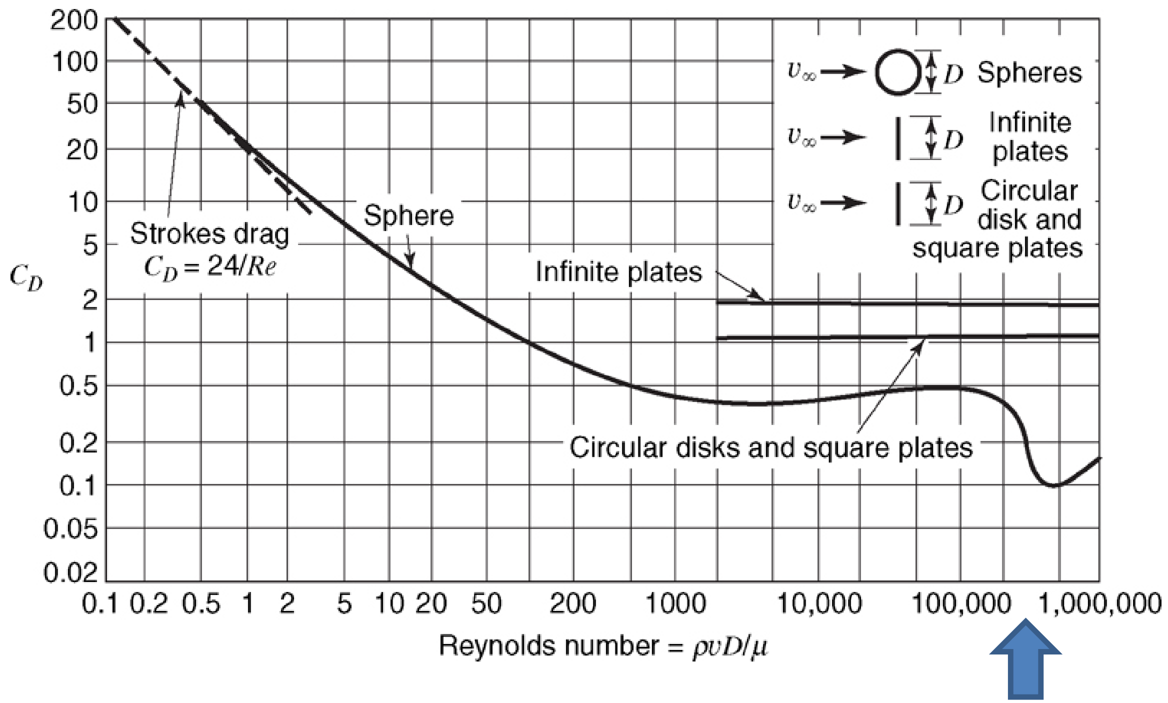
viscous flow

pressure flow ; flow inside a conduit

<u>drag flow</u> ; external flow, flow around a body

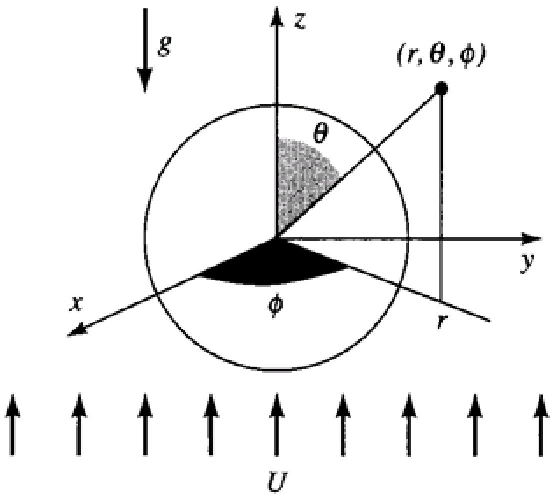
$$\frac{F}{A_P} \equiv C_D \frac{\rho v_\infty^2}{2}$$



change from laminar to turbulent in the boundary layer

$$C_D = \frac{8}{\pi}\frac{F_D}{\rho U^2 D^2} = \frac{24\mu}{\rho U D} = \frac{24}{\mathrm{Re}}$$

Slow flow around a solid sphere



The flow is laminar and symmetric about z-axis

$$\partial/\partial\phi = 0, u_\phi = 0$$

$$\rho\left[u_r\frac{\partial u_r}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_r}{\partial\theta}-\frac{u_\theta^2}{r}\right]=-\frac{\partial p}{\partial r}+\mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2u_r)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u_r}{\partial\theta}\right)\right]$$

$$\rho\left[u_r\frac{\partial u_\theta}{\partial r}+\frac{u_\theta}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{u_ru_\theta}{r}\right]=-\frac{1}{r}\frac{\partial p}{\partial\theta}+\mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u_\theta}{\partial r}\right)\right.$$
$$\left.+\frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(u_\theta\sin\theta)\right)+\frac{2}{r^2}\frac{\partial u_r}{\partial\theta}\right]$$

$$0=\left[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2u_r)+\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta u_\theta)\right]$$

$$s\equiv\frac{r}{R},\quad \tilde{u}_r\equiv\frac{u_r}{U},\quad \tilde{u}_\theta\equiv\frac{u_\theta}{U},\quad \tilde{p}=\frac{p}{\mu U/R}$$

$$\frac{\rho UR}{\mu}\left[\tilde{u}_r\frac{\partial\tilde{u}_r}{\partial s}+\frac{\tilde{u}_\theta}{s}\frac{\partial\tilde{u}_r}{\partial\theta}-\frac{\tilde{u}_\theta^2}{s}\right]=-\frac{\partial\tilde{p}}{\partial s}+\left[\frac{1}{s^2}\frac{\partial^2}{\partial s^2}(s^2\tilde{u}_r)+\frac{1}{s^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tilde{u}_r}{\partial\theta}\right)\right]$$

$$\frac{\rho UR}{\mu}\left[\tilde{u}_r\frac{\partial\tilde{u}_\theta}{\partial s}+\frac{\tilde{u}_\theta}{s}\frac{\partial\tilde{u}_\theta}{\partial\theta}+\frac{\tilde{u}_r\tilde{u}_\theta}{s}\right]=-\frac{1}{s}\frac{\partial\tilde{p}}{\partial\theta}+\left[\frac{1}{s^2}\frac{\partial}{\partial s}\left(s^2\frac{\partial\tilde{u}_\theta}{\partial s}\right)\right.$$
$$\left.+\frac{1}{s^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\tilde{u}_\theta\sin\theta)\right)+\frac{2}{s^2}\frac{\partial\tilde{u}_r}{\partial\theta}\right]$$

Creeping flow

Stokes flow; *Re=0*

BCs $\tilde{u}_\theta = \tilde{u}_r = 0$ on the surface $r = R$

$$\left.\begin{aligned} u_\theta &= -U\sin\theta \\ u_r &= U\cos\theta \end{aligned}\right\} \quad \text{for} \quad r \to \infty \quad \Rightarrow \quad \tilde{u}_\theta = G(s)\sin\theta \quad \text{and} \quad \tilde{u}_r = F(s)\cos\theta$$

Continuity $\left[F(s) + \frac{s}{2}\frac{dF(s)}{ds}\right] + G(s) = 0$

Cross differentiation of NS equations, and continutiy

$$s^4 \frac{d^4F(s)}{ds^4} + 8s^3\frac{d^3F(s)}{ds^3} + 8s^2\frac{d^2F(s)}{ds^2} - 8s\frac{dF(s)}{ds} = 0$$

