Momentum Transfer

The study of the motion of fluids and the forces that produce these motions

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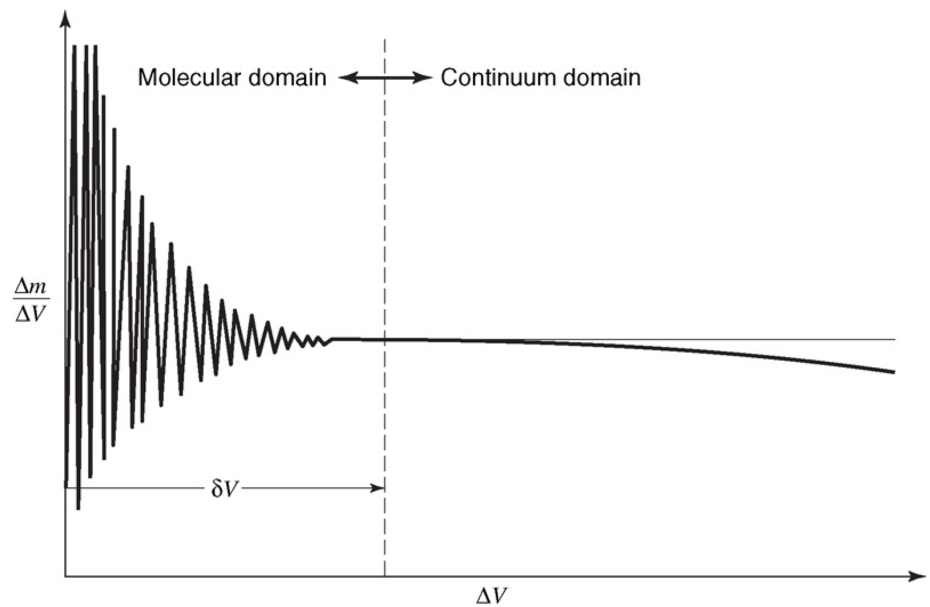
forces ← pressure, shear stress ← molecular transfer of momentum

fluid;
- a substance that deforms continuously under the action of a shear stress
- a continuous distribution of matter (continuum)

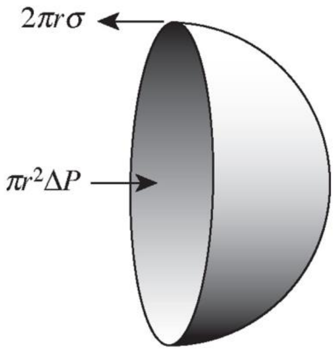
property at a point;

$$\rho = \lim_{\Delta V \to \delta V} \frac{\Delta m}{\Delta V}$$

minimum volume surrounding the point for which statistical averages are meaningful



surface tension; a property of the surface of a liquid that allows it to resist an external force



$$\pi r^2 \Delta P = 2\pi r \sigma \qquad \Delta P = \frac{2\sigma}{r}$$

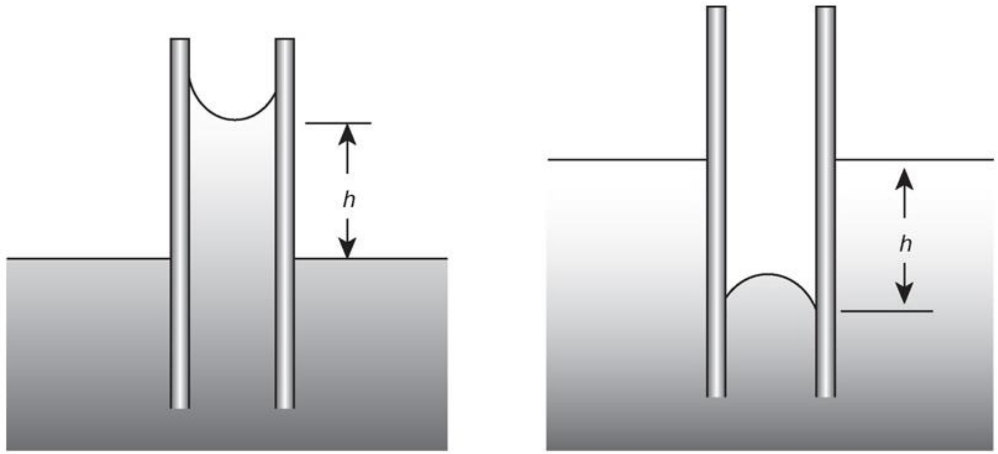
$$\boxed{p_\mathrm{i} - p_\mathrm{o} = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

