

Intro to DB

# CHAPTER 8

# RELATIONAL DB DESIGN

# Chapter 8: Relational Database Design

- Features of Good Relational Design
- Atomic Domains and First Normal Form
- Decomposition Using Functional Dependencies
- Functional Dependency Theory
- Algorithms
- Decomposition Using Multivalued Dependencies
- More Normal Form
- Database-Design Process
- Modeling Temporal Data

# Pitfalls of Relational Database Design

- Relational database design



- $R = (A B C D E)$  <----- single relation schema
- $DB_1 = \{ R_1, \dots, R_n \}$  <----- DB schema (set of relation schemas)

- Design Goals:

- Ensure that relationships among attributes are represented (information content)
- Avoid redundant data
- Facilitate enforcement of database integrity constraints

- A bad design may lead to

- Inability to represent certain information
- Repetition of Information
- Loss of information

# Example

*Lending-schema = (branch-name, branch-city, assets, customer-name, loan-number, amount)*

<i>branch-name</i>	<i>branch-city</i>	<i>assets</i>	<i>customer-name</i>	<i>loan-number</i>	<i>amount</i>
Downtown	Brooklyn	9000000	Jones	L-17	1000
Redwood	Palo Alto	2100000	Smith	L-23	2000
Perryridge	Horseneck	1700000	Hayes	L-15	1500
Downtown	Brooklyn	9000000	Jackson	L-14	1500

- Redundancy:
  - 
  - Wastes space
  - Complicates updating, introducing possibility of inconsistency of *assets* value
- Null values
  - Can use null values, but they are difficult to handle.

# Redundancy creates problems

- Anomalies (by Codd)
  - *Insertion anomaly*: cannot store information about a branch if no loans exist
  - *Deletion anomaly*: lose branch info when that last account for the branch is deleted
  - *Update anomaly*: what happens when you modify asset for a branch in only a single record?




- Solution

decompose schema so that each information content is represented only once (later)

- information content: relationship between attributes

# First Normal Form

- Domain is **atomic** if its elements are considered to be indivisible units
  - Examples of non-atomic domains:
    - set of names, composite attributes
    - identification numbers like CS101 that can be broken up into parts
- A relational schema is in **first normal form (1NF)**  

- Atomicity is actually a property of how the elements of the domain are used
  - Student ID numbers: CS0012, EE1127, ...
- Non-atomic attributes leads to
  - encoding of information in the application program ...
    - ... rather than in the database
  - complication in storage and query processing
- We assume all relations are in first normal form

# Relational Theory

Goal: Devise a theory for the following

- Decide whether a particular relation  $R$  is in “good” form.
- In the case that a relation  $R$  is not in “good” form,

- each relation is in good form
- the decomposition is lossless (preserves the information in the original relation before decomposition)
- Our theory is based on:
  - functional dependencies
  - multivalued dependencies (not covered in this semester)

# Functional Dependencies

- Constraints on the set of legal relations.
- Require that the value for a certain set of attributes determines uniquely the value for another set of attributes.

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- Example
  - *Which attribute's values depend on other attributes?*

*Student=(ID, Name, Dept, Dept\_office, College, Dean, Advisor, Adv\_phone)*

*Supplies=(Supplier, S-contact, Part-ID, Part-Name, Size, Proj-ID, Location, Manager, P-contact, Quantity)*



# Functional Dependencies (Cont.)

- Let  $R$  be a relation schema

$$\alpha \subseteq R \text{ and } \beta \subseteq R$$

- The *functional dependency*  $\alpha \rightarrow \beta$  holds on  $R$  if and only if
  - for any *legal* relations  $r(R)$ ,
  - whenever any two tuples  $t_1$  and  $t_2$  of  $r$  agree on the attributes  $\alpha$ ,
  - they also agree on the attributes  $\beta$ .
  - That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

- Example

- Consider  $r(A,B)$  with the following instance of  $r$

1	4
1	5
3	7

□

# Applications of FD

- $K$  is a *superkey* for relation schema  $R$  if and only if  $K \rightarrow R$
- $K$  is a *candidate key* for  $R$  if and only if
  - $K \rightarrow R$ , and
  - for no  $\alpha \subset K$ ,  $\alpha \rightarrow R$
- Functional dependencies allow us to  ed using superkeys.

*Loan-info-schema = (customer-name, loan-number, branch-name, amount)*

We expect the following functional dependencies to hold:

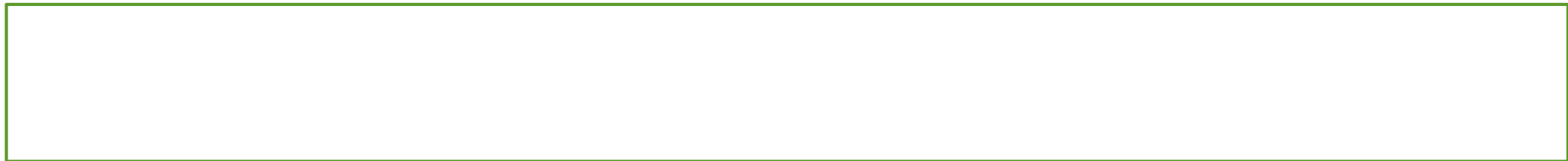
*loan-number  $\rightarrow$  amount*

*loan-number  $\rightarrow$  branch-name*

but would not expect the following to hold:

