

Let the reading on the voltmeter be $V_o = V_1 - V_2$.
Considering the circuit $122'1'$, only the straight portion $2'1'$ cuts the magnetic flux.
From equation (7-24) in the textbook,

$$\begin{aligned} V_o &= V_1 - V_2 \\ &= \oint_C (\vec{u} \times \vec{B}) \cdot d\vec{l}' \\ &= \int_{2'}^{1'} (-\vec{a}_y / 1000 \times \vec{a}_z (0.2)) \cdot \vec{a}_x dl \\ &= (-\vec{a}_x \cdot \vec{a}_x) 200 \cdot (0.1) \\ &= -20 \text{ (V)} \end{aligned}$$

$\therefore V_o = V_1 - V_2 = -20 \text{ V}$

2. Representing time-harmonic \vec{E} -field using vector phasor, calculate the conduction current density and the displacement current density.

① - magnitude of the conduction current density

$$|\vec{J}| = |\sigma \vec{E}| = \sigma |\vec{E}| \quad (1) \quad \text{where } \sigma \text{ is the conductivity of seawater.}$$

② - magnitude of the displacement current density

$$\begin{aligned} \left| \frac{d\vec{D}}{dt} \right| &= \left| \frac{d(\epsilon_0 \epsilon_r \vec{E})}{dt} \right| \quad \text{where } \epsilon_r \text{ is the dielectric constant of seawater} \\ &= \epsilon_0 \epsilon_r \left| \frac{d\vec{E}}{dt} \right| \\ &= \epsilon_0 \epsilon_r |j\omega \vec{E}| = \epsilon_0 \epsilon_r \omega |\vec{E}| \quad (2) \end{aligned}$$



(1) = (2) gives

$$\sigma = \epsilon_0 \epsilon_r \omega = \epsilon_0 \epsilon_r (2\pi \nu)$$

where ν is the frequency of the time-harmonic field.

$$\Rightarrow \nu = \frac{\sigma}{2\pi \epsilon_0 \epsilon_r}$$

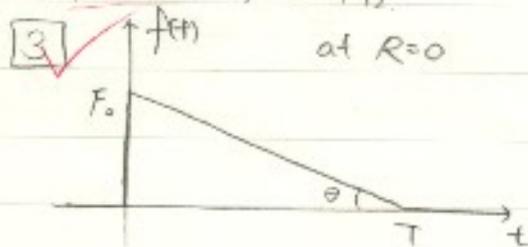
$$= \frac{0.4 \times 10^{-3} \text{ (S/m)}}{2\pi \cdot 81 \cdot (1/36\pi \times 10^{-9}) \text{ (F/m)}}$$

$$= \frac{0.4 \times 10^{-3} \text{ (A/V.m)}}{4.5 \times 10^{-9} \text{ (C/V.m)}}$$

$$= 8.89 \times 10^4 \text{ s}^{-1}$$

$$\therefore \nu = 8.89 \times 10^4 \text{ s}^{-1}$$

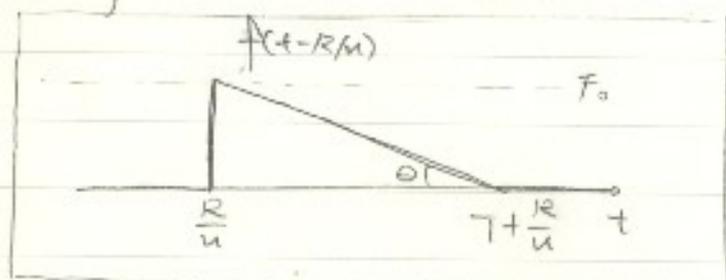
→ 바닷물에서는 time-harmonic \vec{E} -field가 걸리면 이로인한 conduction current와 displacement current가 생기는데, 저주파에서는 displacement current가 conduction current에 비해 무시할만큼 작으나, 고주파에서는 displacement current가 상당한 양이 되기 시작할 수 있게 된다.



$$f(0-0/u) = f(0) = F_0 \quad \therefore f(0) = F_0$$

$$f(T-0/u) = f(T) = 0 \quad \therefore f(T) = 0$$

(a) $f(t - R/u)$ versus t .