Linear homogeneous ode

$$F(s) = s^n \qquad F(s) = as^{-3} + bs^{-1} + cs^0 + ds^2$$

$$\begin{aligned} &F = 0 \quad \text{on} \quad s = 1 && G = 0 \quad \text{on} \quad s = 1 \\ &F = 1 \quad \text{for} \quad s \to \infty && G = -1 \quad \text{for} \quad s \to \infty \end{aligned} \quad \Rightarrow \quad \begin{aligned} &\frac{dF}{ds} = 0 \quad \text{on} \quad s = 1 \\ &\frac{dF}{ds} = 0 \quad \text{for} \quad s \to \infty \end{aligned}$$

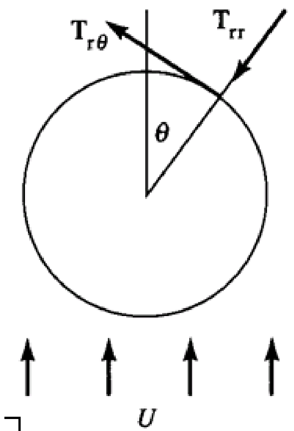
$$F(s) = 1 - \frac{3}{2s} + \frac{1}{2s^3}$$

$$G(s) = -1 + \frac{3}{4s} + \frac{1}{4s^3}$$

Velocity profile *(s=r/R)*

$$\tilde{p} = \tilde{p}_\infty - \frac{3}{2s^2}\cos\theta$$

The stress acting on the surface of the sphere



Force due to the normal stress in the direction of motion

$$F_{\mathrm{P}} = \int_0^{2\pi}\int_0^{\pi} [-T_{rr}(R,\theta)\cos\theta] R^2 \sin\theta \, d\theta \, d\phi = 2\pi \int_0^{\pi} \left[\left(-p + 2\mu \frac{\partial u_r}{\partial r} \right)_R \cos\theta \right] R^2 \sin\theta \, d\theta$$

$$F_{\mathrm{P}} = 2\pi\mu RU$$

Force due to the shear stress in the direction of motion

$$F_{\mathrm{V}} = \int_0^{2\pi}\int_0^{\pi} [T_{r\theta}(R,\theta)\sin\theta] R^2 \sin\theta \, d\theta \, d\phi$$

$$F_{\mathrm{V}} = 4\pi\mu RU$$

Total drag force $$F_{\mathrm{SL}} = \underbrace{F_{\mathrm{P}}}_{\text{Form drag}} + \underbrace{F_{\mathrm{V}}}_{\text{Frictional drag}} = 6\pi\mu RU = 3\pi\mu UD$$

$$\mathrm{Re} = \frac{2\rho_{\mathrm{f}} UR}{\mu} \leq 1 \quad \text{for Stokes' law to hold}$$

$$F_{\mathrm{D}} = 3\pi\mu UD$$

$$C_{\mathrm{D}} = \frac{8}{\pi}\frac{F_D}{\rho U^2 D^2} = \frac{24\mu}{\rho UD} = \frac{24}{\mathrm{Re}}$$

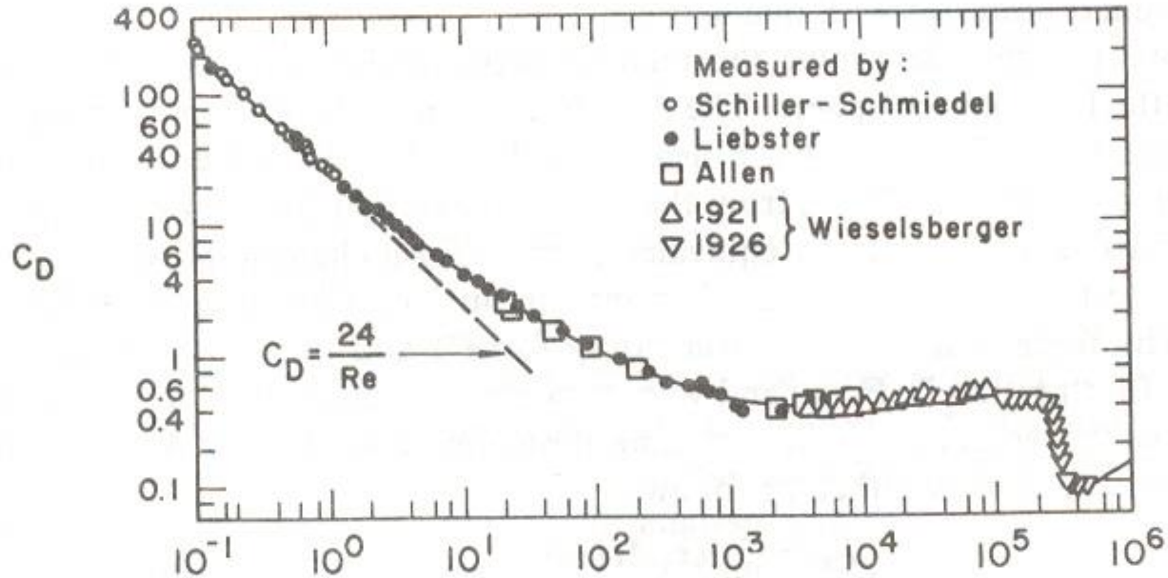


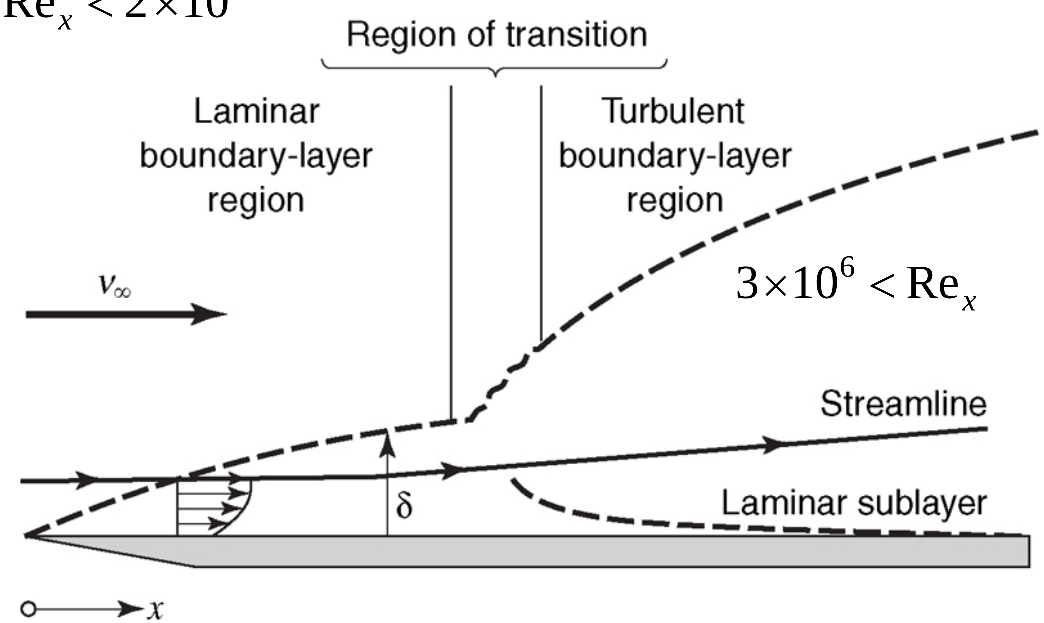
Figure 4-1. Drag coefficient as a function of Reynolds number for flow past a sphere. Reproduced from H. Schlichting, *Boundary Layer Theory*, 6th ed., McGraw-Hill Book Company, New York, 1968, by permission.