# Applications of FD (Cont.)

- Specify constraints on the set of **legal** relations
  - We say that  $F$  **holds on**  $R$  if *all legal* relations on  $R$  satisfy the set of functional dependencies  $F$
- Test relations to see if they are legal under a given set of FDs
  - If a relation  $r$  is legal under a set  $F$  of functional dependencies, we say that  $r$  **satisfies**  $F$ .
- Note:



# What causes redundancy?

*Lending-schema = (b-name, b-city, assets, c-name, loan#, amount)*

<i>branch-name</i>	<i>branch-city</i>	<i>assets</i>	<i>customer-name</i>	<i>loan-number</i>	<i>amount</i>
Downtown	Brooklyn	9000000	Jones	L-17	1000
Redwood	Palo Alto	2100000	Smith	L-23	2000
Perryridge	Horseneck	1700000	Hayes	L-15	1500
Downtown	Brooklyn	9000000	Jackson	L-14	1500

$F = \{ b\text{-name} \rightarrow b\text{-city assets} ; \text{loan\#} \rightarrow \text{amount } b\text{-name} \}, \text{ Key} = \{ c\text{-name, loan\#} \}$

- Redundancy
  - *b-city*
  - *amount, b-name* are repeated for each loan
- Observations
  - Same values repeated for attributes that are functionally dependent on non-key attributes!

# Boyce-Codd Normal Form - informally

- A relation  $R$  is in “good” form IF attributes are only dependent on keys
  - No non-key FDs!
  - Solution: Break  $R$  into smaller relations that hold tightly related attributes!

- Example

$Lending\text{-}schema = (b\text{-}name, b\text{-}city, assets, c\text{-}name, loan\#, amount)$

$F = \{ b\text{-}name \rightarrow b\text{-}city \text{ assets} ; loan\# \rightarrow amount \text{ } b\text{-}name \}$ ,  $Key = \{c\text{-}name, loan\#\}$

=> Decompose

$Branch = (b\text{-}name, b\text{-}city, assets) \quad \{ b\text{-}name \rightarrow b\text{-}city \text{ assets} \}$

$Loan = (loan\#, amount, b\text{-}name) \quad \{ loan\# \rightarrow amount, b\text{-}name \}$

$CustLoan = (c\text{-}name, loan\#)$

# Trivial FD

- A functional dependency is *trivial* if it is satisfied by all instances of a relation
  - *E.g.*
    - *customer-name, loan-number* → *customer-name*
    - *customer-name* → *customer-name*
- Lemma:

# Closure of a Set of FDs

- Given a set  $F$  of FDs, there are other FDs that are *logically implied* by  $F$ 
  - E.g. If  $A \rightarrow B$  and  $B \rightarrow C$ , then we can infer that  $A \rightarrow C$
- **Definition:** The set of all functional dependencies logically implied by  $F$  is the *closure* of  $F$  (denoted  $F^+$  ).
- We can find all of  $F^+$  by applying *Armstrong's Axioms*:
  - if  $\beta \subseteq \alpha$ , then  $\alpha \rightarrow \beta$  (**reflexivity**)
  - if  $\alpha \rightarrow \beta$ , then  $\gamma \alpha \rightarrow \gamma \beta$  (**augmentation**)
  - if  $\alpha \rightarrow \beta$ , and  $\beta \rightarrow \gamma$ , then  $\alpha \rightarrow \gamma$  (**transitivity**)
- These rules are
  - **sound**
  - and
  - **complete**

# Example

- $R = (A, B, C, G, H, I)$

$F =$

$A \rightarrow C$   
 $CG \rightarrow H$   
 $CG \rightarrow I$   
 $B \rightarrow H$  }

- some members of  $F^+$

- $A \rightarrow H$

- by transitivity from  $A \rightarrow B$  and  $B \rightarrow H$

- $AG \rightarrow I$

- 

- $CG \rightarrow HI$

- from  $CG \rightarrow H$  and  $CG \rightarrow I$ : “union rule”
  - can be inferred from definition of functional dependencies, or
  - Augmentation of  $CG \rightarrow I$  to infer  $CG \rightarrow CGI$ , augmentation of  $CG \rightarrow H$  to infer  $CGI \rightarrow HI$ , and then transitivity



# Boyce-Codd Normal Form – formally

- We want a way to decide whether a particular relation  $R$  is in “good” form.
- **Definition:** A relation schema  $R$  is in **BCNF** (with respect to a set  $F$  of FDs) if for each FD  $\alpha \rightarrow \beta$  in  $F^+$  ( $\alpha \subseteq R$  and  $\beta \subseteq R$ ), at least one of the following holds:

□

□

- Example

$$R = (A, B, C), \quad F = \{A \rightarrow B; B \rightarrow C\}, \quad \text{Key} = \{A\}$$

- $R$  is not in BCNF
- Decompose into  $R_1 = (A, B), R_2 = (B, C)$ 
  - $R_1$  and  $R_2$  are in BCNF
- Is the decomposed set of schemas equivalent to the original schema?

