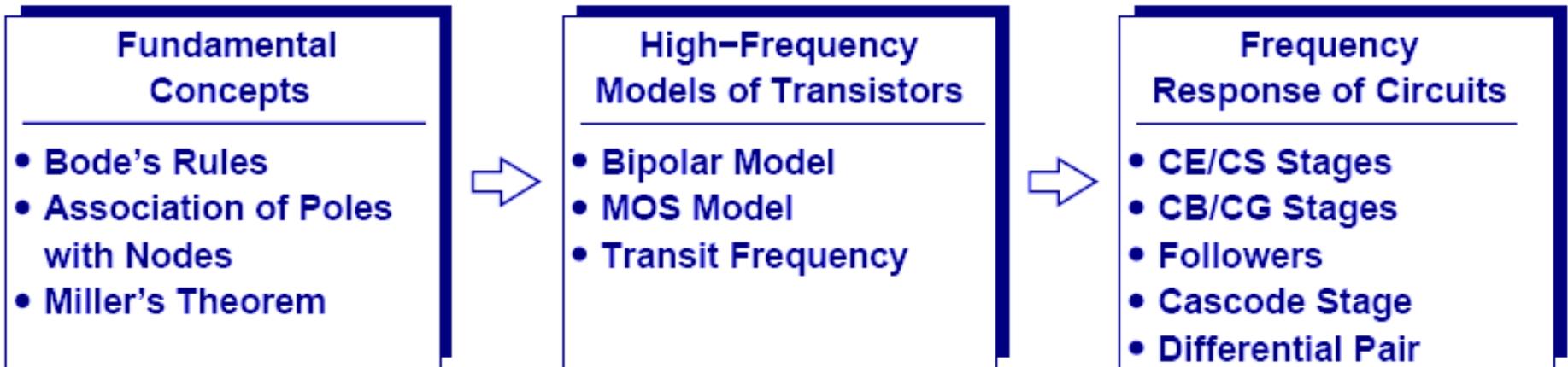


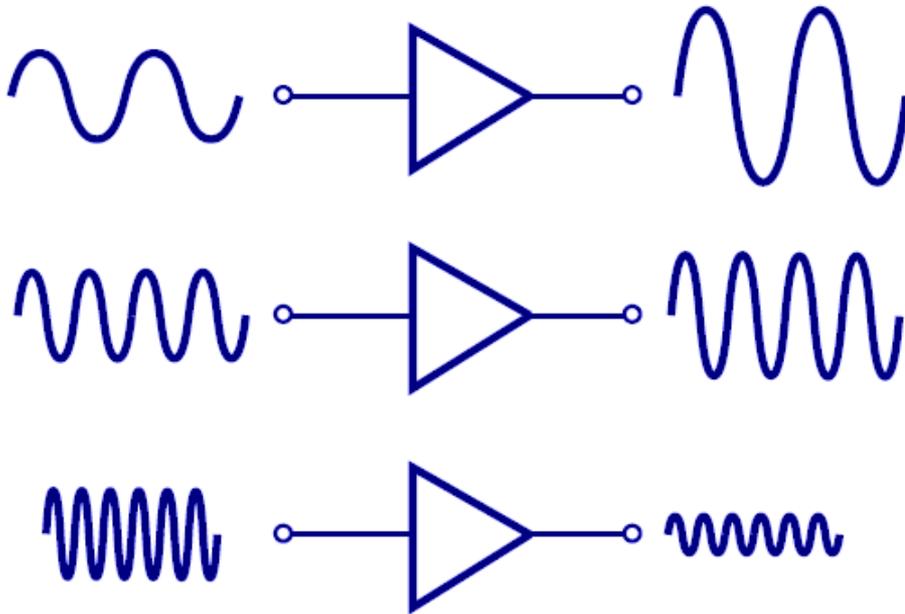
Chapter 11 Frequency Response

- **11.1 Fundamental Concepts**
- **11.2 High-Frequency Models of Transistors**
- **11.3 Analysis Procedure**
- **11.4 Frequency Response of CE and CS Stages**
- **11.5 Frequency Response of CB and CG Stages**
- **11.6 Frequency Response of Followers**
- **11.7 Frequency Response of Cascode Stage**
- **11.8 Frequency Response of Differential Pairs**
- **11.9 Additional Examples**

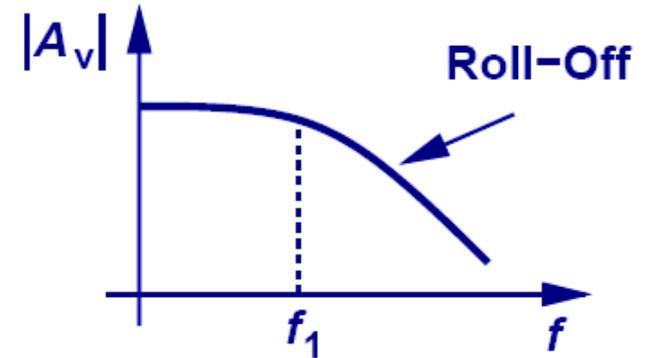
Chapter Outline



High Frequency Roll-off of Amplifier



(a)



(b)

➤ **As frequency of operation increases, the gain of amplifier decreases. This chapter analyzes this problem.**

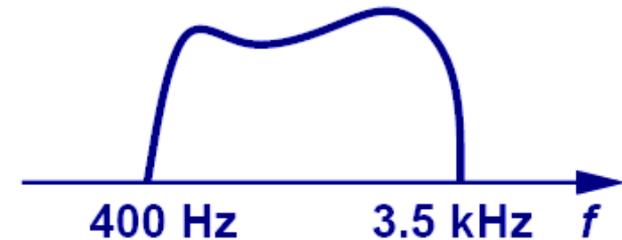
Example: Human Voice I

Natural Voice



(a)

Telephone System



(b)

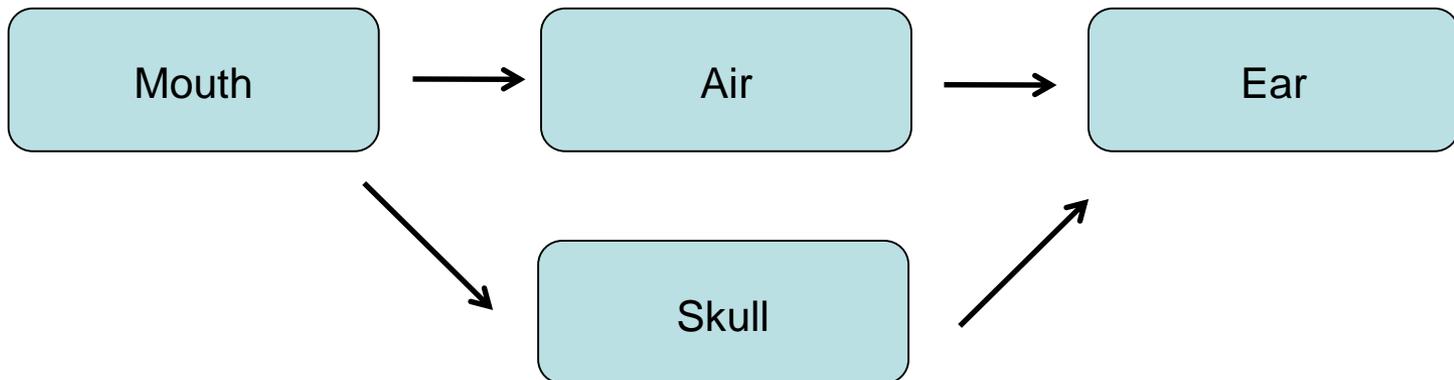
- Natural human voice spans a frequency range from 20Hz to 20KHz, however conventional telephone system passes frequencies from 400Hz to 3.5KHz. Therefore phone conversation differs from face-to-face conversation.

Example: Human Voice II

Path traveled by the human voice to the voice recorder



Path traveled by the human voice to the human ear



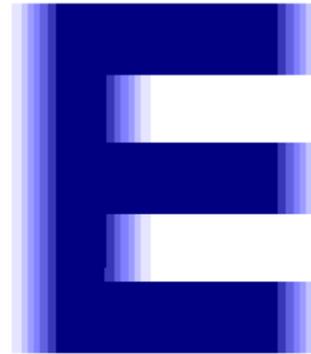
➤ **Since the paths are different, the results will also be different.**

Example: Video Signal



(a)

High Bandwidth

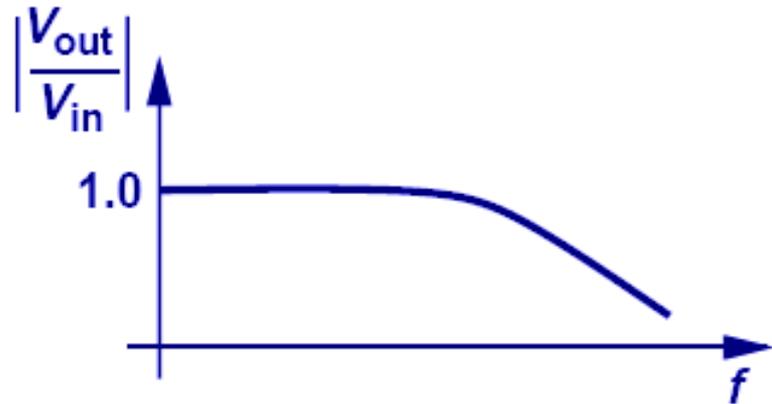
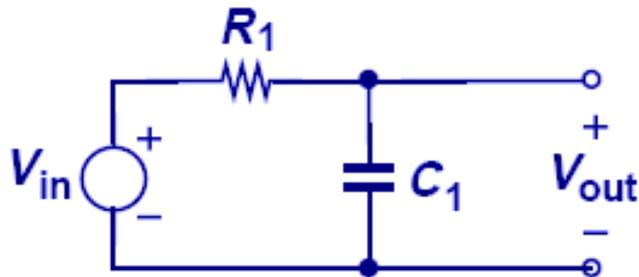


(b)

Low Bandwidth

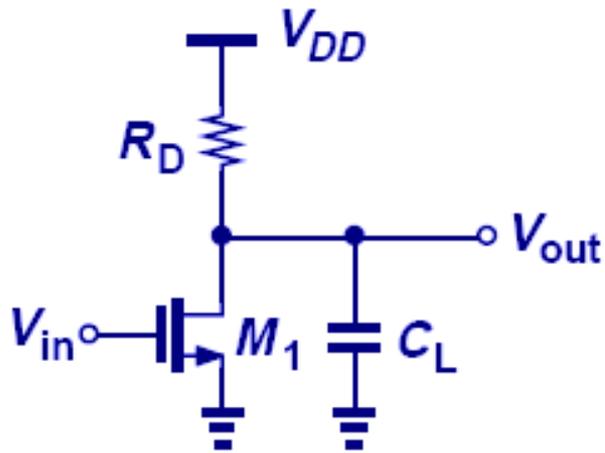
- **Video signals without sufficient bandwidth become fuzzy as they fail to abruptly change the contrast of pictures from complete white into complete black.**

Gain Roll-off: Simple Low-pass Filter

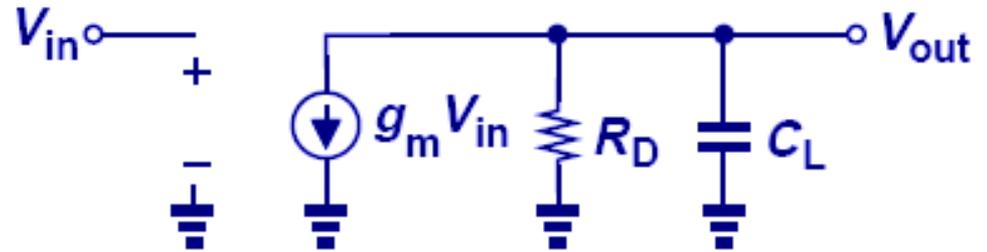


- In this simple example, as frequency increases the impedance of C_1 decreases and the voltage divider consists of C_1 and R_1 attenuates V_{in} to a greater extent at the output.

Gain Roll-off: Common Source



(a)

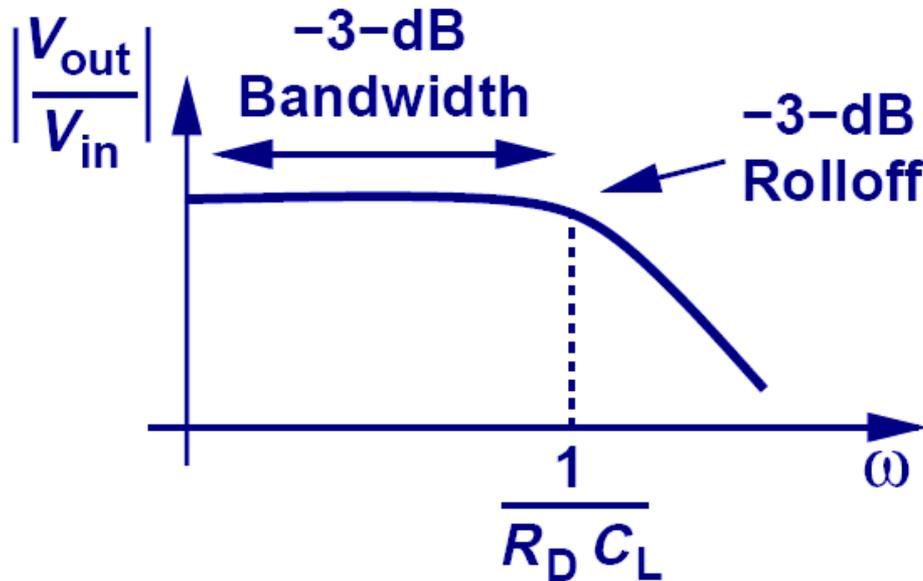


(b)

$$V_{out} = -g_m V_{in} \left(R_D \parallel \frac{1}{C_L s} \right)$$

- The capacitive load, C_L , is the culprit for gain roll-off since at high frequency, it will “steal” away some signal current and shunt it to ground.

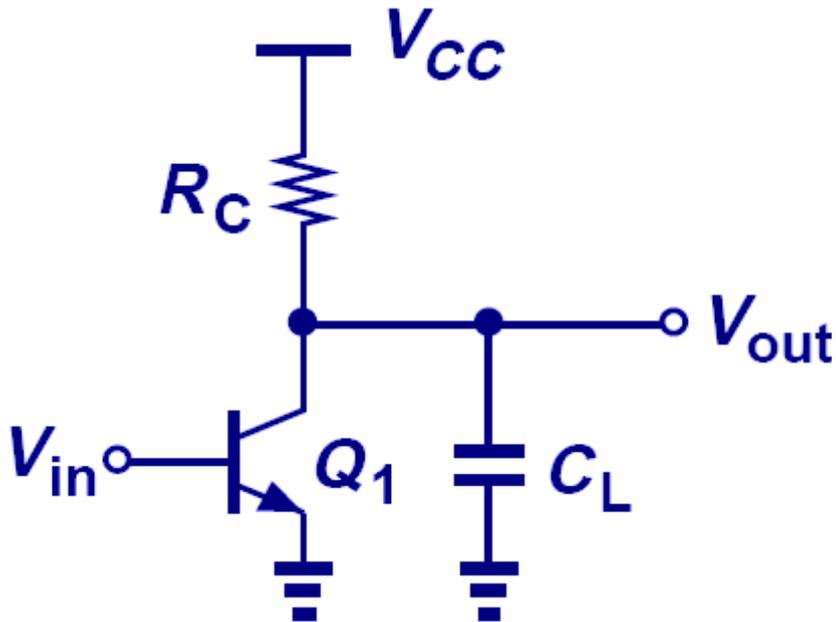
Frequency Response of the CS Stage



$$\begin{aligned} H(s) &= \frac{V_{out}}{V_{in}}(s) \\ &= -g_m \left(R_D \parallel \frac{1}{C_L s} \right) \\ &= \frac{-g_m R_D}{R_D C_L s + 1} \\ \left| \frac{V_{out}}{V_{in}} \right| &= \frac{g_m R_D}{\sqrt{R_D^2 C_L^2 \omega^2 + 1}} \end{aligned}$$

- At low frequency, the capacitor is effectively open and the gain is flat. As frequency increases, the capacitor tends to a short and the gain starts to decrease. A special frequency is $\omega = 1/(R_D C_L)$, where the gain drops by 3dB.

Example: Figure of Merit



$$F.O.M. = \frac{\text{Gain} \times \text{Bandwidth}}{\text{Power Consumption}}$$

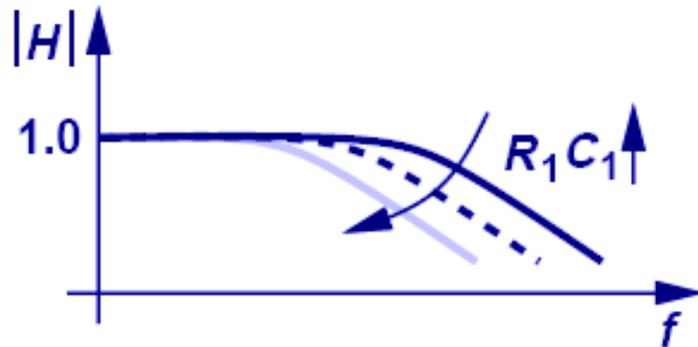
$$= \frac{g_m R_C \times \frac{1}{R_C C_L}}{I_C V_{CC}}$$

$$= \frac{\frac{I_C}{V_T} R_C \times \frac{1}{R_C C_L}}{I_C V_{CC}}$$

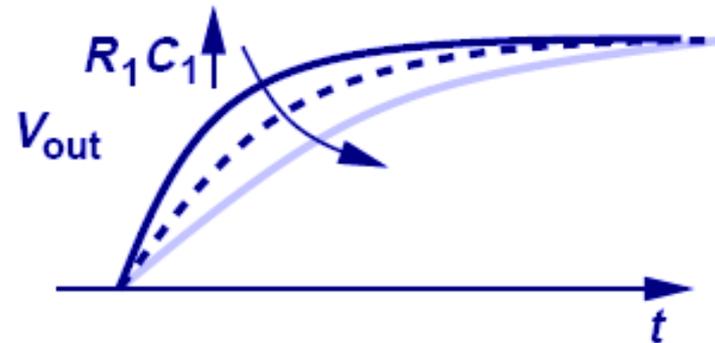
$$= \frac{1}{V_T V_{CC} C_L}$$

- This metric quantifies a circuit's gain, bandwidth, and power dissipation. In the bipolar case, low temperature, supply, and load capacitance mark a superior figure of merit.

Example: Relationship between Frequency Response and Step Response



(a)



(b)

$$|H(s = j\omega)| = \frac{1}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

$$V_{out}(t) = V_0 \left(1 - \exp\left(\frac{-t}{R_1 C_1}\right) \right) u(t)$$

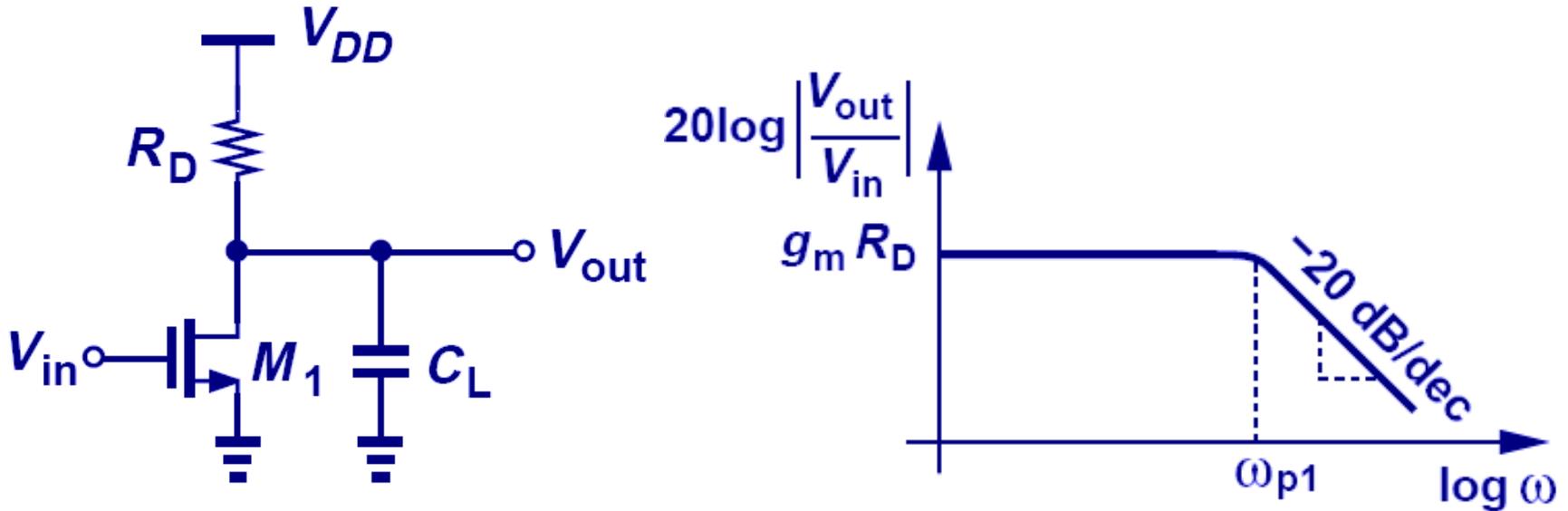
➤ The relationship is such that as $R_1 C_1$ increases, the bandwidth *drops* and the step response becomes *slower*.

Bode Plot

$$H(s) = A_0 \frac{\left(1 + \frac{s}{\omega_{z1}}\right) \left(1 + \frac{s}{\omega_{z2}}\right) \dots}{\left(1 + \frac{s}{\omega_{p1}}\right) \left(1 + \frac{s}{\omega_{p2}}\right) \dots}$$

- When we hit a zero, ω_{zj} , the Bode magnitude rises with a slope of +20dB/dec.
- When we hit a pole, ω_{pj} , the Bode magnitude falls with a slope of -20dB/dec

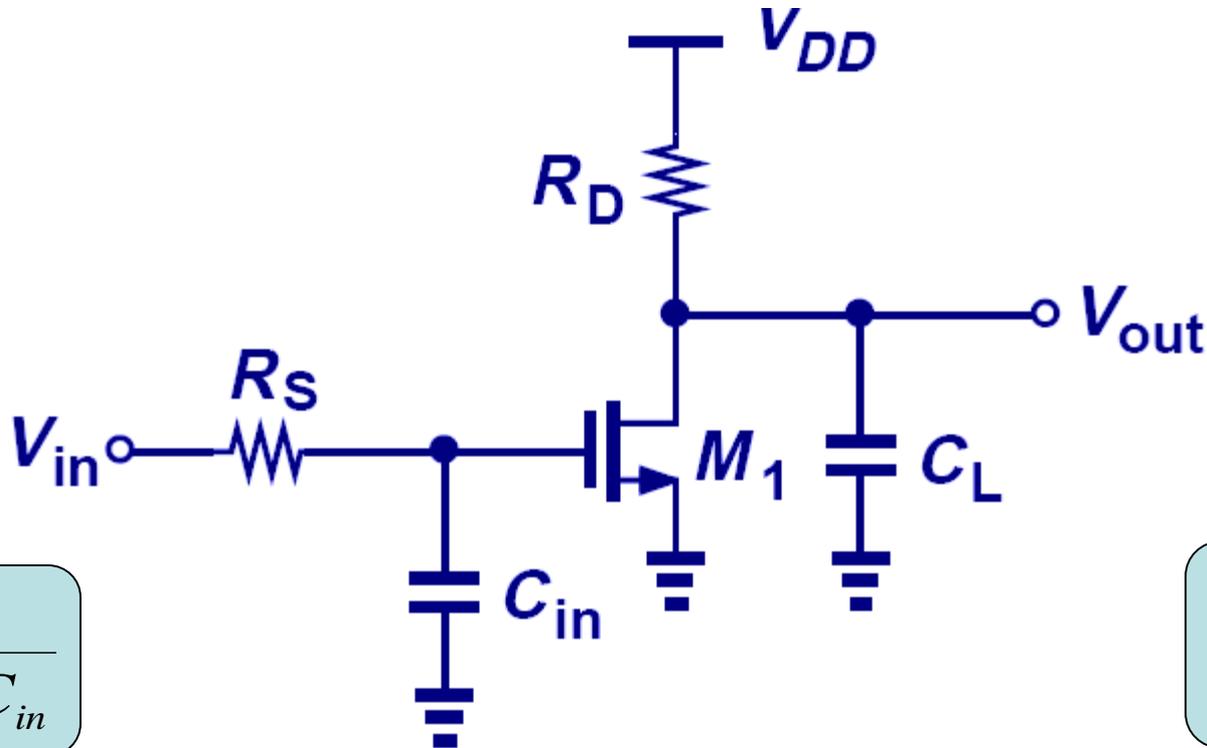
Example: Bode Plot



$$\left| \omega_{p1} \right| = \frac{1}{R_D C_L}$$

- The circuit only has one pole (no zero) at $1/(R_D C_L)$, so the slope drops from 0 to -20 dB/dec as we pass ω_{p1} .

