Homework set 7 selected solution

P. 5-19 The cross section of a long thin metal strip and a parallel wire is shown in Fig. 5-30. Equal and opposite currents I flow in the conductors. Find the force per unit length on the conductors.

Solution)

dB at the wire due to dI on the metal strip :

$$dB(D) = \frac{\mu_0 dI}{2\pi r} = \frac{\mu_0 I dy}{2\pi w\sqrt{D^2 + y^2}}$$

$\mathbf{B}(D) = \int dB$ has only y-component due to symmetry about x-axis(x-components are cancelled)

$$\mathbf{B}(D) = 2\int_0^{w/2} dB \frac{D}{\sqrt{D^2 + y^2}} = \hat{y}\frac{\mu_0 ID}{\pi w} 2\int_0^{w/2} dB \frac{dy}{D^2 + y^2} = \hat{y}\frac{\mu_0 I}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right)$$

Force per unit length on the wire :

$$\mathbf{F}' = \frac{\mathbf{F}}{l} = \mathbf{I}\times\mathbf{B} = (-\hat{z}I)\times\hat{y}\frac{\mu_0 I}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right) = \hat{x}\frac{\mu_0 I^2}{\pi w}\tan^{-1}\left(\frac{w}{2D}\right) \quad (N/m)$$

P.5-21 5-21 A d-c current I =10 (A) flows in a triangular loop in the xy-plane as in Fig. 5-32. Assuming a uniform magnetic flux density $\mathbf{B} = \mathbf{a}_y 6(mT)$ in the region, find the forces and torque on the loop. The dimensions are in (cm).

Solution)

$$\mathbf{F} = I\mathbf{L}\times\mathbf{B}$$

$$\mathbf{F}_{AB} = I\mathbf{L}_{AB}\times\mathbf{B} = -\hat{z}6\times10^{-3}(N)$$

$$\mathbf{F}_{BC} = I\mathbf{L}_{BC}\times\mathbf{B} = -\hat{z}12\times10^{-3}(N)$$

$$\mathbf{F}_{CA} = I\mathbf{L}_{CA}\times\mathbf{B} = -\hat{z}6\times10^{-3}(N)$$

Total force on loop :

$$\mathbf{F}_{total} = \mathbf{F}_{AB} + \mathbf{F}_{BC} + \mathbf{F}_{CA} = 0$$

Torque on loop :

$$\mathbf{T} = \mathbf{m}\times\mathbf{B} = (\hat{z}Is)\times(\hat{y}B) = -\hat{x}1.2\times10^{-3}(N\cdot m)$$