HW#2 - Selected solution

7-3. Obtain a general formula that expresses the phasor $\mathbf{E}(\mathbf{R})$ in terms of the phasor $\mathbf{H}(\mathbf{R})$ of a TEM wave and the intrinsic impedance of the medium, where **R** is the radius vector.

From Faraday's law : $\nabla \times \vec{H} = \varepsilon \frac{\partial \vec{E}}{\partial t}$ in source-free medium.

In phasor domain, $-j\hat{k} \times \vec{H}(R) = j\omega\varepsilon\vec{E}(R)$

$$\Rightarrow \quad \vec{E}(R) = -\frac{k}{\omega\varepsilon}\hat{k} \times \vec{H}(R) = -\eta\hat{k} \times \vec{H}(R)$$

where $\eta = \sqrt{\frac{\mu}{\varepsilon}}$: intrinsic impedence, $\vec{H}(R) = H_0 e^{-j\hat{k}\cdot\vec{R}}$

7-5. The **E**-field of a uniform plane wave propagating in a dielectric medium is given by

$$\mathbf{E}(t,z) = \boldsymbol{a_x} 2\cos(10^8 t - z/\sqrt{3}) - \boldsymbol{a_y}\sin(10^8 t - z/\sqrt{3}) \qquad (V/m)$$

In the cosine-reference phasor domain, $\vec{E}(z) = (\hat{x}\,2 + \hat{y}\,j)e^{-jz/\sqrt{3}}$ $\Rightarrow$ $\vec{E}(t,z) = \mathrm{Re}\left[\vec{E}(z)e^{-j\omega t}\right]$

a) Determine the frequency and wavelength of the wave.

$$\omega = 10^8 \ (rad/s) \longrightarrow f = \frac{\omega}{2\pi} \approx 1.59 \times 10^7 \ (Hz)$$

$$k = \beta = 1/\sqrt{3} \ (rad/m) \longrightarrow \lambda = \frac{2\pi}{k} = 2\sqrt{3}\pi \approx 10.88 \ (m)$$

b) What is the dielectric constant of the medium?

$$u_p = \frac{\omega}{k} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c}{\sqrt{\varepsilon_r}} \ for \ \mu = \mu_0 \longrightarrow \varepsilon_r = \left(\frac{kc}{\omega}\right)^2 = 3$$

c) Describe the polarization of the wave.

$$\frac{E_x}{E_y} = \frac{2}{j} = -j2 \longrightarrow \text{Left elliptic polarization (LEP)}$$

d) Find the corresponding **H**-field.

$$\vec{H}(z) = \frac{1}{\eta}\hat{z}\times\vec{E}(z) = \frac{\sqrt{3}}{120\pi}(\hat{y}\,2 - \hat{x}\,j)e^{-jz/\sqrt{3}}$$

$$\therefore \vec{H}(z,t) = \frac{\sqrt{3}}{120\pi}\hat{y}\,2\cos\left(10^8 t - z/\sqrt{3}\right) + \hat{x}\sin\left(10^8 t - z/\sqrt{3}\right)$$

7-10. There is a continuing discussion on radiation hazards to human health. The following calculations provide a rough comparison.

Average EM power density $= P_{av} = \dfrac{|E|^2}{2\eta_0} = 100\ (W/m^2)\ \ in\ air$

a) The U.S. standard for personal safety in a microwave environment is that the power density be less than 10(mW/cm^2). Calculate the corresponding standard in terms of electric field intensity. In terms of magnetic field intensity.

Electric field intensity: $|E| = \sqrt{2\eta_0\times100} = \sqrt{2\times120\pi\times100} \approx 275\ (V/m)$

Magnetic field intensity: $|H| = |E|/\eta_0 = 275/120\pi \approx 0.728\ (A/m)$

b) It is estimated that the earth receives radiant energy from the sun at a rate of about 1.3(kW/m^2) on a sunny day. Assuming a monochromatic plane wave(which it is not), calculate the equivalent amplitudes of the electric and magnetic field intensity vectors.

$$P_{av} = \frac{|E|^2}{2\eta_0} = 1300\ (W/m^2)$$

$$|E| = \sqrt{2\times120\pi\times1300} \approx 992\ (V/m)$$

$$|H| = 992/120\pi \approx 2.625\ (A/m)$$