Precision Design: Thermal Design

Thermal design:

Thermal deformation error is one of the largest error sources in machine structures, and it is due to the temperature change of machine element; heat sources, heat flow mechanism, thermal deformation, thus thermal design strategy is the key issue.

Material	ν	E (GPa)	ρ (Mg/m^3)	K ($W/m/C^o$)	Cp ($J/kg/C^o$)	K/ρCp ($10^{-6}m^2/s$)	α ($\mu m/m/C^o$)
Aluminum (6061-T651)	0.33	68	2.70	167	896	69	23.6
Aluminum (cast 201)	0.33	71	2.77	121	921	47	19.3
Aluminum oxide (99.9%)	0.22	386	3.96	38.9	880	11.2	8.0
Aluminum oxide (99.5%)	0.22	372	3.89	35.6	880	10.4	8.0
Aluminum oxide (96%)	0.21	303	3.72	27.4	880	8.4	8.2
Beryillium (pure)	0.05	290	1.85	140	190	398	11.6
Copper (OFC)	0.34	117	8.94	391	385	114	17.0
Copper (free machining)	0.34	115	8.94	355	415	96	17.1
Copper (bery.-copper)	0.29	125	8.25	118	420	34	16.7
Copper (brass)	0.34	110	8.53	120	375	38	19.9
Granite	0.1	19	2.6	1.6	820	0.8	6
Iron (Class 40 cast)	0.25	120	7.3	52	420	17	11
Iron (Invar)	0.3	150	8.0	11	515	2.7	0.8
Iron (Super Nilvar)	0.3	150	8.0	11	515	2.7	0
Iron (Nitralloy 135M)	0.29	200	8.0	4.2	481	1.1	11.7
Iron (1018 steel)	0.29	200	7.9	60	465	16	11.7
Iron (303 stainless)	0.3	193	8.0	16.2	500	4.1	17.2
Iron (440C stainless)	0.3	200	7.8	24.2	460	6.7	10.2
Polymer concrete	0.23-0.3	45	2.45	0.83-1.94	1250	0.27-0.63	14
Zerodur	0.24	91	2.53	1.64	821	0.8	0.05
Silicon carbide	0.19	393	3.10	125	-	-	4.3
Silicon nitride (hip)	-	350	3.31	15	700	13	3.1
Tungsten carbide	-	550	14.5	108	-	-	5.1
Zirconia	0.28	173	5.60	2.2	-	-	10.5

Basic Material Properties (Slocum's precision machine design)

(Source: Dornfeld's precision manufacturing)

Heat sources:

Motors, Control electronics, Bearings, Transmission, Hydraulic oil, Gears/Clutch, Pumps and Engine, Guideways, process, chips, coolant system, lubricating systems, external/environmental heats, operator (human)

Power Losses as Heat sources

1) Power losses in Belt transmission

Belt transmission with V-belts is one of high power losses arising from belt slip. Belt transmission with cog-belts is high efficient and stiff, in use for machine tool's steerable axes.

Power losses $\Delta N=(1-\eta)N$ [W]

N=input power

$\eta = \exp(-0.55/M)$ for transmitted torque $M<25Nm$

$=0.978$ for $M>25Nm$

2) Power losses in Ball Screw

The limiting friction moment, considering the return motion of the rolling elements and tension on rolling elements;

$M_B=FP_n\eta/2\pi$ [Nm]

Where F=rolling elements' tension force[N]

P_n=Guiding length [m]

η=Ideal efficiency of system

Then, Power loss=ωM_B [W]

3) Power losses in Slide and Rolling guide joints

$\Delta N=\mu PV$ [W]

μ= Guide's friction coefficient

=0.1 - 0.07 for steel/steel sliding pairs, under lubrication

=0.15-0.18 for steel/steel under other condition

=0.05-0.18 for steel/cast-iron, under lubrication

=0.15-0.19 for steel/cast-iron under other condition

4) power losses in rolling guides

Power losses in rolling elements in linear guide carriages

=$\mu P_w V$; where μ=friction coefficient from 0.001-0.01

P_w=resultant load[N], P_0=preload[N], f_w=Load factor, P_n=changing load[N], k=0.5-0.65

$P_w=P_0+kf_wP_n$

when $f_wP_n/P_0 \le 2.83$

$P_w=f_wP_n$

when $f_wP_n/P_0>2.83$

(by THK and Mekid)

5) Power losses in motors driving machine spindles

$Q_{motor}=\omega M(1-\eta_{mot})$

Q_{motor}:heat generated by motor,

M:motor's torque

η_{mot}:motor's efficiency

6) Power losses in rolling bearings of spindle

Power loss=ωM

M=total friction moment=M_0+M_1

M_0=friction moment in the lubricating film

M_1=friction moment due to loading

$=f_1FD$ [Nmm]

where f_1=friction coefficient

F=Load to bearing

D=motor diameter

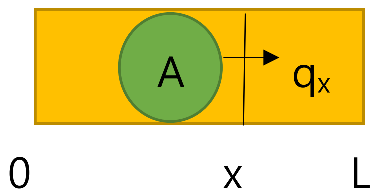
Heat transfer paths:

①Conduction(ambient or vacuum)

: Energy transfer from the high temperature region to low temperature region, when there exists temperature gradient in a body.