# Decomposition

- Decompose schema so that each information content is represented only once
- **Definition:** Let  $R$  be a relation scheme  
 $\{R_1, \dots, R_n\}$  is a *decomposition* of  $R$   
if  $R = R_1 \cup \dots \cup R_n$
- We will deal mostly with binary decomposition:
  - $R$  into  $\{R_1, R_2\}$  where  $R = R_1 \cup R_2$

*student*(*ID*, *name*, *dept*, *dept\_chair*, *dept\_phone*, *year*)

=> *student'*(*ID*, *name*, *year*, *dept*)  
*department*(*dept*, *chair*, *phone*)

*Lending* = (*b\_name*, *asset*, *b\_city*, *loan#*, *c\_name*, *amount*)

=> *Branch* = (*b\_name*, *asset*, *b\_city*)  
*Loan* = (*loan#*, *c\_name*, *amount*)

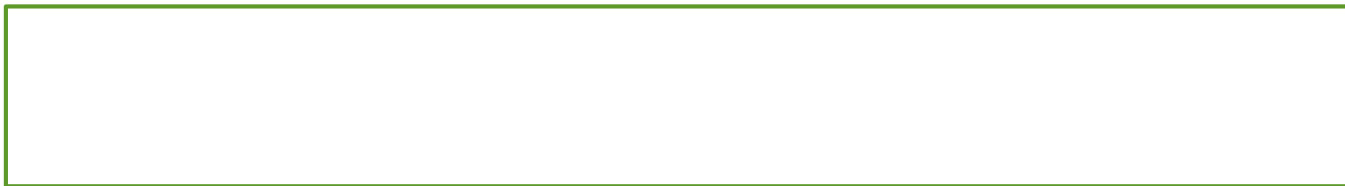
# Lossy Decomposition

- Careless decomposition leads to loss of information: *Lossy decomposition*
- Decompose schema so that each information content is represented only once

$Lending = (b\_name, asset, b\_city, loan\#, c\_name, amount)$

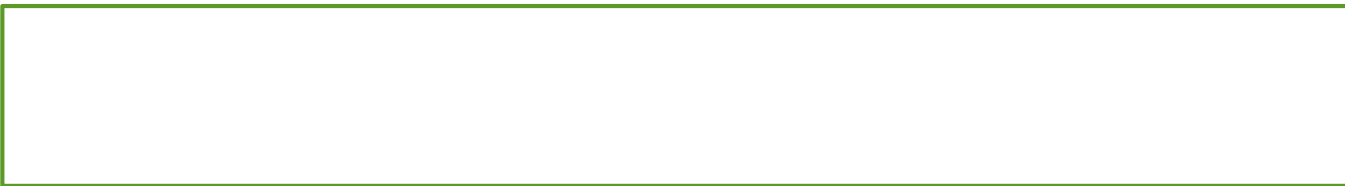
=>  $Branch = (b\_name, asset, b\_city)$

$Loan = (loan\#, c\_name, amount)$



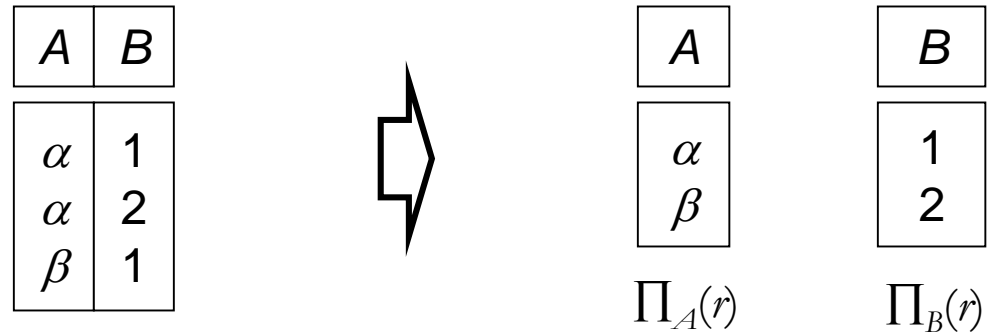
=>  $Branch = (b\_name, asset, b\_city)$

$Loan = (loan\#, c\_name, amount, b\_city)$

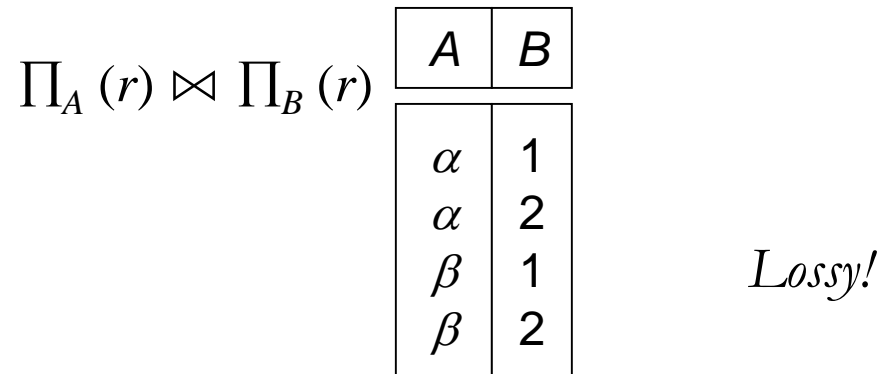


# Lossy Decomposition (cont.)

- Decomposition of  $R = (A, B)$  into  $R_1 = (A)$  and  $R_2 = (B)$



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# Lossless-join Decomposition