boundary layer

; the effects of fluid friction at high Reynolds numbers are limited to a thin layer near the boundary of a body
; no significant pressure change across the boundary layer ->the only unknowns are the velocity components

$$2\times10^5 < \mathrm{Re}_x < 3\times10^6$$

$$\mathrm{Re}_x < 2\times10^5$$



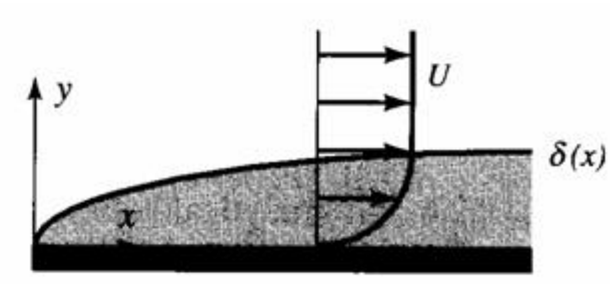
$$3\times10^6 < \mathrm{Re}_x$$

$$\mathrm{Re}_x \equiv \frac{xv\rho}{\mu}$$

thickness ; when the velocity reaches 99% of the free-stream velocity

Laminar boundary layer

No matter how turbulent the flow is far from the surface



$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}=0$$

$$\rho\left(u_x\frac{\partial u_y}{\partial x}+u_y\frac{\partial u_y}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 u_y}{\partial x^2}+\frac{\partial^2 u_y}{\partial y^2}\right)$$

$$u_y=0;\quad \frac{\partial p}{\partial y}=0 \quad \text{within the boundary layer}$$

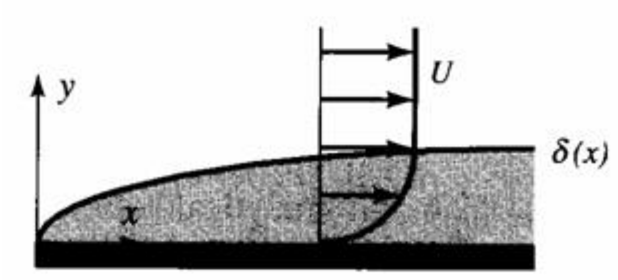
$$\rho\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}\right)=-\cancel{\frac{\partial p}{\partial x}}+\mu\left(\cancel{\frac{\partial^2 u_x}{\partial x^2}}+\frac{\partial^2 u_x}{\partial y^2}\right)$$

Negligible viscous effect outside the boundary layer

$$\frac{\partial u_x}{\partial x}+\frac{\partial u_y}{\partial y}=0 \qquad \rho\left(u_x\frac{\partial u_x}{\partial x}+u_y\frac{\partial u_x}{\partial y}\right)=\mu\frac{\partial^2 u_x}{\partial y^2}$$

$$\boxed{\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=\nu\frac{\partial^3 \psi}{\partial y^3}}$$

Boundary conditions



$$u_x=u_y=0 \quad \text{on} \quad y=0, \text{ for } x>0$$

$$u_x=U \quad \text{for} \quad x<0$$

$$u_x\to U \quad \text{for} \quad y\to\infty$$

$$\frac{\partial \psi}{\partial y}=-\frac{\partial \psi}{\partial x}=0 \quad \text{on} \quad y=0, \text{ for } x>0$$

$$\frac{\partial \psi}{\partial y}\to U \quad \text{for} \quad x<0, \text{ and } y\to\infty$$

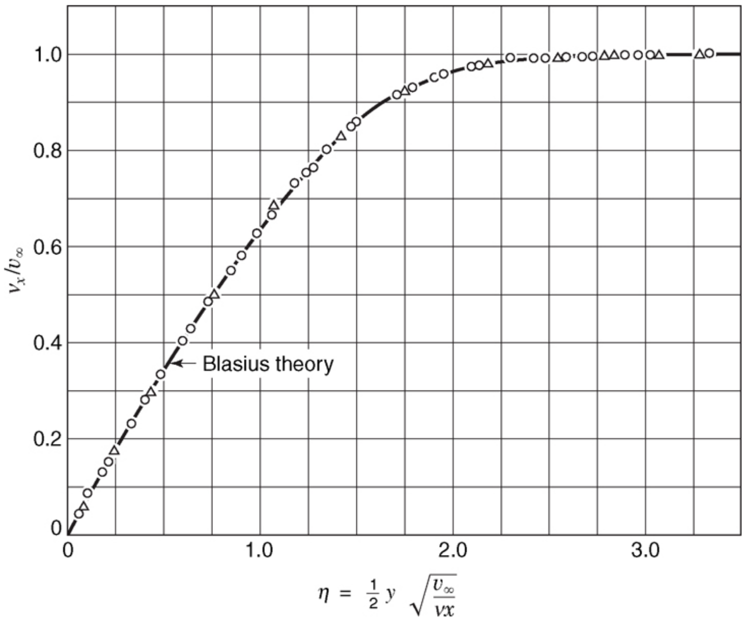
Blasius' similarity transformation

$$f(\eta)\equiv\frac{\psi}{(U\nu x)^{1/2}} \qquad \eta\equiv\frac{y}{2}\left(\frac{U}{\nu x}\right)^{1/2}$$

$$\boxed{f'''+ff''=0}$$

$$f=f'=0 \quad \text{at} \quad \eta=0$$

$$f'\to 2 \quad \text{as} \quad \eta\to\infty$$



U_x becomes nearly U
along the boundary layer $\quad y = \delta(x)$

$$\delta \left(\frac{U}{\nu x} \right)^{1/2} = 5$$

Boundary layer thickness

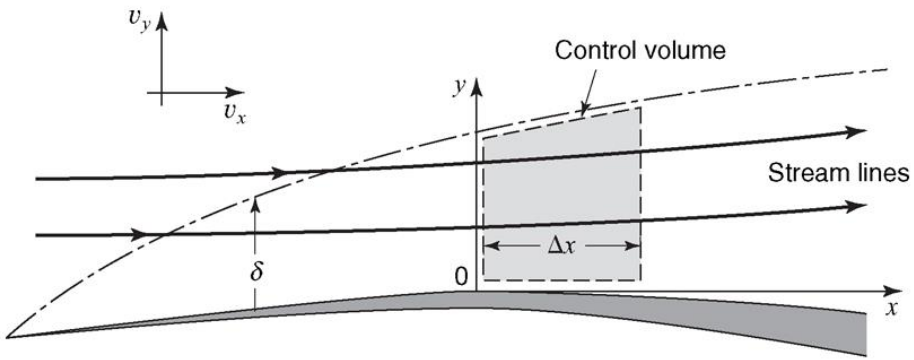
Friction coeff. $\quad C_f \equiv \dfrac{-\mu (\partial u_x / \partial y)_{y=0}}{\frac{1}{2} \rho U^2} \equiv 0.664 \left(\dfrac{\nu}{xU} \right)^{1/2} = \dfrac{0.664}{(\mathrm{Re}_x)^{1/2}}$

Shear force $\quad F_s = W \int_0^L - \mu \left(\dfrac{\partial u_x}{\partial y} \right)_{y=0} dx \qquad \dfrac{F_s / LW}{\frac{1}{2} \rho U^2} \equiv \bar{C}_f = \dfrac{1.328}{(UL/\nu)^{1/2}} = \dfrac{1.328}{(\mathrm{Re}_L)^{1/2}}$