$q=KA\cdot\partial T/\partial x$ [W]

K=thermal conductivity [W/m˚C], A=Cross sectional area[m^2]



$q_x=KA\partial T/\partial x$

$\partial T/\partial x=\Delta T/\Delta x \fallingdotseq (T_L-T_0)/L$

Thermal-conductivity, K[W/m°C]

Copper 385, Al 202, Iron 73, Steel 43, Marble 2.5,

Glass 0.78, Water 0.556, Air 0.024

Conduction is a very fundamental heat transfer mechanism, and the convection is a particular case of conduction.

②Convection(ambient)

:Heat transfer at the wall of temperature change

$q=hA(T_w-T_\infty)$, where h=convection coefficient

T_w and T_∞ are temp. at the wall and free stream

<u>Convection coefficient for modes</u>, [W/m^2K]

4.5 for free convection

12 for forced convection of 2m/s flow

75 for forced convection of 35m/s flow

$\therefore\ q=hA(T-T_\infty)=hA\Delta T$

③Radiation(ambient and vacuum)

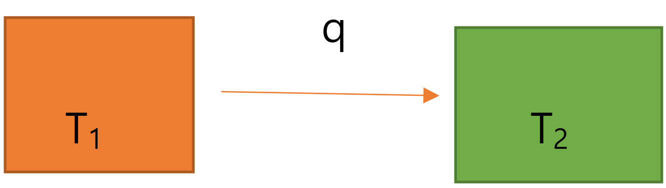
:electro-magnetic radiation due to temperature difference

$q=F_eF_G\sigma A(T_1^4-T_2^4)$;

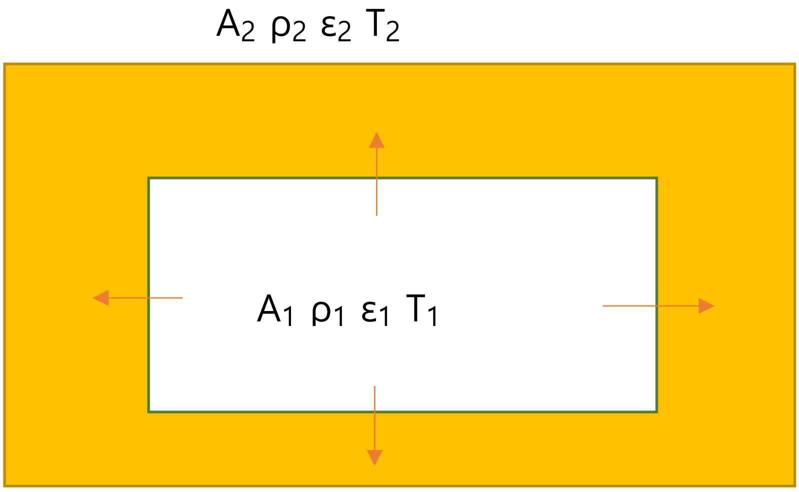
F_e=Emissivity, F_G=View factor

σ=Stefan-Boltzmann Constant=5.669E-8 [W/m^2K^4]

For $T_1 > T2$



Ex) Radiation between body 1 and enclosing body 2



When body1 is enclosed by body2

the heat transfer between 1 and 2 is;

$q_{12}/A_1 = Hr(T_1 - T_2)$

$Hr = \sigma(T_1^2 + T_2^2)(T_1 + T_2)/[1 + (\rho_1/\varepsilon_1) + [(A_1/A_2)(\rho_2/\varepsilon_2)]]$

where A_1,A_2: surface Area; ρ_1, ρ_2: reflectivity; ε_1, ε_2:emissivity

Various Temperature fields in machine elements:

(1) Uniform temperature field other than 20°C, or 68°F

; $\partial/\partial x \doteqdot \partial/\partial y \doteqdot \partial/\partial z \doteqdot 0$

(2) Non-uniform temperature gradient field (static effects)

; $\partial/\partial x \neq 0$, $\partial/\partial y \neq 0$, $\partial/\partial z \neq 0$; but $\partial/\partial t \doteqdot 0$,

(3) Dynamic temperature variation field (dynamic effects)