For a spherical gas bubble $\quad R_1 = R_2 = R \qquad \Delta p = \dfrac{2\sigma}{R}$

For a bubble of radius 1mm $\quad \Delta p = \dfrac{2(0.05)}{0.001} = 100\ \mathrm{N/m^2}$

For a bubble of radius 1μm $\quad \Delta p = \dfrac{2(0.05)}{1\times 10^{-6}} = 1\times 10^5\ \mathrm{N/m^2} \approx 1 atm$

capillarity; the ability of liquid to flow against gravity where liquid spontaneously rises in a narrow space such as a thin tube



$$2\pi r\sigma\cos\theta = \rho g\pi r^2 h \qquad h = \frac{2\sigma\cos\theta}{\rho g r}$$

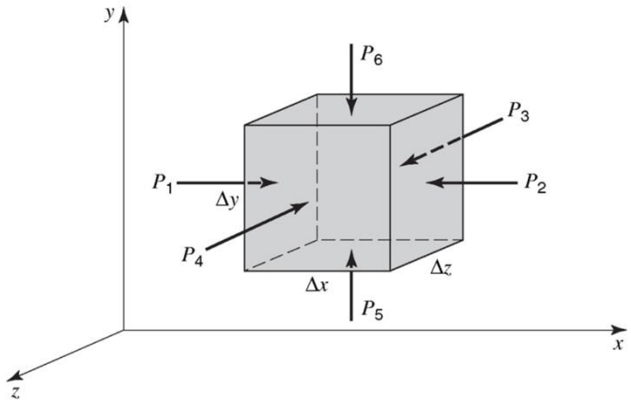
for mercury and glass $\theta = 130°$

for mercury in air $\sigma = 0.44 N/m$

Fluid statics

the only forces acting on the fluid at rest are those due to gravity and pressure

Newton's 2nd law of motion; $$\sum \mathbf{F} = m\mathbf{a}$$



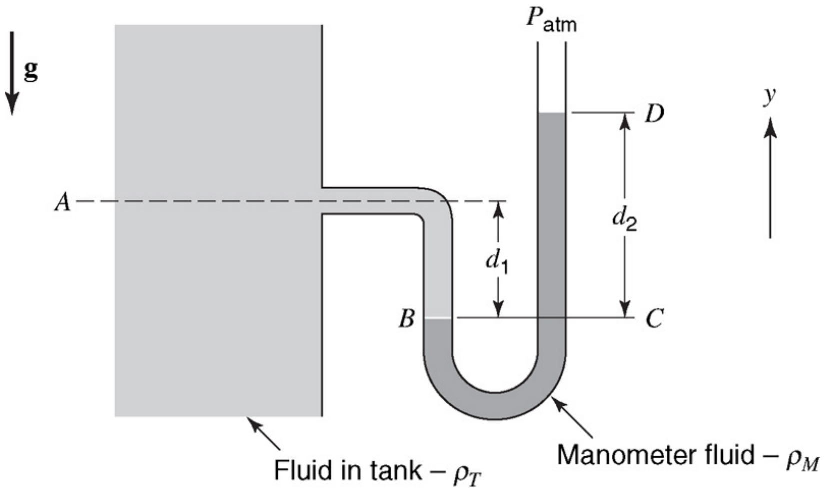
pressure forces acting on
a static fluid element