Pole Identification Example I

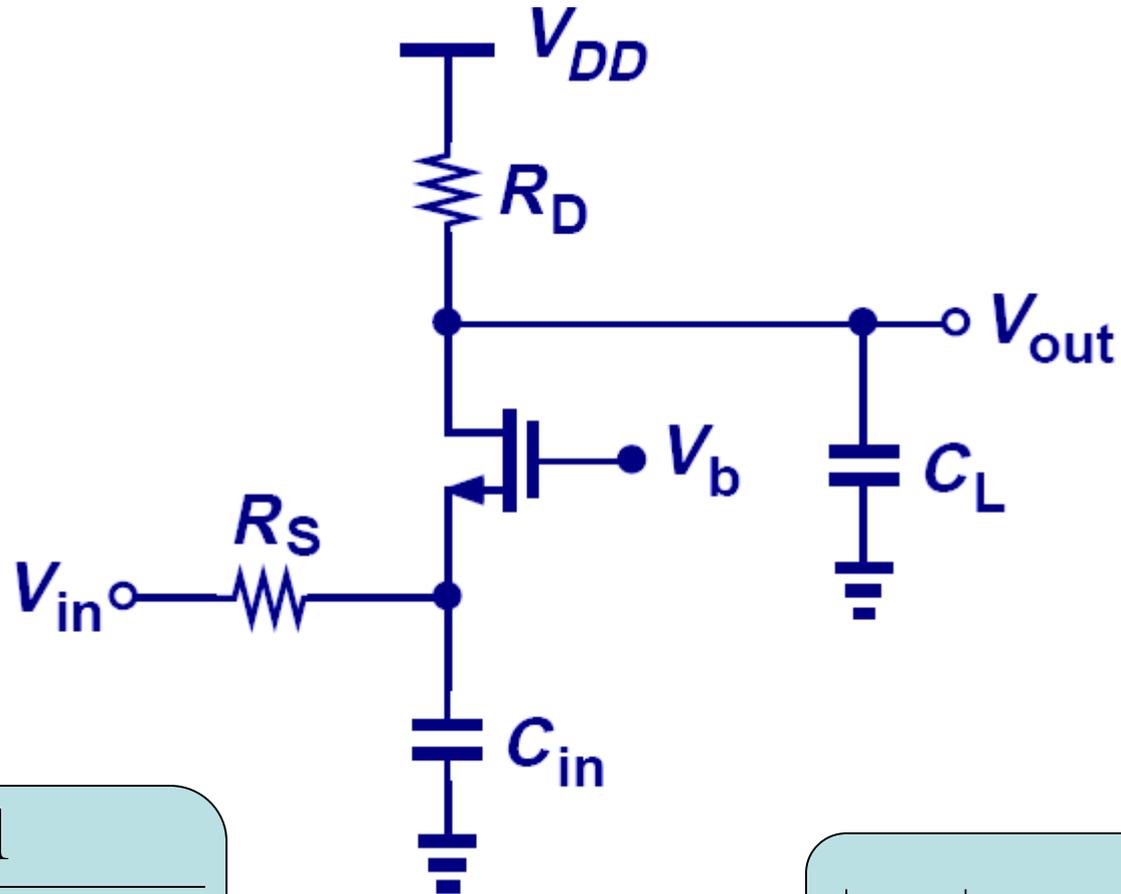


$$|\omega_{p1}| = \frac{1}{R_S C_{in}}$$

$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{g_m R_D}{\sqrt{\left(1 + \omega^2 / \omega_{p1}^2\right) \left(1 + \omega^2 / \omega_{p2}^2\right)}}$$

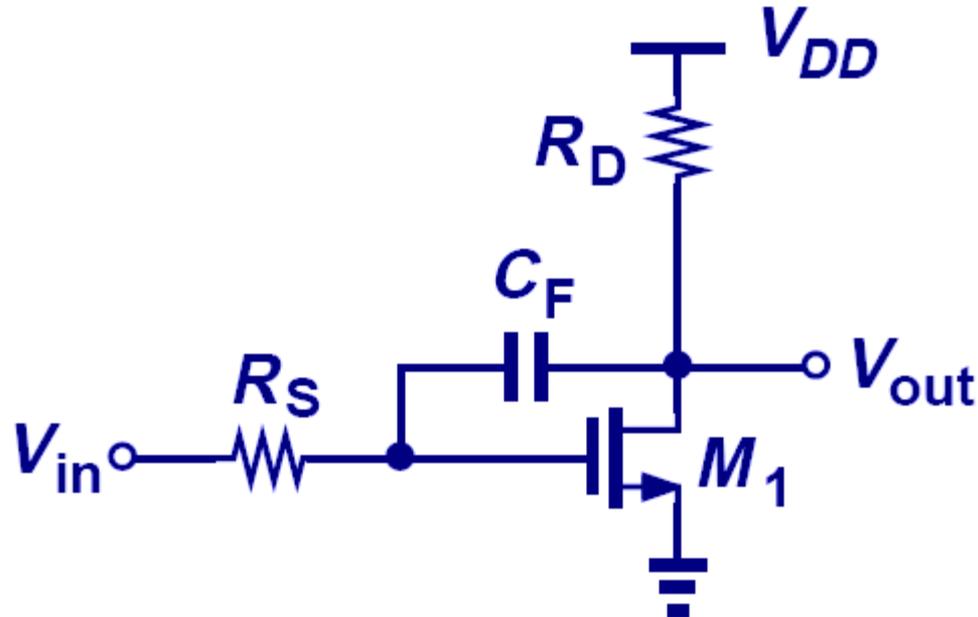
Pole Identification Example II



$$|\omega_{p1}| = \frac{1}{\left(R_S \parallel \frac{1}{g_m} \right) C_{in}}$$

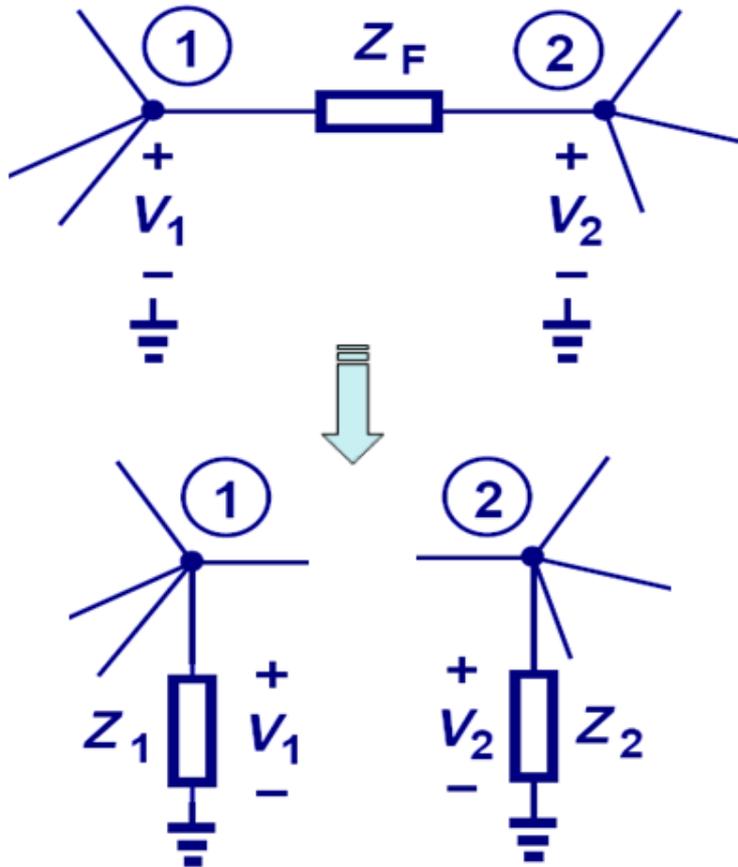
$$|\omega_{p2}| = \frac{1}{R_D C_L}$$

Circuit with Floating Capacitor



- The pole of a circuit is computed by finding the effective resistance and capacitance from a node to GROUND.
- The circuit above creates a problem since neither terminal of C_F is grounded.

Miller's Theorem



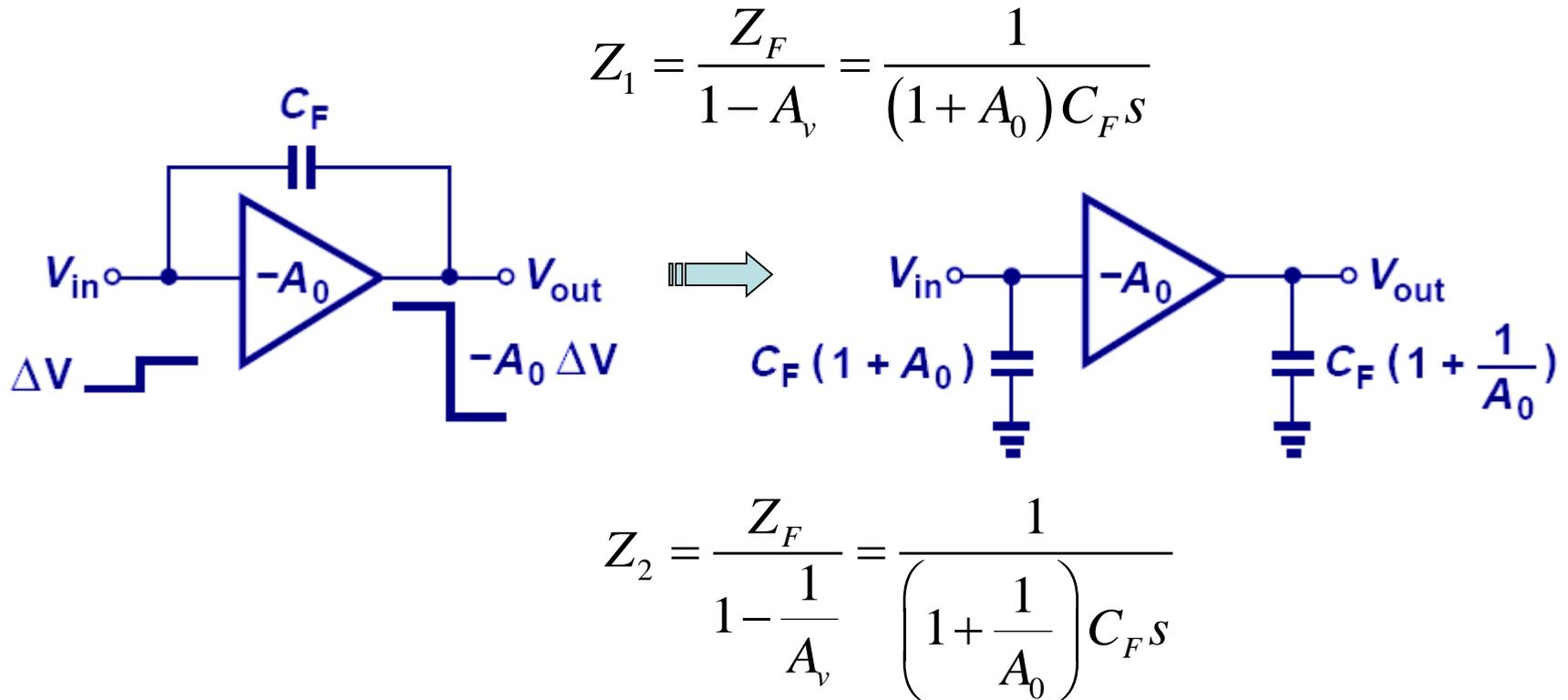
$$\frac{V_1 - V_2}{Z_F} = \frac{V_1}{Z_1}, \quad \frac{V_1 - V_2}{Z_F} = -\frac{V_2}{Z_2}$$

$$\Rightarrow Z_1 = Z_F \frac{V_1}{V_1 - V_2} = \frac{Z_F}{1 - A_v}$$

$$Z_2 = Z_F \frac{-V_2}{V_1 - V_2} = \frac{Z_F}{1 - \frac{1}{A_v}}$$

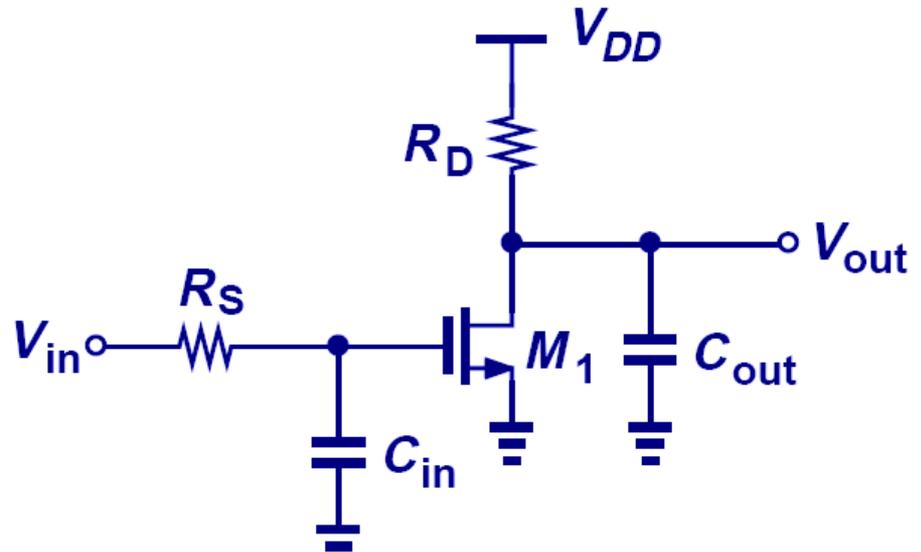
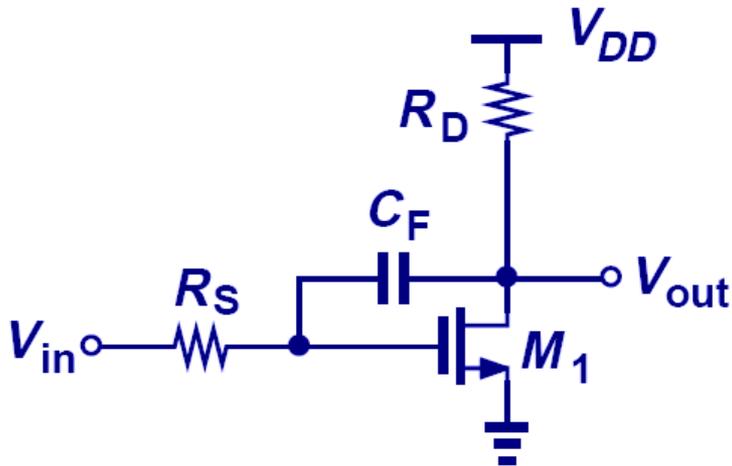
- If A_v is the gain from node 1 to 2, then a floating impedance Z_F can be converted to two grounded impedances Z_1 and Z_2 .

Miller Multiplication



➤ With Miller's theorem, we can separate the floating capacitor. However, the input capacitor is larger than the original floating capacitor. We call this Miller multiplication.

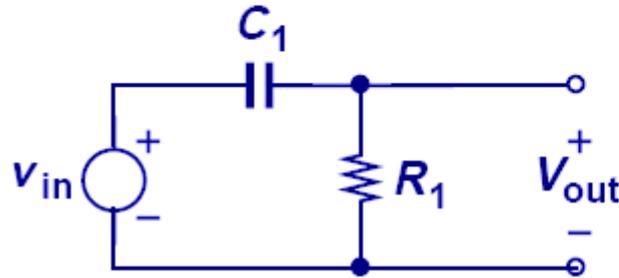
Example: Miller Theorem



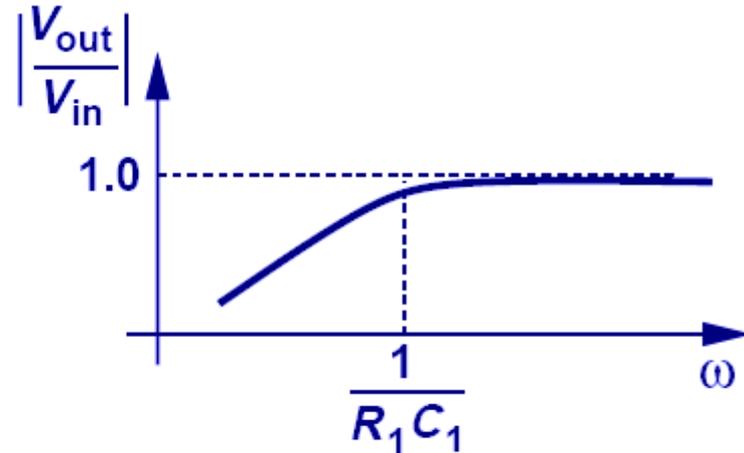
$$\omega_{in} = \frac{1}{R_S (1 + g_m R_D) C_F}$$

$$\omega_{out} = \frac{1}{R_D \left(1 + \frac{1}{g_m R_D} \right) C_F}$$

High-Pass Filter Response



(a)

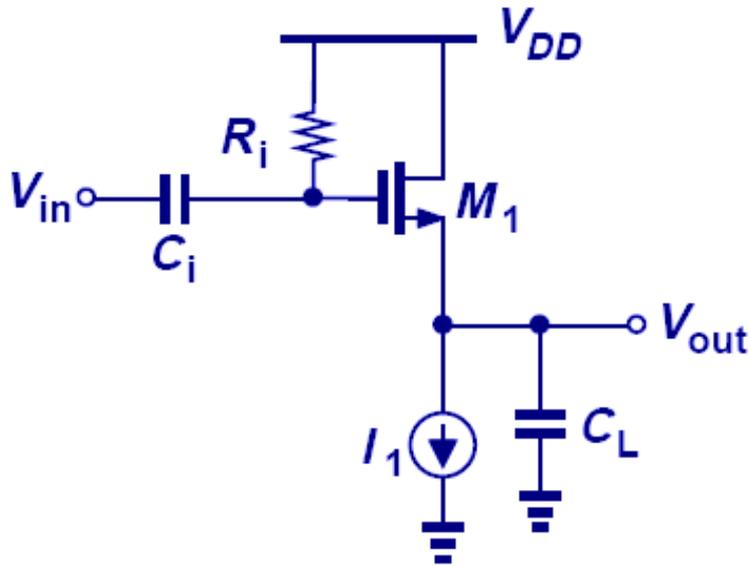


(b)

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{R_1 C_1 \omega}{\sqrt{R_1^2 C_1^2 \omega^2 + 1}}$$

- The voltage division between a resistor and a capacitor can be configured such that the gain at low frequency is reduced.

Example: Audio Amplifier



$$R_i = 100 \text{ k}\Omega$$

$$g_m = 1 / 200 \text{ }\Omega$$

$$\frac{1}{R_i C_i} \leq 2\pi \times (20\text{Hz})$$

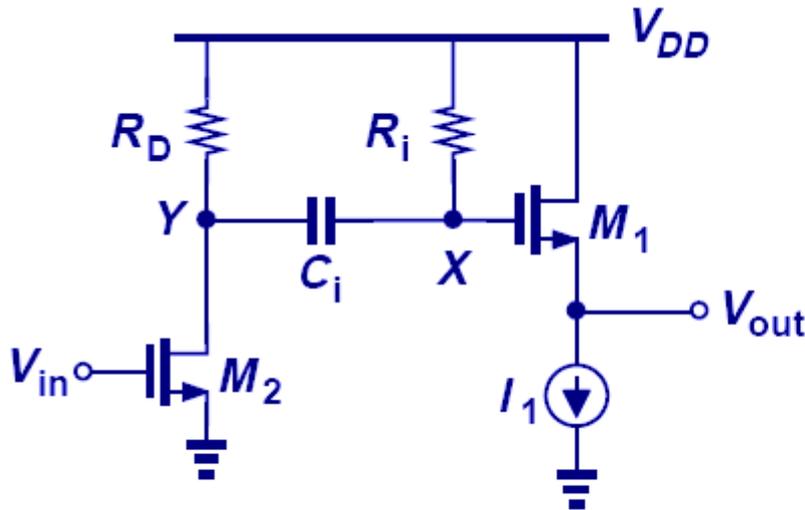
$$\Rightarrow C_i \geq \frac{1}{100k \times 2\pi \times 20} = 79.6\text{nF}$$

$$\omega_{p,out} = \frac{g_m}{C_L} \geq 2\pi \times (20\text{kHz})$$

$$\Rightarrow C_L \leq \frac{1}{200 \times 2\pi \times 20k} = 39.8\text{nF}$$

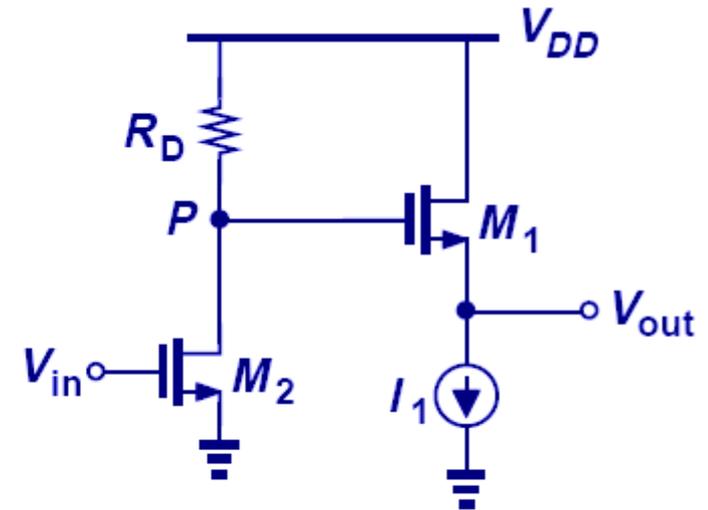
➤ In order to successfully pass audio band frequencies (20 Hz-20 KHz), large input and small output capacitances are needed.

Capacitive Coupling vs. Direct Coupling



(a)

Capacitive Coupling

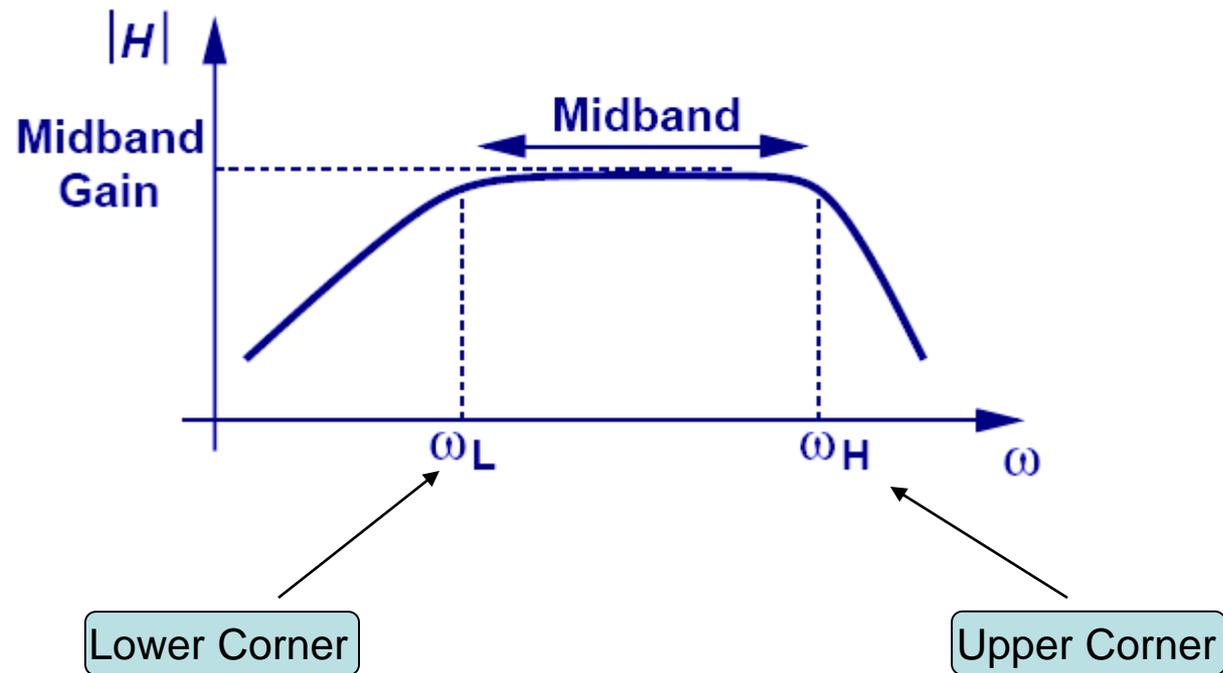


(b)

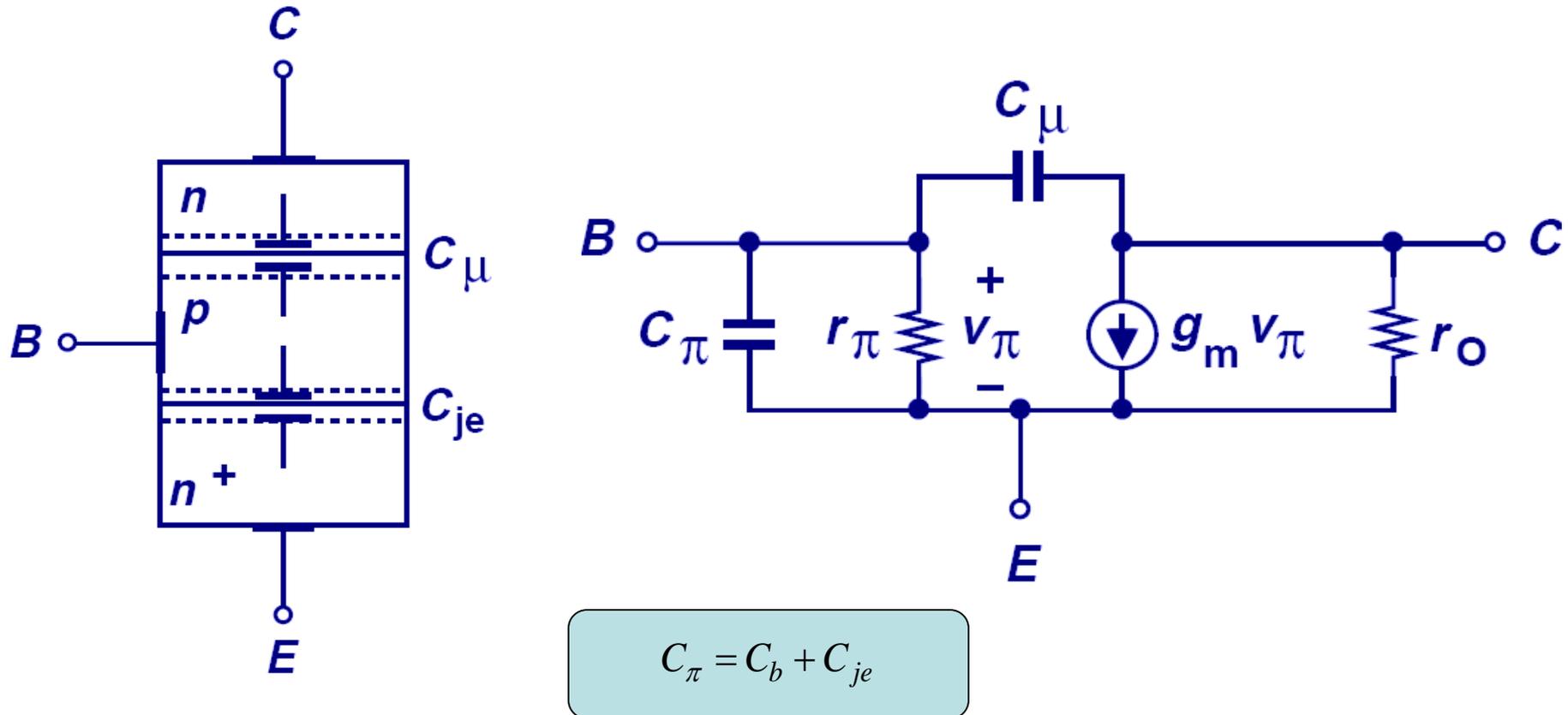
Direct Coupling

- **Capacitive coupling, also known as AC coupling, passes AC signals from Y to X while blocking DC contents.**
- **This technique allows independent bias conditions between stages. Direct coupling does not.**