; $\partial/\partial t \neq 0$

Temperature field and thermal deformation

Under free constraints of structure;

Temperature field, T(x,y,z) generates the thermal strain field

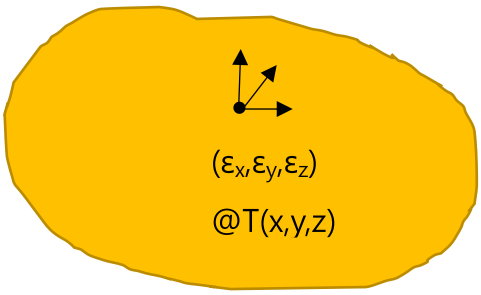
$(\varepsilon_x, \varepsilon_y, \varepsilon_z)$, and

$\varepsilon_x = \alpha_x T(x,y,z)$

$\varepsilon_y = \alpha_y T(x,y,z)$

$\varepsilon_z = \alpha_z T(x,y,z)$

where α_x, α_y, α_z are the thermal expansion coefficients in respective axis



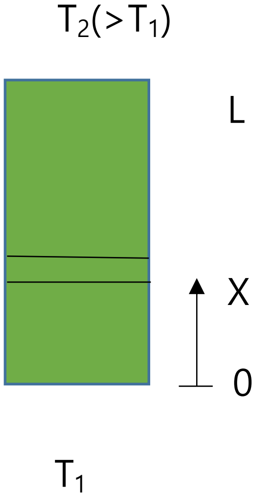
Thermal expansion coefficients (unit: ppm/K)

Al	23.03	Fe	12.3	Ti	8.35
Cu	16.5	Ni	13.3	Zn	25
Sn	21.2	Co	13.3	Cr	6.2
Ag	19.2	Au	14.15	Pt	9
Quartz	0.55	Glass	3.78	SiC	4.4
Carbon	0.7~6	Al_2O_3	8	SiO_2	8.8
SUS304	17.3	SUS316	16	Granite	6.0

Thermal deformation under (1)Temperature gradient, (2)Constant temperature change, (3)Dynamic temperature changes are of interest.

1. Temperature gradient

1) Temperature gradient in longitudinal direction



$T(x)=T_1+X(T_2-T_1)/L$

$=T_1+X\Delta T/L \quad \therefore T(x)-T_1=X\Delta T/L$

Thermal strain at x, $\varepsilon(x)=\alpha T=\alpha X\Delta T/L$

Thermal elongation, ΔL

$\Delta L=\int_0^L \varepsilon(x)dx=\alpha\Delta TL/2= \alpha L\Delta T/2$

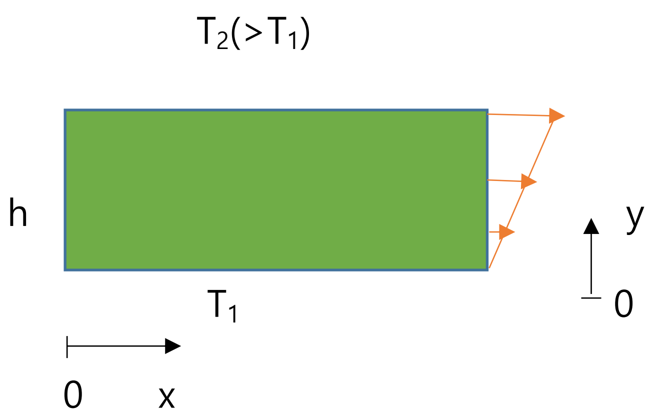
Ex)1m steel structure of 1˚C temperature change, α=12ppm/˚C

$\therefore \Delta L=\alpha\Delta TL/2=6[um]$

When the structure is constrained at two ends, the induced compressive load becomes, when cross sectional area $A=1cm^2$

$\therefore P=AE\varepsilon_T=AE\alpha\Delta T/2 =10^{-4}(200)10^{9}(12)10^{-6}(0.5)=120[N]$

2)Temperature gradient in the transverse direction



$T(y)=T_1+y(T_2-T_1)/h$

$=T_1+y\Delta T/h$

Thermal strain at y, $\varepsilon(y)=\alpha y\Delta T/h$=Bending strain=$y/\rho$

Bending curvature due to temperature change, $1/\rho=\alpha\Delta T/h$

This structure bends, and the bending angle, $\theta(x)$, and deflection, $\delta(x)$, can be obtained from the curvature.

<u>Bending angle $\theta(x)$</u>

$\theta(x)=\int 1/\rho dx=\int_0^x\alpha\Delta T/h dx=\alpha x\Delta T/h$

∴Bending angle between two ends

$=\theta(L)-\theta(0)=\alpha L\Delta T/h=L/\rho$ [urad]

This bending angle frequently contributes to the Abbe offset, making large amplification.