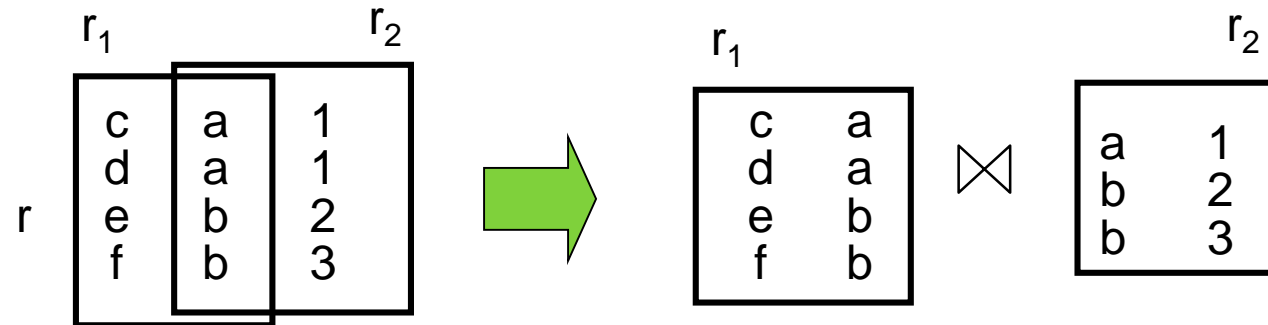
- For  $r(R)$  and decomposition  $\{R_1, R_2\}$ , it is always the case that



- Definition:** Decomposition  $\{R_1, R_2\}$  is a *lossless-join decomposition* of  $R$  if

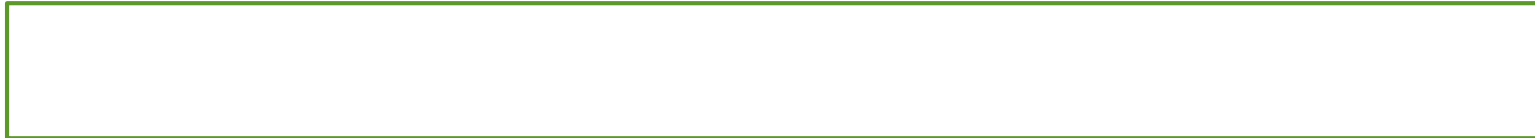
$$r = \Pi_{R_1}(r) \bowtie \Pi_{R_2}(r)$$

- The information content of the original relation  $r$  is always the basis

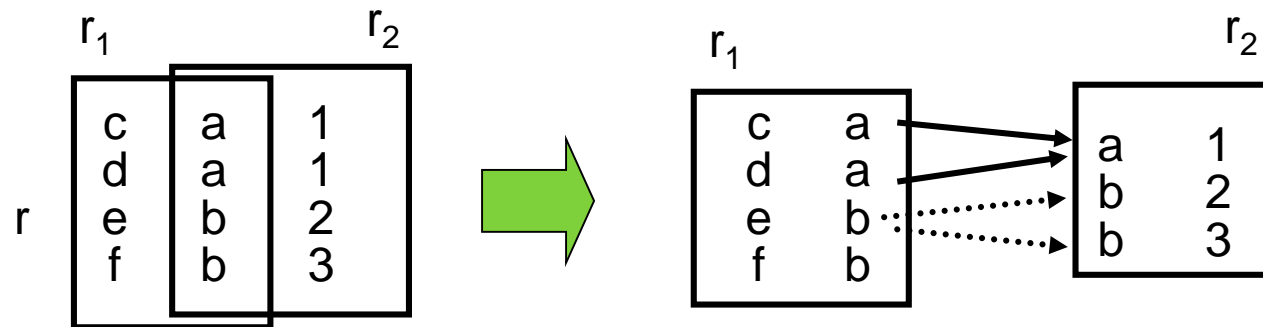


# Lossless-join Decomposition

- **Lemma:**  $\{R_1, R_2\}$  is a *lossless join decomposition* if



- i.e., if one of the two sub-schemas hold the key of the other sub-schema



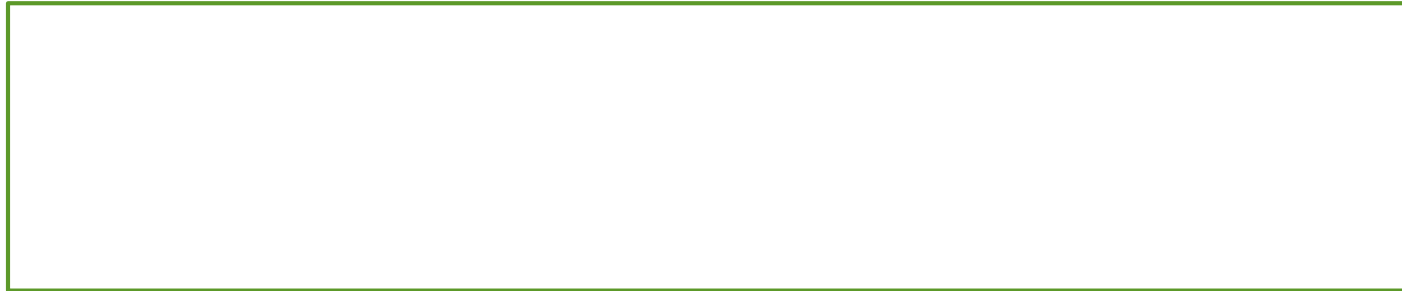
# BCNF Example

- $R = (bname, bcity, assets, cname, loan\#, amount)$   
 $F = \{ bname \rightarrow assets\ bcity ;\ loan\# \rightarrow amount\ bname \}$   
Key =  $\{loan\#, cname\}$

Decomposition

$R_1 = (bname, bcity, assets)$

$R_2 =$



Final decomposition result:  $\{ R_1, R_3, R_4 \}$

# Dependency Preservation

## Example

*student(name, dept, college)*      *name* → *dept, college*  
*dept* → *college*

- Decomposition 1

*student1(name, dept)*      *name* → *dept*  
*department(dept, college)*      *dept* → *college*

- Decomposition 2

*student1(name, dept)*      *name* → *dept*  
*student2(name, college)*      *name* → *college*

- 
-



# Dependency Preservation (cont.)

- **Definition**

Let  $F$ : set of FD on  $R$ .      $\{R_1, \dots, R_n\}$ : decomposition of  $R$ .