Typical Frequency Response

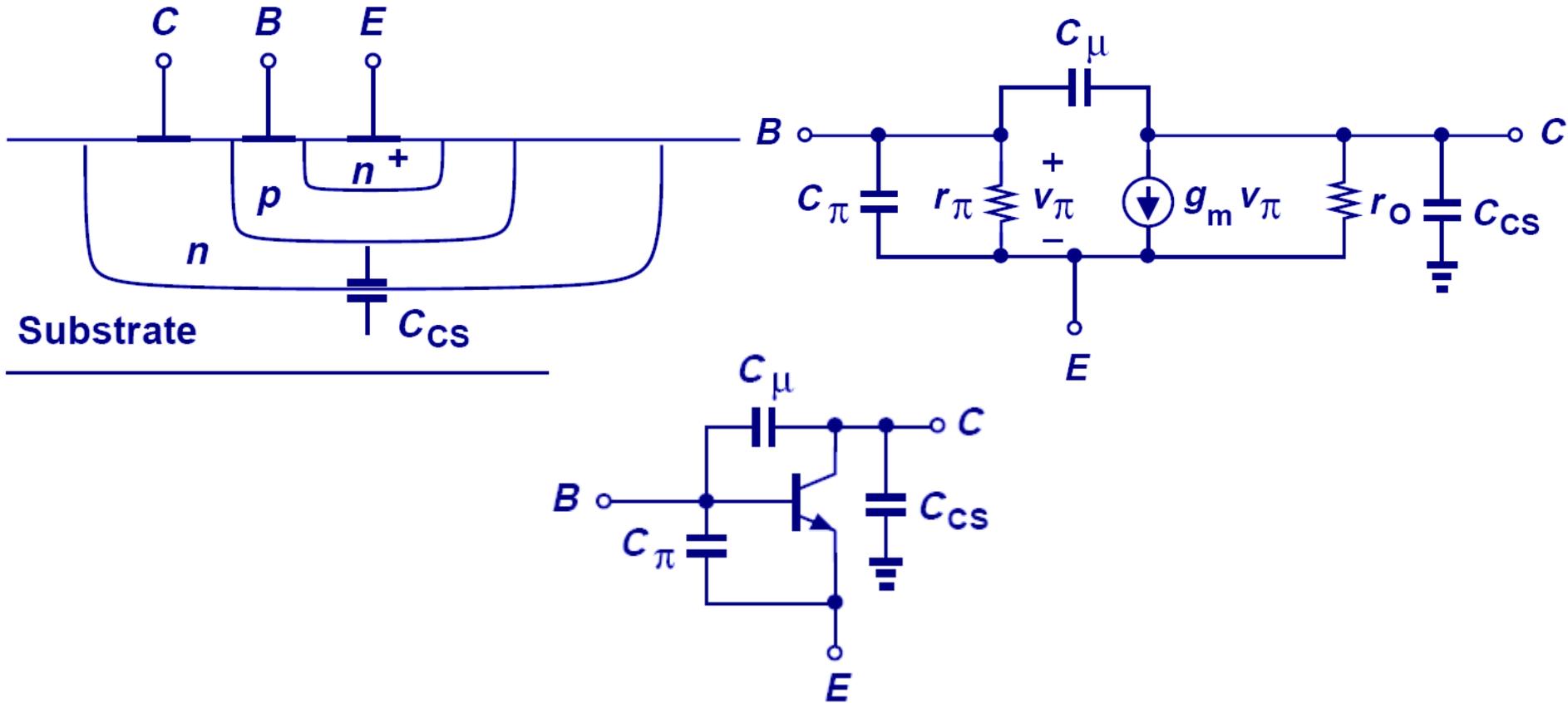


High-Frequency Bipolar Model



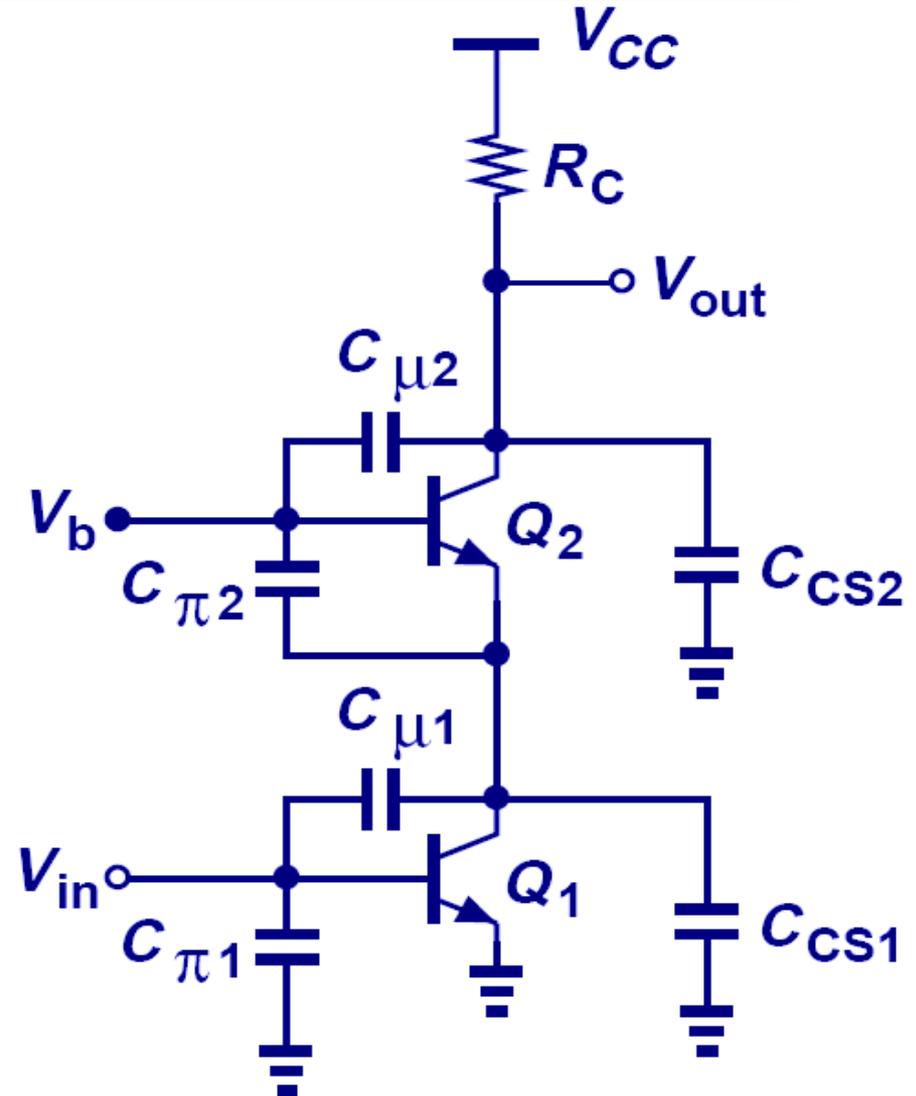
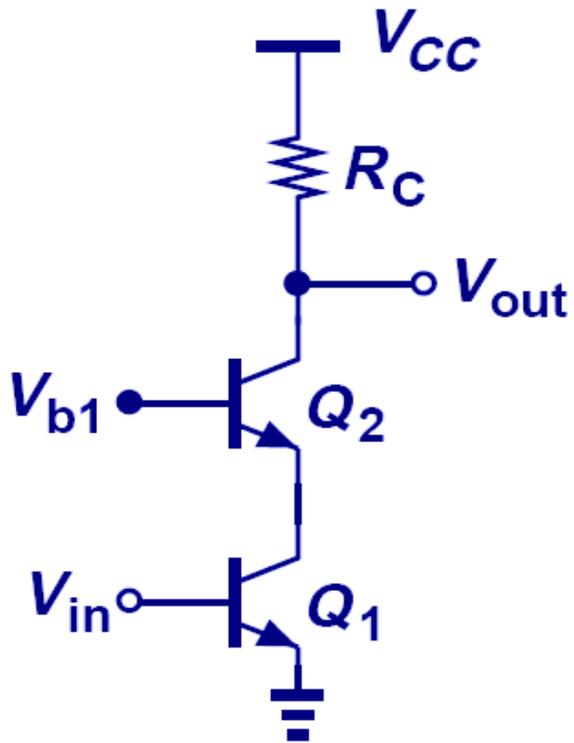
- At high frequency, capacitive effects come into play. C_b represents the base charge, whereas C_{μ} and C_{je} are the junction capacitances.

High-Frequency Model of Integrated Bipolar Transistor

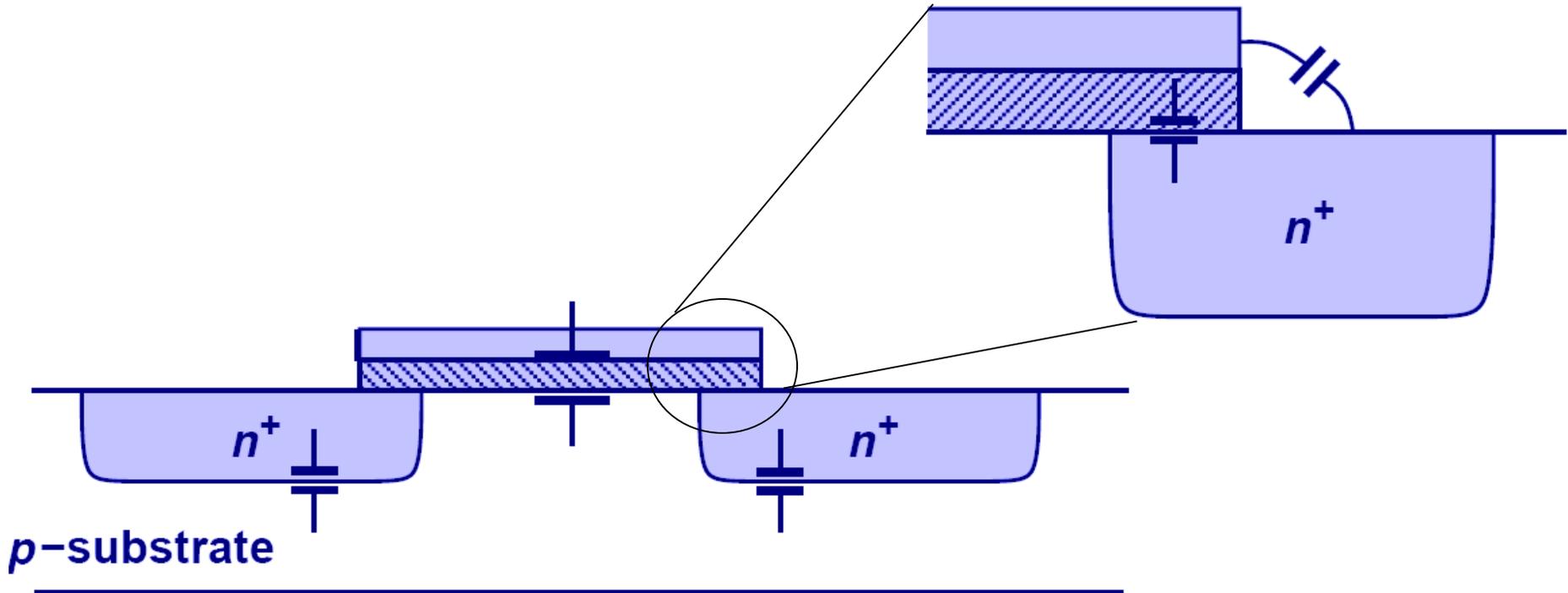


- Since an integrated bipolar circuit is fabricated on top of a substrate, another junction capacitance exists between the collector and substrate, namely C_{CS} .

Example: Capacitance Identification

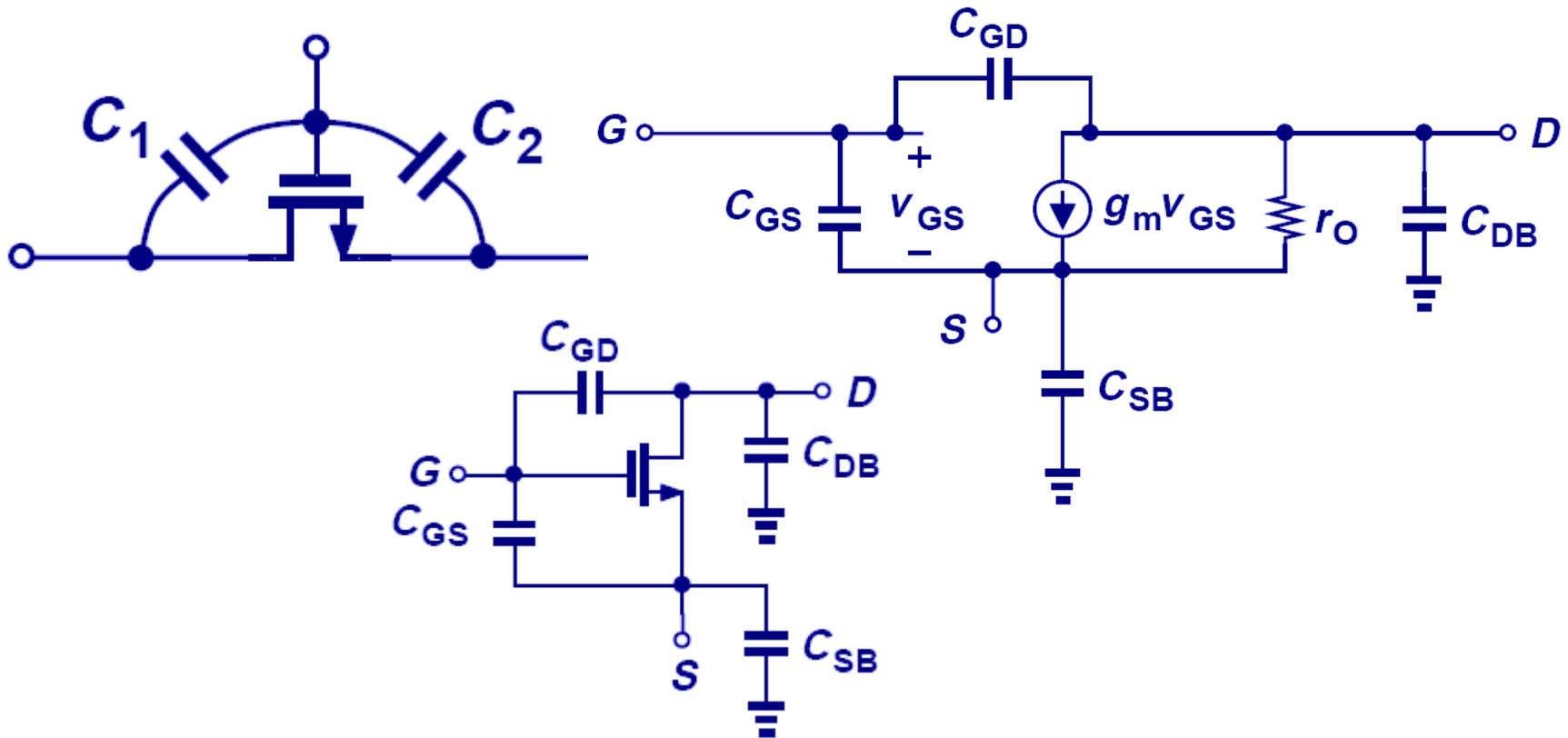


MOS Intrinsic Capacitances



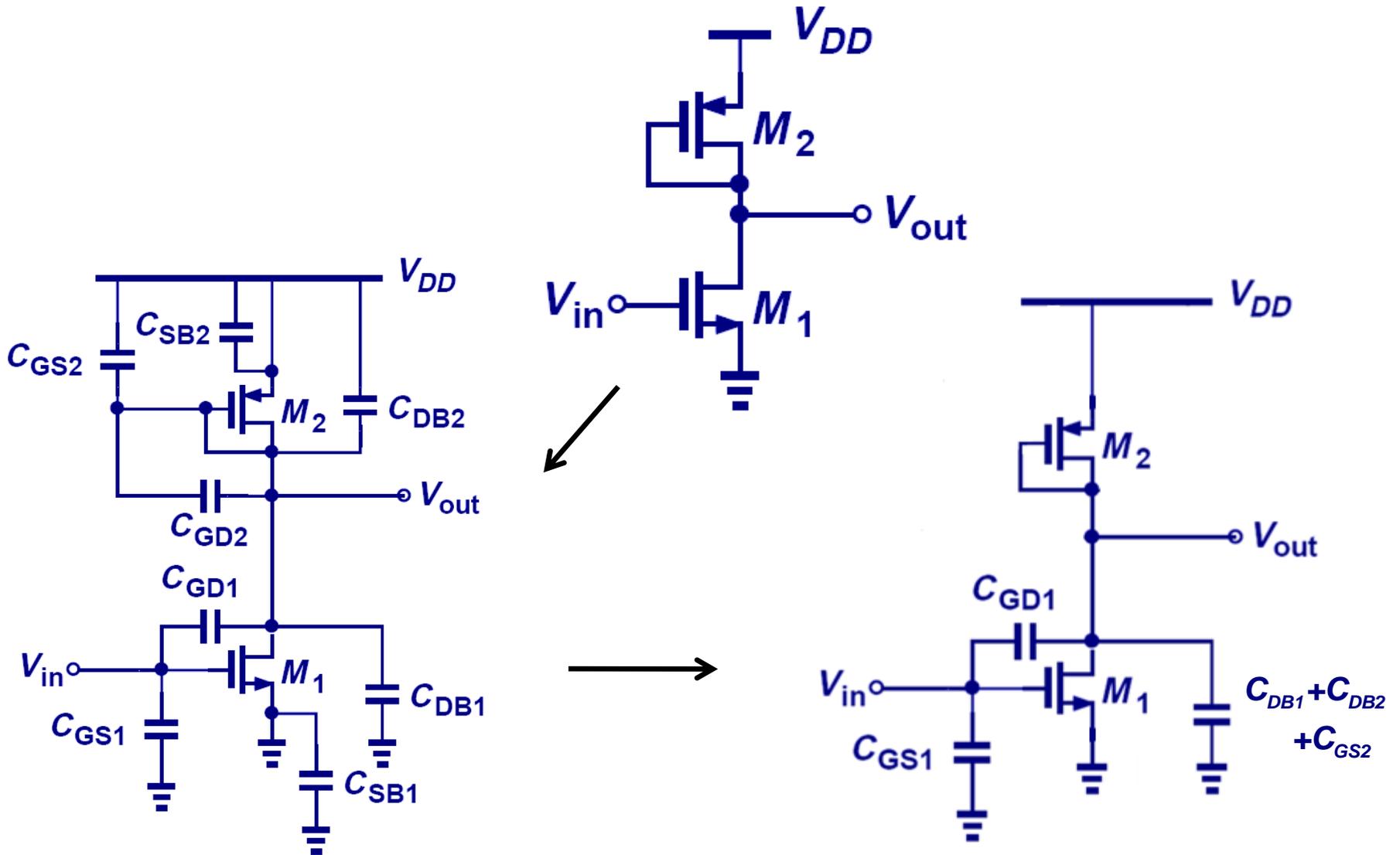
- For a MOS, there exist oxide capacitance from gate to channel, junction capacitances from source/drain to substrate, and overlap capacitance from gate to source/drain.

Gate Oxide Capacitance Partition and Full Model

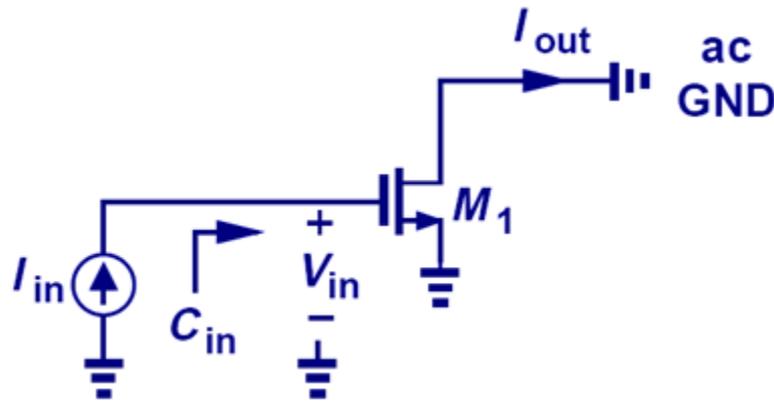
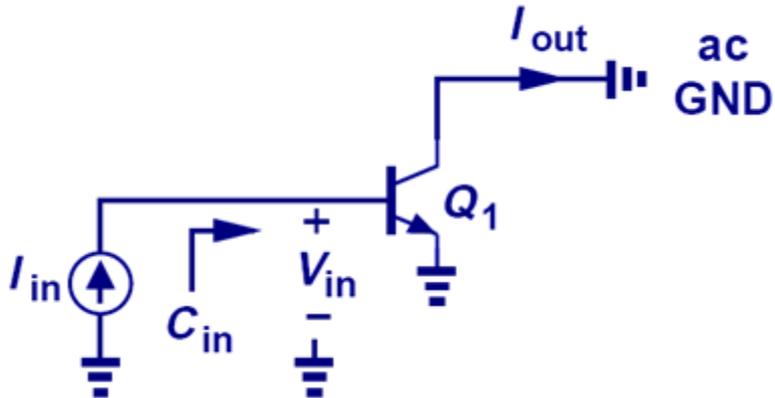


- The gate oxide capacitance is often partitioned between source and drain. In saturation, $C_2 \sim C_{\text{gate}}$, and $C_1 \sim 0$. They are in parallel with the overlap capacitance to form C_{GS} and C_{GD} .

Example: Capacitance Identification



Transit Frequency



$$Z_{in} = \frac{1}{C_{\pi}s} \parallel r_{\pi}, \quad I_{out} = g_m I_{in} Z_{in}$$

$$\Rightarrow \frac{I_{out}}{I_{in}} = \frac{g_m r_{\pi}}{r_{\pi} C_{\pi} s + 1} = \frac{\beta}{r_{\pi} C_{\pi} s + 1}$$

$$\frac{I_{out}}{I_{in}} = 1 \Rightarrow r_{\pi}^2 C_{\pi}^2 \omega_T^2 = \beta^2 - 1 \approx \beta^2$$

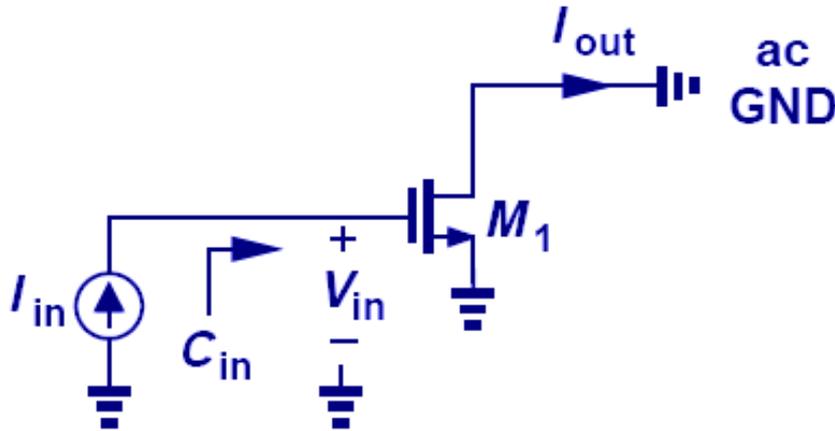
$$\Rightarrow \omega_T = 2\pi f_T \approx \frac{g_m}{C_{\pi}}$$

The transit frequency of MOSFETs is obtained in a similar fashion.

$$\omega_T = 2\pi f_T \approx \frac{g_m}{C_{GS}}$$

➤ **Transit frequency, f_T , is defined as the frequency where the current gain from input to output drops to 1.**

Example: Transit Frequency Calculation



From Problem 11.28,

$$2\pi f_T = \frac{3}{2} \frac{\mu_n}{L^2} (V_{GS} - V_{TH})$$

$$\Rightarrow \frac{f_{T, \text{today}}}{f_{T, 1980s}} = \frac{100}{(65 \times 10^{-9})^2} \bigg/ \frac{400}{(1 \times 10^{-6})^2} \approx 59$$

If $\mu_n = 400 \text{ cm}^2 / (\text{V} \cdot \text{s})$,

$$f_{T, \text{today}} \approx 226 \text{ GHz}$$

- The minimum channel length of MOSFETs has been scaled from $1\mu\text{m}$ in the late 1980s to 65nm today. Also, the inevitable reduction of the supply voltage has reduced the gate-source overdrive voltage from about 400mV to 100mV . By what factor has the f_T of MOSFETs increased?