Ex) Steel beam of L=1m, h=0.1m, under transverse temp change 0.1˚C;

Bending angle=12(1)(0.1)/0.1=12 [urad]

=2.5 [arcsec] (∴Big number!!)

Bending deflection $\delta(x)$

$\delta(x)=\int\theta(x)dx=\int_o^x \alpha x\Delta T/h dx=\alpha x^2\Delta T/2h$

Bending deflection between two ends $=\delta(L)-\delta(0)=\alpha L^2\Delta T/h$

The straightness error due to the bending is

Straightness error $=|\delta(0)-\delta(L/2)|=\alpha L^2\Delta T/8h=L^2/8\rho$

Ex)For the above case, the straightness error induced=

$12(1)^2(0.1)/0.8=1.5$[um] (∴Big number!!)

When the structure is constrained at two ends by clamping the induced bending moment, M, is

$M=EI/\rho=EI\alpha\Delta T/h$,

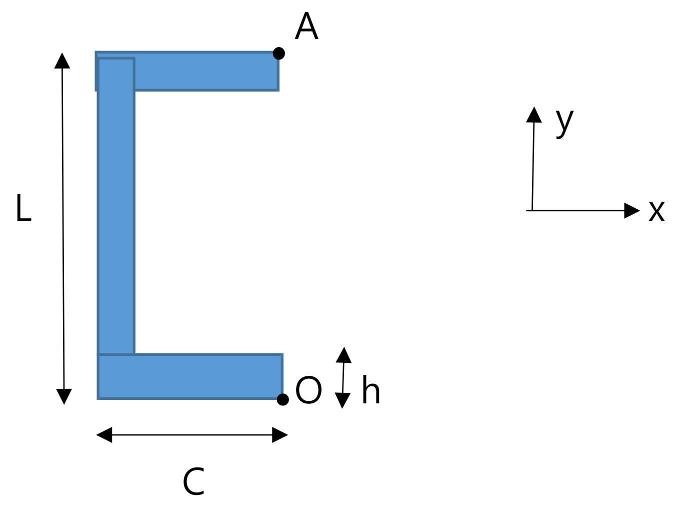
and maximum bending stress=$E\alpha\Delta T/2$

=200E9(12E-6)(0.1)/2=200(12)(1000)/2=120[KPa]

3) General case:

It is the combination the longitudinal temperature gradient and the transverse gradient, thus both cases are summed for the general case.

Ex) C frame Aluminum structure with L=200mm, h=20mm, C=75mm, α=23ppm/˚C. The temperature difference is 2˚C between left and right , and 2˚C between bottom and top.



Under the gradient $\partial T/\partial x$

Horizontal displacement of A w.r.t O $=-\delta_{L\text{-}plate} - \delta_{column} + \delta_{U\text{-}plate}$

$\delta_{column}=L\theta=L\alpha\Delta TL/h=\alpha\Delta TL^2/h=23(2)(0.2)^2/0.02=92[um]$

$\delta_{L\text{-}plate}= \delta_{U\text{-}plate} =\alpha\Delta TC/2=23(2)(0.075)/2=1.725[um]$

$\therefore$Horizontal displacement of A w.r.t O=-92 um; (Quite big!)

For gradient $\partial T/\partial y$

Vertical displacement of A wrt O

$=\delta_{column}= \alpha\Delta TL/2=(23)(2)(0.2)/2=4.6[um]$

For combining cases,

$\therefore$ δx=-92 um, δy=4.6 um at A w.r.t O

Therefore, in order to minimize the thermal deformation

①Material selection of lower thermal expansion is preferred.

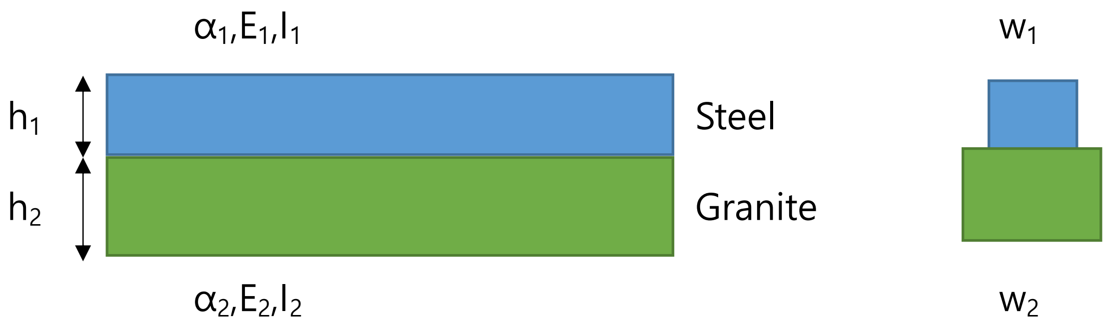
②Symmetric design w.r.t the heat source is preferred

