HW#4 - Selected solution

8-5. The following characteristics have been measured on a lossy transmission line at 100 (MHz):

$$Z_0 = 50 + j0 \ \ (\Omega)$$

$$\alpha = 0.01 \ \ (\mathrm{dB/m})$$

$$\beta = 0.8\pi \ \ (\mathrm{rad/m}).$$

Determine *R, L, G,* and *C* for the line.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0 \ , \ \alpha = 0.01(dB/m) = 0.01/8.69 = 0.00115(Np/m)$$

Since $Z_0 = R_0 = real \longrightarrow \mathrm{Im}(Z_0) = X_0 = 0$

This lossy line is distortionless: $R_0 = \sqrt{\frac{L}{C}} \ , \ X_0 = 0$ i.e., $\frac{R}{L} = \frac{G}{C}$

In this case,

$$\alpha = R\sqrt{C/L} \ , \ \beta = \omega\sqrt{LC} \ , \ Z_0 = \sqrt{L/C}$$

$$\therefore R = \frac{\alpha}{\sqrt{C/L}} = \alpha Z_0 = 0.00115 \times 50 = 0.0575(\Omega/m)$$

$$\beta Z_0 = \omega L \longrightarrow L = \frac{\beta Z_0}{\omega} = \frac{0.8\pi \times 50}{2\pi \times 10^8} = 0.2 \times 10^{-6}(H/m) = 0.2(\mu H/m)$$

$$G = \frac{RC}{L} = \frac{\alpha}{Z_0} = \frac{0.00115}{50} = 23 \times 10^{-6}(s/m) = 23(\mu H/m)$$

$$C = \frac{LG}{R} = \frac{L}{Z_0^2} = \frac{0.2 \times 10^{-6}}{50^2} = 80 \times 10^{-12}(F/m) = 80(pF/m)$$

8-11. A 2-(m) lossless air-spaced transmission line having a characteristic impedance $50\ (\Omega)$ is terminated with an impedance $40 + j30\ (\Omega)$ at an operating frequency of 200 (MHz). Find the input impedance.

(8-79): $Z_i = R_0 \dfrac{Z_L + jR_0 \tan \beta l}{R_0 + jZ_L \tan \beta l}$

$$\beta l = \frac{2\pi}{\lambda} l = \frac{2\pi l}{c/f} = \frac{2\pi \times 2}{\left(3\times10^8\right)/\left(2\times10^8\right)} = \frac{8\pi}{3} = 480^\circ$$

$$\tan \beta l = \tan\frac{8\pi}{3} = \tan\left(3\pi - \frac{\pi}{3}\right) = -\tan\frac{\pi}{3} = -\sqrt{3}$$

$$\therefore Z_i = 50\frac{(40 + j30) + j50\left(-\sqrt{3}\right)}{50 + j(40 + j30)\left(-\sqrt{3}\right)}$$

$$\approx 26.3 - j9.87(\Omega)$$