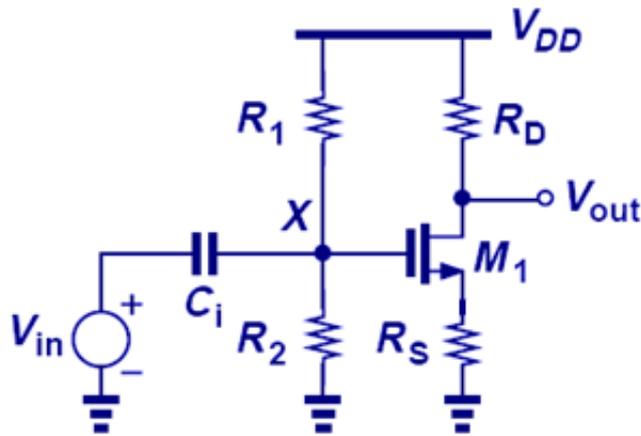
Analysis Summary

- **The frequency response refers to the magnitude of the transfer function.**
- **Bode's approximation simplifies the plotting of the frequency response if poles and zeros are known.**
- **In general, it is possible to associate a pole with each node in the signal path.**
- **Miller's theorem helps to decompose floating capacitors into grounded elements.**
- **Bipolar and MOS devices exhibit various capacitances that limit the speed of circuits.**

High Frequency Circuit Analysis Procedure

- **Determine which capacitor impact the low-frequency region of the response and calculate the low-frequency pole (neglect transistor capacitance).**
- **Calculate the midband gain by replacing the capacitors with short circuits (neglect transistor capacitance).**
- **Include transistor capacitances.**
- **Merge capacitors connected to AC grounds and omit those that play no role in the circuit.**
- **Determine the high-frequency poles and zeros.**
- **Plot the frequency response using Bode's rules or exact analysis.**

Frequency Response of CS Stage

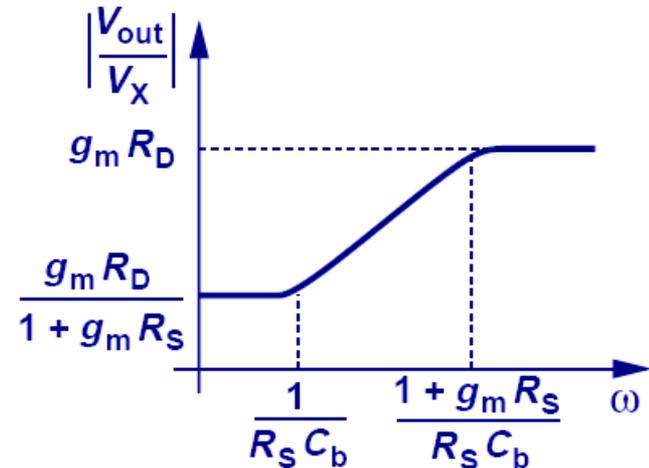
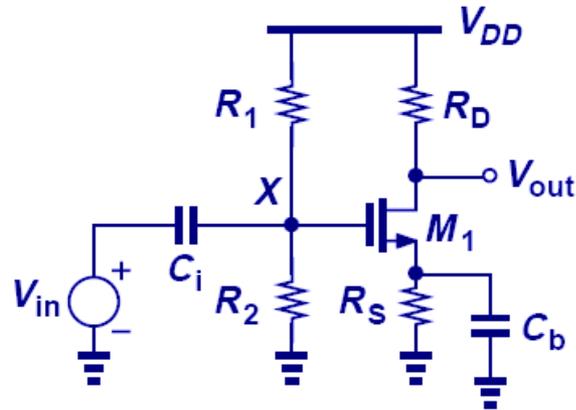
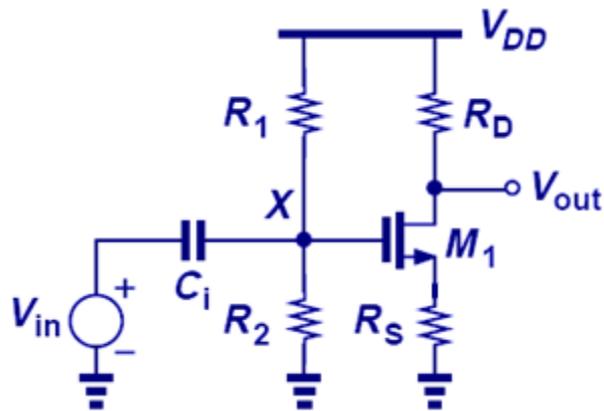


$$\frac{V_X}{V_{in}}(s) = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + \frac{1}{C_i s}} = \frac{(R_1 \parallel R_2) C_i s}{(R_1 \parallel R_2) C_i s + 1}$$

$$\text{Thus, } \frac{1}{2\pi [(R_1 \parallel R_2) C_i]} < f_{sig, min}$$

- C_i acts as a high pass filter.
- Lower cut-off frequency must be lower than the lowest signal frequency $f_{sig, min}$ (20 Hz in audio applications).

Frequency Response of CS Stage with Bypassed Degeneration

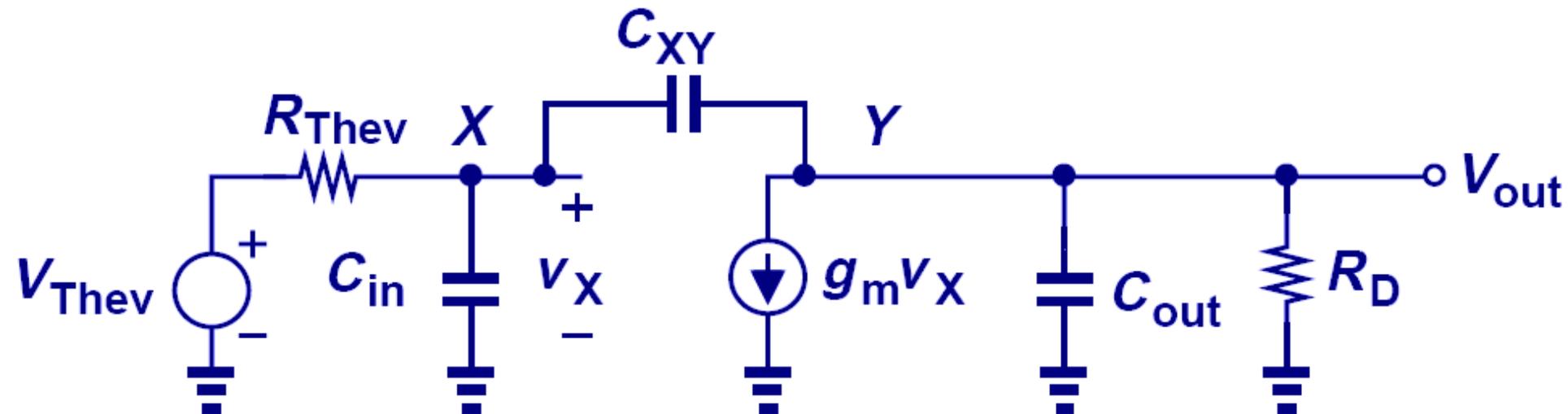
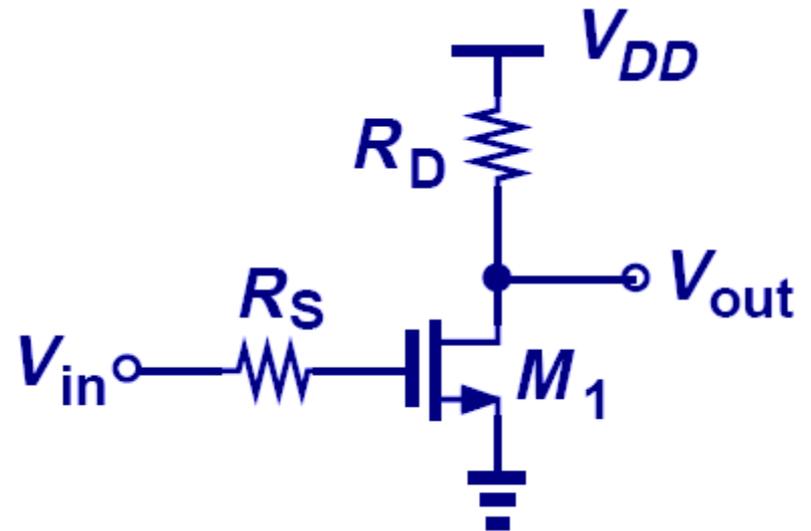
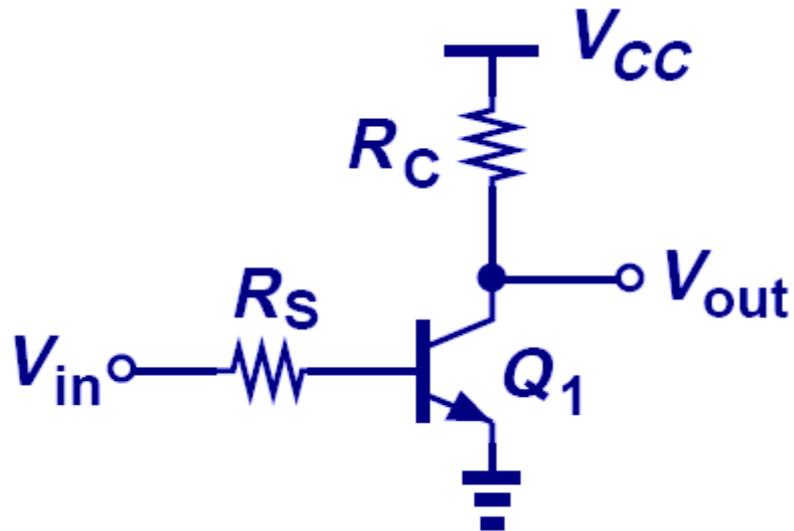


$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S + \frac{1}{g_m}}$$

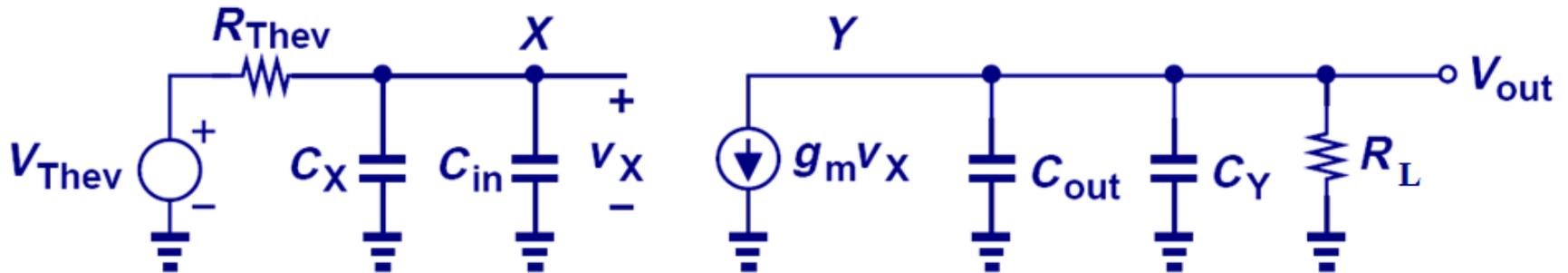
$$\frac{V_{out}}{V_X}(s) = \frac{-R_D}{R_S \left[\frac{1}{C_b s} + \frac{1}{g_m} \right]} = \frac{-g_m R_D (R_S C_b s + 1)}{R_S C_b s + g_m R_S + 1}$$

- In order to increase the midband gain, a capacitor C_b is placed in parallel with R_S .
- The pole frequency must be well below the lowest signal frequency to avoid the effect of degeneration.

Unified Model for CE and CS Stages



Unified Model Using Miller's Theorem



$$|\omega_{p,in}| = \frac{1}{R_{Thev} [C_{in} + (1 + g_m R_L) C_{XY}]}$$

$$|\omega_{p,out}| = \frac{1}{R_L [C_{out} + \left(1 + \frac{1}{g_m R_L}\right) C_{XY}]}$$

CE Stage

$$V_{Thev} = V_{in} \frac{r_{\pi}}{r_{\pi} + R_S}$$

$$R_{Thev} = R_S \parallel r_{\pi}$$

$$C_X = C_{\mu} (1 + g_m R_L)$$

$$C_Y = C_{\mu} \left(1 + \frac{1}{g_m R_L}\right)$$

CS Stage

$$V_{Thev} = V_{in}$$

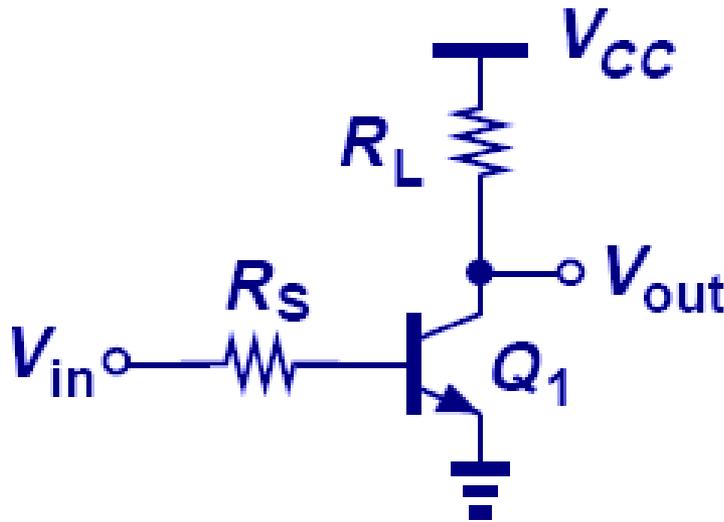
$$R_{Thev} = R_S$$

$$C_X = C_{GD} (1 + g_m R_L)$$

$$C_Y = C_{GD} \left(1 + \frac{1}{g_m R_L}\right)$$

Example: CE Stage

- (a) Calculate the input and output poles if $R_L=2\text{ k}\Omega$. Which node appears as the speed bottleneck?



$$|\omega_{p,in}| = \frac{1}{(R_S \parallel r_\pi) [C_\pi + (1 + g_m R_L) C_\mu]}$$

$$|\omega_{p,out}| = \frac{1}{R_L \left[C_{CS} + \left(1 + \frac{1}{g_m R_L} \right) C_\mu \right]}$$

$$R_S = 200\ \Omega, \quad I_C = 1\ \text{mA}$$

$$\beta = 100, \quad C_\pi = 100\ \text{fF}$$

$$C_\mu = 20\ \text{fF}, \quad C_{CS} = 30\ \text{fF}$$

$$|\omega_{p,in}| = 2\pi \times (516\ \text{MHz})$$

$$|\omega_{p,out}| = 2\pi \times (1.59\ \text{GHz})$$

Example: CE Stage – cont'd

- (b) Is it possible to choose R_L such that the output pole limits the bandwidth?

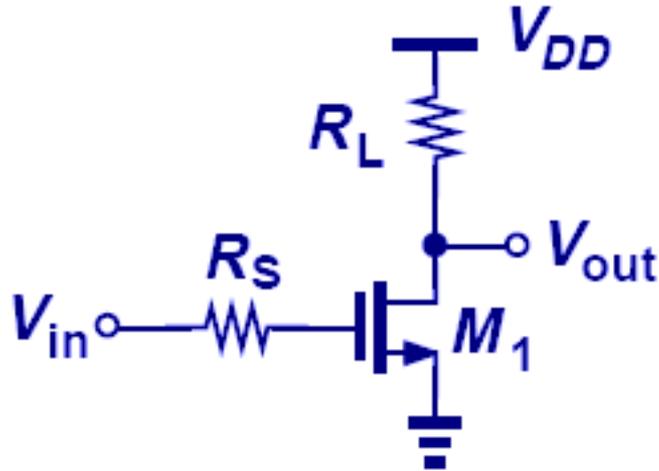
$$\begin{aligned} |\omega_{p,in}| &> |\omega_{p,out}| \\ \Rightarrow \frac{1}{(R_S \parallel r_\pi) [C_\pi + (1 + g_m R_L) C_\mu]} &> \frac{1}{R_L \left[C_{CS} + \left(1 + \frac{1}{g_m R_L} \right) C_\mu \right]} \end{aligned}$$

If $g_m R_L \ll 1$,

$$\Rightarrow \left[C_{CS} + C_\mu - g_m (R_S \parallel r_\pi) C_\mu \right] R_L > (R_S \parallel r_\pi) C_\pi$$

With the values assumed in this example, the left-hand side is negative, implying that no solution exists. Thus, the input pole remains the speed bottleneck.

Example: Half Width CS Stage



$$|\omega_{p,in}| = \frac{1}{R_S \left[\frac{C_{in}}{2} + \left(1 + \frac{g_m R_L}{2} \right) \frac{C_{XY}}{2} \right]}$$

$$|\omega_{p,out}| = \frac{1}{R_L \left[\frac{C_{out}}{2} + \left(1 + \frac{2}{g_m R_L} \right) \frac{C_{XY}}{2} \right]}$$

$$W \downarrow 2X$$

$$\text{bias current} \downarrow 2X$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \downarrow 2X$$

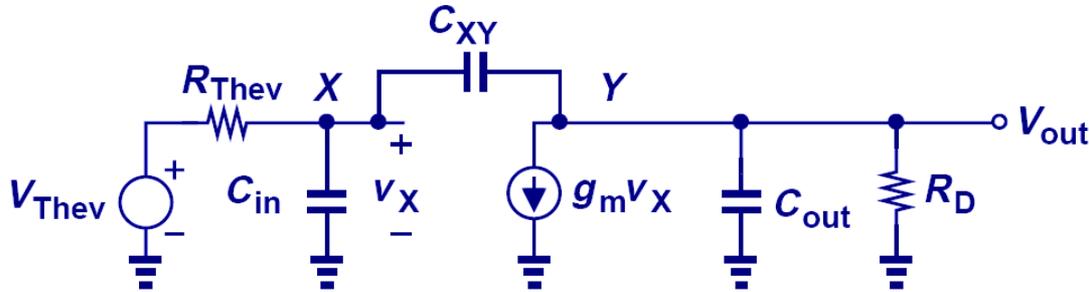
$$\text{capacitances} \downarrow 2X$$

$$\text{bandwidth} \uparrow 2X$$

$$\text{gain} \downarrow 2X$$

$$\text{gain} \times \text{bandwidth} \rightarrow \text{constant}$$

Direct Analysis of CE and CS Stages



$$\text{At Node Y: } (V_X - V_{out}) C_{XY} s = g_m V_X + V_{out} \left(\frac{1}{R_L} + C_{out} s \right) \Rightarrow V_X = V_{out} \frac{C_{XY} s + \frac{1}{R_L} + C_{out} s}{C_{XY} s - g_m}$$

$$\begin{aligned} \text{At Node X: } (V_{out} - V_X) C_{XY} s &= V_X C_{in} s + \frac{V_X - V_{Thev}}{R_{Thev}} \\ \Rightarrow V_{out} C_{XY} s - \left(C_{XY} s + C_{in} s + \frac{1}{R_{Thev}} \right) \frac{C_{XY} s + \frac{1}{R_L} + C_{out} s}{C_{XY} s - g_m} V_{out} &= \frac{-V_{Thev}}{R_{Thev}} \end{aligned}$$

$$\Rightarrow \frac{V_{out}}{V_{Thev}}(s) = \frac{(C_{XY} s - g_m) R_L}{as^2 + bs + 1} \quad \text{where } a = R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out}),$$

$$b = (1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})$$

Direct Analysis of CE and CS Stages – cont'd

$$|\omega_z| = \frac{g_m}{C_{XY}}$$

$$as^2 + bs + 1 = \left(\frac{s}{\omega_{p1}} + 1 \right) \left(\frac{s}{\omega_{p2}} + 1 \right) = \frac{s^2}{\omega_{p1}\omega_{p2}} + \left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} \right) s + 1$$

$$\text{if } \omega_{p2} \square \omega_{p1} \Rightarrow \omega_{p1}^{-1} + \omega_{p2}^{-1} \approx \omega_{p1}^{-1}$$

Dominant-pole approximation

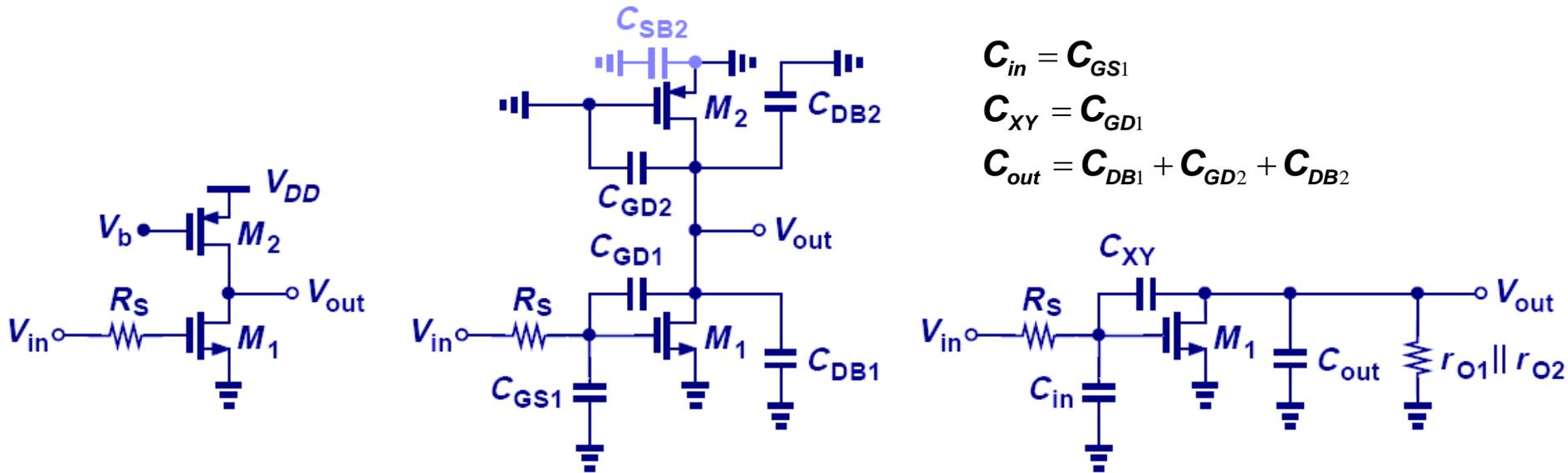
$$\Rightarrow b = \frac{1}{\omega_{p1}}$$

$$|\omega_{p1}| = \frac{1}{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}$$

$$|\omega_{p2}| = \frac{b}{a} = \frac{(1 + g_m R_L) C_{XY} R_{Thev} + R_{Thev} C_{in} + R_L (C_{XY} + C_{out})}{R_{Thev} R_L (C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

➤ Direct analysis yields different pole locations and an extra zero.

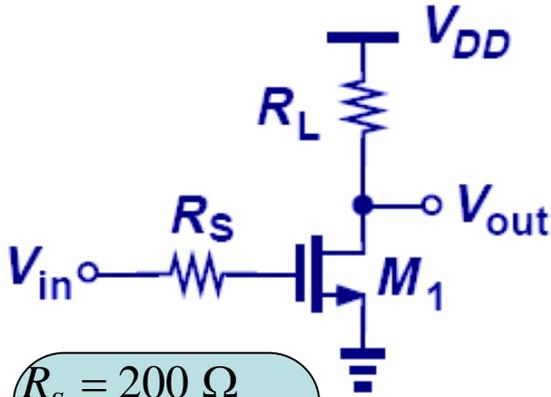
Example: Dominant-pole approximation



$$\omega_{p1} \approx \frac{1}{[1 + g_{m1}(r_{O1} \parallel r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{out})}$$

$$\omega_{p2} \approx \frac{[1 + g_{m1}(r_{O1} \parallel r_{O2})]C_{XY}R_S + R_S C_{in} + (r_{O1} \parallel r_{O2})(C_{XY} + C_{out})}{R_S (r_{O1} \parallel r_{O2})(C_{in} C_{XY} + C_{out} C_{XY} + C_{in} C_{out})}$$

Example: Comparison Between Different Methods



$$R_S = 200 \Omega$$

$$C_{GS} = 250 \text{ fF}$$

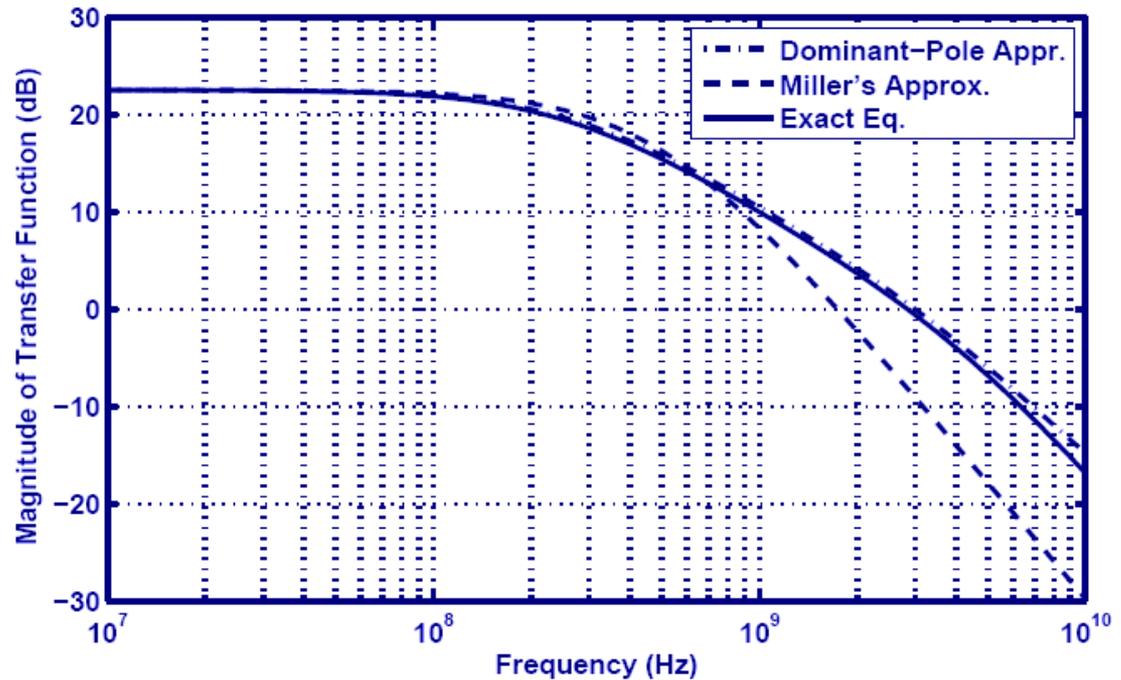
$$C_{GD} = 80 \text{ fF}$$

$$C_{DB} = 100 \text{ fF}$$

$$g_m = (150 \Omega)^{-1}$$

$$\lambda = 0$$

$$R_L = 2 \text{ k}\Omega$$



This error arises because we have multiplied C_{GD} by the midband gain $(1+g_m R_L)$ rather than the gain at high frequencies.