(b) $f(t - R/u)$ versus R for $t > T$

$$\Rightarrow f(t - R/u) = F_0 \Leftrightarrow t - R/u = 0$$

$$\Leftrightarrow R = ut$$

$$\therefore f(t - R/u) = F_0 \text{ at } R = ut$$

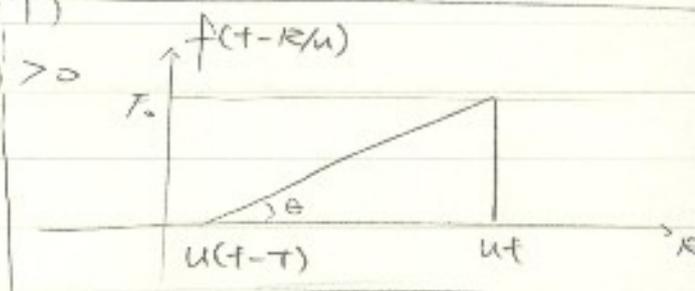


$$\Rightarrow f(t-R/u) = 0 \Leftrightarrow t-R/u = T$$

$$\Leftrightarrow R = u(t-T) \quad f(t-R/u) = 0 \text{ at } R = u(t-T)$$

since $t > t-T > 0$ ($\because T > 0$ & $t > T$)

if $u > 0$ assumed, $ut > u(t-T) > 0$



4 source-free polarized medium where $\rho = 0, \vec{J} = 0, \mu = \mu_0$
 volume density of polarization \vec{P}
 a single vector potential $\vec{\pi}_e$ is defined $\vec{H} = j\omega\epsilon_0 \vec{\nabla} \times \vec{\pi}_e -$

$$(a) \vec{\nabla} \times \vec{E} = -j\omega\mu_0 \vec{H} \quad \text{by equation (7-94a)}$$

$$= -j\omega\mu_0 (j\omega\epsilon_0 \vec{\nabla} \times \vec{\pi}_e)$$

$$= \omega^2\mu_0\epsilon_0 (\vec{\nabla} \times \vec{\pi}_e)$$

$$\Rightarrow \vec{\nabla} \times (\vec{E} - \omega^2\mu_0\epsilon_0 \vec{\pi}_e) = \vec{\nabla} \times (\vec{E} - k_0^2 \vec{\pi}_e) = 0, \text{ where } k_0 = \omega\sqrt{\mu_0\epsilon_0}$$

Since $(\vec{E} - k_0^2 \vec{\pi}_e)$ is curl-free, there exist scalar potential $\stackrel{d}{=} V_e$
 s.t $\vec{E} - k_0^2 \vec{\pi}_e = \vec{\nabla} V_e \quad - (2)$

$$\text{Also, } \vec{\nabla} \times \vec{H} = \vec{J} + j\omega\vec{D} \quad \text{by equation (7-94b)}$$

$$= j\omega(\epsilon_0 \vec{E} + \vec{P}) \quad \text{by equation (3-97)}$$

$$= j\omega\epsilon_0 \left(\vec{E} + \frac{\vec{P}}{\epsilon_0} \right) \quad - (3)$$

By additional curl-operation of (1).

$$\vec{\nabla} \times \vec{H} = j\omega\epsilon_0 (\vec{\nabla} \times \vec{\nabla} \times \vec{\pi}_e)$$

$$= j\omega\epsilon_0 (\vec{\nabla}(\vec{\nabla} \cdot \vec{\pi}_e) - \vec{\nabla}^2 \vec{\pi}_e) \quad - (4) \quad \text{by 'back-cap rule'}$$



$$= j\omega\epsilon_0 \left(k_0^2 \vec{\kappa}_e + \nabla V_e + \frac{\vec{P}}{\epsilon_0} \right) \quad \text{--- (5)} \quad \text{by (1), (2)}$$

Since $\nabla \cdot \vec{\kappa}_e$ is not yet defined, we can choose $\nabla \cdot \vec{\kappa}_e = V_e$,
 then from (4), (5),

the nonhomogeneous Helmholtz's equation is derived

$$\boxed{\nabla^2 \vec{\kappa}_e + k_0^2 \vec{\kappa}_e = -\frac{\vec{P}}{\epsilon_0}} \quad \text{--- (6)}$$

Now, from (2),

$$\vec{E} = k_0^2 \vec{\kappa}_e + \nabla V_e$$

$$= k_0^2 \vec{\kappa}_e + \nabla(\nabla \cdot \vec{\kappa}_e)$$

$$= k_0^2 \vec{\kappa}_e + (\nabla^2 \vec{\kappa}_e + \nabla \times \nabla \times \vec{\kappa}_e) \quad \text{by 'back-cap rule'}$$

$$= \boxed{\nabla \times \nabla \times \vec{\kappa}_e - \frac{\vec{P}}{\epsilon_0}} \quad \text{by (6)}$$