

## Homework No. 2

Due Date: April 07 (Mon) 6:30 PM

1.

By writing out the stress-strain equations as matrices, show the relationship between the 21 independent compliances in contracted notation  $S_{mn}$  ( $m, n = 1$  to 6) and the compliances in tensor notation  $S_{ijkl}$  ( $i, j, k, l = 1$  to 3).

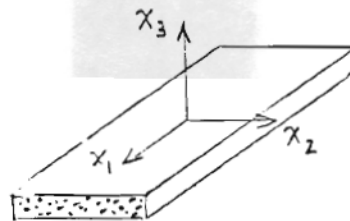
2.

Show that the 3-D compliances  $S_{ij}$  ( $i, j = 1$  to 6) can be used without modification (with  $i, j = 1, 2, 6$ ) for 2-D plane stress calculations. Show (but do not do the algebra) that this is not the case for the stiffnesses  $C_{ij}$ .

3.

Given a transversely isotropic material about the  $x_1$  axis.

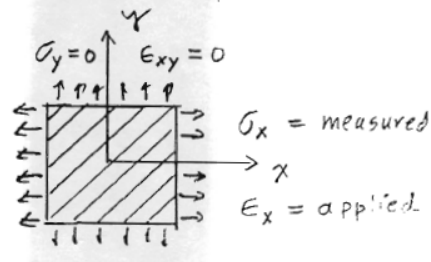
- Write out the 3-D compliances  $S_{ij}$  for this material. How many constants are required to characterize it?
- If the material is in plane stress along the  $x_1, x_2$  axes ( $\sigma_3 = \sigma_4 = \sigma_5 = 0$ ), what are the corresponding 2-D compliances  $S_{ij}^*$ ? How many constants are required to characterize it?

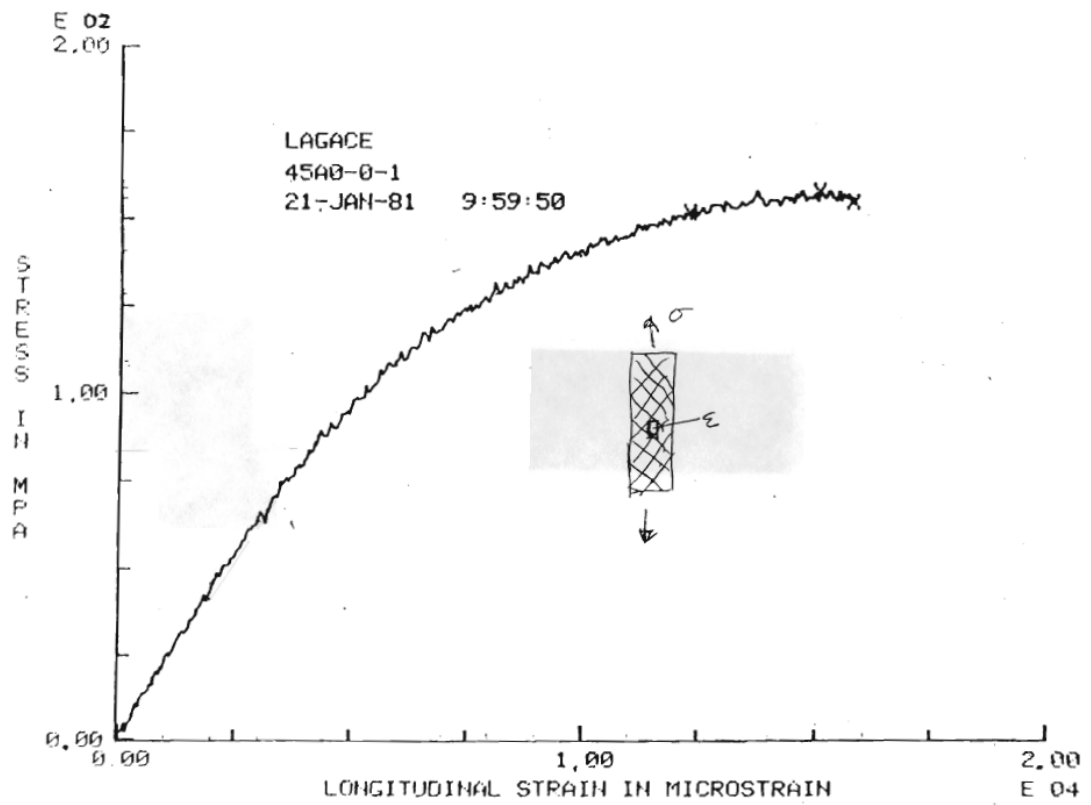
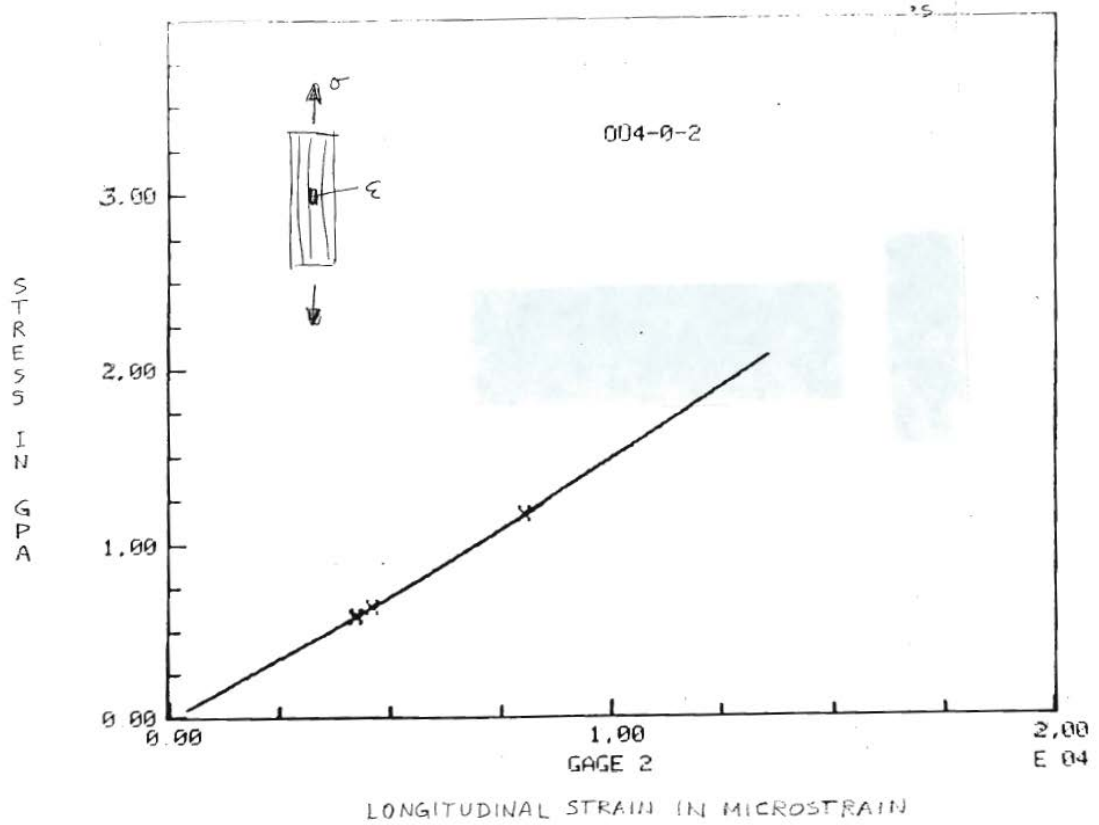