Miller's

$$|\omega_{p,in}| = 2\pi \times (571 \text{ MHz})$$

$$|\omega_{p,out}| = 2\pi \times (428 \text{ MHz})$$

Exact

$$|\omega_{p,in}| = 2\pi \times (264 \text{ MHz})$$

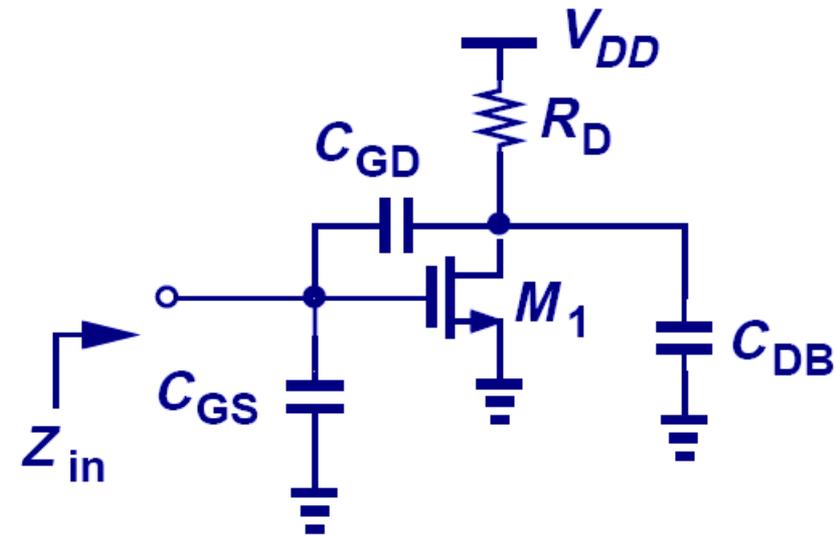
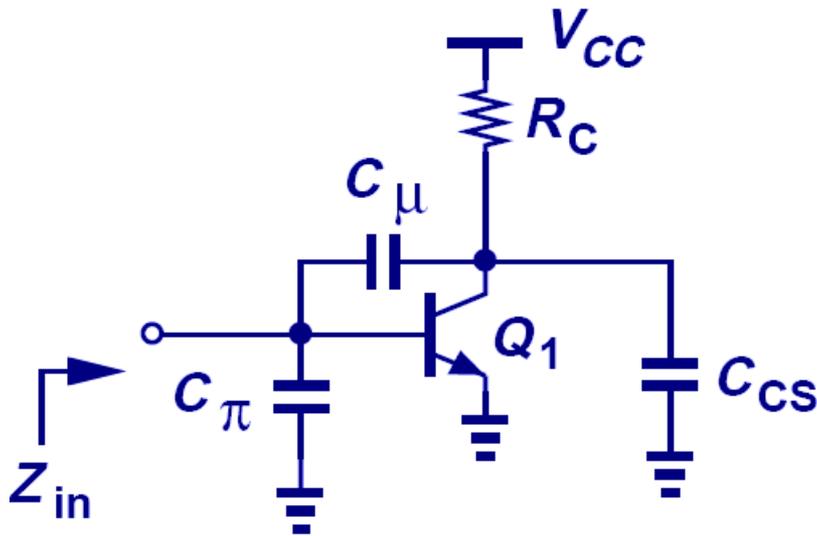
$$|\omega_{p,out}| = 2\pi \times (4.53 \text{ GHz})$$

Dominant Pole

$$|\omega_{p,in}| = 2\pi \times (249 \text{ MHz})$$

$$|\omega_{p,out}| = 2\pi \times (4.79 \text{ GHz})$$

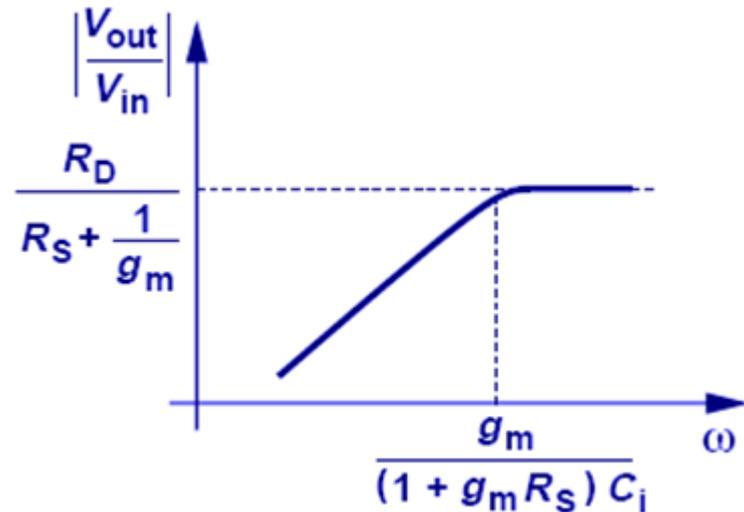
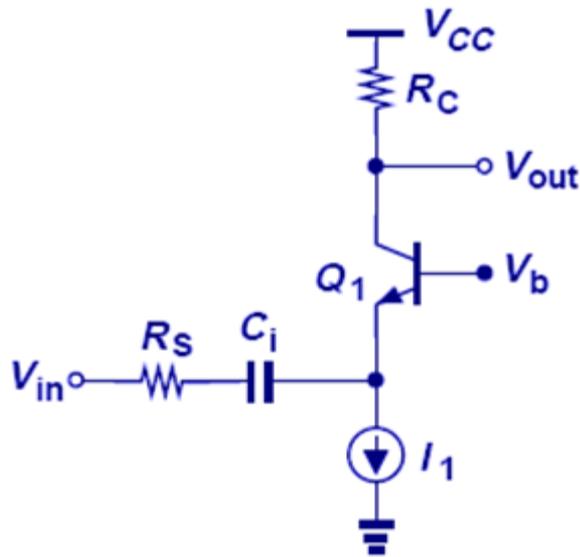
Input Impedance of CE and CS Stages



$$Z_{in} \approx \frac{1}{[C_{\pi} + (1 + g_m R_C)C_{\mu}]s} \parallel r_{\pi}$$

$$Z_{in} \approx \frac{1}{[C_{GS} + (1 + g_m R_D)C_{GD}]s}$$

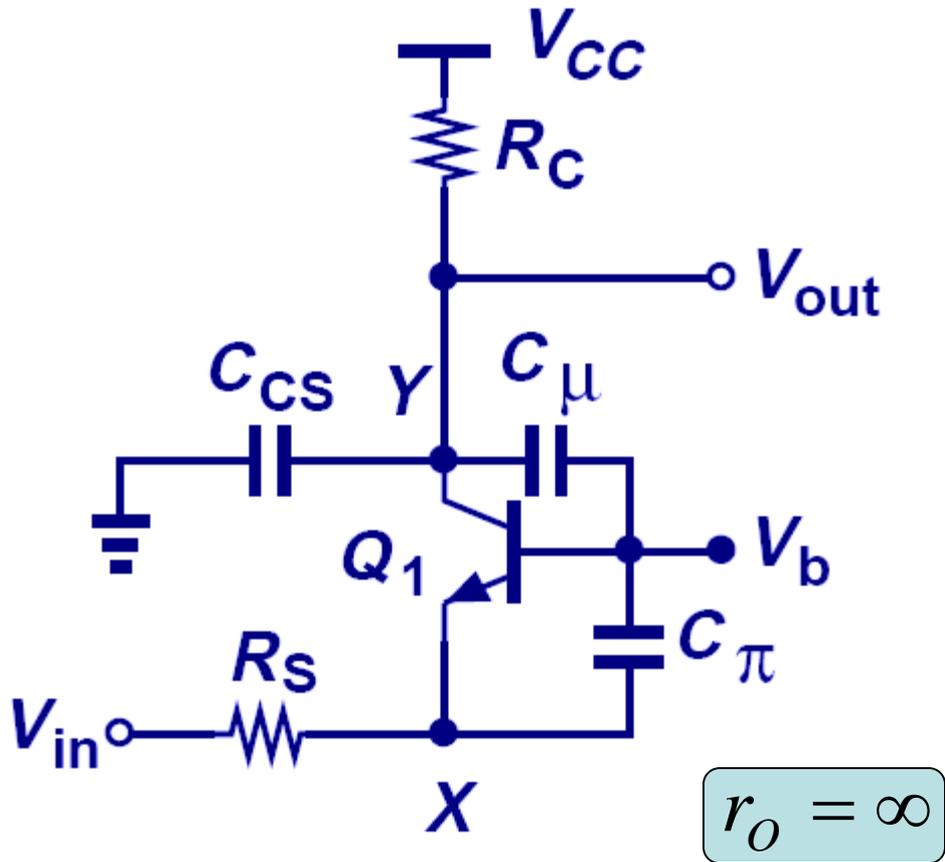
Low Frequency Response of CB and CG Stages



$$\frac{V_{out}}{V_{in}}(s) = \frac{R_C}{R_S + (C_i s)^{-1} + 1/g_m} = \frac{g_m R_C C_i s}{(1 + g_m R_S) C_i s + g_m}$$

➤ As with CE and CS stages, the use of capacitive coupling leads to low-frequency roll-off in CB and CG stages (although a CB stage is shown above, a CG stage is similar).

Frequency Response of CB Stage



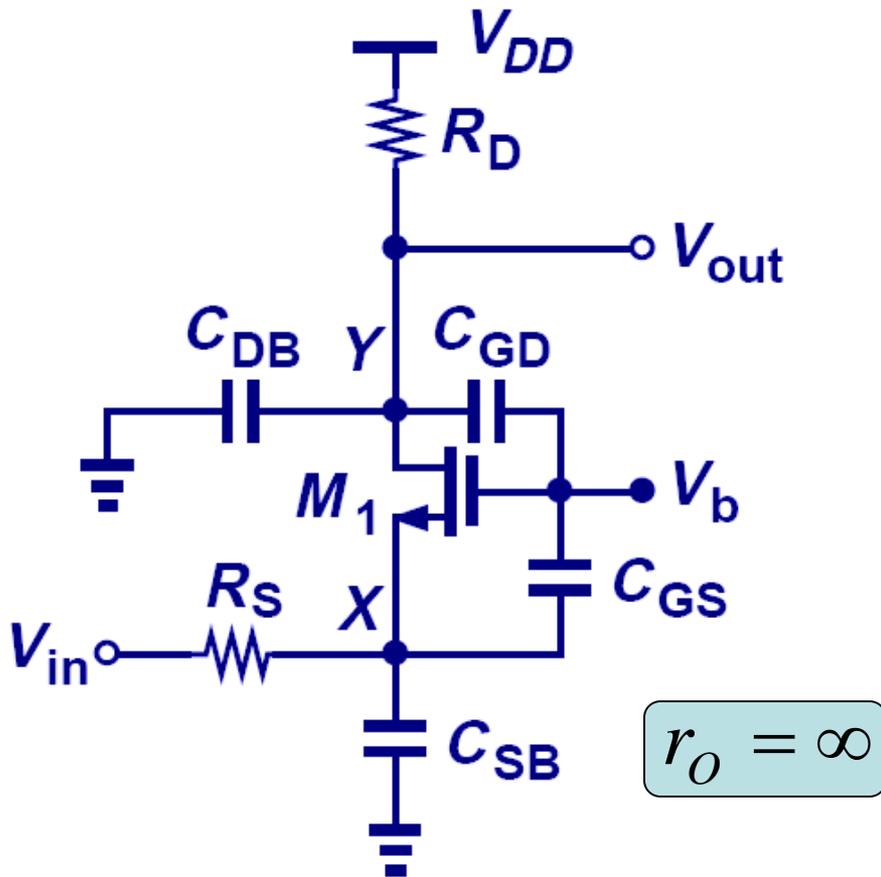
$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m} \right) C_X}$$

$$C_X = C_\pi$$

$$\omega_{p,Y} = \frac{1}{R_C C_Y}$$

$$C_Y = C_\mu + C_{CS}$$

Frequency Response of CG Stage



$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_m} \right) C_X}$$

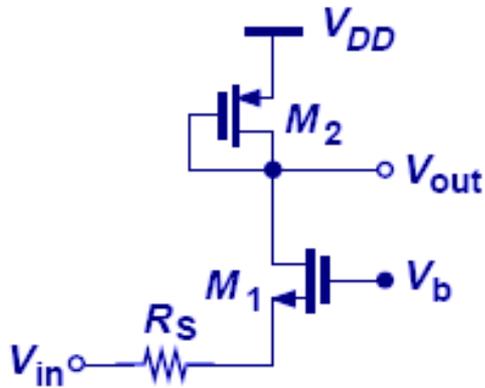
$$C_X = C_{GS} + C_{SB}$$

$$\omega_{p,Y} = \frac{1}{R_D C_Y}$$

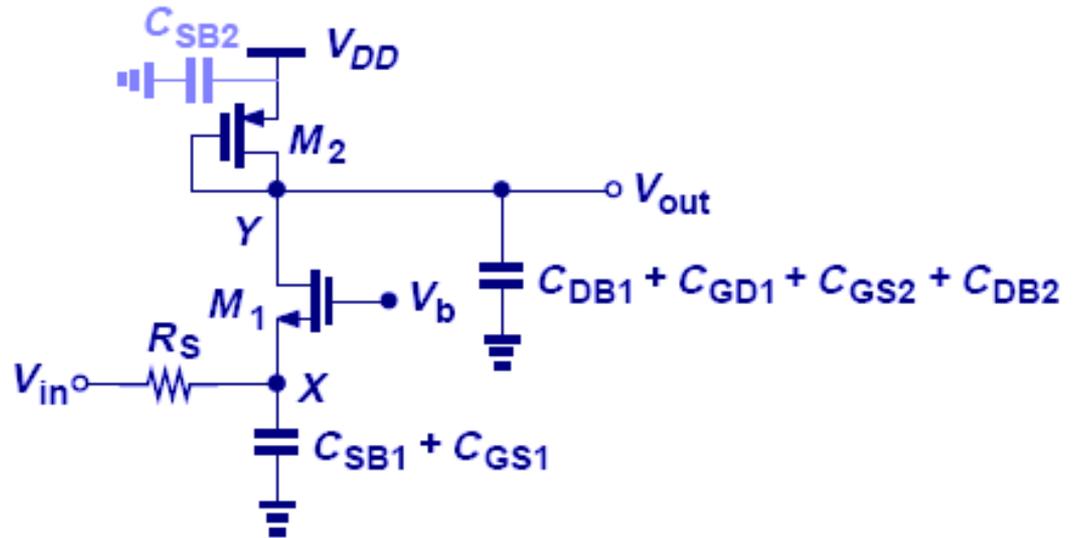
$$C_Y = C_{GD} + C_{DB}$$

- Similar to a CB stage, the input pole is on the order of f_T , so rarely a speed bottleneck.

Example: CG Stage Pole Identification



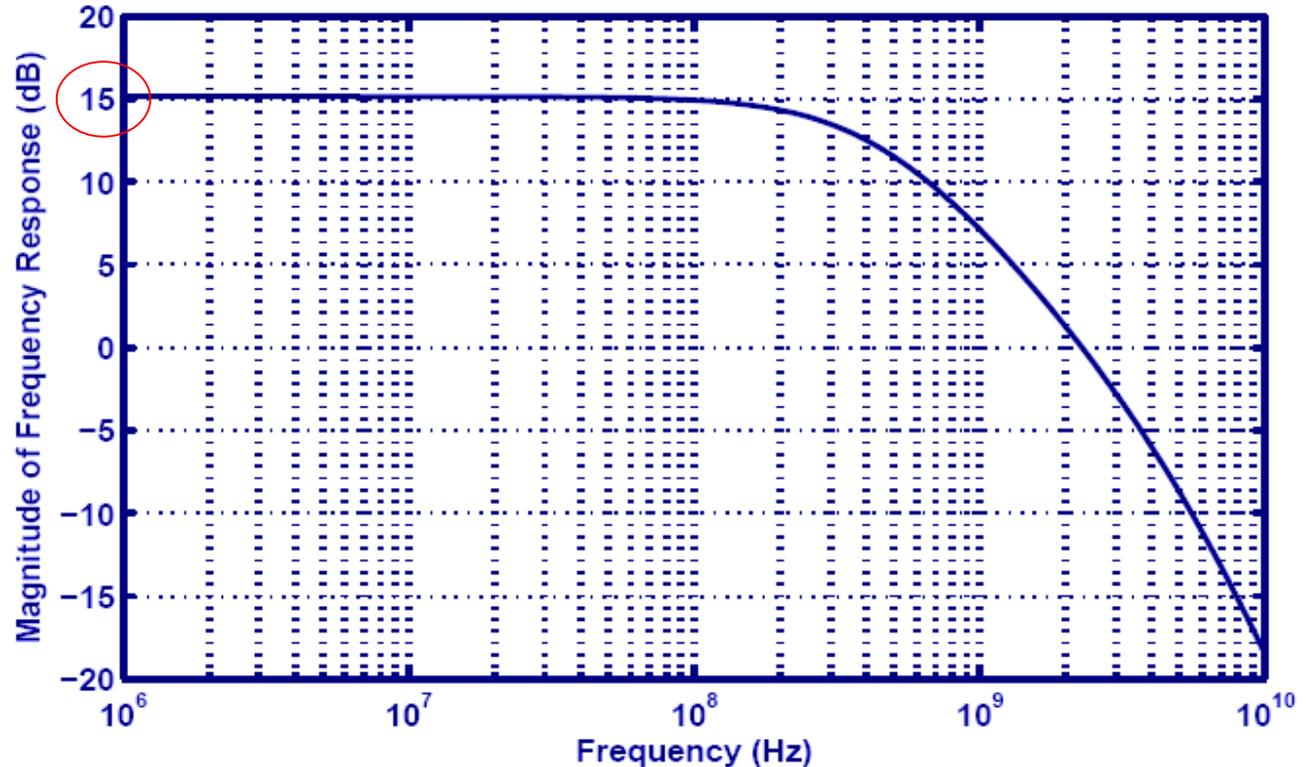
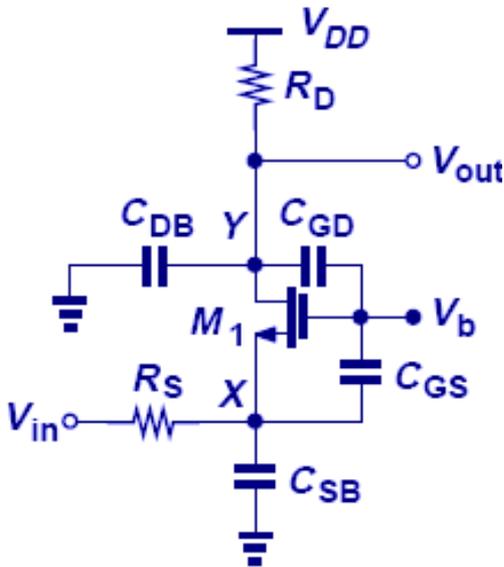
(a)



(b)

$$\omega_{p,X} = \frac{1}{\left(R_S \parallel \frac{1}{g_{m1}} \right) (C_{SB1} + C_{GS1})} \quad \omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}} (C_{DB1} + C_{GD1} + C_{GS2} + C_{DB2})}$$

Example: Frequency Response of CG Stage



$$R_S = 200 \, \Omega$$

$$C_{GS} = 250 \, \text{fF}$$

$$C_{GD} = 80 \, \text{fF}$$

$$C_{DB} = 100 \, \text{fF}$$

$$g_m = (150 \, \Omega)^{-1}$$

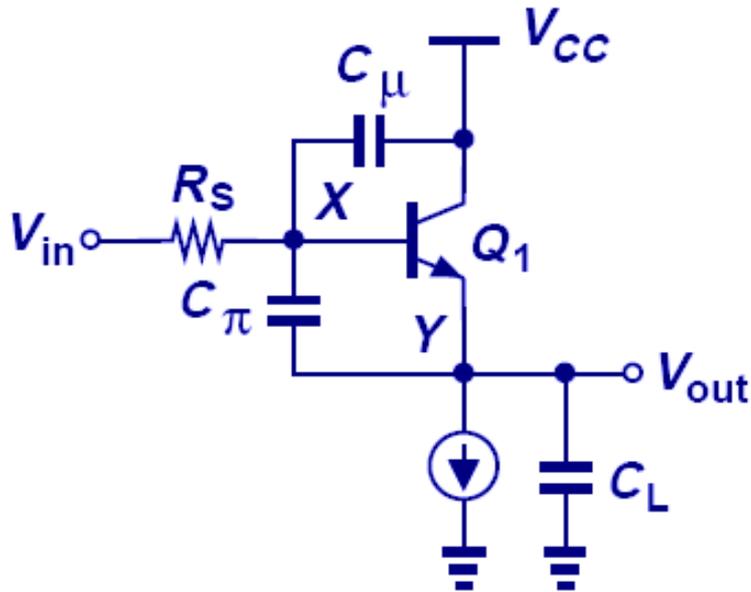
$$\lambda = 0$$

$$R_D = 2 \, \text{k}\Omega$$

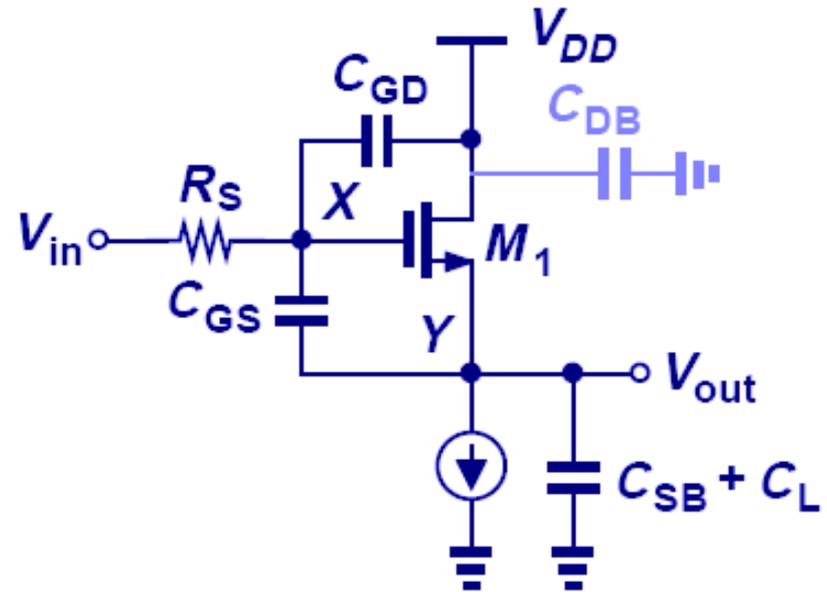
$$|\omega_{p,X}| = 1 / \left(R_S \parallel \frac{1}{g_m} \right) C_X = 2\pi \times (5.31 \, \text{GHz})$$

$$|\omega_{p,Y}| = R_L / C_Y = 2\pi \times (442 \, \text{MHz})$$

Emitter and Source Followers



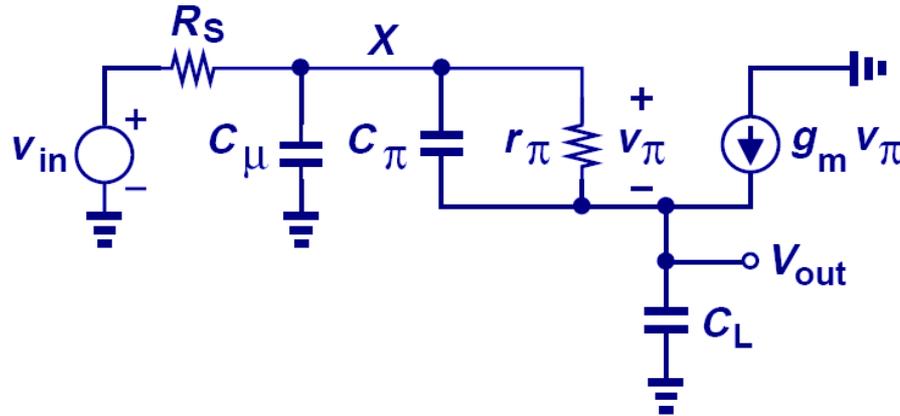
(a)



(b)

- The following will discuss the frequency response of emitter and source followers using direct analysis.
- Emitter follower is treated first and source follower is derived easily by allowing r_{π} to go to infinity.

