# Chapter 4. Light Waves

# Light waves

- Electromagnetic waves
- Velocity ~ $3\times10^8$ m/s in a vacuum
- The spectrum of electromagnetic radiation
  - Visible light

$$\begin{array}{ccccccc} \text{UV} & \leftarrow & 400\ nm & \sim & 700\ nm & \rightarrow & \text{IR} \\ & & \text{blue} & & \text{red} & & \\ & & 3.1\text{ eV} & & 1.7\text{ eV} & & \end{array}$$

  - γ-ray ($10^{-3}$ nm), X-ray (0.1 nm), UV ($10^2$ nm), IR ($10^4$ nm), radar ($10^8$ nm)

# Interactions between EM waves and solids

- Refraction — Index of refraction
- Reflection
- Absorption — Absorption coefficient=absorption of light with distance in the material
  Penetration depth=how far the light will travel before being reduced by a factor of *e*.

Dielectric constant: polarization
Permeability: magnetization
Free carrier absorption: electrical conductivity

# Maxwell's equations

**TABLE 4.1** Maxwell's Equations

| | Gaussian Units | SI Units |
|---|---|---|
| 1. | $\nabla \cdot \mathbf{D} = 4\pi\rho$ | $\nabla \cdot \mathbf{D} = \rho$ |
| 2. | $\nabla \cdot \mathbf{B} = 0$ | $\nabla \cdot \mathbf{B} = 0$ |
| 3. | $\nabla \times \mathcal{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$ | $\nabla \times \mathcal{E} = -\frac{\partial \mathbf{B}}{\partial t}$ |
| 4. | $\nabla \times \mathbf{H} = \frac{1}{c}\frac{\partial \mathcal{E}}{\partial t} + \frac{4\pi\mathbf{J}}{c}$ | $\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$ |

**D**—electric displacement

$\mathbf{D} = \varepsilon_r \mathcal{E}$ $\qquad$ $\mathbf{D} = \varepsilon_r \varepsilon_o \mathcal{E}$

$\varepsilon_r$—dielectric constant; $\varepsilon_o = (36\pi \times 10^9)^{-1}$ F/m—permittivity of free space

$\mathcal{E}$—electric field

$\rho$—charge density

**B**—magnetic induction

$\mathbf{B} = \mu_r \mathbf{H}$ $\qquad$ $\mathbf{B} = \mu_r \mu_o \mathbf{H}$

$\mu_r$—permeability; $\mu_0 = 4\pi \times 10^{-7}$ H/m—permeability of free space

**H**—magnetic field

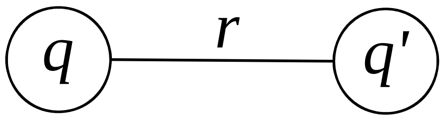
$\mathbf{J} = \sigma\mathcal{E}$—current density; $\sigma$—electrical conductivity

Mathematical generalization of an experimentally observed phenomenon.

# The first Maxwell equation

- Maxwell's first equation: $\nabla \cdot D = \rho$

  – Coulomb's law
  : force on a charge *q'* due to a charge *q* separated by "free spacing"




$$F = \frac{qq'}{4\pi\varepsilon_0 r^2} \quad \text{(SI)}$$

$\varepsilon_0 =$ the permittivity of free space

$\varepsilon_0 = 8.85 \times 10^{-12}\ F/m$

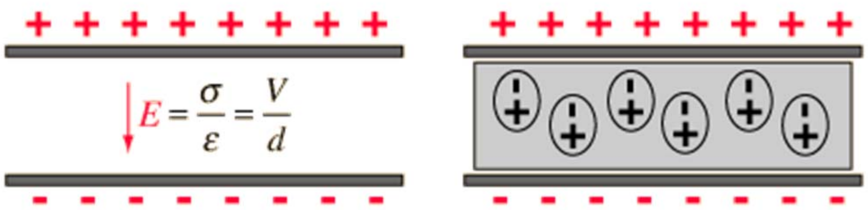
# The first Maxwell equation

- Electric field in free space

$$F = q'E_q \quad (E_q\text{: electric field of } q)$$

$$\rightarrow \; E_q = \frac{q}{4\pi\varepsilon_0 r^2}$$

- In a polarizable (not free space) medium, polarization should be considered.
- Polarization: ability to produced a local separation of charges in the material (formation of dipoles) due to interaction with the electric field $\rightarrow$ dielectric constant ($\varepsilon_r$)



# The first Maxwell equation

- Polarization can arise from several sources including the contribution from the lattice ions and electron themselves.
- The electric field in the medium now is modified to include the polarization effect.

$$E_q = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r^2} \quad \text{(SI)}$$

$\varepsilon_r$ = relative permittivity = relative dielectric constant

- The polarization $P$ is defined as the electric dipole moment per unit volume induced by the electric field.

$$P = Np = Nqd$$

$N$: volume density of dipoles
$p$: dipole moment
$d$: distance between charges

# The first Maxwell equation

- The polarization $P$ in an isotropic homogeneous medium is proportional to $E$.

$P = \chi E = \varepsilon_0 \chi E$ (SI) $\chi$: dielectric susceptiblity

- Let's define the electric displacement $D$ to take into account effects on the electric field $E$ caused by the polarization $P$.

$D = \varepsilon_0 E + P$ (SI) $\because D = \varepsilon_r \varepsilon_0 E$

$$D = \varepsilon_r \varepsilon_0 E = \varepsilon_0 E + P$$
$$= \varepsilon_0 E + \varepsilon_0 \chi E$$

$\varepsilon_r$: dielectric constant

$$\varepsilon_r = 1 + \chi$$

$$\varepsilon_r = 1 + \sum_i \chi_i \quad \text{(SI)}$$

$\chi_L$~polariztion of lattice atoms,
$\chi_e$~polariztion of electron distribution

# The first Maxwell equation

- The polarization mechanisms depends on the frequency.