2. Uniform Temperature Change in all direction

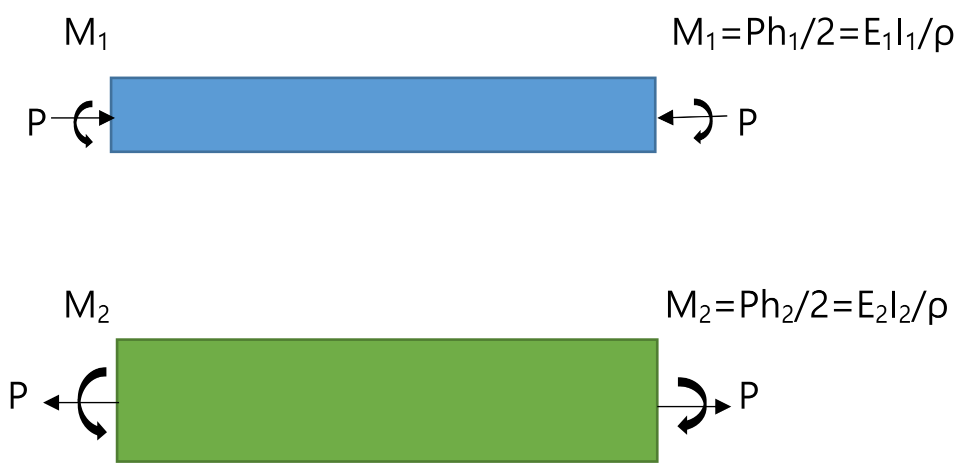
:Uniform thermal expansion for free constrained structure

Structures consisting of different materials is of critical importance such as guideway elements with Steel on Granite that tightly bolted together.

Under ΔT change;



Free body diagram



$M_1+M_2=Ph_1/2+Ph_2/2=E_1I_1/\rho+E_2I_2/\rho= (E_1I_1+E_2I_2)/\rho$

$\therefore$ $1/\rho=P(h_1+h_2)/2(E_1I_1+E_2I_2)$ eq(1)

From no slip at interface,

ε_1 at bottom= ε_2 at top

$\therefore$ $\alpha_1\Delta T-P/E_1A_1-(0.5)h_1/\rho=\alpha_2\Delta T+P/E_2A_2+(0.5)h_2/\rho$ eq(2)

Eq(1),(2) give solutions for two unknowns; P and $1/\rho$

$\therefore 1/\rho=(\alpha_1-\alpha_2)\Delta T/[(h_1+h_2)/2+2(1/E_1A_1+1/E_2A_2)(E_1I_1+E_2I_2)/(h_1+h_2)]$

and $P=2(1/\rho)(E_1I_1+E_2I_2)/(h_1+h_2)$

Maximum straightness error experienced in the middle=$L^2/8\rho$

Bending angle due to temperature=L/ρ

For practical calculation under $\Delta T=1^\circ C$

$L=2[m]$

$\alpha_1=12ppm/^\circ C$, $E_1=200GPa=2.0E11$, $A_1=h_1w_1=(0.01)(0.1)$

$I_1=(0.1)(0.01)^3/12$

$\alpha_2=6ppm/^\circ C$, $E_2=20GPa=2.0E10$, $A_2=h_2w_2=(0.3)(0.3)$

$I_2=(0.3)(0.01)^3/12$

Therefore, bending curvature

$1/\rho=(\alpha_1-\alpha_2)\Delta T/[(h_1+h_2)/2+2(1/E_1A_1+1/E_2A_2)(E_1I_1+E_2I_2)/(h_1+h_2)]$

$=(6E-6)(1)/[0.5(0.01+0.3)+2[1/[(2E11)(0.001)]+1/[(2E10)(0.09)]$

$=9.391E-6$

Straightness error $=L^2/8\rho=(9.391)4/8=4.695$ [um]

(This is quite big under just 1˚C change!)

Bending angle= $L/\rho=(9.691)(2)=18.782$ [urad]=3.913 [arcsec]

$1/\rho=P(h_1+h_2)/2(E_1I_1+E_2I_2)$

Compressive Load, $P=2(E_1I_1+E_2I_2)/[(h_1+h_2)\rho]$

$=2(0.0135E9)(9.391E-6)/0.301=842$[N]

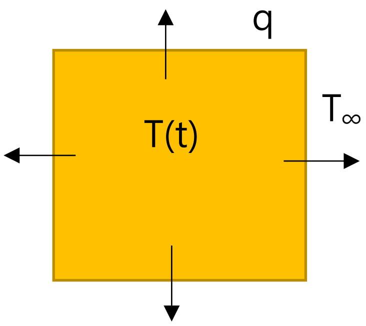
Bending moment induced, $M_1+M_2=(E_1I_1+E_2I_2)/\rho=130.5$[Nm]

Therefore materials with small difference in thermal expansion coeffcients are preferred in multi-layered structures, in order to minimize thermal deformation

3. Dynamic temperature change

1) Soak-out time

When a machine element of initial temperature, T_0, is exposed to environmental temperature of T_∞, by convection to air, then the temperature of element, T(t), is obtained by the heat transfer equation.