The **restriction** of  $F$  to  $R_i$ , denoted  $F_i$ , is the set of all FDs in  $F^+$  that include only attributes of  $R_i$

- **Definition**

Let  $F = F_1 \cup \dots \cup F_n$ .

The decomposition is **dependency-preserving** if  $F^+ = F^+$

- *Motivation:*

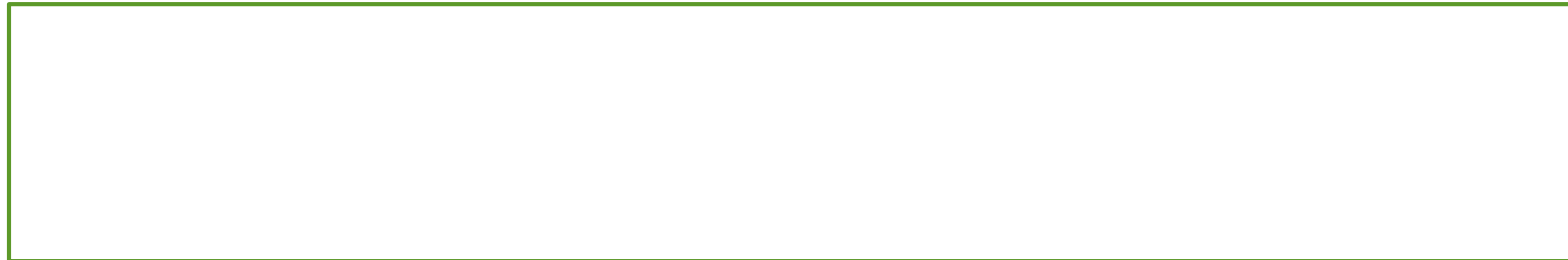


- Accessing multiple tables can be expensive
- SQL does not provide a direct way of specifying functional dependencies other than superkeys
- (Assertions can be ad hoc and expensive)

# Example

- $R = (A, B, C)$   
 $F = \{ A \rightarrow B, B \rightarrow C \}$
- $R_1 = (A, B), R_2 = (B, C)$ 
  - Lossless-join decomposition:  $R_1 \cap R_2 = \{ B \}$  and  $B \rightarrow BC$
  - Dependency preserving
- $R_1 = (A, B), R_2 = (A, C)$ 
  - Lossless-join decomposition:  $R_1 \cap R_2 = \{ A \}$  and  $A \rightarrow AB$

▫



# BCNF and Dependency Preservation

- $R = ( Street, City, Zip )$   
 $F = \{ Street\ City \rightarrow Zip; Zip \rightarrow City \}$   
Two candidate keys:  $Street\ City$  and  $Street\ Zip$ 
  - $R$  is not in BCNF
  - Any decomposition of  $R$  will fail to preserve  $Street\ City \rightarrow Zip$

$St$	$Zp$	$C$
$s_1$	$z_1$	$c_1$
$s_2$	$z_1$	$c_1$
$s_3$	$z_2$	$c_1$
$null$	$z_3$	$c_2$

- 
- There are some situations where
  - BCNF is not dependency preserving, and
  - efficient checking for FD violation on updates is important $\Rightarrow$  *solution*: define a weaker normal form

# BCNF and Dependency Preservation (cont.)

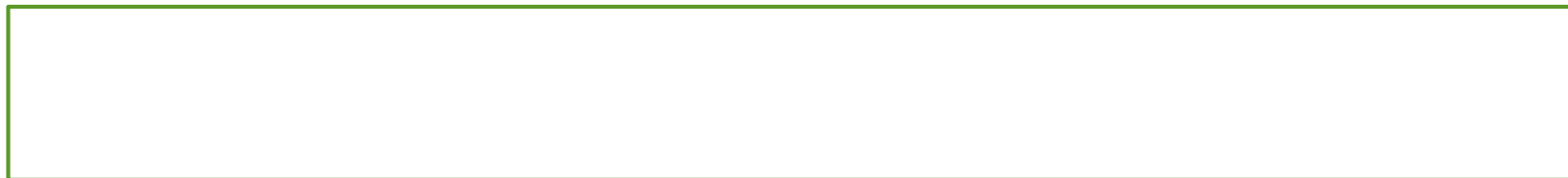
- BCNF decomposition has
  - $R_1(\textit{Street}, \textit{Zip})$
  - $R_2(\textit{Zip}, \textit{City})$
  
- $R_1, R_2$  are in BCNF
  - but not dependency-preserving
  - ⇒ Testing for  $\textit{Street City} \rightarrow \textit{Zip}$  requires a join

