Homework set 8 selected solution

P.6-4 In Fig. 6-10 assume a constant current $i_1 = I_0$, but that the rectangular loop moves away with a constant velocity $\mathbf{u} = \mathbf{a}_y u_0$. Determine i_2 when the loop is at a position as shown.

Solution)

B_1 In loop due to wire current $i_1 = I_0$:

$$\mathbf{B}_1 = \hat{x}\frac{\mu_0 I_0}{2\pi y}$$

Motional emf in loop moving $\vec{u} = \hat{y}u_0$:

$$u_2 = \oint (\mathbf{u} \times \mathbf{B}_1) \cdot d\mathbf{l} = \frac{\mu_0 I_0 u_0}{2\pi}\left(\frac{1}{d} - \frac{1}{d+w}\right) = \frac{\mu_0 I_0 u_0 hw}{2\pi d(d+w)}$$

Induced current flowing around loop :

$$i_2 = -\frac{u_2}{R} = -\frac{\mu_0 I_0 u_0 hw}{2\pi d(d+w)R}$$

: counter clockwise direction to increase Φ in loop to oppose the decrease in Φ due to loop movement..

P. 6-7 Determine the frequency at which a time-harmonic electric field intensity causes a conduction current density and a displacement current density of equal magnitude in seawater and moist soil.

Solution)

Time harmonic electric field : $E = Ee^{j\omega t} \rightarrow E\cos\omega t$

$$J_C = \sigma E = \sigma E e^{j\omega t}$$

$$J_D = \varepsilon\frac{\partial E}{\partial t} = \varepsilon_0\varepsilon_r(j\omega E)e^{j\omega t}$$

$$J_C = J_D \Rightarrow \sigma = \varepsilon_0\varepsilon_r\omega = 2\pi\varepsilon_0\varepsilon_r f$$

$$\Rightarrow f = \frac{\sigma}{2\pi\varepsilon_0\varepsilon_r} = 2\times 9\times 10^9\left(\frac{\sigma}{\varepsilon_r}\right)\ (Hz)$$

a) Seawater $(\varepsilon_r = 72,\ \sigma = 4)$: $f = 18\times 10^9\left(\frac{4}{72}\right) = 10^9\,Hz = 1GHz$

b) Moist soil $(\varepsilon_r = 2.5,\ \sigma = 10^{-3})$: $f = 18\times 10^9\left(\frac{10^{-3}}{2.5}\right) = 7.2\times 10^6\,Hz = 7.2MHz$