addition of U-235 by 2.52g (14.5g × 4/23=2.52g). Channel #1 & 5 is monitored again to read saturated cpm. These records are sent to the working table to log on the spread sheet and make two dots on the graph.
7. As the 7th step, a coarse control rod CR is inserted more by 4cm as an addition of U-235 by 2.52g (14.5g × 4/23=2.52g). Channel #1 & 5 is monitored again to read saturated cpm. These records are sent to the working table to log on the spread sheet and make two dots on the graph. From now on, all students should discuss whether experiment may proceed or not. Decision should be made based on the extrapolation prediction on the graph.

Search for the Critical Mass

- Students are asked to make a table for A_0/A_i at each step-i. X-axis for a plot should be a mass of U-235. Mass of U-235 should be also calculated for each step-i.



Different Searching Scenarios

Your choice?

Plan A

	SR#1	SR#2	CR	FR
0	0	0	0	0
1	23	0	0	0
2	23	23	0	0
3	23	23	0	23
4	23	23	5	23
5	23	23	10	23
6	23	23	15	23
7	23	23	17	23
8	23	23	19	23

Plan B

	SR#1	SR#2	CR	FR
0	23	23	0	23
1	23	23	3	23
2	23	23	6	23
3	23	23	9	23
4	23	23	12	23
5	23	23	15	23
6	23	23	16	23
7	23	23	17	23
8	23	23	18	23

Plan C

	SR#1	SR#2	CR	FR
0	23	0	0	0
1	23	23	0	0
2	23	23	3	3
3	23	23	6	6
4	23	23	9	9
5	23	23	12	12
6	23	23	15	15
7	23	23	17	17
8	23	23	19	19

Video for Exp. #3

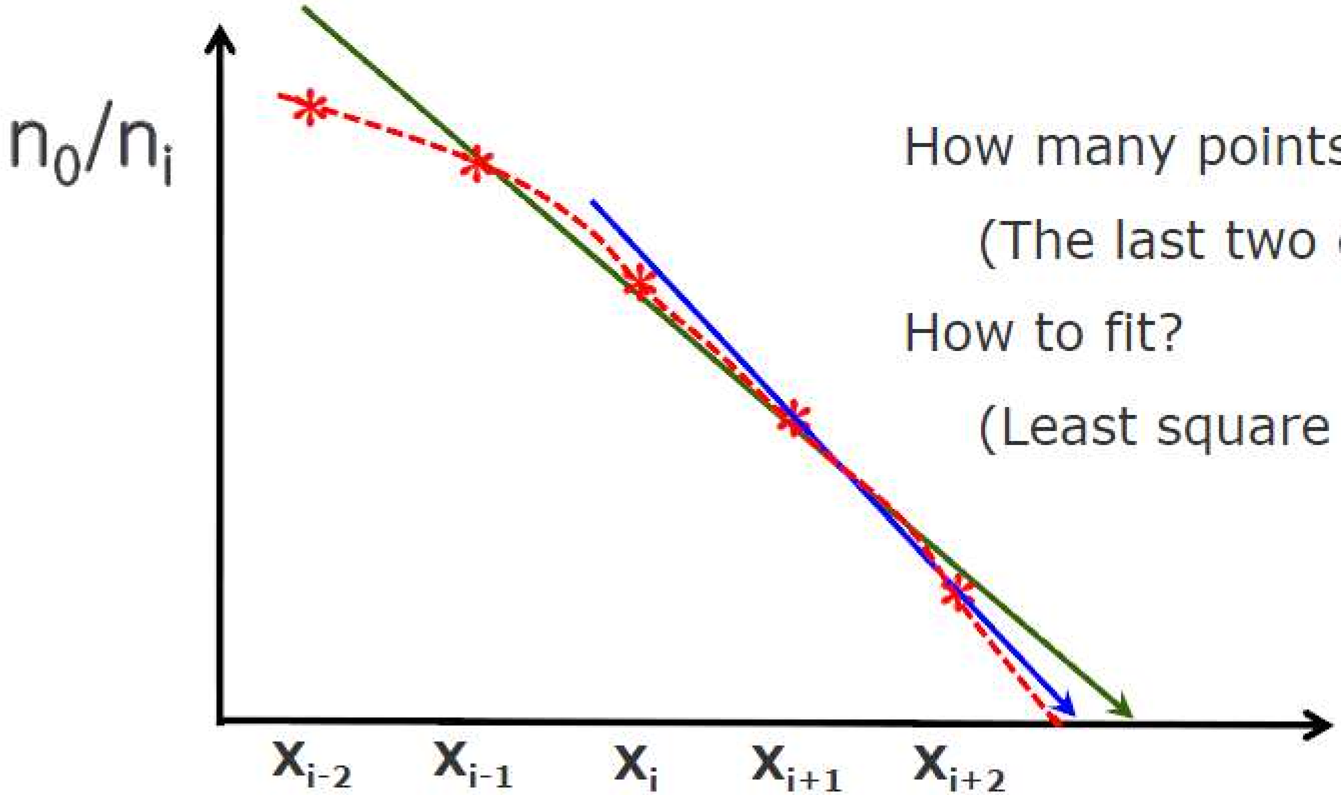
Static State w/ Source → Source Out → Source In