$\varepsilon_{r(lo)} = 1 + \chi_L + \chi_e$ at low frequencies ex) atoms are charged and binding is ionic.

$\varepsilon_{r(hi)} = 1 + \chi_e$ at high frequencies

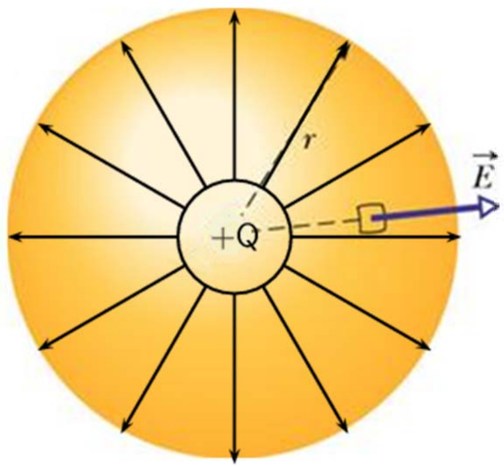
→ We can compare $\varepsilon_{r(lo)}$ and $\varepsilon_{r(hi)}$ to find out the type of bonding.
$\chi_L$ is large for ionic bonding and small for covalent bonding.

$$\varepsilon_r\Big|_{low\ \omega} \gg \varepsilon_r\Big|_{high\ \omega} \quad \text{for ionic bonding}$$

$$\varepsilon_r\Big|_{low\ \omega} \simeq \varepsilon_r\Big|_{high\ \omega} \quad \text{for covalent bonding}$$

# The first Maxwell equation

- Consider the sphere with radius $r$ of which the charge $q$ in located at the center.




$E_q$: lines of force radiating out radially from the charge $q$ with spherical symmetry.
$E_{q(r)}$: surface density of lines of force passing through a sphere with radius $r$.

$$E_q = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r^2}$$

- We can figure out the electronic field as the charge density per unit area at the distance $r$ from the charge.

$$E_{q(r)} \propto \text{surface density of line of force } (c.f.\, F = q'E_q)$$
$$\propto \frac{1}{r^2} \quad \text{because area is proportional to } r^2$$

# The first Maxwell equation

$$E_q = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r^2} \rightarrow \underline{4\pi r^2 E_q} = \frac{q}{\varepsilon_r\varepsilon_0}$$

Integral of the normal component of $E_q$ over the area of a sphere

$$\int E \cdot ds = \frac{q}{\varepsilon_r\varepsilon_0}$$

$$\int \nabla \cdot E \, dV = \int \frac{\rho}{\varepsilon_r\varepsilon_0} \, dV$$

from the divergence theorem (volume integral)
($\rho$: charge density)

$$\nabla \cdot E = \frac{\rho}{\varepsilon_r\varepsilon_0}$$

for homogeneous isotropic materials

$$\because D = \varepsilon_r\varepsilon_0 E$$

$$\boxed{\nabla \cdot D = \rho \quad \text{(SI)}}$$

[1st Maxwell equation]

$$E = -\nabla\phi$$

$\phi$: electrostatic potential (volt)

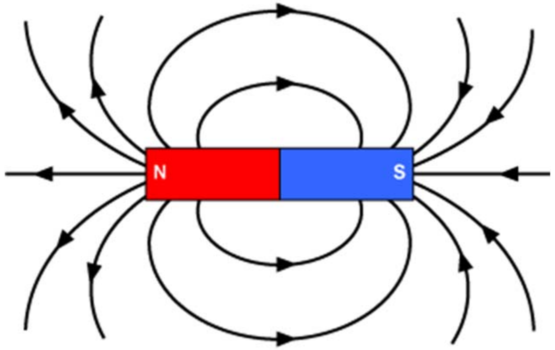
: Spatial dependence of electrostatic potential (or electric field) in the presence of charge density

$$\left.\begin{aligned} E &= -\frac{d\phi}{dx} \\ \frac{dE}{dx} &= \frac{\rho}{\varepsilon_r\varepsilon} \end{aligned}\right\} \quad \boxed{\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon_r\varepsilon_0} \quad \text{or} \quad \nabla^2\phi = -\frac{\rho}{\varepsilon_r\varepsilon}}$$

[Poisson's equation]

# The second Maxwell equation

- This equation is on the magnetic field.




$$\boxed{\nabla \cdot B = 0}$$

$B$ : Magnetic inductance

- The above equation is simply an expression of the fact that there are no isolated magnetic poles (unlike isolated electric charges).
- All lines of forces are closed.
- Similarity between electric field & magnetic field
  - Electric field $D, E, P, \chi, \varepsilon_r$
  - Magnetic field $B, H, \mathrm{M}, \kappa\ \mu_r$

# The second Maxwell equation

- Application of magnetic field to a material induces magnetization.

$M = \kappa H \qquad \text{(SI)}$
$B = \mu_r \mu_0 H \quad \text{(SI)}$
$\mu_r =$ relative permeability
$\mu_0 =$ permeability of free space $= 4\pi \times 10^{-7}$ H/m

$B = \mu_0 (H + M) \qquad \text{(SI)}$
$\mu_r = 1 + \kappa \qquad \text{(SI)}$

$\nabla \cdot B = 0$
$\nabla \cdot H = 0$ for isotropic, homogeneous materials