Where ρ=density, C=specific heat capacity, V=volume, A=exposed area to environment, h=convection coefficient

$-hA(T-T_\infty)=\rho CV\partial T/\partial t$

$\therefore T(t)=T_\infty+(T_0-T_\infty)\exp(-hAt/\rho CV)$

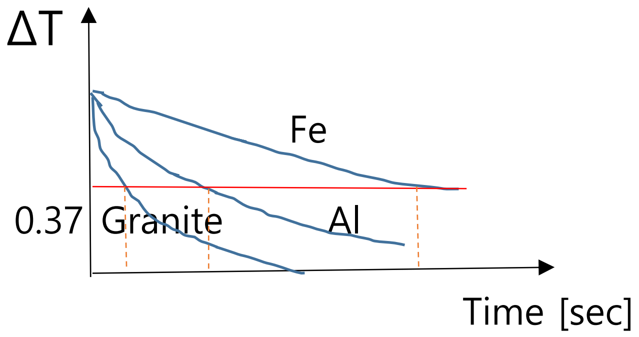
$= T_\infty+(T_0-T_\infty)\exp(-t/\tau)$

$\therefore \Delta T=(T_0-T_\infty) \exp(-t/\tau)$

$\tau=\rho CV/hA$=time constant, and indicates the time required for reaching 37% of initial temperature change, τ for Fe=(7897)(0.465)=3672 (for per volume)

τ for Al=(2707)(0.896)=2424 (for per volume)

τ for Granite=(2640)(0.82)=2164 (per volume)



Design strategy with τ(=ρCV/hA)

①Faster thermal equilibrium (with shorter τ)

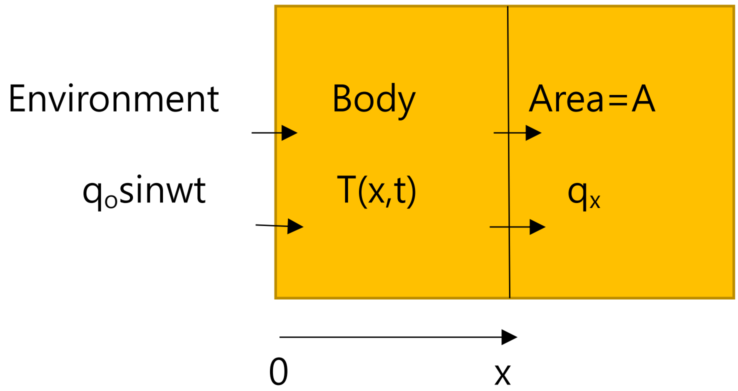
Granite>Al>Fe

②Slower thermal equilibrium (with longer τ)

Fe>Al>Granite

2) Dynamic change in environmental temperature

When machine element is under the dynamic change of environmental temperature, the temperature distribution inside the element is obtained as;



Assume the environment inputs to the body, a sinusoidal heat transfer, $q_o \sin wt$;

<u>Energy conservation</u>:

$q_0 \sin wt - q_x = \rho C V \partial T/\partial t$, and

$q_x = kA\partial T/\partial x$=transfer by conduction at x

$\therefore\ q_0 \sin wt - kA\partial T/\partial x = \rho C V \partial T/\partial t$ eq(1)

Assuming $T(x,t)=U(x)\sin(wt+\Phi)$, eq(1) becomes

$q_0 \sin wt - kA\partial U/\partial x \sin(wt+\Phi) = w\rho C V U(x)\cos(wt+\Phi)$

When $t=t_0$, let $\sin(wt_0+\Phi)=\beta$, $\cos(wt_0+\Phi)=\gamma$, the above equation is

$q_0 \sin wt_0 - \beta kA \partial U/\partial x = \gamma w \rho CVU(x)$

$\therefore U(x) + \beta KA/\gamma w \rho CV\ \partial U/\partial x = q_0/\gamma w \rho CV \sin wt_0$

<u>Thus $U(x) = T_0 \exp(-\lambda x)$</u>

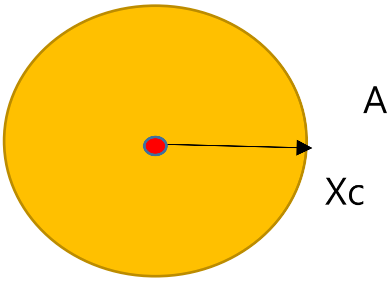
<u>and $T(x,t) = T_0 \exp(-\lambda x)\sin(wt+\Phi) = U(x)\sin(wt+\Phi)$</u>

where $\lambda = \beta KA/\gamma w \rho CV$ and $T_0 = q_0/\gamma w \rho CV \sin wt_0$

$\therefore T(x,t)$ is also a sinusoidal function of t, where U(x) is the amplitude.