Source Out

Source In

Discussion Points

Technical Issue #1: How to extrapolate?



How many points?

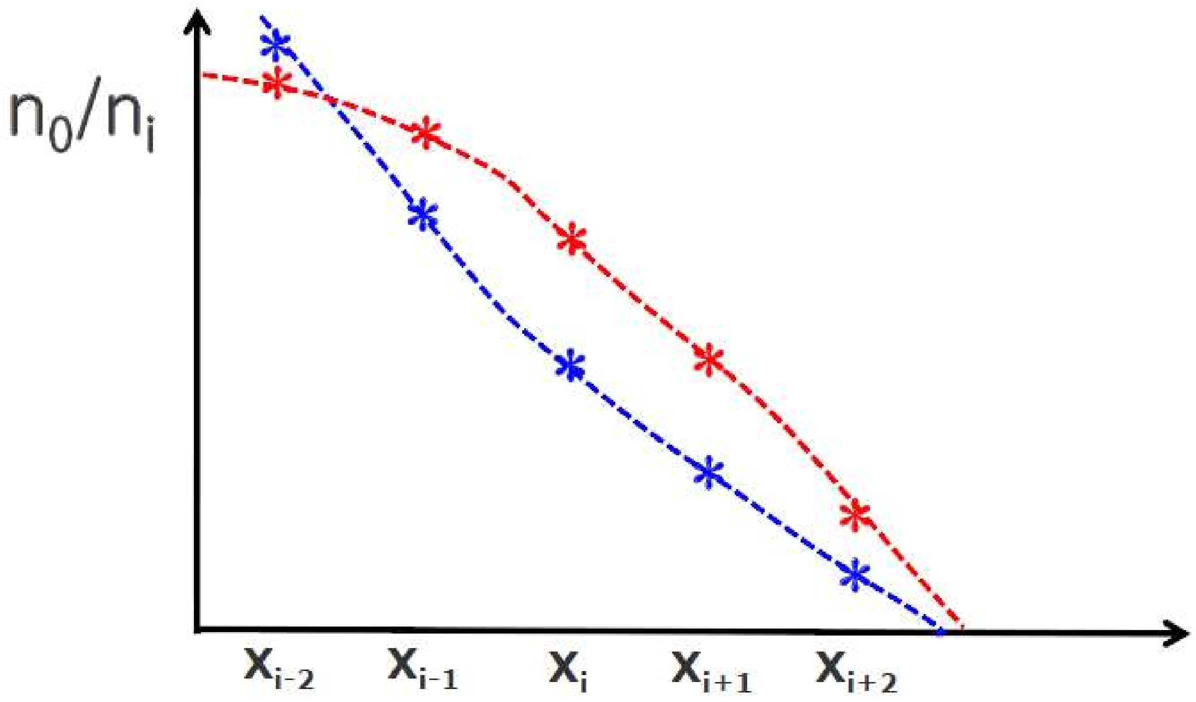
(The last two or All)

How to fit?