Local Reynolds number $\quad \mathrm{Re}_x \equiv \dfrac{xU}{\nu} \qquad \mathrm{Re}_x^{\mathrm{trans}} = 5 \times 10^5$

Blasius solution; applies only to the laminar boundary layer over a flat surface

von Karman momentum integral analysis



$$\sum \mathrm{F_x} = \iint_{\mathrm{c.s.}} v_{\mathrm{x}} \rho (\mathrm{v} \cdot \mathrm{n}) \mathrm{dA} + \frac{\partial}{\partial \mathrm{t}} \iiint_{\mathrm{c.v.}} v_{\mathrm{x}} \rho \, \mathrm{dV}$$

$$\sum F_x = P\delta\big|_x - P\delta\big|_{x+\Delta x} + \left(P\big|_x + \frac{P\big|_{x+\Delta x} - P\big|_x}{2} \right)\left(\delta\big|_{x+\Delta x} - \delta\big|_x\right) - \underline{\tau_0 \Delta x}$$

frictional force on the bottom

$$\iint_{\mathrm{c.s.}} v_{\mathrm{x}} \rho (\mathrm{v} \cdot \mathrm{n}) \mathrm{dA} = \int_0^{\delta} \rho v_{\mathrm{x}}^2 \mathrm{dy}\Big|_{\mathrm{x+\Delta x}} - \int_0^{\delta} \rho v_{\mathrm{x}}^2 \mathrm{dy}\Big|_{\mathrm{x}} - \underline{v_{\infty} \dot{\mathrm{m}}_{\mathrm{top}}}$$

from mass balance

von Karman momentum integral expression

$$\frac{\tau_0}{\rho}=\left(\frac{d}{dx}v_\infty\right)\int_0^\delta (v_\infty - v_x)\,dy+\frac{d}{dx}\int_0^\delta v_x(v_\infty - v_x)\,dy$$

laminar flow over a flat plate

$$\frac{\tau_0}{\rho}=\frac{d}{dx}\int_0^\delta v_x(v_\infty - v_x)\,dy \qquad v_x = a+by+cy^2+dy^3$$

$$v_x = 0 \quad \text{at } y=0$$

$$v_x = v_\infty \quad \text{at } y=\delta$$

$$\frac{\partial v_x}{\partial y}=0 \quad \text{at } y=\delta$$

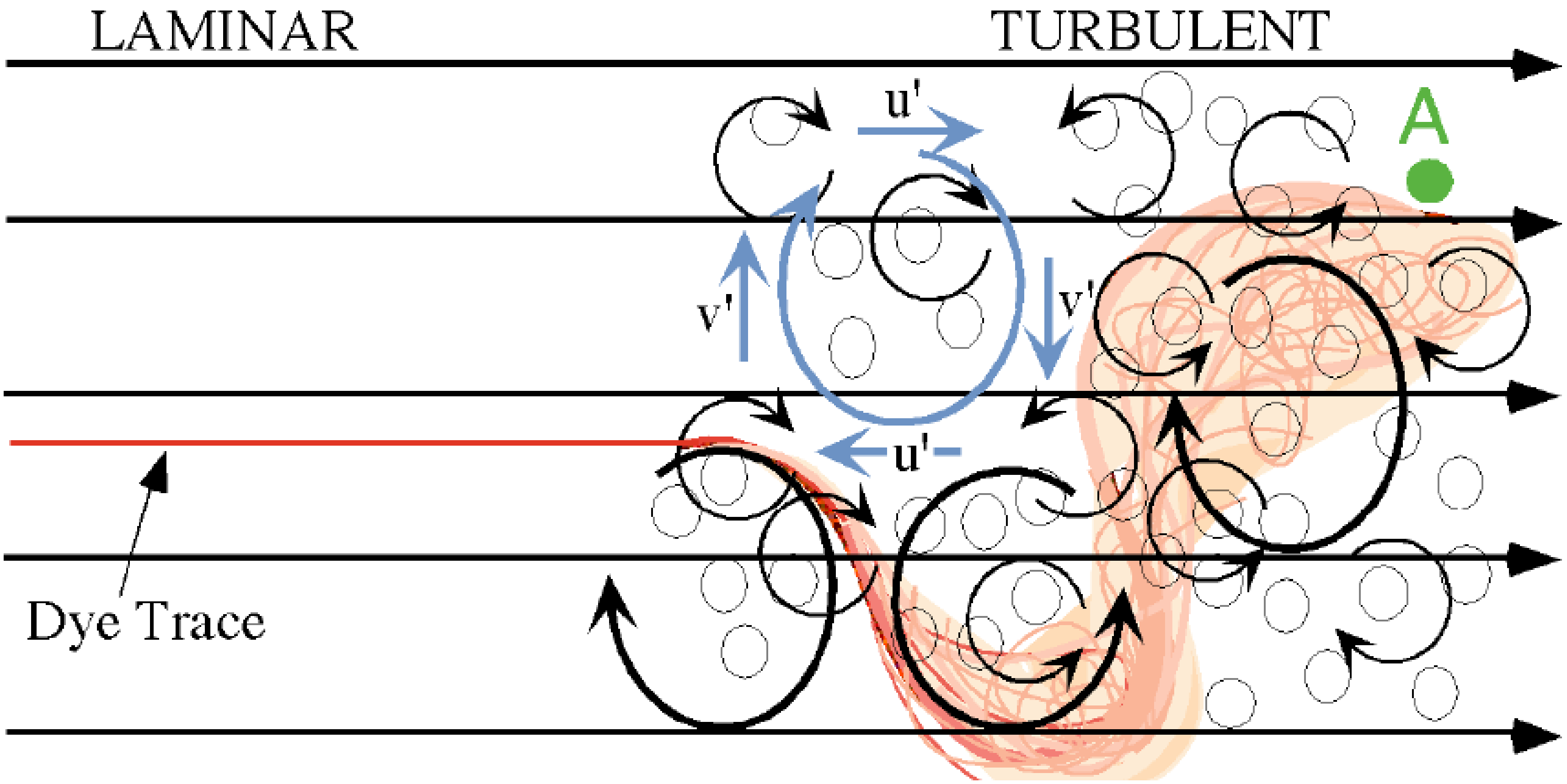
$$\frac{\partial^2 v_x}{\partial y^2}=0 \quad \text{at } y=0$$

$$\frac{v_x}{v_\infty}=\frac{3}{2}\left(\frac{y}{\delta}\right)-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