4.

Attached are test results for two tests of laminates made from AS4/3501-6 material. The applied stress (in GPa or MPa) is plotted against the longitudinal strain measured with a strain gage (in  $\mu\text{strain}$ , i.e., strain times  $10^{-6}$ ). The computer in some cases has printed "EOX" at the end of an axis -- this means multiplying the values shown on that axis by  $10^{-6}$ .

- The first graph is the results from a longitudinal tensile test. Use this information to find  $E_L$  for this material. Show how you derived your value from the test results. Your value may not agree exactly with published values due to test scatter.
- From other tests, we find  $E_T = 9.8 \text{ GPa}$  and  $\nu_{LT} = .30$ . The second graph shows results of testing a  $[+45/-45]_S$  laminate. Use this information, and the material properties you already know, to find  $G_{LT}$  for this material. Again show your work. To do this, consider a  $+45$  ply as shown below. Assume that the boundary conditions on this ply are  $\sigma_x = \text{applied stress}$  from the graph,  $\sigma_y = 0$ , and  $\epsilon_{xy} = 0$  (not  $\sigma_{xy} = 0$ ). Given that you know  $\epsilon_x$  (from the strain gage), use the rotated Hooke's law  $\bar{\sigma} = \bar{Q}\bar{\epsilon}$  to find the unknowns  $\epsilon_y$ ,  $\sigma_{xy}$ , and  $G_{LT}$ . Use the early, linear region of the plot.
- Can you justify the assumptions used for the boundary conditions in b)?
- Compute the  $\bar{Q}$  and  $\bar{S}$  matrices for this material. Compare  $Q_{11}$  to  $E_L$ .





5.

For a ply rotated by an angle  $\theta$ , compute the rotated  $\underline{Q}$  matrix,  $\underline{Q}_\theta$  for AS4/3501-6. Plot  $Q_{11}$ ,  $Q_{12}$ ,  $Q_{16}$  and  $Q_{66}$  for  $\theta$  from 0 to 90 degrees

6.

Consider a stack of 3 plies of AS4/3501-6, 0.125 mm thick each, not glued together, in a  $[0/90/0]_T$  arrangement. We apply to each ply individually, the boundary conditions of

$$\epsilon_x = .001 \quad \sigma_y = 0 \quad \sigma_{xy} = 0$$

- a) What are the stresses and strains  $\sigma_x, \epsilon_y, \epsilon_{xy}$  in each of the plies?
- b) Why is what we just did, not a solution of the laminate problem (in which the plies are glued to each other)?

Note: For doing these calculations, and for later use, produce convenient computer subroutines, MATLAB routines, etc., for your personal use.

Suggested routines are,

- a) Compute the  $\underline{Q}$  matrix from given  $E_L, E_T, \nu_{LT}, G_{LT}$ .
- b) Rotate  $\underline{Q}$  by  $\theta$
- c) Invert  $\underline{Q}$  (get  $\underline{S}$ ) and extract effective engineering constants.