(Least square fit or Lagrange fit)

Discussion Points (Contd.)

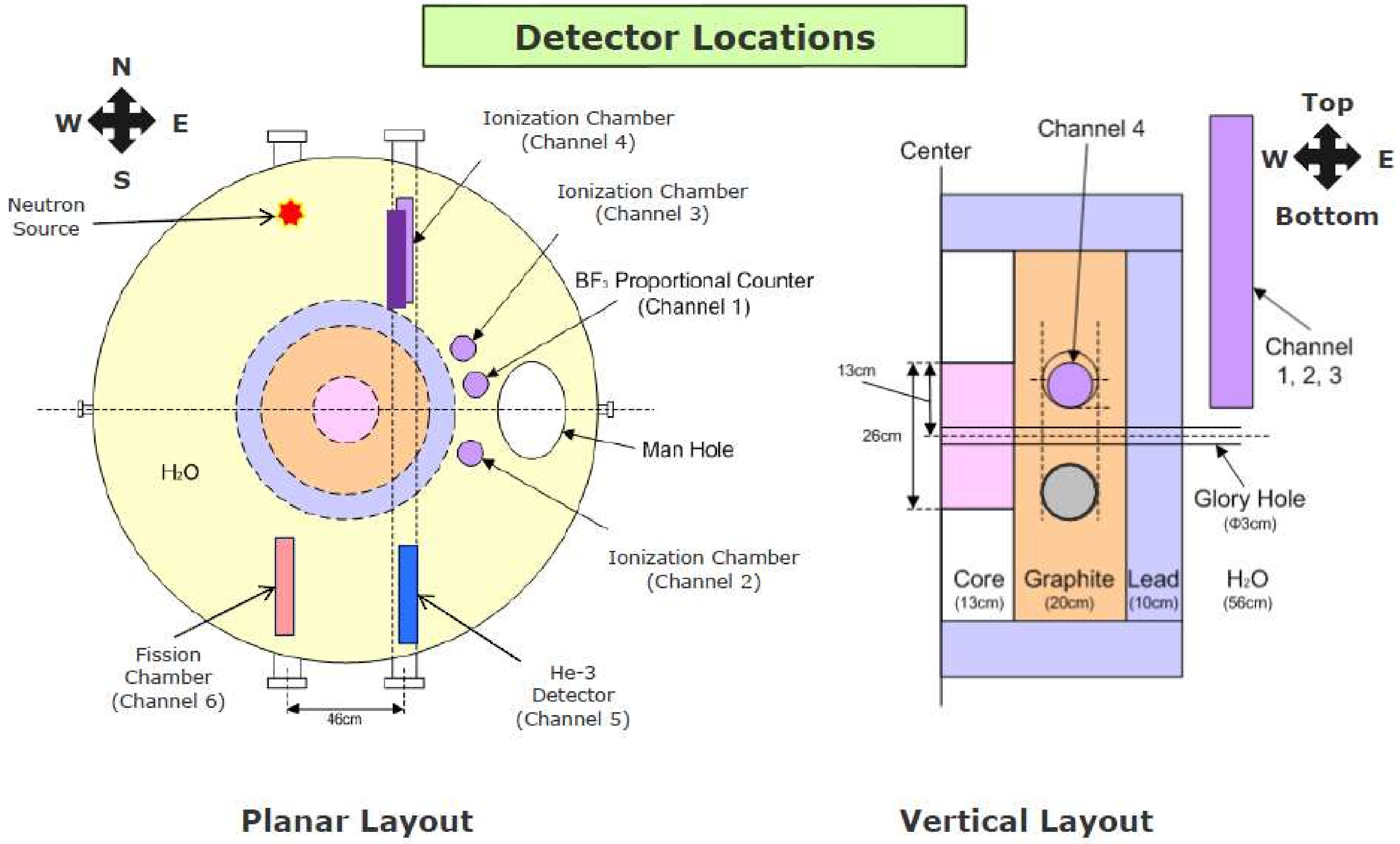
Technical Issue #2: Convex or Concave?



Question #2

Which effects make the fitting curve to concave curve instead of convex?

Consider Detector Positions for Question #2



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