<i>St</i>	<i>Zp</i>	<i>C</i>
$s_1$	$z_1$	$c_1$
$s_2$	$z_1$	$c_1$
$s_3$	$z_2$	$c_1$
<i>null</i>	$z_3$	$c_2$

<i>St</i>	<i>Zp</i>	<i>Zp</i>	<i>C</i>
$s_1$	$z_1$	$z_1$	$c_1$
$s_2$	$z_1$	$z_2$	$c_1$
$s_3$	$z_2$	$z_3$	$c_2$

# Third Normal Form

- Third Normal Form
  - Allows some redundancy (with resultant problems)
  - But FDs can be checked on individual relations without a join
  - There is always a lossless-join, dependency-preserving decomposition into 3NF
- A relation schema  $R$  is in **third normal form (3NF)** if for all  $\alpha \rightarrow \beta$  in  $F^+$  at least one of the following holds:
  - $\alpha \rightarrow \beta$  is trivial (i.e.,  $\beta \in \alpha$ )
  - $\alpha$  is a superkey for  $R$
  - Each attribute  $A$  in  $\beta - \alpha$  is contained in a candidate key for  $R$ .  
(NOTE: each attribute may be in a different candidate key)



# Example

$R = ( \textit{Street}, \textit{City}, \textit{Zip} )$

$F = \{ \textit{Street City} \rightarrow \textit{Zip}; \textit{Zip} \rightarrow \textit{City} \}$

- Two candidate keys: *Street City* and *Street Zip*

- $R$  is in 3NF

$\textit{Street City} \rightarrow \textit{Zip}$  : *Street City* is a superkey

$\textit{Zip} \rightarrow \textit{City}$  : *City* is contained in a candidate key

- But not in BCNF (nontrivial & *zip* is not key)

- There is some redundancy in this schema

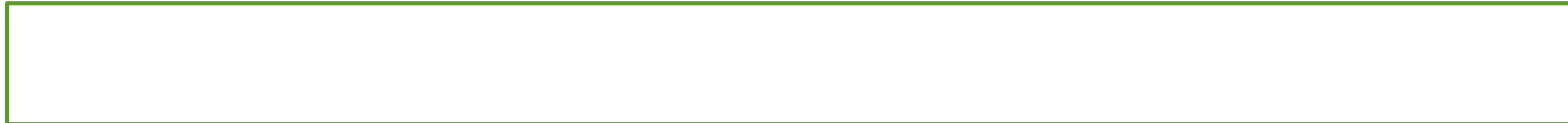
- repetition of information (e.g., the relationship  $z_1, c_1$ )
- need to use null values (e.g., to represent the relationship  $z_3, c_2$  where there is no corresponding value for *St*)

<i>St</i>	<i>Zp</i>	<i>C</i>
$s_1$	$z_1$	$c_1$
$s_2$	$z_1$	$c_1$
$s_3$	$z_2$	$c_1$
<i>null</i>	$z_3$	$c_2$

# Comparison of BCNF and 3NF

- It is always possible to decompose a relation into relations in 3NF and
  - the decomposition is lossless
  - the dependencies are preserved
  
- It is always possible to decompose a relation into relations in BCNF and
  - the decomposition is lossless

- 



# Design Goals

- When we decompose a relation schema  $R$  with a set of functional dependencies  $F$  into  $R_1, R_2, \dots, R_n$  we want
  1. Lossless decomposition
  2. No redundancy
  3. Dependency preservation
- First, try to achieve
  - BCNF
  - Lossless join
  - Dependency preservation
- If we cannot achieve this, we accept one of
  - 
  -



# Algorithms

- Testing for BCNF
- BCNF Decomposition
- Testing for 3NF
- 3NF Decomposition
  
- Closure of FDs
- Closure of attributes
- Cover
- Canonical cover

# Closure of Attribute Sets

- **Definition:** Given a set of attributes  $\alpha$ , the *closure* of  $\alpha$  under  $F$  (denoted by  $\alpha^+$ ) is the set of attributes that are functionally determined by  $\alpha$  under  $F$ :



- Algorithm to compute  $\alpha^+$

```
result :=  $\alpha$ ;  
while (changes to result) do  
  for each  $\beta \rightarrow \gamma$  in  $F$  do  
    begin  
      if  $\beta \subseteq \text{result}$  then  $\text{result} := \text{result} \cup \gamma$   
    end
```

# Example

- $R = (A, B, C, G, H, I)$
- $F = \{ A \rightarrow B; A \rightarrow C; \quad CG \rightarrow H; \quad CG \rightarrow I; \quad B \rightarrow H \}$

- $(AG)^+$

1. result =

2. result =

3. result =

4. result =

- Is  $AG$  a candidate key?
  - Is  $AG$  a super key?
    - Does  $AG \rightarrow R$ ?
  - Is any subset of  $AG$  a superkey?
    - Does  $A^+ \rightarrow R$ ?
    - Does  $G^+ \rightarrow R$ ?

# Uses of Attribute Closure

There are several uses of the attribute closure algorithm:

- **Testing for superkey:** “is  $\alpha$  a superkey?”

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- **Testing functional dependencies:** “does  $\alpha \rightarrow \beta$  hold?”

