HW#1 - Selected solution

6-17. A short conducting wire carrying a time-harmonic current is a source of electromagnetic waves. Assuming that a uniform current $i(t) = I_0 \cos\omega t$ flows in a very short wire dl placed along the z-axis,

a) determine the phasor retarded vector potential **A** at a distance *R* in spherical coordinates,

Determine the retarded **A** at R

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{v'} \frac{\vec{J}e^{-jkR}}{R} dv' = \hat{z}\frac{\mu_0 Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right) \quad \text{where,} \quad \beta = k = \omega\sqrt{\mu_0\varepsilon_0} = \omega / c = 2\pi / \lambda$$

$$Jdv = Idl$$

In spherical coordinates: $\vec{A} = \hat{R}\,A_R + \hat{\theta}\,A_\theta + \hat{\phi}\,A_\phi$

where $$A_R = A_z \cos\theta = \frac{\mu_0 Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\cos\theta$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu_0 Idl}{4\pi}\left(\frac{e^{-j\beta R}}{R}\right)\sin\theta$$

$$A_\phi = 0$$

b) find the magnetic field intensity **H** from **A**.

$$\vec{H} = \frac{1}{\mu_0}\nabla\times\vec{A} = \hat{\phi}\frac{1}{\mu_0 R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right] = -\hat{\phi}\frac{Idl}{4\pi}\beta^2\sin\theta\left[\frac{1}{j\beta R} + \frac{1}{(j\beta R)^2}\right]e^{-j\beta R}$$

6-19. It is known that the electric field intensity of a spherical wave in free space is

$$\mathbf{E}(R,\theta;t) = \boldsymbol{a_\theta}\frac{10^{-3}}{R}\sin\theta\cos(2\pi 10^9 t - kR) \qquad (V/m)$$

Determine the magnetic field intensity $\mathbf{H}(R,\theta;t)$ and the value of k.

$$\vec{E}(R,\theta;t) = \hat{\theta}\frac{10^{-3}}{R}\sin\theta\cos\left(2\pi 10^9 t - kR\right) = \hat{\theta}\,\mathrm{Re}\left[E_0(R,\theta)e^{j(\omega t - kR - k_\theta\theta)}\right]$$

where $E_0(R,\theta) = \frac{10^{-3}}{R}\sin\theta,\ \omega = 2\pi\times 10^9 t,\ k_\theta = 0,\ k = \frac{\omega}{c} = \frac{2\pi\times 10^9}{3\times 10^8} = \frac{20\pi}{3}$

$$\Rightarrow \quad \vec{E}(R,\theta) = \hat{\theta}\frac{10^{-3}}{R}\sin\theta\, e^{-jkR}$$

$$\vec{H}(R,\theta) = \frac{j}{\omega\mu_0}\nabla\times\vec{E}(R,\theta) = \frac{j}{\omega\mu_0}\frac{1}{R^2\sin\theta}\begin{vmatrix} \hat{R} & \hat{\theta}R & \hat{\phi}R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & 10^{-3}\sin\theta\, e^{-jkR} & 0 \end{vmatrix}$$

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$$= \hat{\phi}\frac{j}{\omega\mu_0}\frac{1}{R^2\sin\theta}\left(10^{-3}\sin\theta\frac{\partial e^{-jkR}}{\partial R}R\sin\theta\right)$$

$$= \hat{\phi}\frac{k}{\omega\mu_0}\frac{10^{-3}}{R}\sin\theta\, e^{-jkR}$$

$$\frac{\sqrt{\mu_0\varepsilon_0}}{\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}} = \frac{1}{\eta_0} = \frac{1}{120\pi}$$

$$= \hat{\phi}\frac{10^{-3}}{120\pi R}\sin\theta\, e^{-j20\pi R/3}$$

$$\therefore \vec{H}(R,\theta;t) = \mathrm{Re}\left[\vec{H}(R,\theta)e^{j\omega t}\right]$$

$$= \hat{\phi}\frac{10^{-3}}{120\pi R}\sin\theta\cos(2\pi\times 10^9 t - 20\pi R/3) \qquad (A/m)$$