Direct Analysis of Emitter Follower



At node X:
$$\frac{V_{out} + V_{\pi} - V_{in}}{R_S} + (V_{out} + V_{\pi})C_{\mu}s + \frac{V_{\pi}}{r_{\pi}} + V_{\pi}C_{\pi}s = 0$$

At output node:
$$\frac{V_{\pi}}{r_{\pi}} + V_{\pi}C_{\pi}s + g_m V_{\pi} = V_{out}C_Ls \Rightarrow V_{\pi} = \frac{V_{out}C_Ls}{\frac{1}{r_{\pi}} + C_{\pi}s + g_m}$$

$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{\pi}}{g_m}s}{as^2 + bs + 1}$$

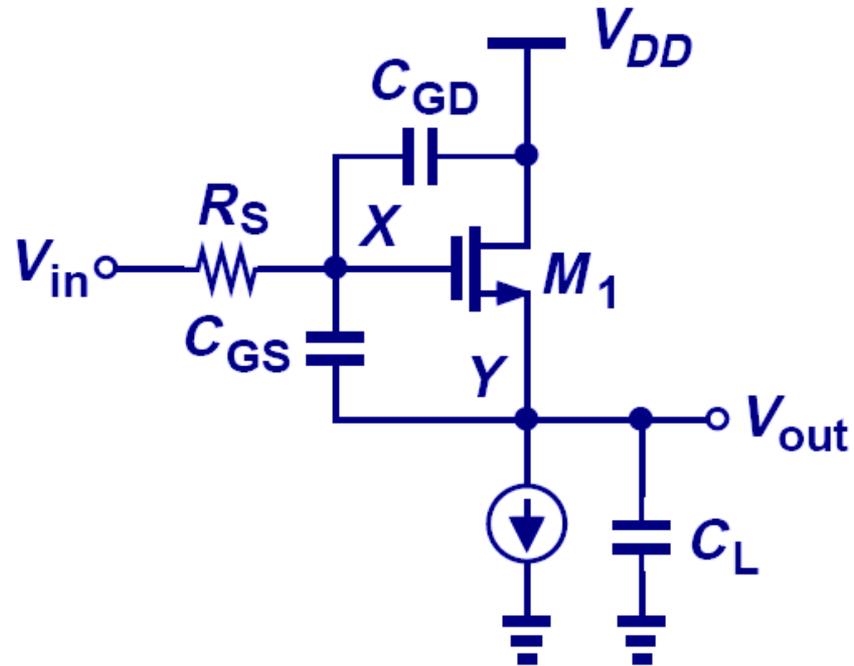
with $r_{\pi} \square g_m^{-1}$

where $a = \frac{R_S}{g_m} (C_{\mu}C_{\pi} + C_{\mu}C_L + C_{\pi}C_L)$

$$b = R_S C_{\mu} + \frac{C_{\pi}}{g_m} + \left(1 + \frac{R_S}{r_{\pi}}\right) \frac{C_L}{g_m}$$

$$|\omega_z| = \frac{g_m}{C_{\pi}} \approx f_T$$

Direct Analysis of Source Follower Stage

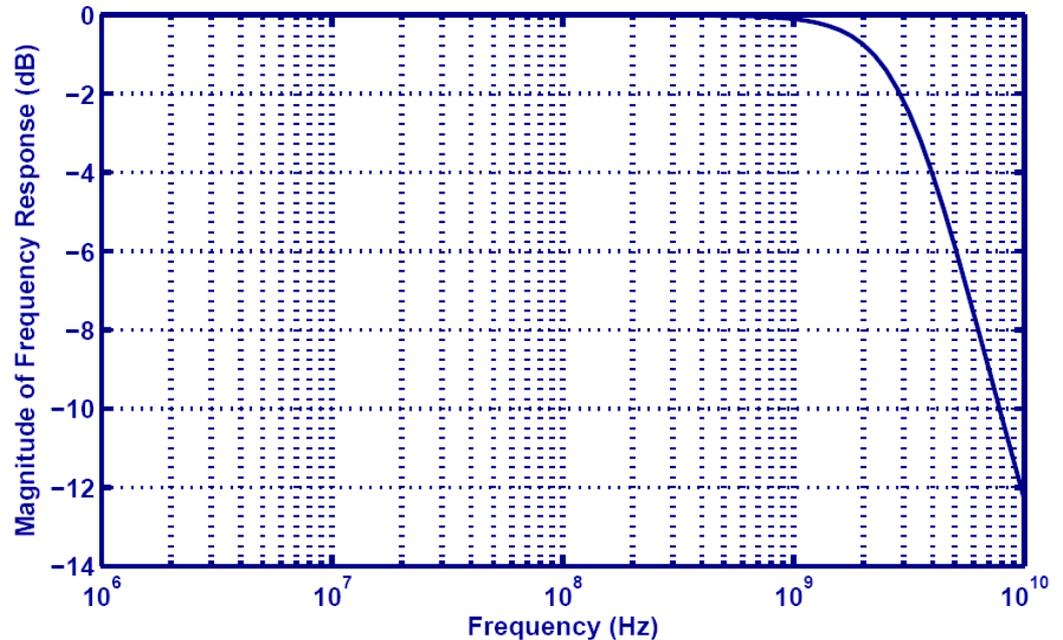
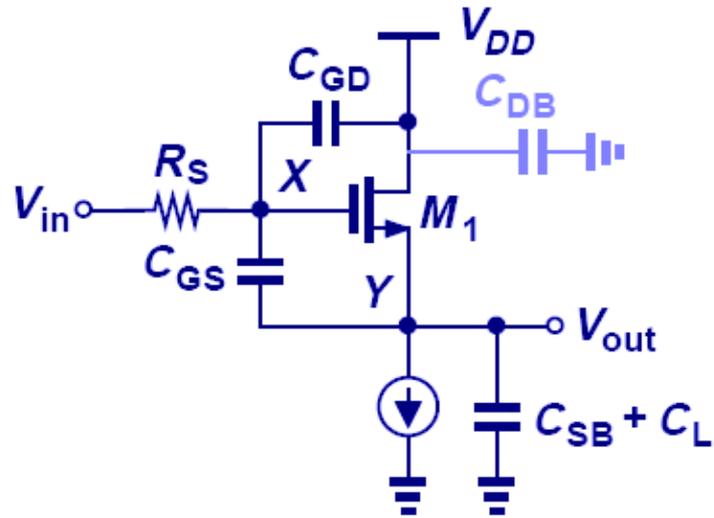


$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m} s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_m} (C_{GD} C_{GS} + C_{GD} (C_{SB} + C_L) + C_{GS} (C_{SB} + C_L))$$

$$b = R_S C_{GD} + \frac{C_{GD} + C_{SB} + C_L}{g_m}$$

Example: Frequency Response of Source Follower



$$R_S = 200 \Omega$$

$$C_L = 100 \text{ fF}$$

$$C_{GS} = 250 \text{ fF}$$

$$C_{GD} = 80 \text{ fF}$$

$$C_{DB} = 100 \text{ fF}$$

$$g_m = (150 \Omega)^{-1}$$

$$\lambda = 0$$

$$a = 2.58 \times 10^{-21} \text{ s}^{-2}$$

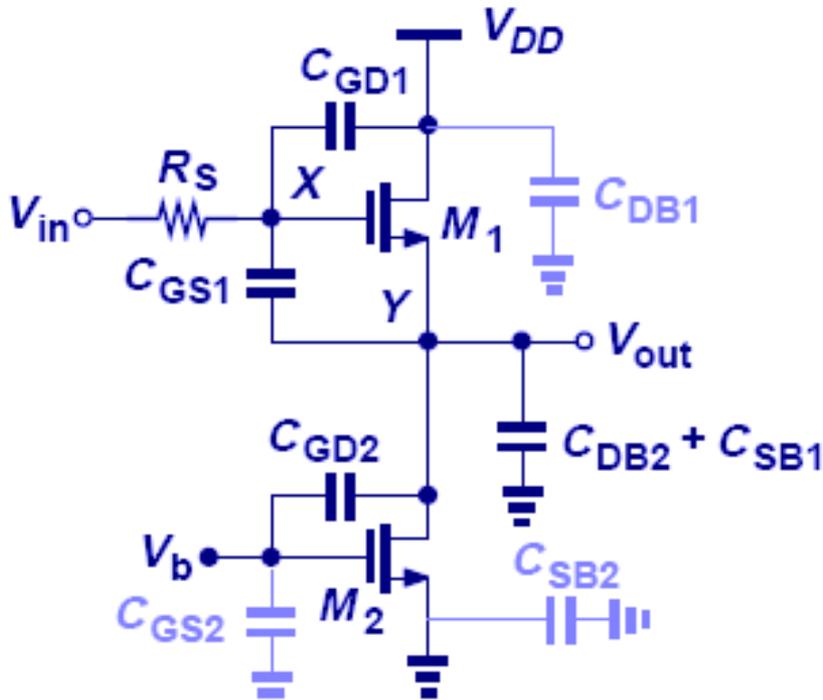
$$b = 5.8 \times 10^{-11} \text{ s}$$

$$\omega_z = g_m / C_{GS} = 2\pi \times (4.24 \text{ GHz})$$

$$\omega_{p1} = 2\pi \left[-1.79 \text{ GHz} + j(2.57 \text{ GHz}) \right]$$

$$\omega_{p2} = 2\pi \left[-1.79 \text{ GHz} - j(2.57 \text{ GHz}) \right]$$

Example: Source Follower

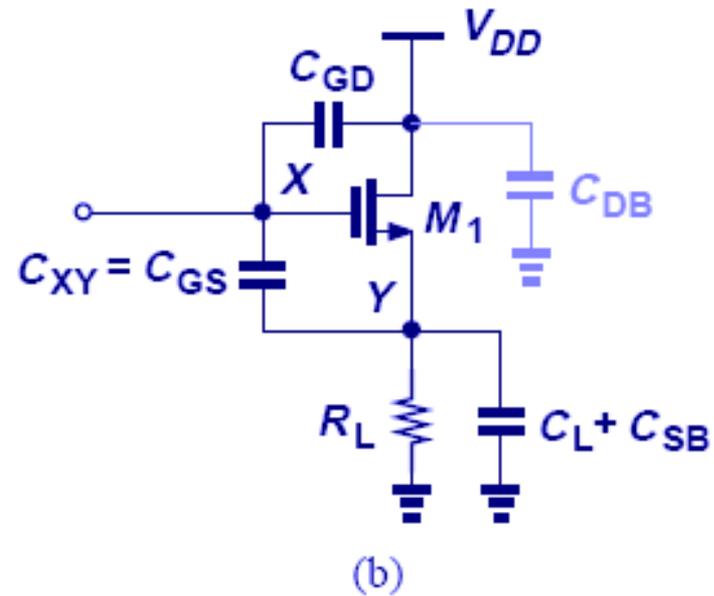
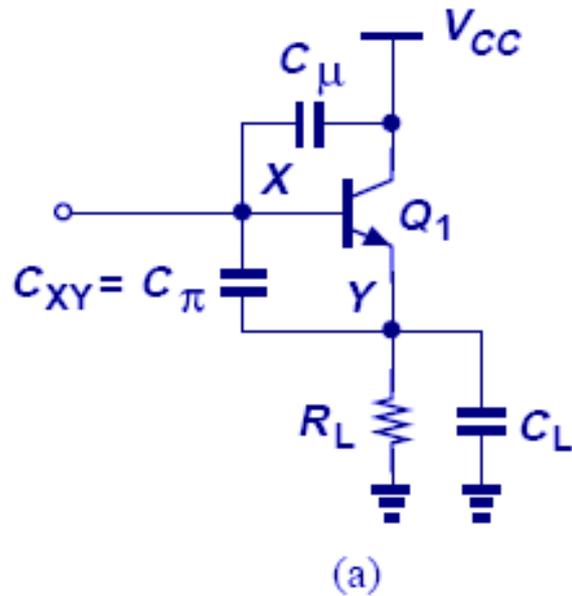


$$\frac{V_{out}}{V_{in}} = \frac{1 + \frac{C_{GS}}{g_m} s}{as^2 + bs + 1}$$

$$a = \frac{R_S}{g_{m1}} \left[C_{GD1} C_{GS1} + (C_{GD1} + C_{GS1})(C_{SB1} + C_{GD2} + C_{DB2}) \right]$$

$$b = R_S C_{GD1} + \frac{C_{GD1} + C_{SB1} + C_{GD2} + C_{DB2}}{g_{m1}}$$

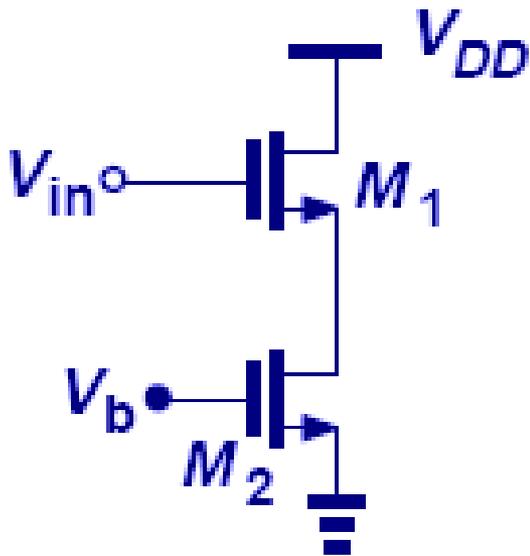
Input Capacitance of Emitter/Source Follower



$$A_v = \frac{R_L}{R_L + \frac{1}{g_m}} \Rightarrow C_X = (1 - A_v) C_{XY} = \frac{1}{1 + g_m R_L} C_{XY}$$

$$\therefore C_{in} = (C_\mu \text{ or } C_{GD}) + \frac{C_{XY}}{1 + g_m R_L}$$

Example: Source Follower Input Capacitance

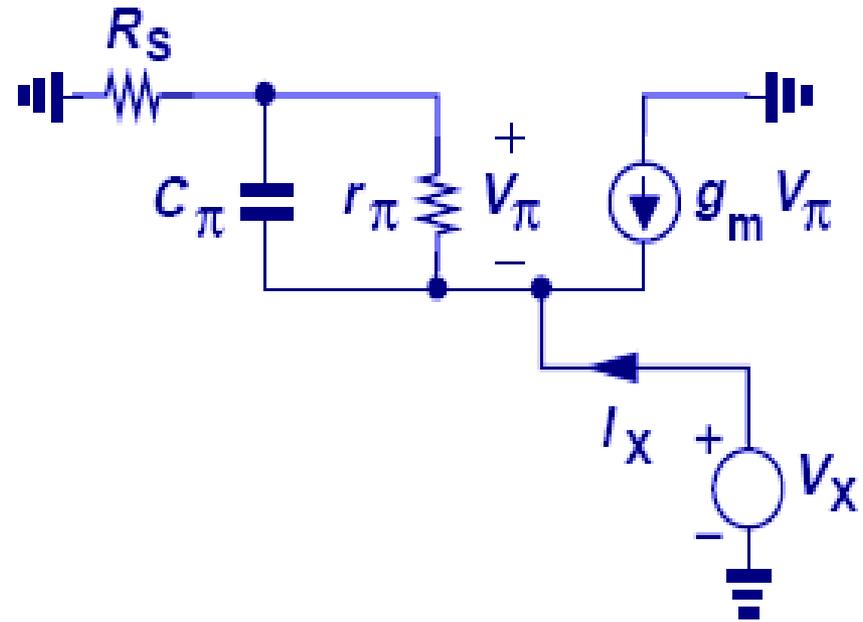
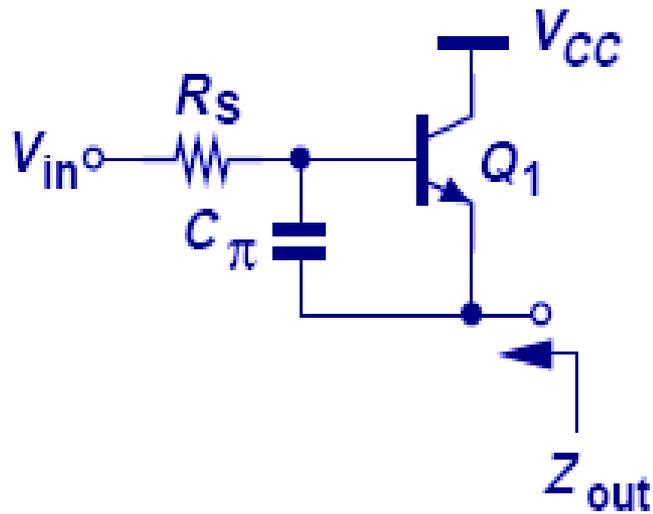


$$A_v = \frac{r_{O1} \parallel r_{O2}}{r_{O1} \parallel r_{O2} + \frac{1}{g_{m1}}}$$

$$\Rightarrow C_{in} = C_{GD1} + (1 - A_v) C_{GS1}$$

$$= C_{GD1} + \frac{1}{1 + g_{m1} (r_{O1} \parallel r_{O2})} C_{GS1}$$

Output Impedance of Emitter Follower



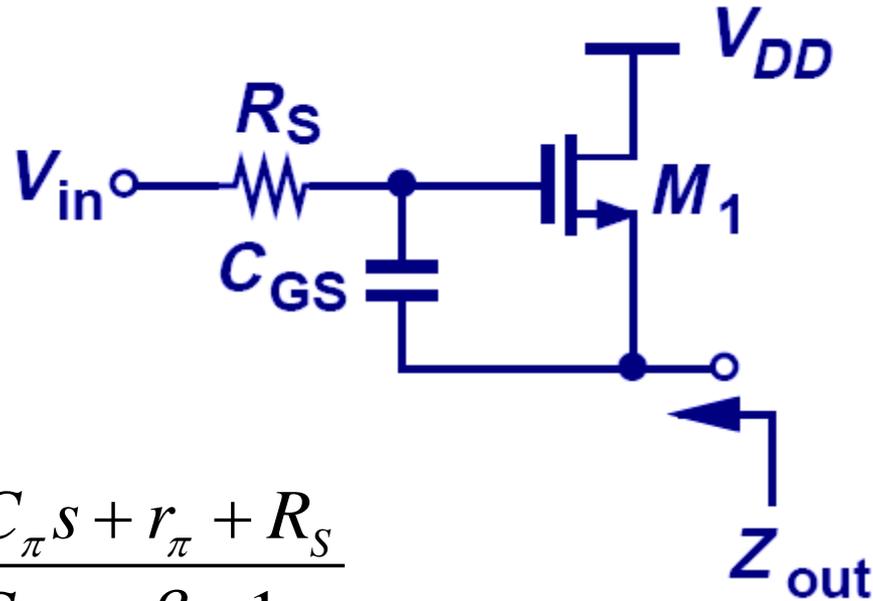
$$(I_X + g_m V_\pi) \left(r_\pi \parallel \frac{1}{C_\pi s} \right) = -V_\pi$$

$$\Rightarrow V_\pi = -I_X \frac{r_\pi}{r_\pi C_\pi s + \beta + 1}$$

$$(I_X + g_m V_\pi) R_S - V_\pi = V_X$$

$$\Rightarrow \frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

Output Impedance of Source Follower

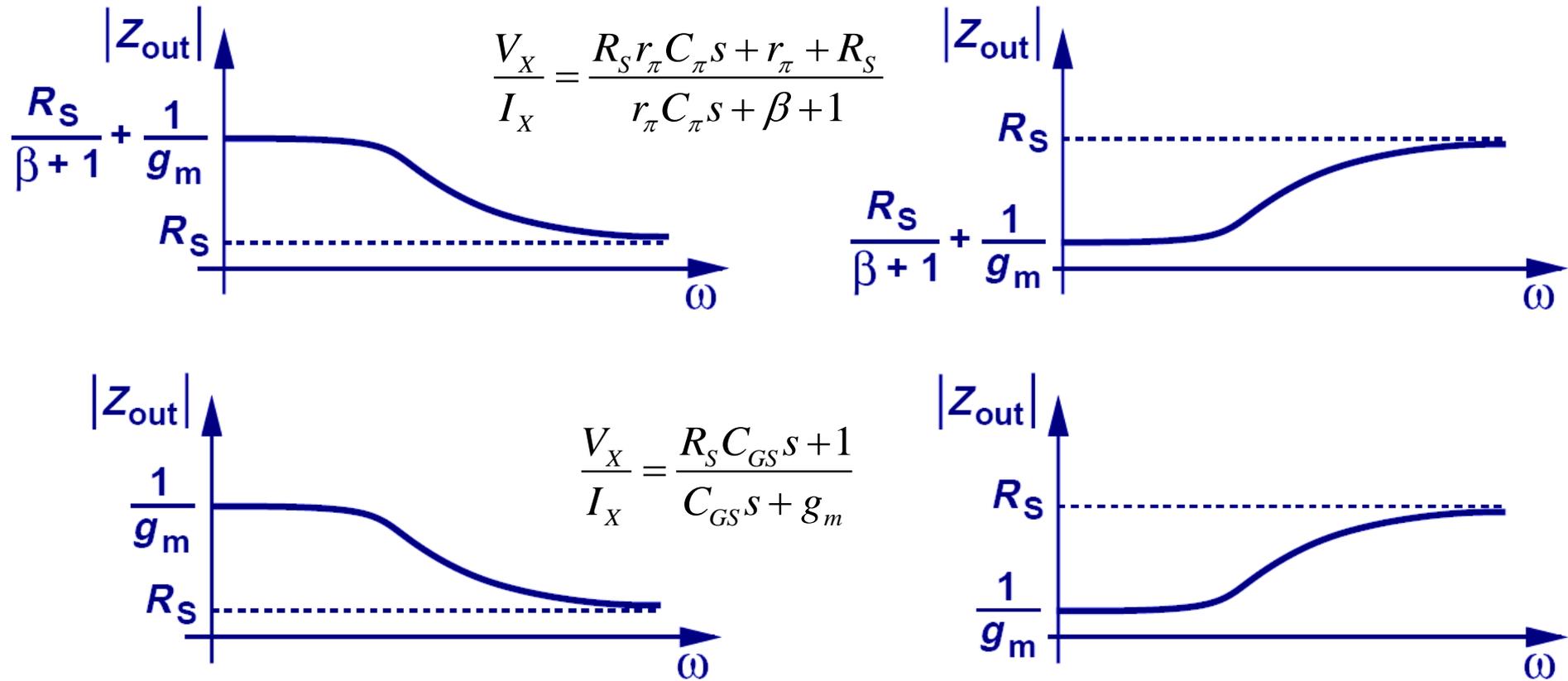


$$\frac{V_X}{I_X} = \frac{R_S r_\pi C_\pi s + r_\pi + R_S}{r_\pi C_\pi s + \beta + 1}$$

with $g_m \cdot r_\pi = \beta \rightarrow \infty$

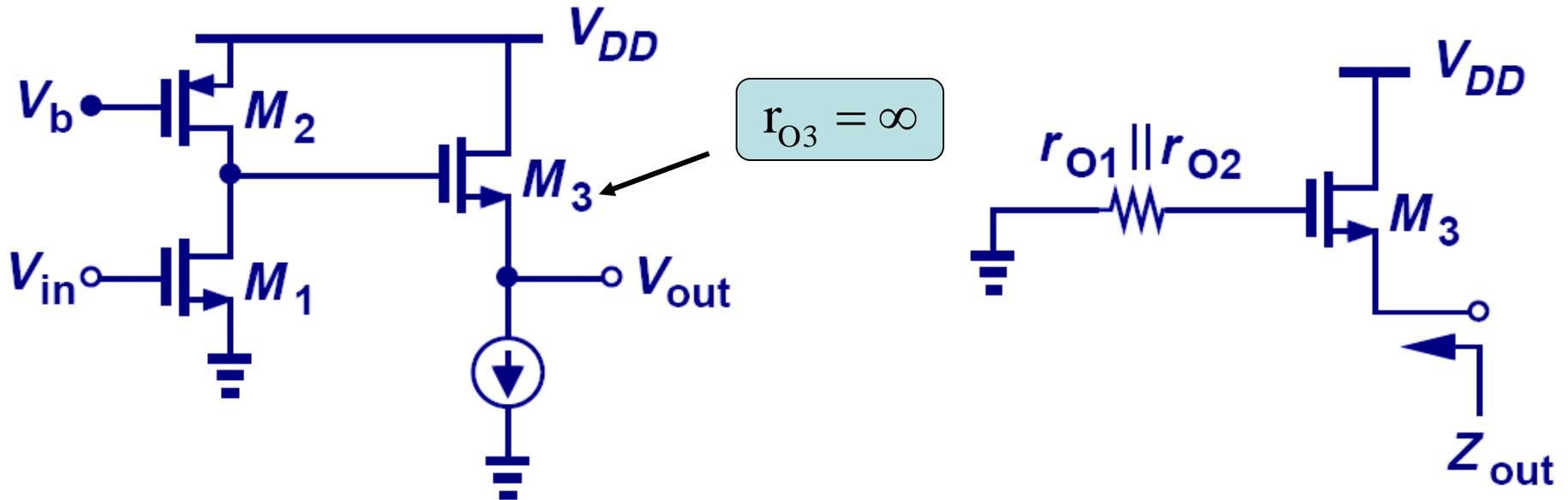
$$\frac{V_X}{I_X} = \frac{R_S C_{GS} s + 1}{C_{GS} s + g_m}$$

Active Inductor



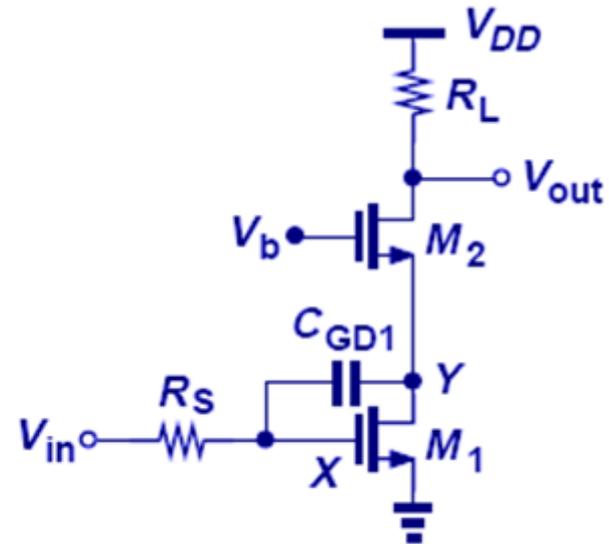
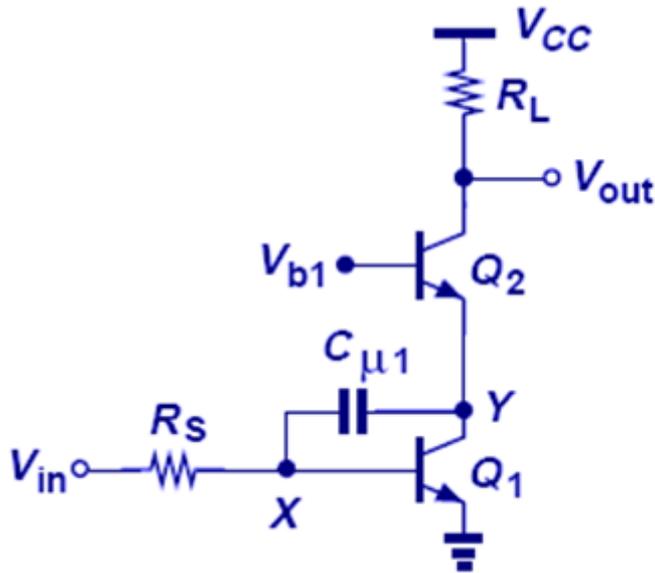
➤ The plot above shows the output impedance of emitter and source followers. Since a follower's primary duty is to lower the driving impedance ($R_S > 1/g_m$), the "active inductor" characteristic on the right is usually observed.