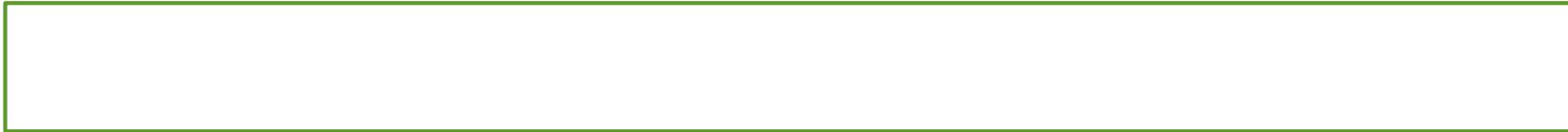
- Or, in other words, is  $\alpha \rightarrow \beta$  in  $F^+$
  - Just check if  $\beta \subseteq \alpha^+$ .
  - Is a very useful simple test

- **Computing the closure of  $F$ :  $F^+$**

- For each  $\gamma \subseteq R$ , we find the closure  $\gamma^+$ , and
  - for each  $S \subseteq \gamma^+$ , we output a functional dependency  $\gamma \rightarrow S$ .

# Testing for BCNF

- **Check if  $\alpha \rightarrow \beta$  cause a violation of BCNF**
  1. compute  $\alpha^+$  (the attribute closure of  $\alpha$ ), and
  2. verify that it includes all attributes of  $R$  (i.e., it is a superkey of  $R$ )
- **Check if  $R$  is in BCNF (w.r.t.  $F$ )**



- It can be shown that if none of the dependencies in  $F$  causes a violation of BCNF, then none of the dependencies in  $F^+$  will cause a violation of BCNF either

# Testing for BCNF (cont.)

- However, using only  $F$  is *incorrect* when testing a relation in a *decomposition of  $R$*

- Example

Consider  $R(A, B, C, D)$  with  $F = \{A \rightarrow B, B \rightarrow C\}$

- Decompose into  $R_1(A, B)$  and  $R_2(A, C, D)$
- Neither of the dependencies in  $F$  contain only attributes from  $(A, C, D)$  so we might be misled into thinking  $R_2$  satisfies BCNF.
- In fact, dependency  $A \rightarrow C$  in  $F^+$  shows  $R_2$  is not in BCNF.

# BCNF Decomposition Algorithm

*result* := {*R*};    *done* := false

compute  $F^+$

**while (not *done*) do**

**if** (there is a schema  $R_i$  in *result* that is not in BCNF)

**then begin**

  let  $\alpha \rightarrow \beta$  ( $\alpha \cap \beta = \emptyset$ )

  be a nontrivial FD

  that holds on  $R_i$ , and

$\alpha \rightarrow R_i$  is not in  $F^+$

*result* := (*result* –  $R_i$ )  $\cup$   
          ( $R_i - \beta$ )  $\cup$  ( $\alpha, \beta$ );

**end**

**else *done* := true;**

\* Each  $R_i$  in *result* is in BCNF, and decomposition is lossless-join.

$R = (bname, bcity, assets, cname, loan\#, amount)$

$F = \{ bname \rightarrow assets \ bcity;$

$loan\# \rightarrow amount \ bname \}$

Key={ $loan\#, cname$ }

Decomposition

$R_1 = (bname, bcity, assets),$

$R_2 = (bname, cname, loan\#, amount)$

$R_3 = (bname, loan\#, amount)$

$R_4 = (cname, loan\#)$

Final decomposition result:

$R_1, R_3, R_4$

# Overall Database Design Process

- We have assumed schema  $R$  is given

$R$  could have been

- a single relation containing *all* attributes that are of interest
  - called ***universal relation***
  - Normalization breaks  $R$  into smaller relations.

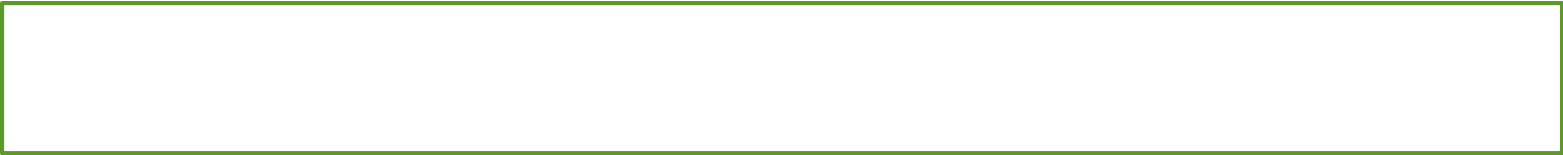
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- the result of some ad hoc design of relations, which we then test/convert to normal form.



# Denormalization for Performance

- May want to use non-normalized schema for performance
  - E.g. displaying *customer-name* along with *account-number* and *balance* requires join of *account* with *depositor*
- Alternative 1: Use **denormalized** relation containing attributes of *account* as well as *depositor* with all above attributes
  - faster lookup
  - extra space and extra execution time for updates
  - extra coding work for programmer and possibility of error in extra code
- Alternative 2: use a **materialized view** defined as  
*account* ⋈ *depositor*
  - benefits and drawbacks same as above
  - 

# Other Design Issues

- Some aspects of database design are not caught by normalization

Instead of *earnings(company-id, year, amount)*, use

- *earnings-2000, earnings-2001, earnings-2002, ...*
  - all on the schema (*company-id, earnings*).
  - above are in BCNF
  - but make querying across years difficult and
  - needs new table each year
- *company-year(comp-id, earnings2000, earnings2001, earnings2002)*
  - Also in BCNF
  - makes querying across years difficult and
  - requires new attribute each year.
  - Is an example of a ***crosstab***, where values for one attribute become column names => used in spreadsheets and data analysis tools