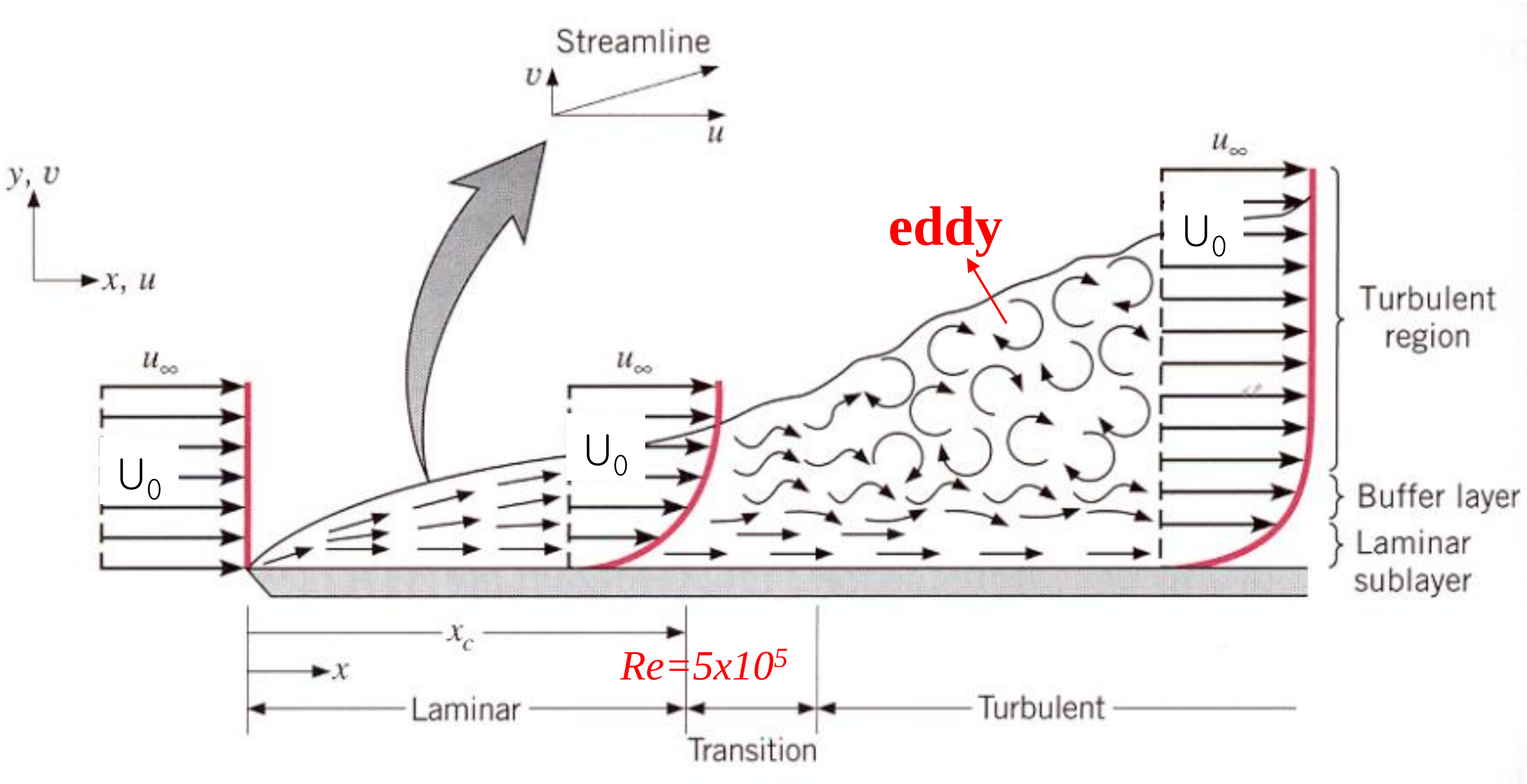
von Karman	Blasius
$\frac{\delta}{x}=\frac{4.64}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x}=\frac{5}{\sqrt{\mathrm{Re}_x}}$
$C_{fx}=\frac{0.646}{\sqrt{\mathrm{Re}_x}}$	$C_{fx}=\frac{0.664}{\sqrt{\mathrm{Re}_x}}$
$C_{fL}=\frac{1.292}{\sqrt{\mathrm{Re}_L}}$	$C_{fL}=\frac{1.328}{\sqrt{\mathrm{Re}_L}}$

turbulent flow

The fluctuations in velocity components of turbulent flow



turbulent flow



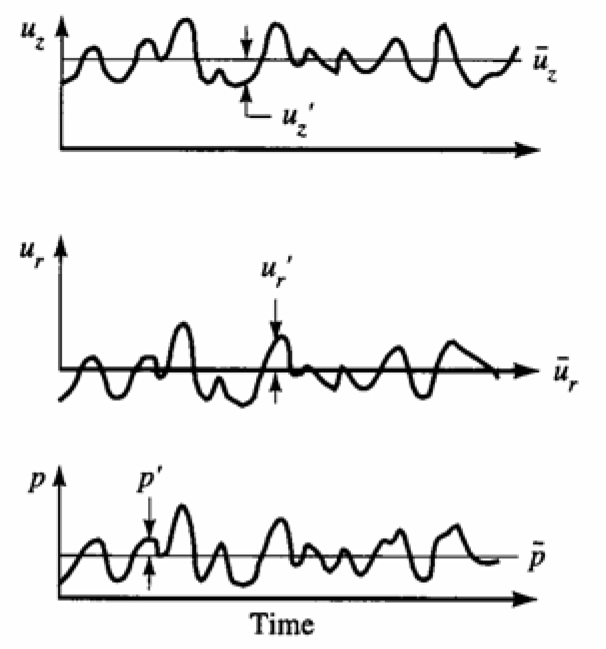
laminar

turbulent

$$u_z\big|_{r=0} = \frac{2Q}{\pi R^2}$$

$$u_r\big|_{r=0} = 0$$

$$p\big|_{z=L/2} = \frac{4\mu LQ}{\pi R^4}$$



Random fluctuating funtions of time

Characterized by mean, intensity, frequency spectrum

$$\bar{u}_z \equiv \frac{1}{\tau}\int_t^{t+\tau} u_z \, dt$$

Time averaging

$$\overline{S} \equiv \frac{1}{\tau}\int_t^{t+\tau} S\,dt \qquad \overline{S} \equiv \frac{1}{\tau}\int_t^{t+\tau} (\overline{S}+S')\,dt = \overline{\overline{S}} + \overline{S'} \qquad \overline{S'} \equiv 0$$

$$\overline{TS} \equiv \frac{1}{\tau}\int_t^{t+\tau} (\overline{T}+T')(\overline{S}+S')\,dt = \overline{T}\,\overline{S} + \overline{T'\overline{S}} + \overline{\overline{T}S'} + \overline{T'S'}$$

$$= \overline{T}\,\overline{S} + \overline{T'S'}$$

$$u_z = \overline{u}_z + u'_z$$

$$u_x = \overline{u}_x + u'_x$$

$$u_y = \overline{u}_y + u'_y$$

$$p = \overline{p} + p'$$

$$\frac{\partial \overline{u}_x}{\partial x} + \frac{\partial \overline{u}_y}{\partial y} + \frac{\partial \overline{u}_z}{\partial z} = 0$$

$$\frac{\partial}{\partial t}\rho\overline{u}_z + \frac{\partial}{\partial x}\rho\overline{u}_z\overline{u}_x + \frac{\partial}{\partial y}\rho\overline{u}_z\overline{u}_y + \frac{\partial}{\partial z}\rho\overline{u}_z\overline{u}_z = -\frac{\partial \overline{p}}{\partial z} + \mu\nabla^2\overline{u}_z + \rho g_z$$

