Homework No. 3

Due Date: April 28 (Mon) 6:30 PM

1.

a) Show that the rotated matrix element $\bar{Q}_{66}$ can be represented as,

$$\bar{Q}_{66}(\theta) = \bar{Q}_{12}(\theta) + C_a$$

where C_a is a constant. Determine this constant C_a

b) Show that if $Q_{22} = Q_{11}$, and $Q_{66} = (Q_{11} - Q_{12})/2$, all the rotated matrix elements $\bar{Q}_{ij}$ become independent of θ, i.e., the material is isotropic.

2.

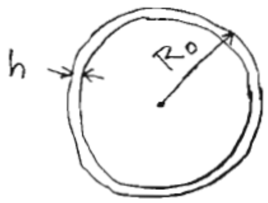
Below is a table of eng'g properties for commonly used composite materials:

	E_L (GPa)	E_T (GPa)	ν_{LT}	G_{LT} (GPa)	Cost ($/lb)
AS4/3501-6 G/E	142	9.8	0.30	6.0	40
Kevlar/E	75	5.5	0.34	2.3	40
P75/934 G/E	310	6.2	0.28	4.8	100

All the composites can be assumed to have a density of about 0.06 lbs/in^3, and to have a ply thickness of .005 inches.

It is desired to compare stiffnesses and costs of composite tubes made from the above materials. The tubes are each 20" long with a 2" diameter. Two tubes are made from each material, one with a quasi-isotropic $[0/\pm45/90]_s$ layup, and another with a $[0_2/\pm30]_s$ layup.

Determine the extensional stiffness EA, the bending stiffness EI, the torsional stiffness GJ, and the cost of each of these six tubes. Do all calculations with English units, i.e., inches, msi, etc. Compare $\bar{E}_x = 1/a_{11}h$ with $E_{11}^{eq} = A_{11}/h$, and $\bar{G}_{xy}$ with E_{66}^{eq} for each case. Which would you use for the EA, EI, and GJ ? Note, for section properties of a thin tube, one has,



$$A \approx 2\pi R_o h$$

$$I_{xx} \approx \pi R_o^3 h$$

$$J \approx 2\pi R_o^3 h$$

$h << R_o$

3.

Consider coupon test specimens 2" wide, made of the following layups of AS4/3501-6 plies. The ply thicknesses are all 0.005",with fiber fractions of 60%.

$[\pm 30]_s$ $[\pm 45]_s$ $[\pm 60]_s$ $[0/90]_s$

Estimate the maximum tensile load P (lbs) each coupon can support, and the corresponding maximum tensile stress σ_x (lbs/in^2) for the coupon. Use both the Maximum Stress and the Tsai-Wu criteria. What kind of first-ply failure would you expect for each laminate?

Also estimate the engineering stiffness $\bar{E}_x$ and the Poisson's ratio $\bar{\nu}_{xy}$ for each laminate. The engineering properties for this material are,

E_L (Msi)	E_T (Msi)	ν_{LT}	G_{LT} (Msi)
20.6	1.42	.30	.87

X_t (Ksi)	X_c (Ksi)	Y_t (Ksi)	Y_c (Ksi)	S (Ksi)
330	-180	7.5	-35	14