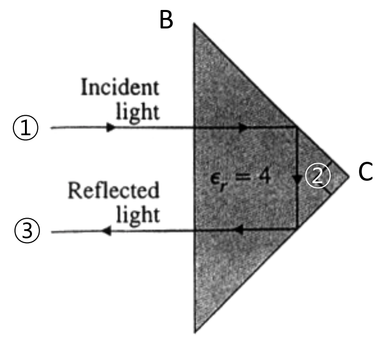
HW#3 - Selected solution

7-30. Glass isosceles triangular prisms shown in following fig. are used in optical instruments. Assuming $\epsilon_r = 4$ for glass, calculate the percentage of the incident light power reflected back by the prism.



For normal incident light ① on the interface AB, $\theta_i = \theta_t = 0^\circ$

(7-95): $\tau_{②} = \dfrac{2\eta_g}{\eta_g + \eta_0}$

(7-95)*: $T_{②} = \dfrac{(P_{av})_{t②}}{(P_{av})_{i①}} = \dfrac{\eta_0}{\eta_g}\tau_{②}^2 \longrightarrow (P_{av})_{t②} = \dfrac{\eta_0}{\eta_g}\tau_{②}^2 (P_{av})_{i①}$

Inside the prism, at both interface BC and CA, $\theta_i = \theta_t = \pi/4 \quad > \quad \theta_c = \sin^{-1}\sqrt{\dfrac{\varepsilon_0}{\varepsilon_g}} = \sin^{-1}\sqrt{\dfrac{1}{\varepsilon_r}} = \dfrac{\pi}{6}$

$\longrightarrow \theta_i > \theta_c \longrightarrow$ Total internal reflection

i.e. $(P_{av})_{t②}$ is unchanged by total reflection inside the prism.

For the exit light ③ at the interface AB,

$\tau_{③} = \dfrac{2\eta_0}{\eta_g + \eta_0}$, $T_{③} = \dfrac{(P_{av})_{t③}}{(P_{av})_{i②}} = \dfrac{(P_{av})_{t③}}{(P_{av})_{t②}} = \dfrac{\eta_g}{\eta_0}\tau_{③}^2$

$$\longrightarrow \quad (P_{av})_{t③} = \frac{\eta_g}{\eta_0}\tau_{③}^2 (P_{av})_{t②} = \frac{\eta_g}{\eta_0}\left(\frac{2\eta_0}{\eta_g + \eta_0}\right)^2 \left[\frac{\eta_0}{\eta_g}\left(\frac{2\eta_0}{\eta_g + \eta_0}\right)^2 (P_{av})_{i①}\right]$$

$$\longrightarrow \quad \frac{(P_{av})_{t③}}{(P_{av})_{i①}} = \left[\frac{4\eta_0\eta_g}{(\eta_g + \eta_0)^2}\right]^2 = \left[\frac{4/\sqrt{\varepsilon_r}}{(1 + 1/\sqrt{\varepsilon_r})^2}\right]^2 = \left[\frac{4/\sqrt{4}}{(1 + 1/\sqrt{4})^2}\right]^2 = \left(\frac{8}{9}\right)^2 = 0.79 = 79\%$$

$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}\ ,\ \eta_g = \sqrt{\frac{\mu_0}{\varepsilon_0\varepsilon_r}} = \eta_0/\sqrt{\varepsilon_r}$$

7-33. For an incident wave with parallel polarization, fine the relation between the critical angle θ_c and the Brewster angle $\theta_{B||}$ for two contiguous media of equal permeability.

From (7-120), $\theta_c = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$: Critical angle

From (7-164), $\theta_{B//} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$: Brewster angle

$$\longrightarrow \quad \sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}} = \tan\theta_{B//}$$