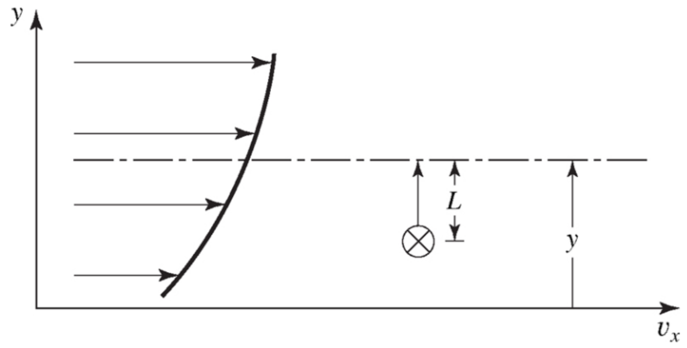
$$-\frac{\partial}{\partial x}\rho\overline{u'_z u'_x} - \frac{\partial}{\partial y}\rho\overline{u'_z u'_y} - \frac{\partial}{\partial z}\rho\overline{u'_z u'_z}$$

Reynolds stress

Convection of momentum by velocity fluctuations

$$\tau_{yx} = \mu\frac{d\overline{v}_x}{dy} - \overline{\rho v'_x v'_y}$$

mixing length theory



velocity fluctuation is hypothesized as being due to the y-directional motion of a lump of fluid through a distance L

$$\bar{v}_x\Big|_{y\pm L}-\bar{v}_x\Big|_{y}=\pm L\frac{d\bar{v}_x}{dy} \qquad v'_x=\pm L\frac{d\bar{v}_x}{dy}$$

$$\overline{v'_x v'_y}=-(\text{constant})L^2\left|\frac{d\bar{v}_x}{dy}\right|\frac{d\bar{v}_x}{dy} \qquad \overline{v'_x v'_y}=-L^2\left|\frac{d\bar{v}_x}{dy}\right|\frac{d\bar{v}_x}{dy}$$

$$L=Ky$$

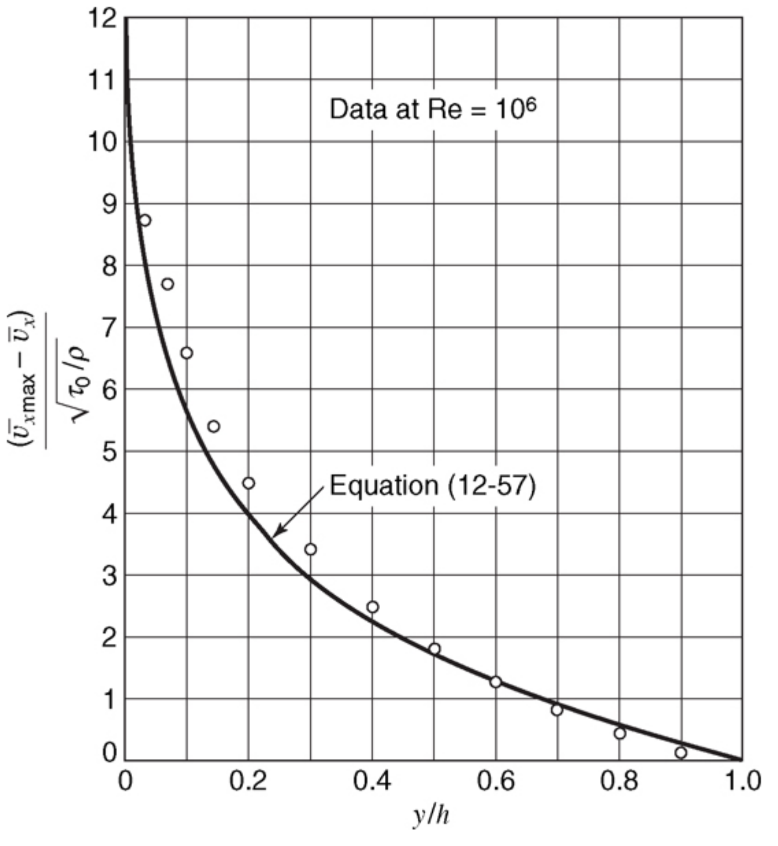
close to the wall, the mixing length is assumed to vary directly with y

$$\tau_{yx}^{\ turb}=-\overline{\rho v'_x v'_y}=\rho K^2 y^2\left(\frac{d\bar{v}_x}{dy}\right)^2=\tau_0(\text{constant})$$

$$\tau_{yx}^{turb} = -\overline{\rho v'_x v'_y} = \rho K^2 y^2 \left(\frac{d\bar{v}_x}{dy} \right)^2 = \tau_0 \left(\text{constant} \right)$$

$$\frac{d\bar{v}_x}{dy} = \frac{\sqrt{\tau_0 / \rho}}{Ky} \qquad \bar{v}_x = \frac{\sqrt{\tau_0 / \rho}}{K} \ln y + C \qquad \bar{v}_x = \bar{v}_{x,\max} \; at \; y = h$$

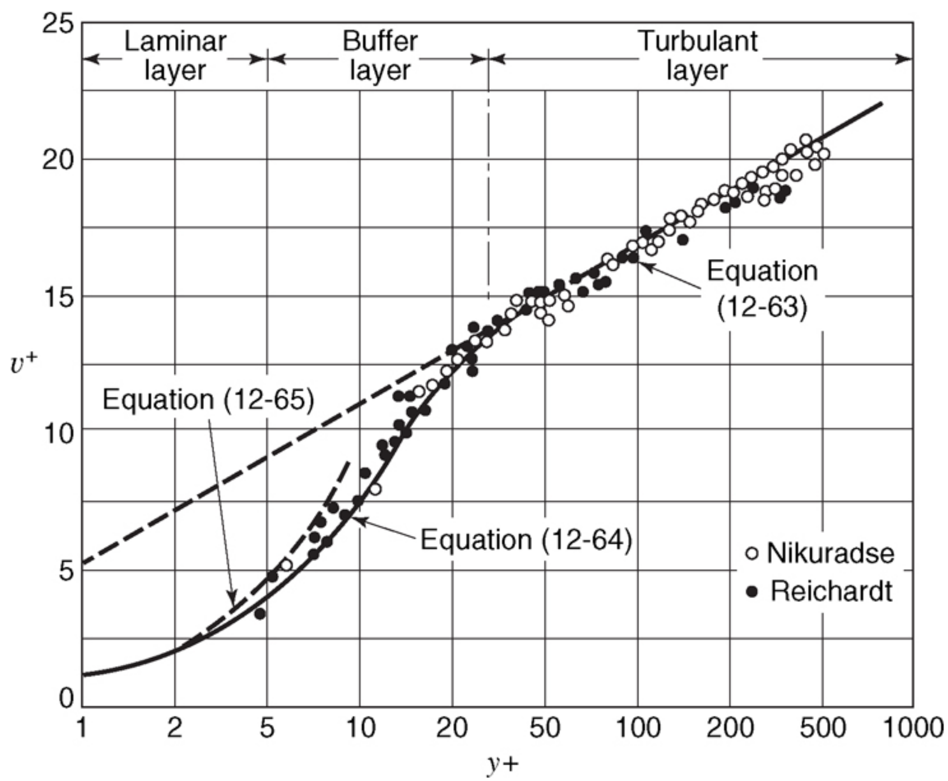
$$\frac{\bar{v}_{x\max} - \bar{v}_x}{\sqrt{\tau_0 / \rho}} = -\frac{1}{K} \left[\ln \frac{y}{h} \right]$$



turbulent flow in smooth tubes

$$\bar{v}_x = \frac{\sqrt{\tau_0/\rho}}{K} \ln y + C \qquad v^+ \equiv \frac{\bar{v}_x}{\sqrt{\tau_0/\rho}} \qquad v^+ = \frac{1}{K}[\ln y] + C$$