Thermal diffusivity, $D=K/\rho C$ [m^2/sec], the rate of a heat transfer in a material, and it can be interpreted as speed of transfer, i.e, speed of transfer area per second.

Let us introduce the critical distance, Xc, which is the distance from the heat source in the material, where 36.7% of amplitude is observed during the period P of the sinusoidal heat source.



Area of transfer observing 36.7% amplitude=DP

$DP=\pi X^2c$ $\therefore$ $Xc=(DP/\pi)^{1/2}$ [m]

$U(Xc)/U(0)=\exp(-\lambda Xc)=0.367=\exp(-1)$

Thus $\lambda=1/Xc$

and $T(x,t)=T_0 \exp(-X/Xc)\sin(wt+\Phi)$ eq(2)

D, Thermal Diffusivity for Materials [unit: 10^{-4} m^2/sec]

Cu:1.1 Al:0.88 Fe:0.13 Granite:0.061

Invar:0.30 Zerodour:0.0079

Xc for Materials under P=1 day=24hr=86,400 sec [unit: m]

Cu:1.74 Al:1.56 Fe:0.60 Granite:0.41

Invar:0.91 Zerodour:0.15

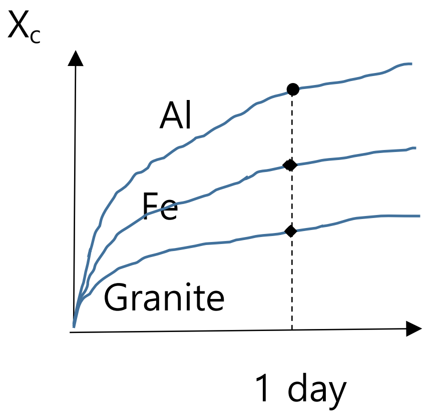
Under dynamic environmental temperature change, the dynamic response of material is:

①Sinusoidal temperature change

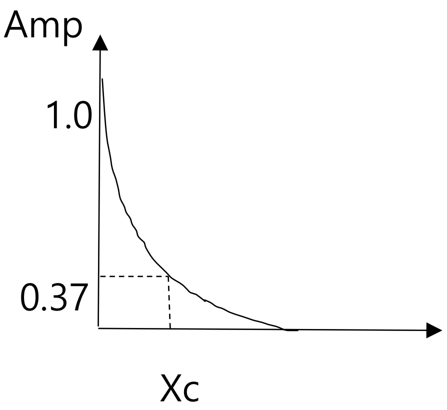
②Amplitude decreases with exp(-X/Xc), where Xc is the distance into the material such that 36.7% of initial amplitude is observed

③Design principle and Xc

Granite>Fe>Al is preferred for minimum effect of dynamic temperature change



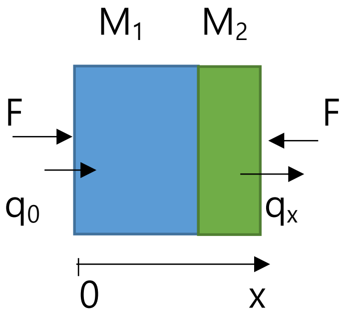
(Period, P)



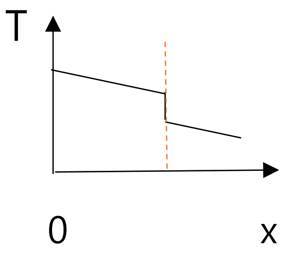
(Dist. into Material)

Therefore material selection of faster/longer thermal equilibrium for thermal soaking, and material selection of longer/shorter Xc for diffusivity are preferred.

<u>Thermal contact conductance across joints or contact surface</u>



F=Applied load at the contact area, A



∴ Temperature drop at contact area, A

∴ $\Delta T = q/hs/A$

Thermal conduction is also sensitive to the contact pressure, surface roughness, hardness (or Young's modulus), manufacturing processes.