Example: Output Impedance



$$\frac{V_X}{I_X} = \frac{(r_{O1} \parallel r_{O2})C_{GS3}s + 1}{C_{GS3}s + g_{m3}}$$

Frequency Response of Cascode Stage



Assuming $r_o = \infty$ for all transistors,

$$A_{v,XY} = \frac{-g_{m1}}{g_{m2}} \approx -1$$

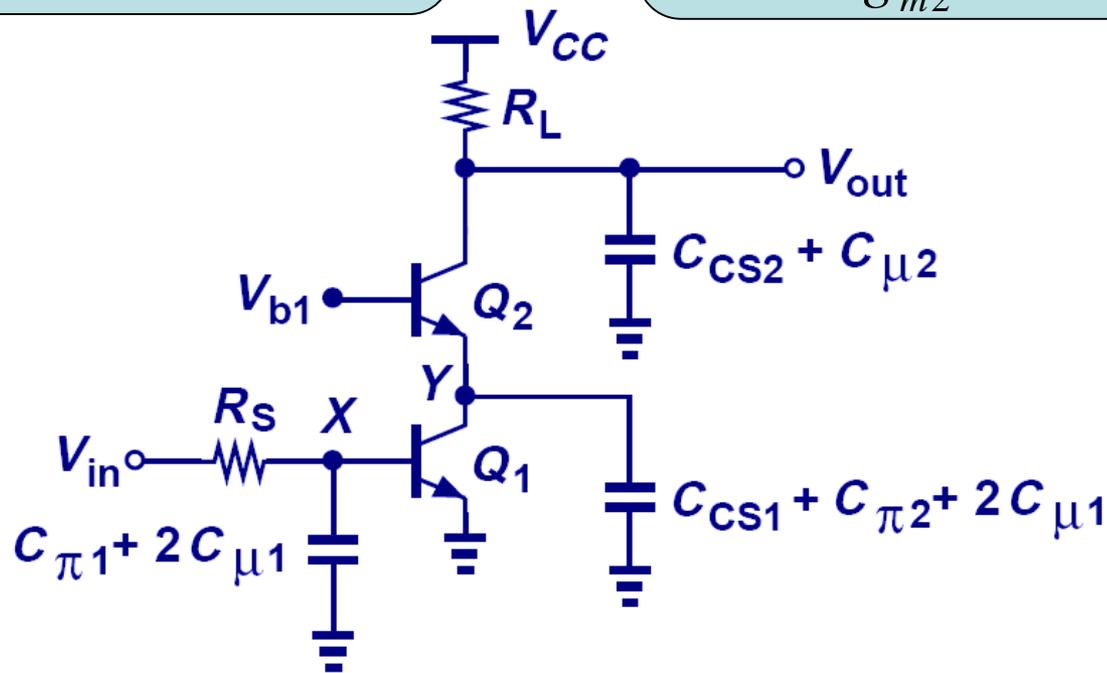
$$C_x = (1 - A_{v,XY}) C_{XY} \\ \approx 2 \cdot C_{XY}$$

- For cascode stages, there are three poles and Miller multiplication is smaller than in the CE/CS stage.

Poles of Bipolar Cascode

$$\omega_{p,X} = \frac{1}{(R_S \parallel r_{\pi 1})(C_{\pi 1} + 2C_{\mu 1})}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}}(C_{CS1} + C_{\pi 2} + 2C_{\mu 1})}$$

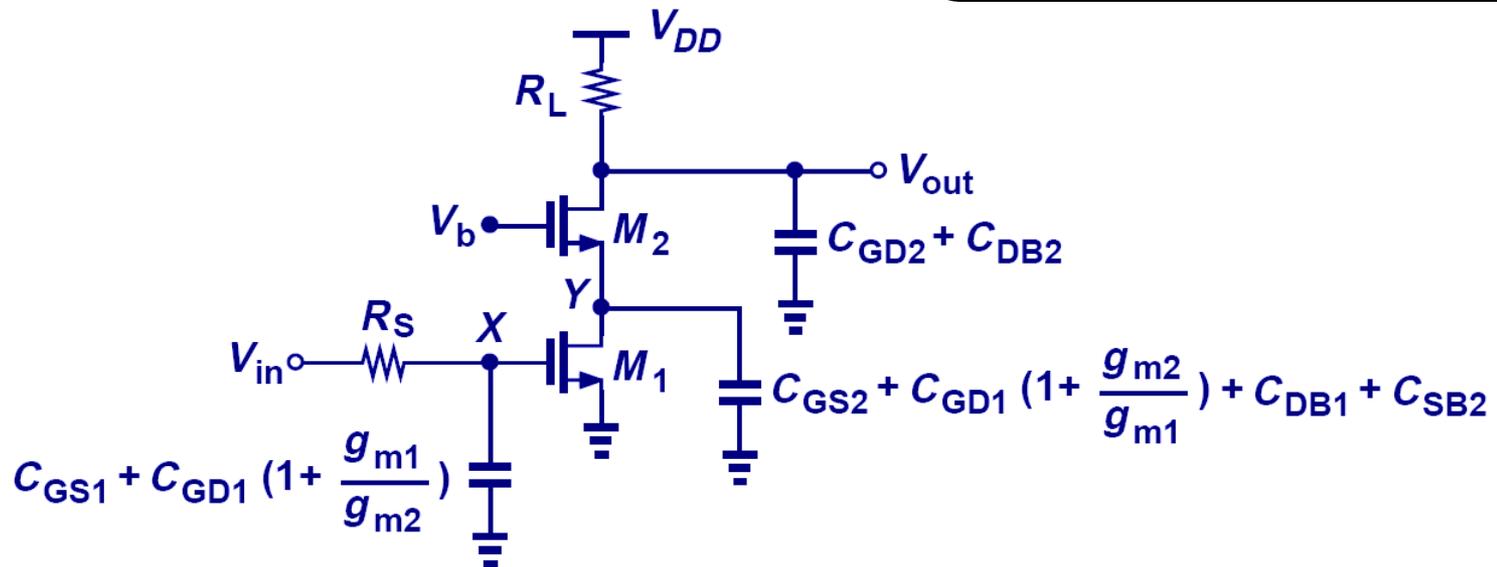


$$\omega_{p,out} = \frac{1}{R_L(C_{CS2} + C_{\mu 2})}$$

Poles of MOS Cascode

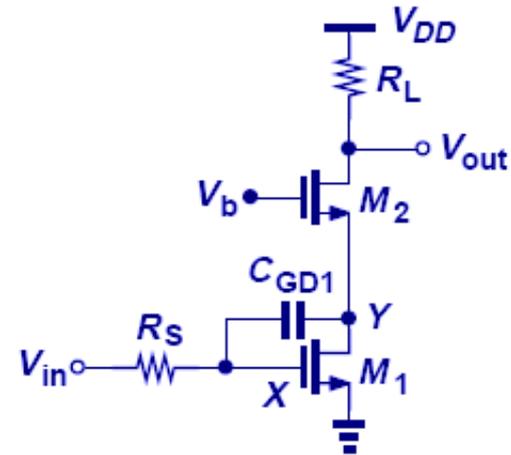
$$\omega_{p,X} = \frac{1}{R_S \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}} \right) C_{GD1} \right]}$$

$$\omega_{p,out} = \frac{1}{R_L (C_{DB2} + C_{GD2})}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}} \left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}} \right) C_{GD1} \right]}$$

Example: Frequency Response of Cascode



$$R_S = 200 \, \Omega$$

$$C_{GS} = 250 \, \text{fF}$$

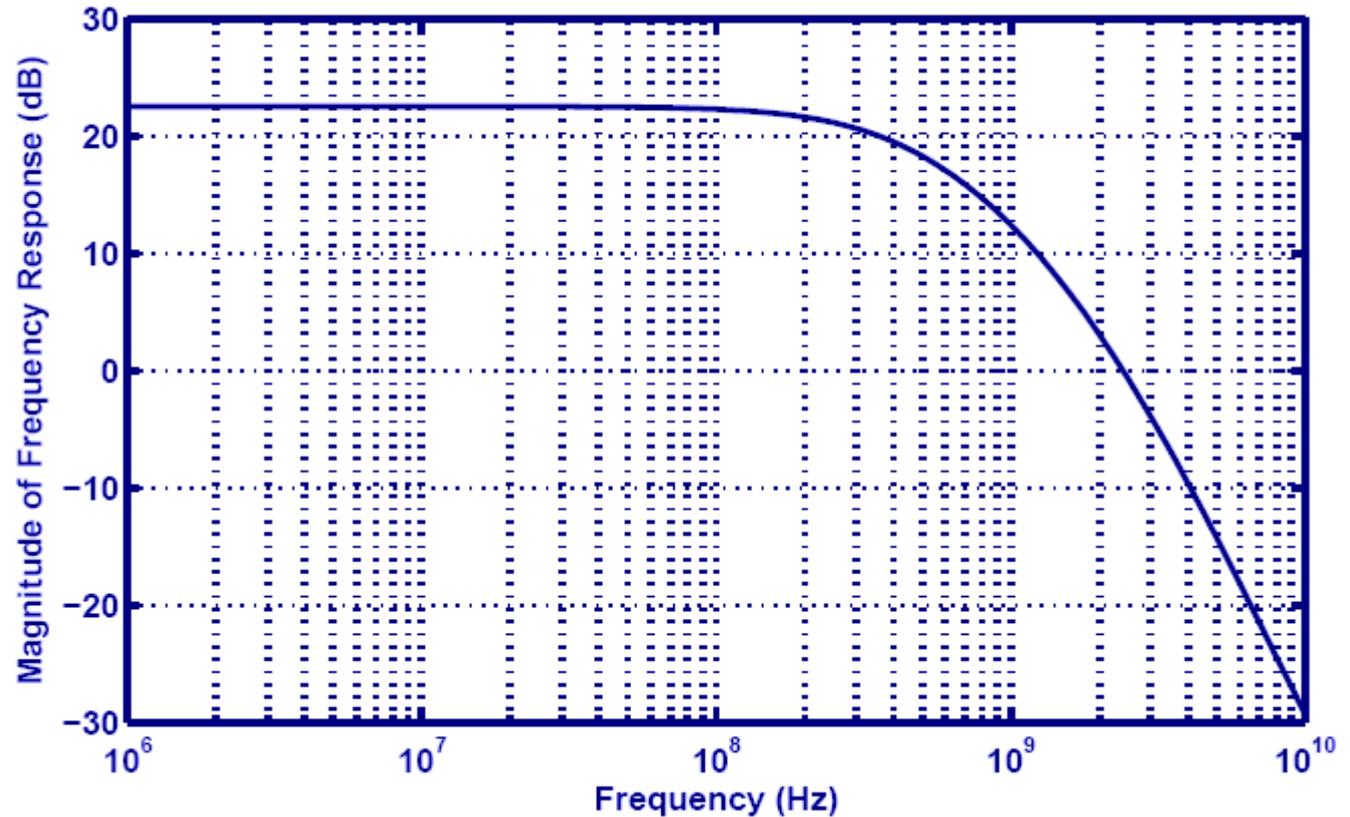
$$C_{GD} = 80 \, \text{fF}$$

$$C_{DB} = 100 \, \text{fF}$$

$$g_m = (150 \, \Omega)^{-1}$$

$$\lambda = 0$$

$$R_L = 2 \, \text{k}\Omega$$



$$|\omega_{p,X}| = 2\pi \times (1.95 \, \text{GHz})$$

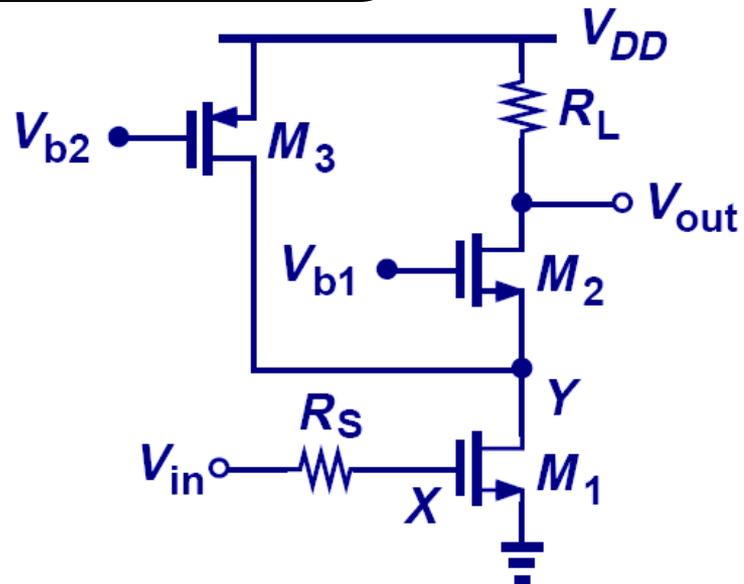
$$|\omega_{p,Y}| = 2\pi \times (1.73 \, \text{GHz})$$

$$|\omega_{p,out}| = 2\pi \times (442 \, \text{MHz})$$

MOS Cascode Example

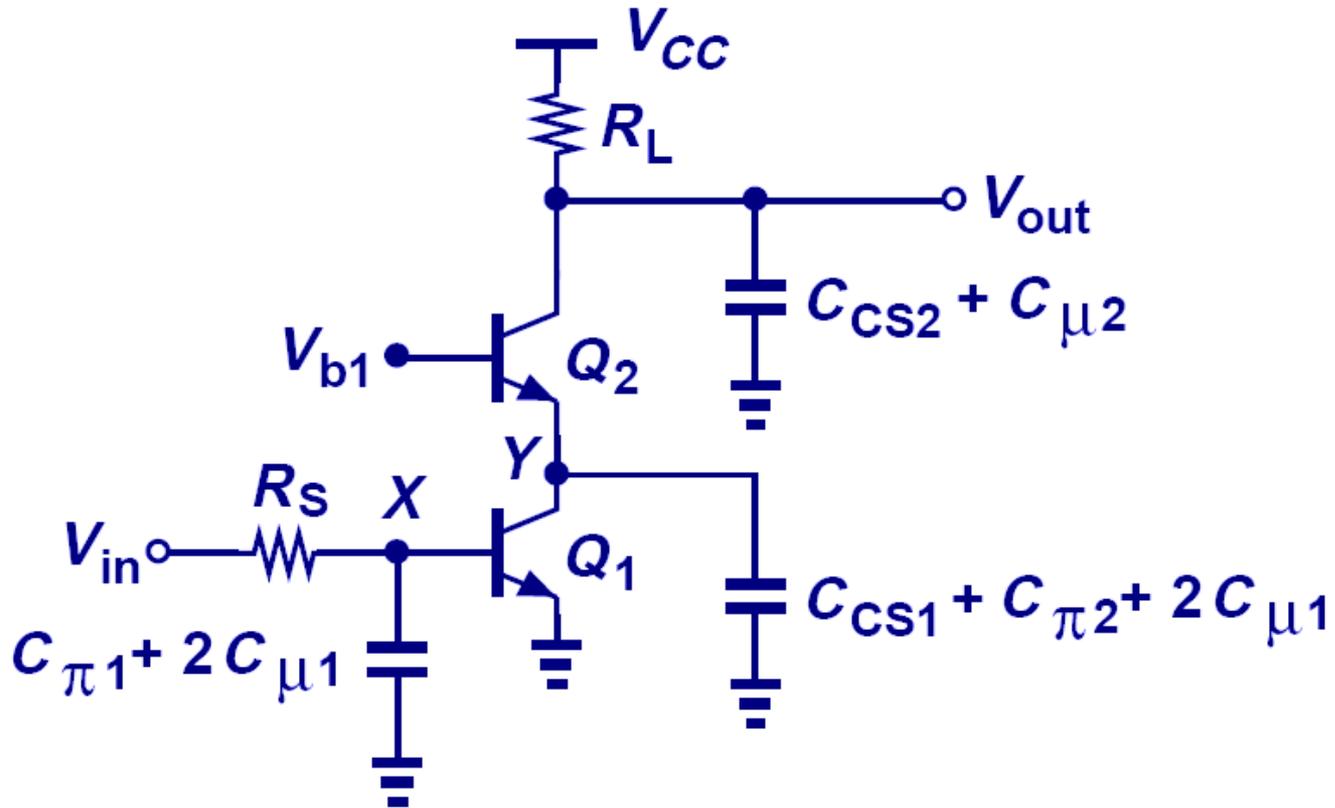
$$\omega_{p,X} = \frac{1}{R_S \left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}} \right) C_{GD1} \right]}$$

$$\omega_{p,out} = \frac{1}{R_L (C_{DB2} + C_{GD2})}$$



$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m2}} \left[C_{DB1} + C_{GS2} + \left(1 + \frac{g_{m2}}{g_{m1}} \right) C_{GD1} + C_{SB2} + C_{GD3} + C_{DB3} \right]}$$

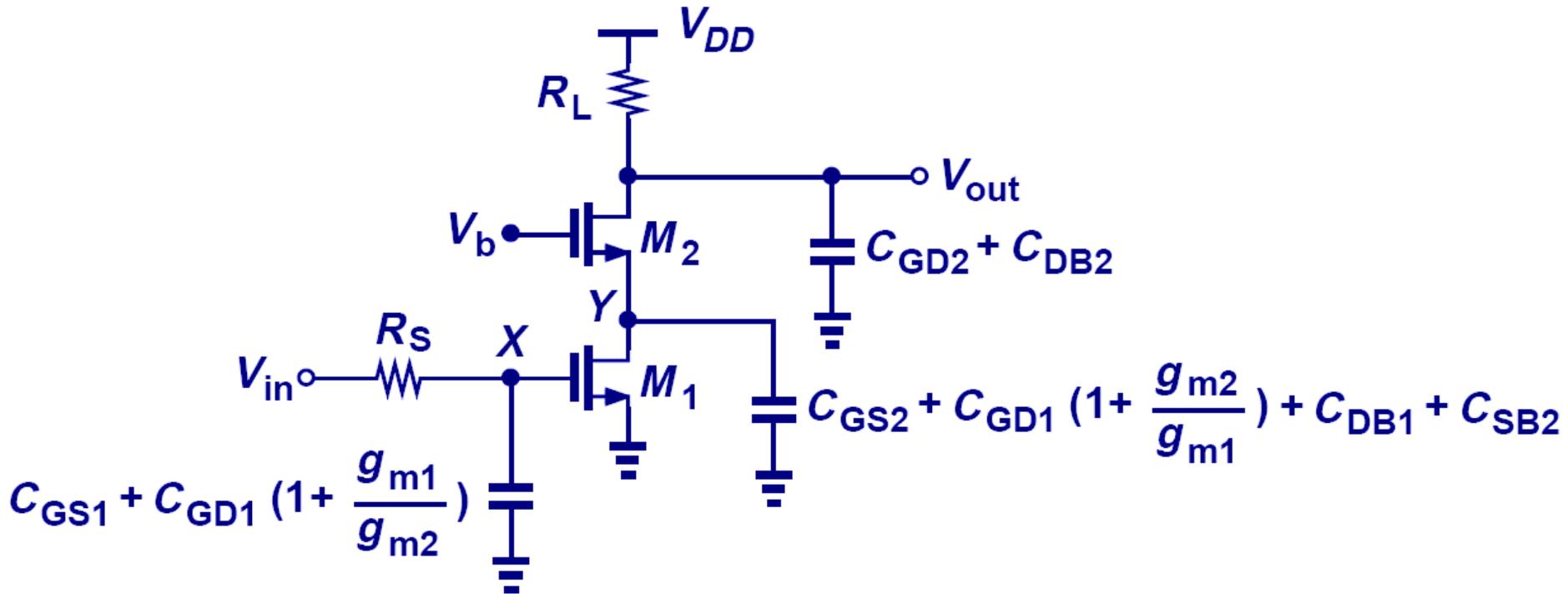
I/O Impedance of Bipolar Cascode



$$Z_{in} = r_{\pi 1} \parallel \frac{1}{(C_{\pi 1} + 2C_{\mu 1})s}$$

$$Z_{out} = R_L \parallel \frac{1}{(C_{\mu 2} + C_{CS2})s}$$

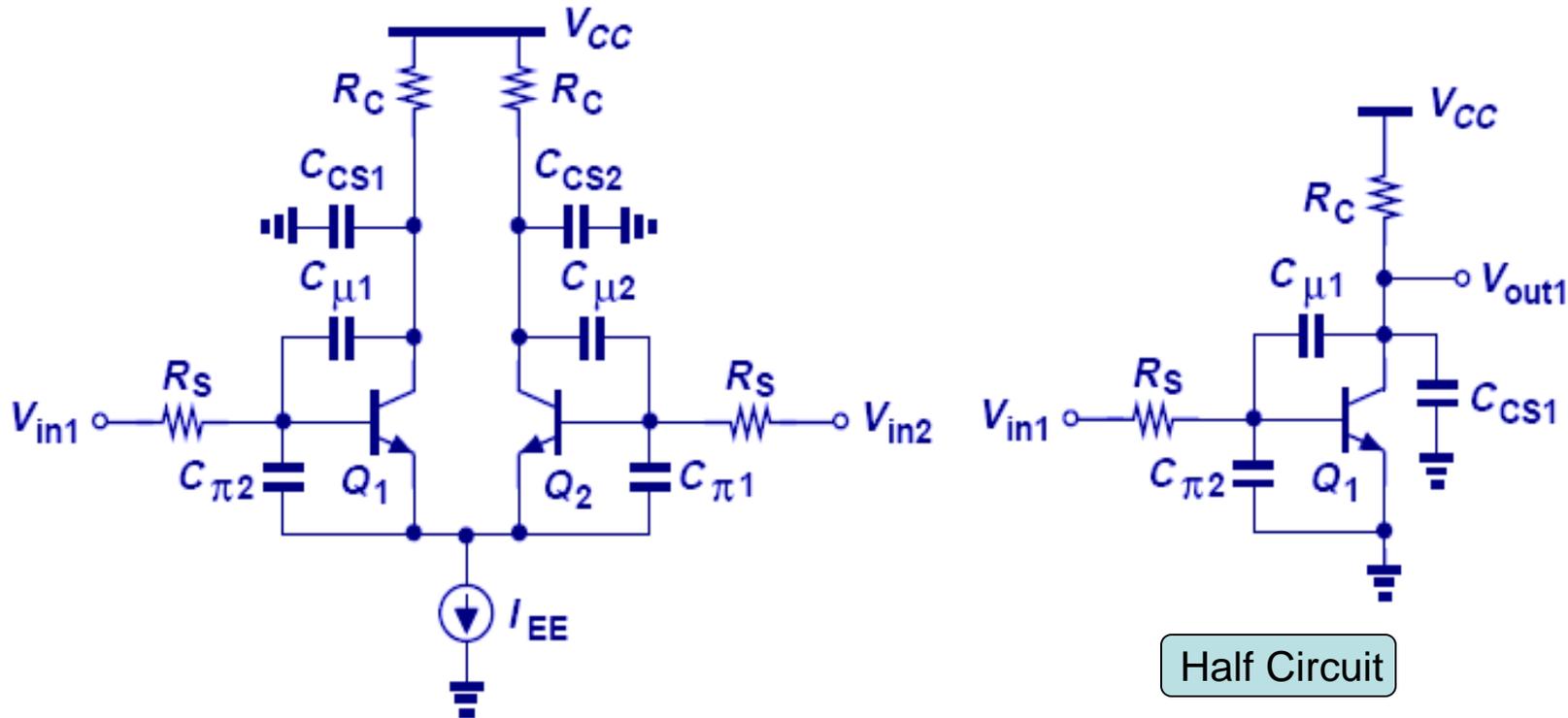
I/O Impedance of MOS Cascode



$$Z_{in} = \frac{1}{\left[C_{GS1} + \left(1 + \frac{g_{m1}}{g_{m2}} \right) C_{GD1} \right] s}$$