$$\rho\mathbf{g}\left(\Delta x\ \Delta y\ \Delta z\right)+\left(P\big|_x - P\big|_{x+\Delta x}\right)\Delta y\Delta z\mathbf{e_x}$$
$$+\left(P\big|_y - P\big|_{y+\Delta y}\right)\Delta x\Delta z\ \mathbf{e_y}+\left(P\big|_z - P\big|_{z+\Delta z}\right)\Delta x\Delta y\ \mathbf{e_z}=\mathbf{0}$$

$$\boxed{\rho\mathbf{g}=\nabla P}$$

$$\rho\frac{\mathrm{D}\mathbf{v}}{\mathrm{Dt}}=\rho\mathbf{g}-\nabla\mathrm{P}+\mu\nabla^2\mathbf{v}$$

U-tube manometer



$$\boxed{\rho \mathbf{g} = \nabla P}$$

$$\frac{dP}{dy} = -\rho g$$

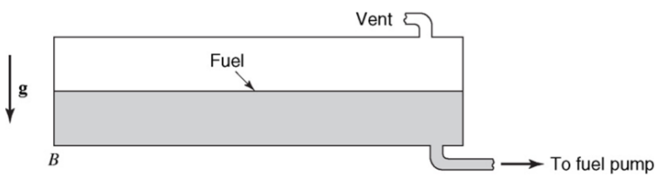
integrating between C and D in manometer fluid $\quad P_{atm} - P_C = -\rho_m g d_2$

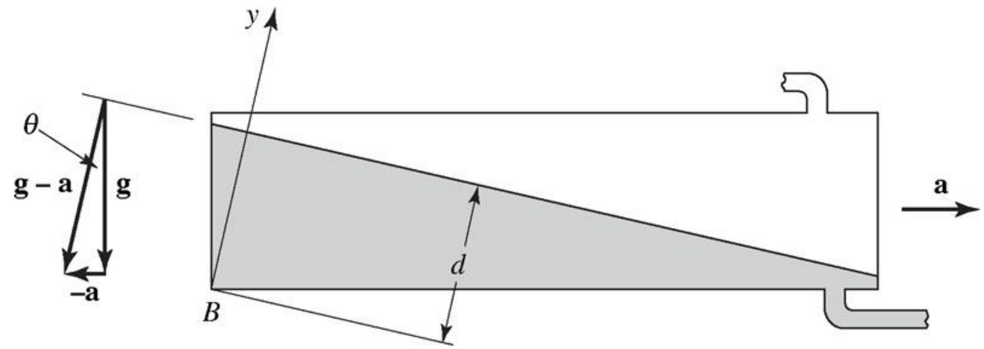
integrating between B and A in tank fluid $\quad P_A - P_B = -\rho_T g d_1$

as elevations B and C are equal, $\quad \underbrace{P_A - P_{atm}}_{\text{gage pressure}} = \rho_m g d_2 - \rho_T g d_1$

uniform rectilinear acceleration

$$\nabla P = \rho(\mathrm{g} - \mathrm{a})$$





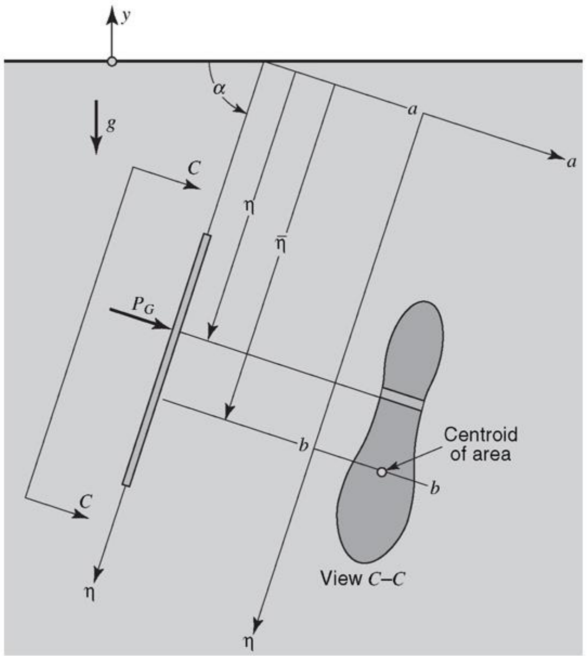
$$\frac{\mathrm{dP}}{\mathrm{dy}} = -\rho|\mathbf{g} - \mathbf{a}| = -\rho\sqrt{\mathrm{g}^2 + \mathrm{a}^2}$$