$$y^+ \equiv \frac{\sqrt{\tau_0/\rho}}{\nu} y \qquad v^+ = \frac{1}{K} \ln \frac{\nu y^+}{\sqrt{\tau_0/\rho}} + C = \frac{1}{K}\left(\ln y^+ + \ln \beta\right) \qquad v^+ = f(y^+)$$



universal velocity distribution

$5 > y^+ > 0$

$$v^+ = y^+$$

$30 \geq y^+ \geq 5$

$$v^+ = -3.05 + 5 \ln y^+$$

$y^+ \geq 30$

$$v^+ = 5.5 + 2.5 \ln y^+$$

von Karman (laminar)	Blasius (laminar)	von Karman (turbulent)
$\dfrac{\delta}{x} = \dfrac{4.64}{\sqrt{\mathrm{Re}_x}}$	$\dfrac{\delta}{x} = \dfrac{5}{\sqrt{\mathrm{Re}_x}}$	$\dfrac{\delta}{x} = \dfrac{0.376}{\mathrm{Re}_x^{1/5}}$
$C_{fx} = \dfrac{0.646}{\sqrt{\mathrm{Re}_x}}$	$C_{fx} = \dfrac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{fx} = \dfrac{0.0576}{\mathrm{Re}_x^{1/5}}$
$C_{fL} = \dfrac{1.292}{\sqrt{\mathrm{Re}_L}}$	$C_{fL} = \dfrac{1.328}{\sqrt{\mathrm{Re}_L}}$	

flow in closed conduits

dimensional analysis

variables
; viscosity, density, pipe diameter, length, roughness, velocity, pressure drop

Buckingham pi theorem
; # of independent dimensionless groups = 7-3 = 4

core group v, D, ρ

$$\pi_1 = v^a D^b \rho^c \Delta P$$
$$\pi_2 = v^d D^e \rho^f L$$
$$\pi_3 = v^g D^h \rho^i e$$
$$\pi_4 = v^j D^k \rho^l \mu$$

$$\pi_1 = \frac{\Delta P}{\rho v^2} = \frac{h_L}{v^2 / g}$$ Euler number

$$\pi_2 = \frac{L}{D}$$

$$\pi_3 = \frac{e}{D}$$

$$\pi_4 = \frac{v D \rho}{\mu}$$ Reynolds number

$$\frac{h_L}{v^2 / g} = \phi_1\left(\frac{L}{D}, \frac{e}{D}, \mathrm{Re}\right) \qquad \frac{h_L}{v^2 / g} = \frac{L}{D}\phi_2\left(\frac{e}{D}, \mathrm{Re}\right) \qquad h_L = 2 f_f \frac{L}{D}\frac{v^2}{g}$$

fanning friction factor
skin friction coefficient

friction factor in laminar flow

$$v_x = v_{\max}\left[1-\left(\frac{r}{R}\right)^2\right] \qquad v_{avg} = -\left(\frac{dP}{dx}\right)\frac{R^2}{8\mu} \qquad -\frac{dP}{dx} = \frac{8\mu v_{avg}}{R^2} = \frac{32\mu v_{avg}}{D^2}$$

$$-\int_{P_0}^{P} dP = 32\frac{\mu v_{avg}}{D^2}\int_0^L dx \qquad \Delta P = 32\frac{\mu v_{avg} L}{D^2}$$

$$h_L = \frac{\Delta P}{\rho g} = 32\frac{\mu v_{avg} L}{g\rho D^2} = 2 f_f \frac{L}{D}\frac{v^2}{g} \qquad \boxed{f_f = 16\frac{\mu}{D v_{avg}\rho} = \frac{16}{\mathrm{Re}}}$$

friction factor in turbulent flow

$$v^{+} = 5.5 + 2.5 \ln y^{+} \qquad v^{+} \equiv \frac{\bar{v}}{\sqrt{\tau_0 / \rho}} \qquad y^{+} \equiv \frac{\sqrt{\tau_0 / \rho}}{\nu} y$$

$$v_{avg} = \frac{\int_0^A \bar{v}\, dA}{A} = \frac{\sqrt{\tau_0 / \rho} \int_0^R \left(2.5 \ln \left\{ \frac{\sqrt{\tau_0 / \rho}\, y}{\nu} \right\} + 5.5 \right) 2\pi r\, dr}{\pi R^2} \qquad y = R - r$$

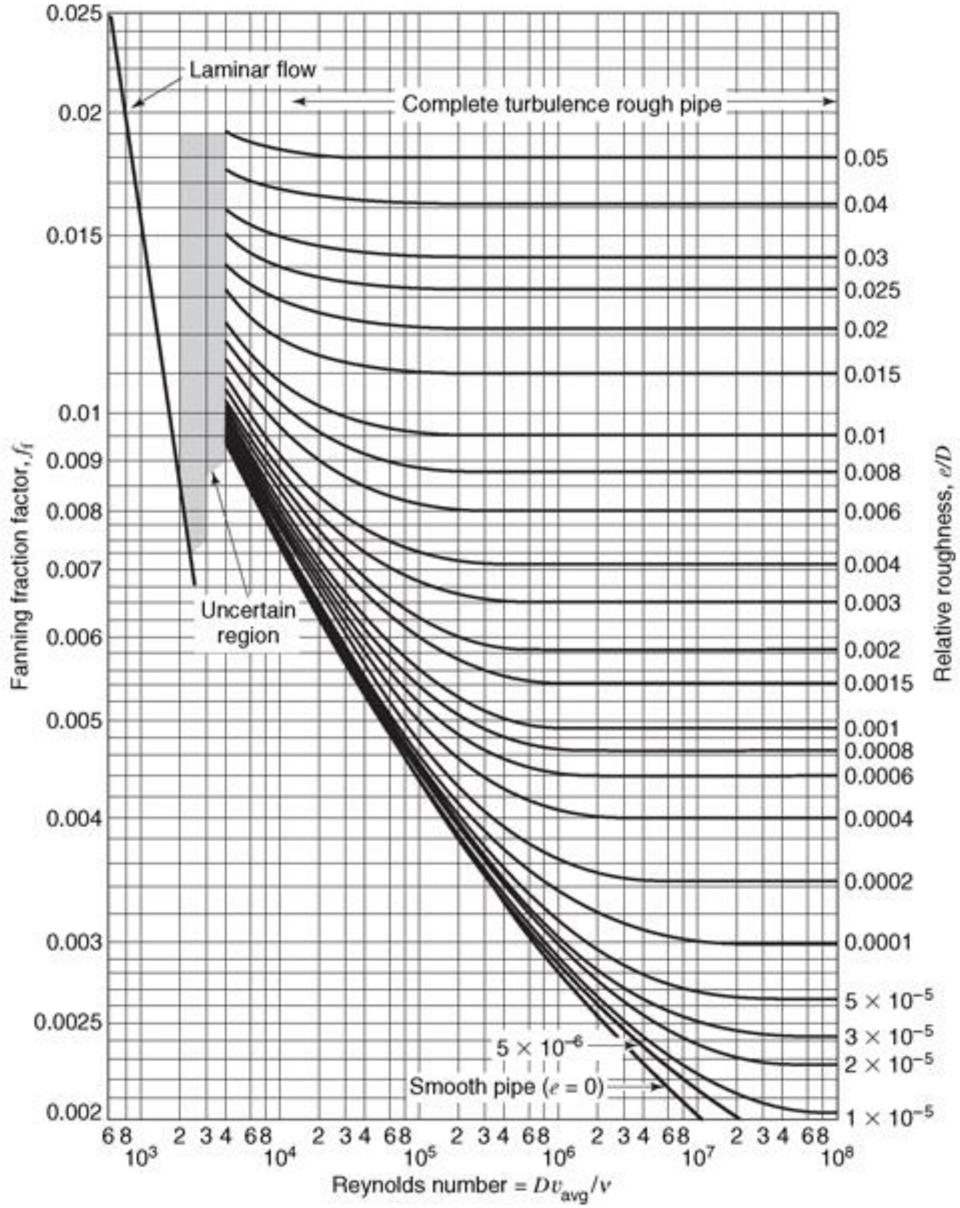
$$v_{avg} = 2.5 \sqrt{\tau_0 / \rho} \ln \left\{ \frac{\sqrt{\tau_0 / \rho}\, R}{\nu} \right\} + 1.75 \sqrt{\tau_0 / \rho} \qquad \frac{v_{avg}}{\sqrt{\tau_0 / \rho}} = \frac{1}{\sqrt{f_f / 2}}$$