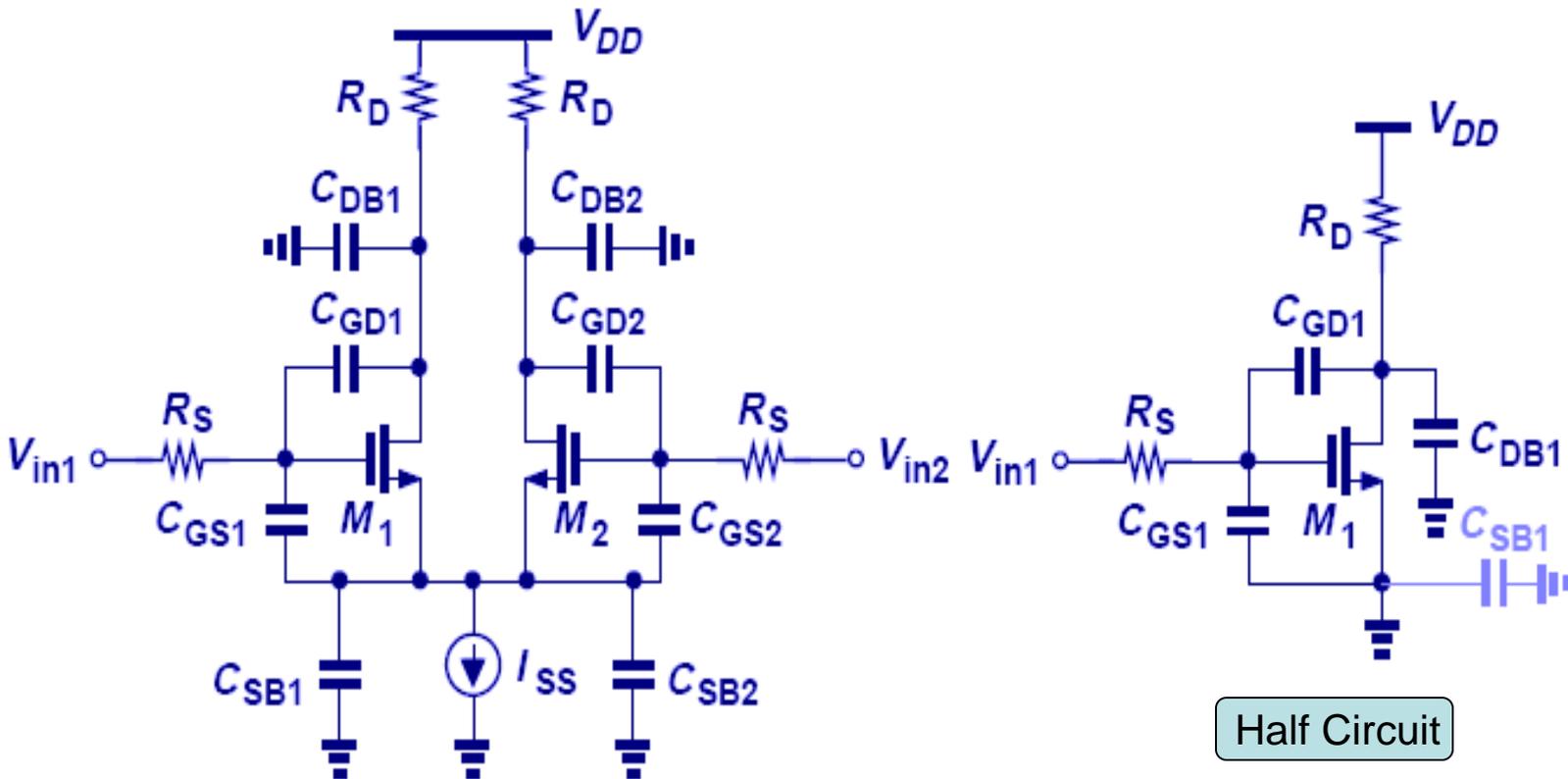
$$Z_{out} = R_L \parallel \frac{1}{(C_{GD2} + C_{DB2})s}$$

Bipolar Differential Pair Frequency Response



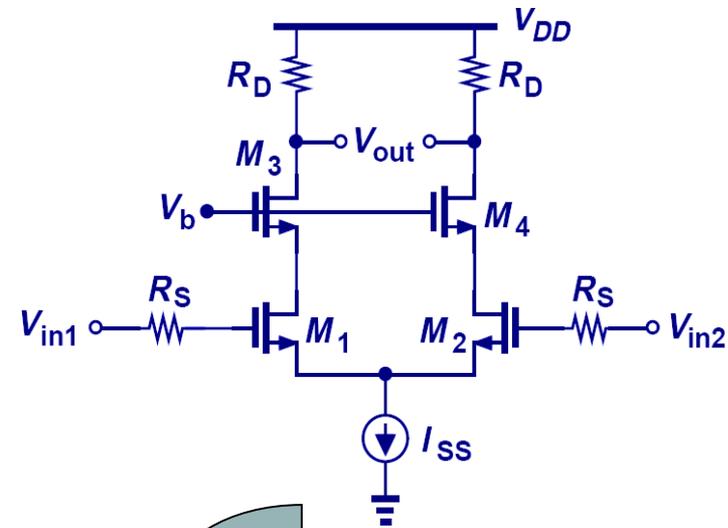
- Since bipolar differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

MOS Differential Pair Frequency Response



- Since MOS differential pair can be analyzed using half-circuit, its transfer function, I/O impedances, locations of poles/zeros are the same as that of the half circuit's.

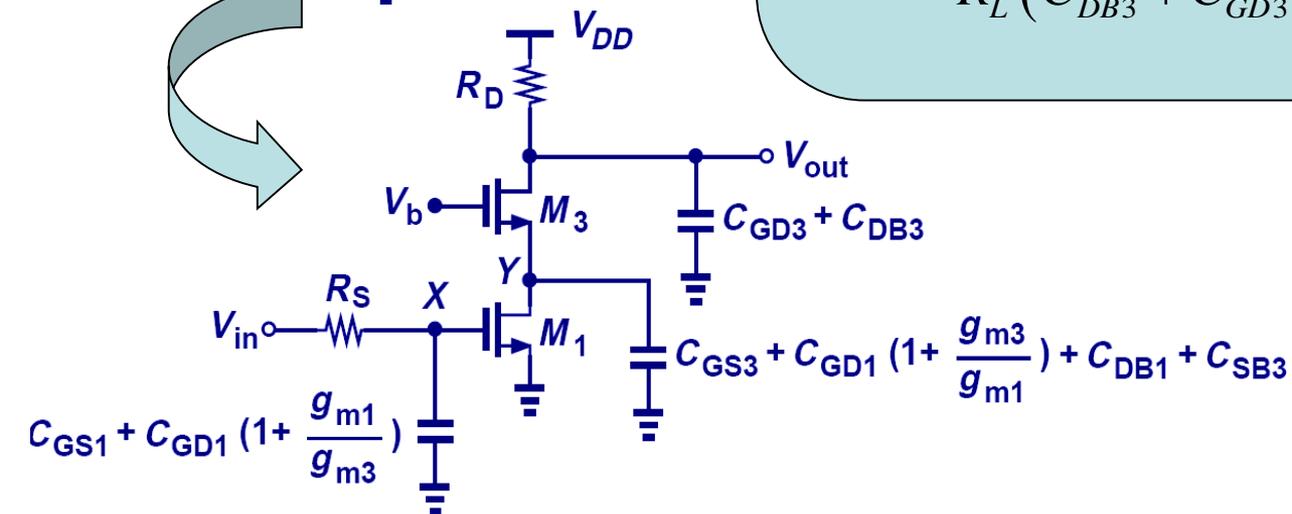
Example: MOS Differential Pair



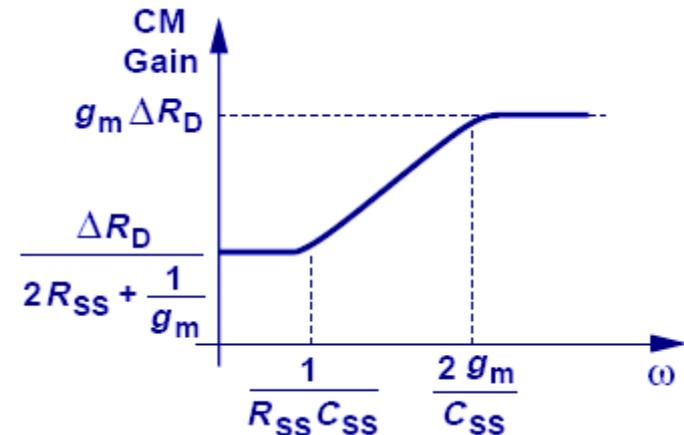
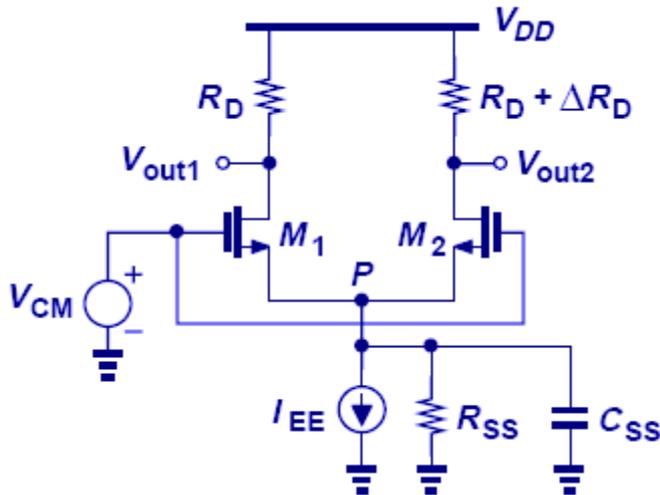
$$\omega_{p,X} = \frac{1}{R_S [C_{GS1} + (1 + g_{m1} / g_{m3}) C_{GD1}]}$$

$$\omega_{p,Y} = \frac{1}{\frac{1}{g_{m3}} \left[C_{DB1} + C_{GS3} + C_{SB3} + \left(1 + \frac{g_{m3}}{g_{m1}} \right) C_{GD1} \right]}$$

$$\omega_{p,out} = \frac{1}{R_L (C_{DB3} + C_{GD3})}$$



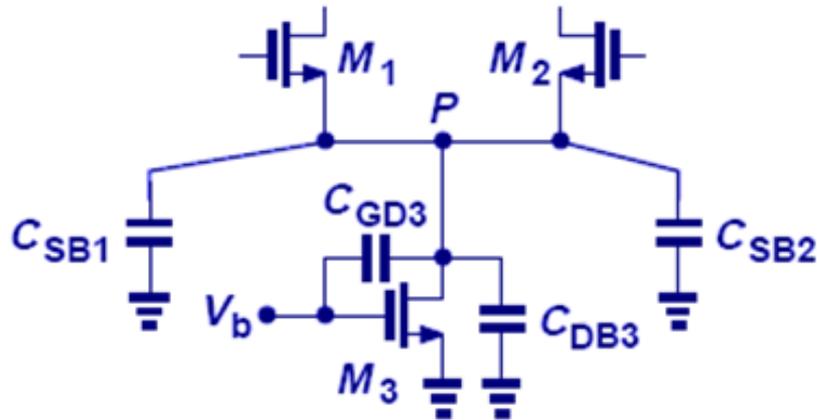
Common Mode Frequency Response



$$\left| \frac{\Delta V_{out}}{\Delta V_{CM}} \right| = \frac{\Delta R_D}{\frac{1}{g_m} + 2 \left(R_{SS} \parallel \frac{1}{C_{SS}s} \right)} = \frac{g_m \Delta R_D (R_{SS} C_{SS} s + 1)}{R_{SS} C_{SS} s + 2g_m R_{SS} + 1}$$

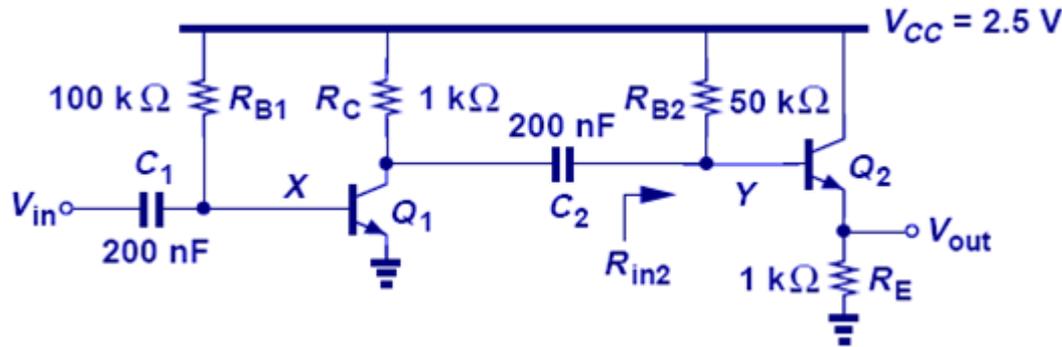
➤ C_{SS} will lower the total impedance between point P to ground at high frequency, leading to higher CM gain which degrades the CM rejection ratio.

Tail Node Capacitance Contribution



- Source-Body Capacitance of M_1, M_2
- Drain-Body Capacitance of M_3
- Gate-Drain Capacitance of M_3

Example: Capacitive Coupling



$$I_S = 5 \times 10^{-16} \text{ A}$$

$$\beta = 100$$

$$V_A = \infty$$

For Q_1 , assuming $V_{BE1} = 800 \text{ mV}$,

$$I_{C1} = \beta \frac{V_{CC} - V_{BE1}}{R_{B1}} = 1.7 \text{ mA}$$

$$\Rightarrow V_{BE1} = V_T \ln(I_{C1} / I_{S1}) = 748 \text{ mV}$$

$$\Rightarrow I_{C1} = 1.75 \text{ mA} \Rightarrow g_{m1} = (14.9 \text{ } \Omega)^{-1}$$

$$\Rightarrow r_{\pi1} = 14.9 \text{ k}\Omega$$

For Q_2 , assuming $V_{BE2} = 800 \text{ mV}$,

$$V_{CC} = I_{B2} R_{B2} + V_{BE2} + R_E I_{C2}$$

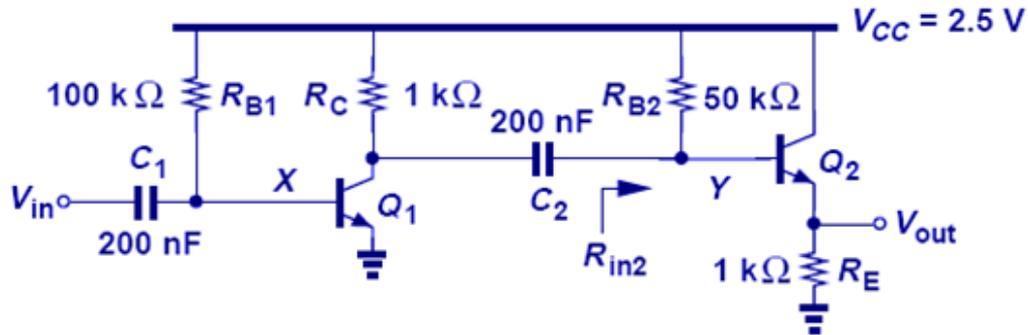
$$\Rightarrow I_{C2} = \frac{V_{CC} - V_{BE2}}{R_{B2} / \beta + R_E} = 1.13 \text{ mA}$$

Iteration yields

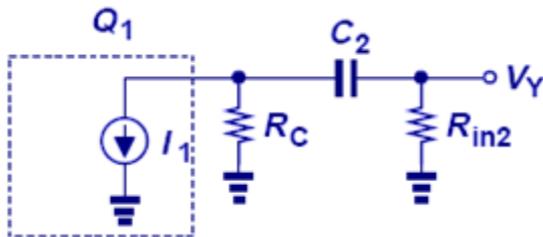
$$I_{C2} = 1.17 \text{ mA}, g_{m2} = (22.2 \text{ } \Omega)^{-1}$$

$$\Rightarrow r_{\pi2} = 2.22 \text{ k}\Omega$$

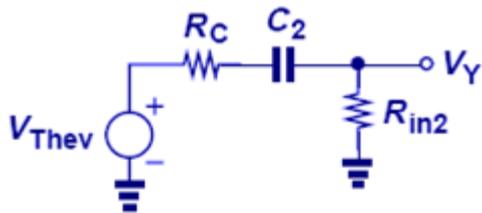
Example: Capacitive Coupling – cont'd



$$\omega_{L1} = \frac{1}{(r_{\pi 1} \parallel R_{B1}) C_1} = 2\pi \times (542 \text{ Hz})$$

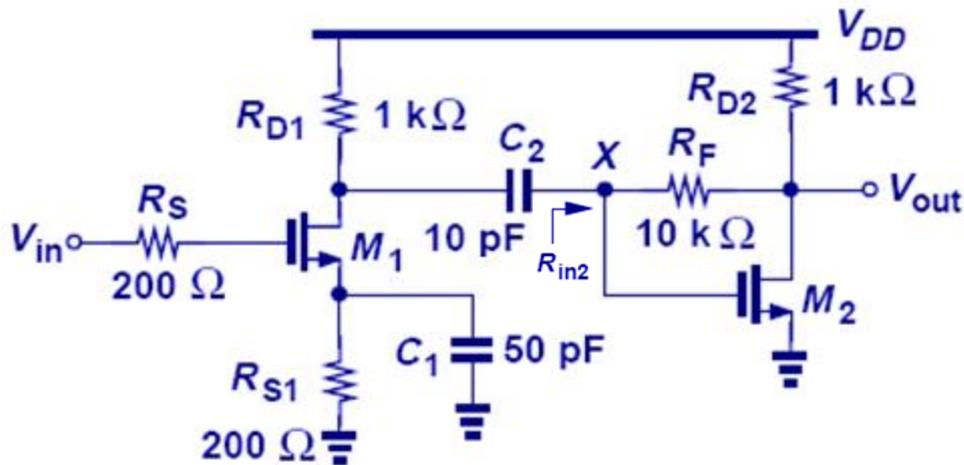


$$R_{in2} = R_{B2} \parallel [r_{\pi 2} + (\beta + 1)R_E]$$



$$\omega_{L2} = \frac{1}{(R_C + R_{in2}) C_2} = \pi \times (22.9 \text{ Hz})$$

Example: IC Amplifier – Low Frequency Behavior



$$R_{in2} = \frac{R_F}{1 - A_{v2}}$$

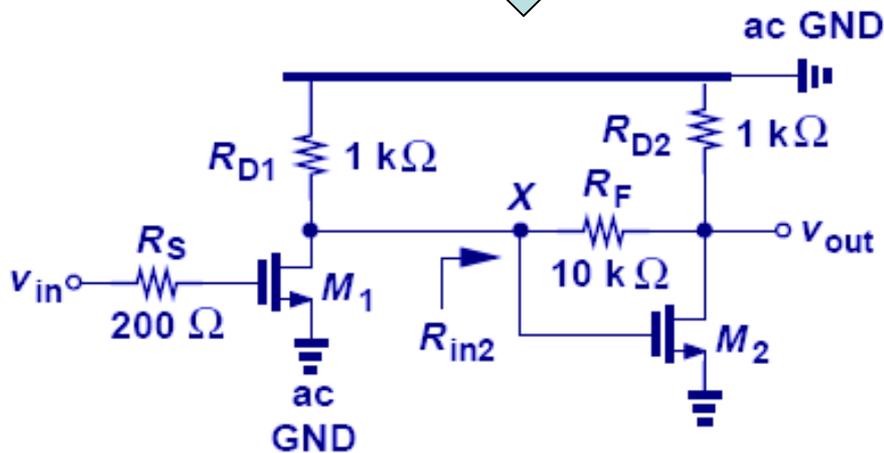
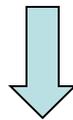
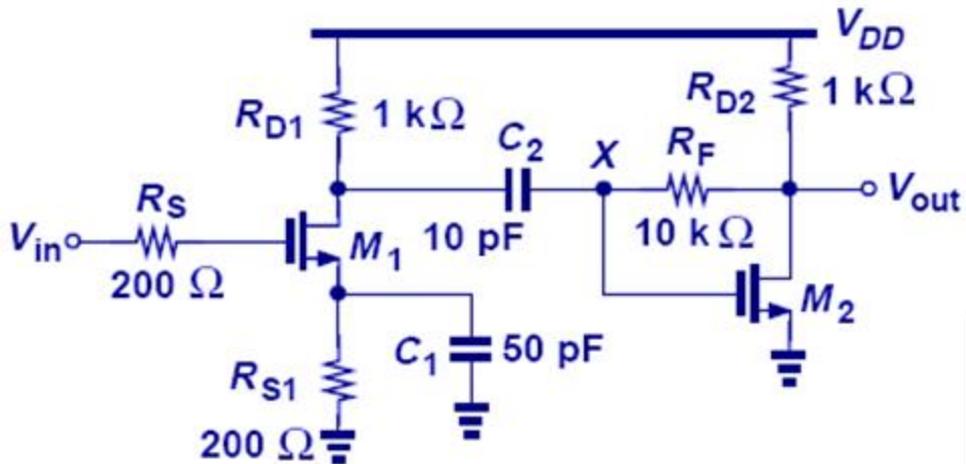
$$A_{v2} \approx -g_{m2} R_{D2} = -6.67$$

$$\Rightarrow R_{in2} = 1.30 \text{ k}\Omega$$

$$\begin{aligned} \omega_{L1} &= \frac{1}{\left(R_{S1} \parallel \frac{1}{g_{m1}} \right) C_1} \\ &= \frac{g_{m1} R_{S1} + 1}{R_{S1} C_1} \\ &= 2\pi \times (42.4 \text{ MHz}) \end{aligned}$$

$$\begin{aligned} \omega_{L2} &= \frac{1}{(R_{D1} + R_{in2}) C_2} \\ &= 2\pi \times (6.92 \text{ MHz}) \end{aligned}$$

Example: IC Amplifier – Midband Behavior



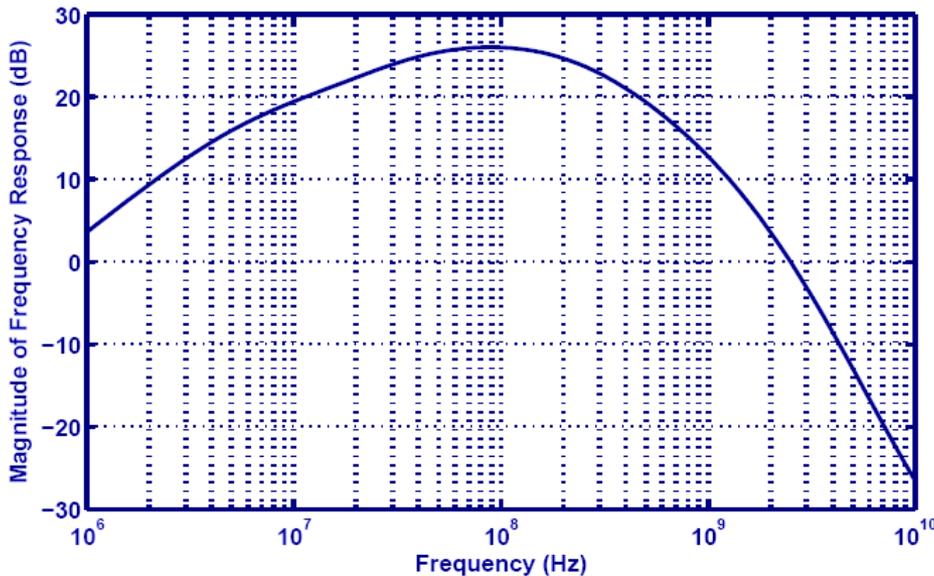
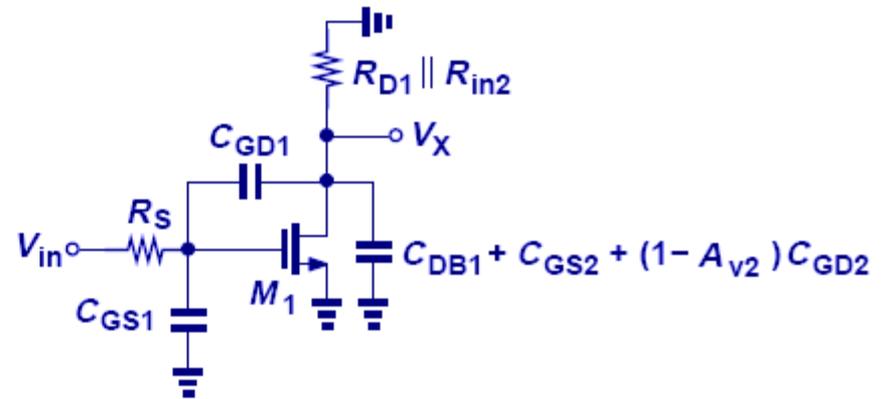
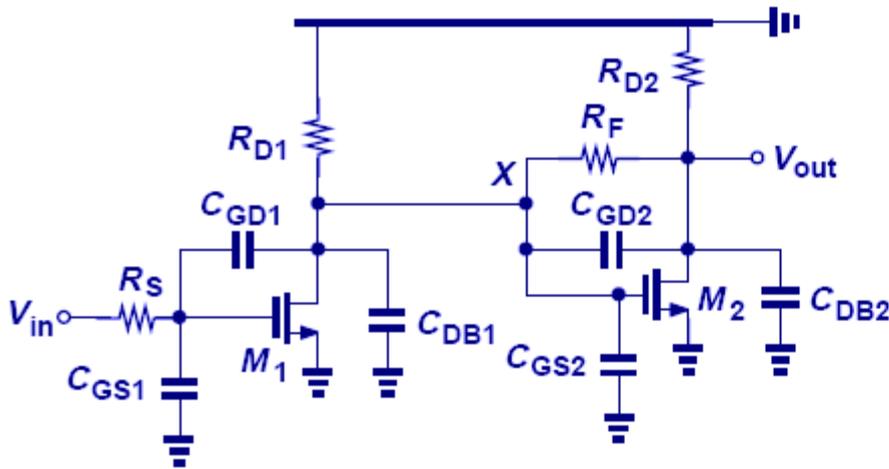
$$\frac{v_{out}}{v_{in}} = \left(\frac{v_X}{v_{in}} \right) \times \left(\frac{v_{out}}{v_X} \right)$$

$$\frac{v_X}{v_{in}} = -g_{m1} (R_{D1} \parallel R_{in2}) = -3.77$$

$$\frac{v_{out}}{v_X} \approx -g_{m2} R_{D2} = -6.67$$

$$\Rightarrow \frac{v_{out}}{v_{in}} \approx 25.1$$

Example: IC Amplifier – High Frequency Behavior



CH 11 Frequency Response

$$|\omega_{p1}| = 2\pi \times (308 \text{ MHz})$$

$$|\omega_{p2}| = 2\pi \times (2.15 \text{ GHz})$$

With Miller effect,

$$(1 - A_{v2}^{-1}) C_{GD2} \approx 1.15 \cdot C_{GD2}$$

$$|\omega_{p3}| = \frac{1}{R_{L2}(1.15 \cdot C_{GD2} + C_{DB2})} = 2\pi \times (1.21 \text{ GHz})$$