integrating between y=0 and y=d

$$\mathrm{P}_{\mathrm{atm}} - \mathrm{P}_{\mathrm{B}} = \rho\sqrt{\mathrm{g}^2 + \mathrm{a}^2}(-\mathrm{d})$$

$$\mathrm{P}_{\mathrm{B}} - \mathrm{P}_{\mathrm{atm}} = \rho\sqrt{\mathrm{g}^2 + \mathrm{a}^2}(\mathrm{d})$$

forces on submerged surfaces



we must specify;

1. the magnitude of the force
2. the direction of the force
3. the line of action of the force

$$P_G = -\rho g y = \rho g \eta \sin\alpha$$

$$dF = \rho g \eta \sin\alpha \; dA$$

centroid of area

$$F = \rho g \; \sin\alpha \int_A \eta \; dA = \underline{\rho g \; \sin\alpha\, \bar{\eta}} A \qquad \bar{\eta} \equiv \frac{1}{A}\int_A \eta \; dA$$

pressure at centroid

the point at which this force acts (center of pressure) is not the centroid of the area total force on the plate must be concentrated in order to produce the same moment as the distributed pressure

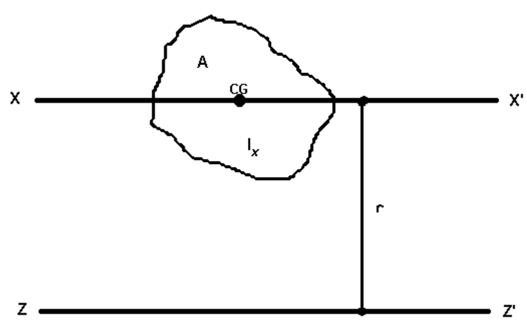
$$F\eta_{c.p.} = \int_A \eta P_G \ dA$$

$$F\eta_{c.p.} = \int_A \rho g \sin\alpha \eta^2 \ dA \qquad F = \rho g \ \sin\alpha \bar{\eta} A$$

$$\eta_{c.p.} = \frac{1}{A\bar{\eta}} \int_A \eta^2 dA = \frac{I_{aa}}{A\bar{\eta}}$$

moment of area about the surface

parallel axis theorem



$$I_z = I_x + Ar^2,$$

where:

I_z is the area moment of inertia of D relative to the parallel axis;

I_x is the area moment of inertia of D relative to its centroid;

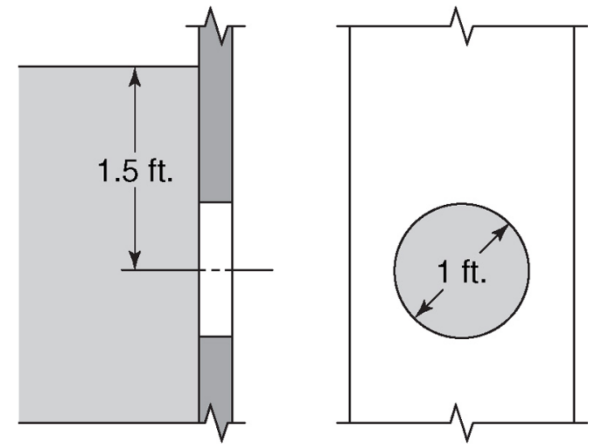
A is the area of the plane region D;

r is the distance from the new axis z to the centroid of the plane region D.