# Testing for 3NF

- Need to check only FDs in  $F$  (not  $F^+$ )
- For each dependency  $\alpha \rightarrow \beta$ ,
  - Check if  $\alpha$  is a superkey (attribute closure check)
- If  $\alpha$  is not a superkey
  - we have to verify if each attribute in  $\beta$  is contained in a candidate key
  - this test is rather more expensive, since it involves finding candidate keys
- Testing for
- Interestingly, decomposition into third normal form can be done in polynomial time

# Cover

- Sets of functional dependencies may have redundant dependencies that can be inferred from the others
  - $A \rightarrow C$  is redundant in:  $\{A \rightarrow B, B \rightarrow C, A \rightarrow C\}$
  - Parts of a functional dependency may be redundant
    - E.g. on RHS:  $\{A \rightarrow B, B \rightarrow C, A \rightarrow CD\}$  can be simplified to  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
    - E.g. on LHS:  $\{A \rightarrow B, B \rightarrow C, AC \rightarrow D\}$  can be simplified to  $\{A \rightarrow B, B \rightarrow C, A \rightarrow D\}$
- 
- A FD  $g \in F$  is *redundant* if  $(F - \{g\})^+ = F^+$  or  $g \in (F - \{g\})^+$
- $F'$  is a *nonredundant (minimal) cover* of  $F$  if
  - $F'^+ = F^+$  and
  - $F'$  contains no redundant FD

# Extraneous Attributes

- Let  $\alpha \rightarrow \beta$  in  $F$ .
  - $A \in \alpha$  is extraneous if  $F \Rightarrow (F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$
  - $A \in \beta$  is extraneous if  $F \Leftarrow (F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$
- *Note:* implication in the opposite direction is trivial, since a “stronger” functional dependency always implies a weaker one
  
- Example
  - Given  $F = \{A \rightarrow C, AB \rightarrow C\}$
  - $B$  is extraneous in  $AB \rightarrow C$  because
    -
  
- Example
  - Given  $F = \{A \rightarrow C, AB \rightarrow CD\}$
  - $C$  is extraneous in  $AB \rightarrow CD$  since
    -

# Testing if an Attribute is Extraneous

$$\alpha \rightarrow \beta \in F$$

- To test if attribute  $A \in \alpha$  is extraneous in  $\alpha$ 
  1. compute  $(\{\alpha\} - A)^+$  using the dependencies in  $F$
  2.  $A$  is extraneous if  $(\{\alpha\} - A)^+$  contains  $\beta$
  
- To test if attribute  $B \in \beta$  is extraneous in  $\beta$ 
  1. compute  $\alpha^+$  using only the dependencies in
  2.  $B$  is extraneous if  $\alpha^+$  contains  $B$ ,

# Canonical Cover

- **Definition:**

A *canonical cover* for  $F$  is a set of dependencies  $F_c$  such that

- $F_c^+ = F^+$
- No FD in  $F_c$  contains an extraneous attribute
- Each left side of a FD in  $F_c$  is unique

- Intuitively,  $F_c$  is

- a “minimal” set equivalent to  $F$

- 

- 


# Algorithm for Canonical Cover

**repeat**

replace any  $\alpha_1 \rightarrow \beta_1$  and  $\alpha_1 \rightarrow \beta_2$   
with  $\alpha_1 \rightarrow \beta_1 \beta_2$  (union rule)

Find  $\alpha \rightarrow \beta$  with extraneous attribute either in  $\alpha$   
or  $\beta$  and delete the extraneous attribute from  $\alpha$   
 $\rightarrow \beta$

**until**  $F$  does not change

- Union rule may become applicable after some extraneous attributes have been deleted, so it has to be re-applied
- $O(n^2)$

$R = (A, B, C)$

$F = \{ A \rightarrow BC$

$B \rightarrow C$

$A \rightarrow B$

$AB \rightarrow C \}$

- Combine  $A \rightarrow BC$  and  $A \rightarrow B$  into  $A \rightarrow BC$

- $A$  is extraneous in  $AB \rightarrow C$

- $B \rightarrow C$  logically implies  $AB \rightarrow C$ .

- $C$  is extraneous in  $A \rightarrow BC$

- $A \rightarrow BC$  is logically implied by  $A \rightarrow B$  and  $B \rightarrow C$ .

- The canonical cover:



# 3NF Decomposition Algorithm

$F_c$  (canonical cover for  $F$ )

$i := 0$

**for each** FD  $\alpha \rightarrow \beta$  in  $F_c$  **do**

**if** no  $R_j$ ,  $1 \leq j \leq i$  contains  $\alpha \beta$

**then** {  $i := i + 1$

$R_i := \alpha \beta$  }

**if** no  $R_j$ ,  $1 \leq j \leq i$  contains a candidate key for  $R$  **then**

  {  $i := i + 1$ ;

$R_i :=$  any candidate key

    for  $R$  }

**return** ( $R_1, R_2, \dots, R_i$ )

$Banker = (branch, cname, banker, office\#)$

$F = \{ banker \rightarrow branch\ office\#$

$cname\ branch \rightarrow banker \}$

Key = {  $cname, branch$  }

Follow the algorithm

$B1 =$

$B2 =$

Since  $B2$  contains a candidate key, we are done.

**END OF CHAPTER 8**