$$\frac{1}{\sqrt{f_f / 2}} = 2.5 \ln \left\{ \frac{R}{\nu} v_{avg} \sqrt{f_f / 2} \right\} + 1.75$$

$$\frac{1}{\sqrt{f_f}} = 4.06 \log_{10} \left\{ \mathrm{Re} \sqrt{f_f} \right\} - 0.60$$

$$\boxed{\frac{1}{\sqrt{f_f}} = 4.0 \log_{10} \left\{ \mathrm{Re} \sqrt{f_f} \right\} - 0.40} \qquad \frac{1}{\sqrt{f_f}} = 4.0 \log_{10} \frac{D}{e} + 2.28$$

for rough pipes



0.025
0.02
0.015
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.0025
0.002
Laminar flow
Complete turbulence rough pipe
Uncertain region
5 × 10⁻⁶
Smooth pipe (e = 0)
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.008
0.006
0.004
0.003
0.002
0.0015
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
5 × 10⁻⁵
3 × 10⁻⁵
2 × 10⁻⁵
1 × 10⁻⁵
Fanning fraction factor, f_f
Relative roughness, e/D
6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8 2 3 4 6 8
10^3 10^4 10^5 10^6 10^7 10^8
Reynolds number = Dv_{avg}/ν

head loss

$$h_L = \frac{\Delta P}{\rho g} = K\frac{v^2}{2g} \qquad h_L = 2 f_f \frac{L_{eq}}{D}\frac{v^2}{g}$$

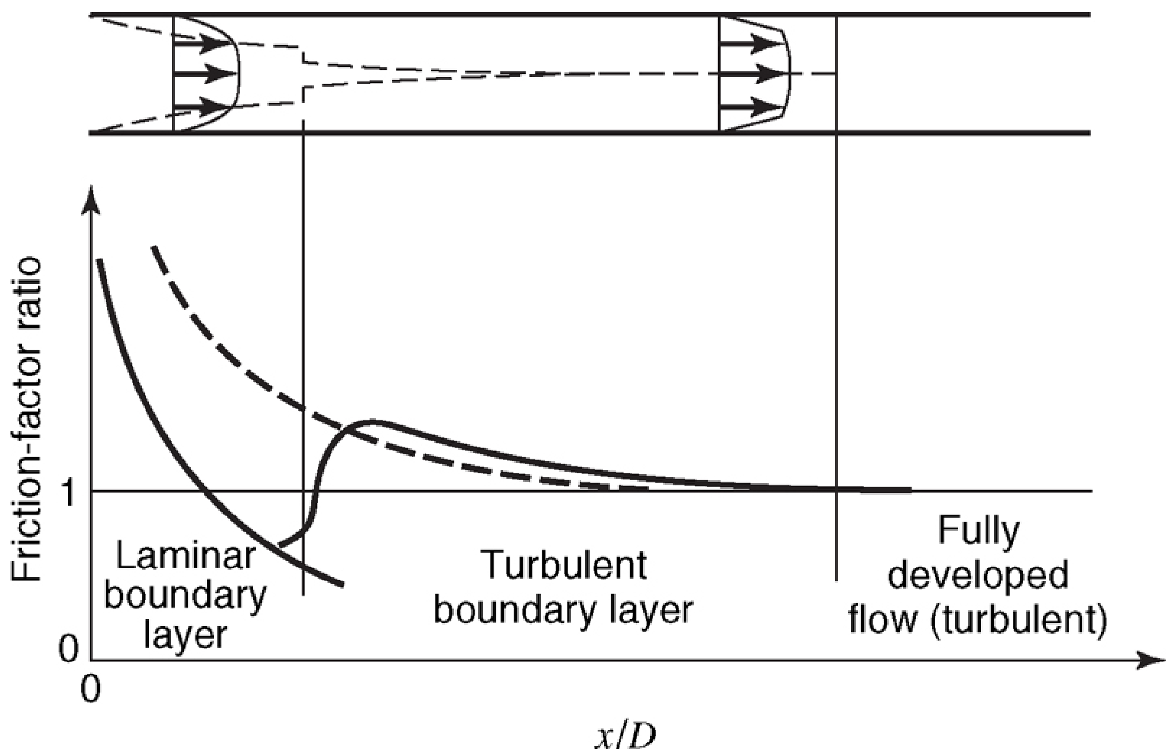
equivalent diameter

$$D_{eq} = 4 \times \frac{cross\,\mathrm{sec}tional\ \ area\ \ of\ \ flow}{wetted\ \ perimeter}$$

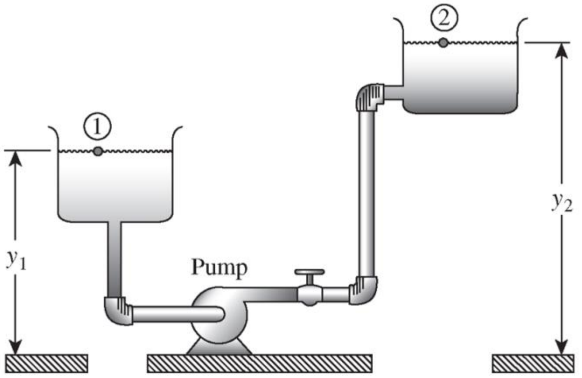
entrance length

for laminar flow

$$\frac{L_e}{D} = 0.0575\,\mathrm{Re}$$



combined pump and system performance



$$\frac{\delta Q}{dt} - \frac{\delta W_s}{dt} = \iint_{c.s.} \left(e + \frac{P}{\rho} \right) \rho (v \cdot n) dA + \frac{\partial}{\partial t} \iiint_{c.v.} e \rho d\nu + \frac{\delta W_\mu}{dt}$$

$$-\frac{\delta W_s}{dt} = \dot{m} \left[u_2 - u_1 + \frac{\nu_2^2 - \nu_1^2}{2} + g(y_2 - y_1) + \frac{P_2 - P_1}{\rho} \right]$$

$$-\frac{\dot{W}}{\dot{m}} = g(y_2 - y_1) + \frac{P_2 - P_1}{\rho} + (u_2 - u_1)$$

$$-\frac{\dot{W}}{\dot{m}g} = y_2 - y_1 + \sum h_L \qquad \sum h_L = \sum K \frac{\nu^2}{2g} \qquad -\frac{\dot{W}}{\dot{m}g} = y_2 - y_1 + \sum K \frac{\nu^2}{2g}$$