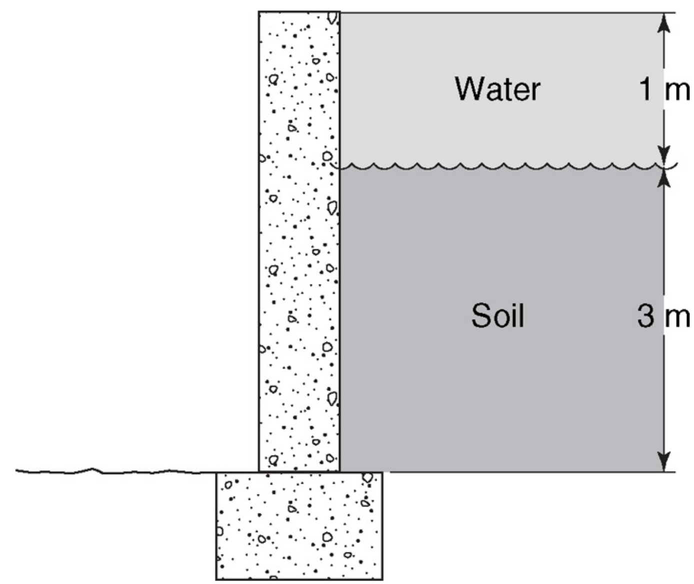
$$I_{aa} = I_{bb} + \bar{\eta}^2 A \qquad \boxed{\eta_{c.p.} - \bar{\eta} = \frac{I_{bb}}{A\bar{\eta}}}$$

the center of pressure is located below the centroid

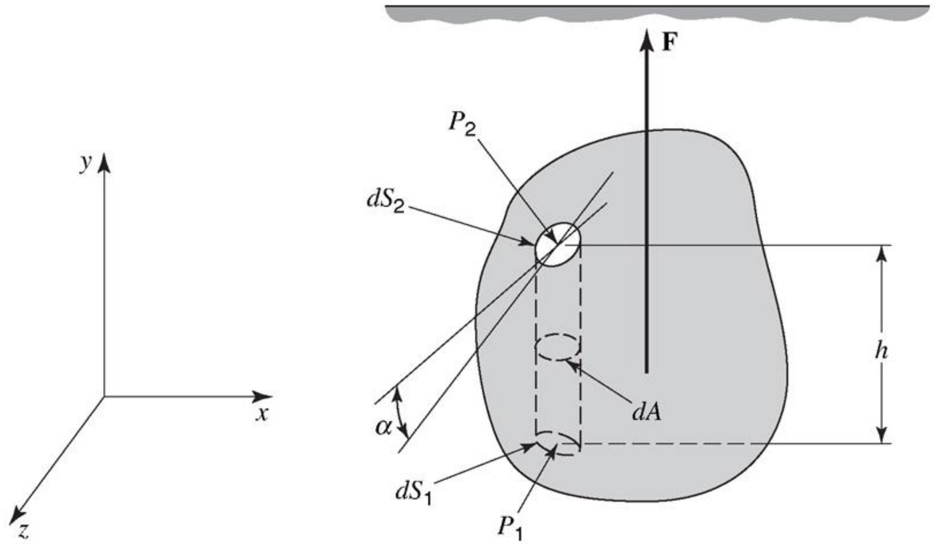
Ex 4; circular viewing port is located below the surface of the tank -> find the magnitude and location of the force acting on the window



Ex 5; rainwater collector; determine the force and center of pressure on the wall



buoyancy



$$d\mathbf{F} = \left(P_1 - P_2\right) dA\, \mathbf{e}_y - \rho_B g h \;\; dA\, \mathbf{e}_y$$

$$d\mathbf{F} = (\rho - \rho_B) g h dA \mathbf{e}_y$$

$$\mathbf{F} = (\rho - \rho_B) g V \mathbf{e}_y$$

fundamental physical laws

except relativistic and nuclear phenomena

1. the law of conservation of mass -> continuity equation
2. Newton's second law of motion ; momentum balance equation
3. the first law of thermodynamics ; energy balance equation