Thermal conductance across contact surface, hs [$w/m^2{}^\circ K$], has been proposed by several models, and one of simpler forms in the elastic region is given by:

[source AKJ Hasselstrom's 'Thermal contact conductance in bolted joints']

$h_s=1.55K_s\ m_s\ P\sqrt{2}/(E_eR_sm_s)^{0.94}$; or

$h_s=1.9K_s(P/Ee)^{0.94}/Rs$; in much simpler form.

where $1/K_s=0.5(1/K_1+1/K_2)$, and

K_1, K_2 are the thermal conductivity of material 1, 2; [W/m˚C]

P=Pressure applied between the surface

=F/A [$Pa=N/m^2$]

E_e=effective Young's modulus of elasticity [Pa]

$1/E_e=1/[(1-\nu_1^2)/E_1+(1-\nu_2^2)/E_2]$

R_s=Mean roughness of surfaces of 1, 2

$=(R_1^2+R_2^2)^{1/2}$ in [m]

where R_1,R_2=RMS roughness of surface 1, 2, respectively

m_s=Mean slope of surfaces 1, 2$=(m_1^2+m_2^2)^{1/2}$

where m_1=RMS slope of surface 1

$=[\int_0^L(dz_1/dx)^2dx/L]^{1/2}$

m_2= RMS slope of surfaces 2

$=[\int_0^L(dz_2/dx)^2dx/L]^{1/2}$

The above equation provides a very important knowledge on the thermal conductivity at the contact surface ;

(1) Thermal conductance across contact surface, $h_{s,}$ is functioning similar to the convection coefficient

(2) Higher Pressure(P) of contact gives higher h_s

(3) Lower R_s (finer surface) gives higher h_s

(4) Higher E_e (or harder material) gives lower h_s

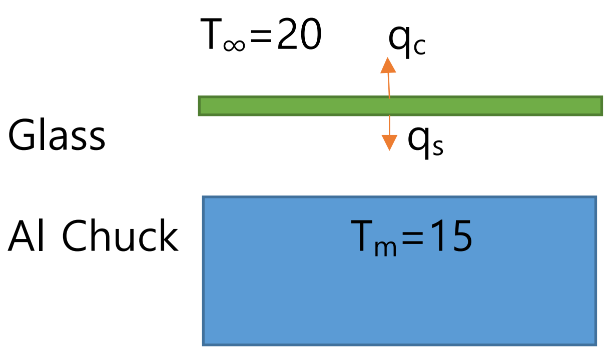
Ex) Glass soaking in contact to Aluminum chuck

1t Glass of 1m^2, initially T_0=50˚C

Bottom side chucking to Aluminum of 15˚C

with 0.5 bar pressure(50KPa)

Top side: Free convection to 20˚C environment



Energy Conservation:

$\rho VCdT/dt=-hA(T-T_\infty)-h_sA(T-T_m)$ eq(1)

Top side $h=4.5\ W/m^2/^\circ C$ for free convection

Bottom side $h_s=1.9K_s(P/E_e)^{0.94}/R_s$

$P=0.5bar=50KPa=0.5E5[Pa]$

For Glass; $K_g=0.78W/m/^\circ C$, $E_g=70GPa$, $\nu_g=0.24$

For Al; $K_a=7202\ W/m/^\circ C$, $E_a=68GPa$, $\nu_a=0.33$

$1/K_s=0.5(1/K_g+1/K_a)=0.5(K_g+K_a)/(K_gK_a)$

$\fallingdotseq 0.5K_a/(K_gK_a)=0.5/K_g$ $\therefore K_s=K_g/0.5=2K_g=1.56[W/m/^\circ C]$

$1/E_s=1/[(1-\nu_g^2)/E_g+(1-\nu_a^2)/E_a]=1/[0.9424/E_g+0.8911/E_a]$

$\therefore E_s=E_gE_a/[0.9424E_a+0.8911E_g]$

$=4760E18/[0.9424(68E9)+0.8911(70E9)]$

$=37.64E9\ [Pa]=37.64\ [GPa]$

Glass; $Rq_1=0.01[um]=0.1E-7\ [m]$

Al; $Rq_2=0.1[um]$ $\therefore R_s=(Rq_1^2+Rq_2^2)^{1/2}$

$\fallingdotseq(0.1)=0.1\ [um]=1.0E-7\ [m]$

$\therefore hs=1.9K_s(P/E_s)^{0.94}/R_s=(1.9)(1.56)[0.5*E5/37.64E9]^{0.94}/1.4E-7$

$=88.6$ [W/m^2/°C] $\gg$ h=4.5

From eq(1)

$T(t)=T_m+(T_0-T_m)\exp(-t/\tau)$

Where $\tau=\rho VC/h_sA$

$=(2600)(1)(1)(0.001)(712)/[88.6(1)]=20.8$ [sec]

:Time, τ, for 36.7% decrease

Therefore, the time constant, τ, can be shortened by increasing P (chucking pressure), or fining surface roughness of Aluminum chuck.