SEOUL NATIONAL UNIVERSITY
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

SYSTEM CONTROL **Fall 2014**

HW.#1 Laplace Transformation

Assigned: September 11 (Th)
Due: September 18 (Th)

[1] Compute Laplace Transform

(1) unit step $L[1(t)] = \frac{1}{s}$

(2) $L[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}$

(3) $L[\frac{1}{a}\left(1 - e^{-at}\right)] = \frac{1}{s(s+a)}$

(4) $L[e^{-at} \cos \omega t] = \frac{s+a}{\left(s+a\right)^2 + \omega^2}$

[2] Find the Laplace transforms of

(1)

$$\begin{aligned}
F(s) &= \int f(t) e^{-st} dt \\
&= L\left[2\left(t-2\right)\cdot 1(t-2) - 2\left(t-4\right)\cdot 1(t-4)\right] \\
&= L\left[2\left(t-2\right)\cdot 1(t-2)\right] - L\left[2\left(t-4\right)\cdot 1(t-4)\right] \\
&= \frac{2}{s^2} e^{-2s} - \frac{2}{s^2} e^{-4s} \\
&= \frac{2}{s^2}\left(1 - e^{-2s}\right) e^{-2s}
\end{aligned}$$

(2)

$$\begin{aligned}
F(s) &= L\left[\left\{t - t1\left(t-1\right)\right\}\right] + L\left[\left\{\left(t-1\right) - \left(t-1\right)1\left(t-2\right)\right\}1\left(t-1\right)\right] + \dots \\
&= L\left[\left\{t - \left(t-1\right)1\left(t-1\right) - 1\left(t-1\right)\right\}\right] + \dots \\
&= \left(\frac{1}{s^2} - \frac{1}{s^2} e^{-s} - \frac{1}{s} e^{-s}\right) + \dots \\
&= \left(\frac{1 - e^{-s} - se^{-s}}{s^2}\right) + \dots \\
&= \left(\frac{1 - e^{-s} - se^{-s}}{s^2}\right)\left(1 + e^{-s} + e^{-2s} + \dots\right) \\
&= \frac{1 - e^{-s} - se^{-s}}{s^2 (1 - e^{-s})}
\end{aligned}$$

[3] Obtain the inverse Laplace Transform of followings:

(1) $L^{-1}[\frac{1}{s+a}] = e^{-at}$

(2) $L^{-1}[\frac{1}{(s+a)^2}] = te^{-at}$

(3) $L^{-1}[\frac{1}{s^2-\omega^2}] = \frac{1}{\omega}\sinh \omega t$

(4) $L^{-1}[\frac{1}{(s+2)^2(s+3)}] = e^{-3t} - [1-t]e^{-2t}$

[4] Show that:

$$L\left\{\frac{d}{dt}f(t)\right\} = sF(s) - f(0) \text{ where } F(s) = L\{f(t)\}$$

$$\int_0^\infty f(t)e^{-st}dt = f(t)\frac{e^{-st}}{-s}\bigg|_0^\infty - \int_0^\infty \left[\frac{d}{dt}f(t)\right]\frac{e^{-st}}{-s}dt$$
$$F(s) = \frac{f(0)}{s} + \frac{1}{s}L\left[\frac{d}{dt}f(t)\right]$$
$$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$$

[5] Show that:

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

$$\lim_{s\to 0}\int_0^\infty \left[\frac{d}{dt}f(t)\right]e^{-st}dt = \lim_{s\to 0}\left[sF(s) - f(0)\right]$$
$$\int_0^\infty \left[\frac{d}{dt}f(t)\right]dt = f(t)\big|_0^\infty = f(\infty) - f(0)$$
$$= \lim_{s\to 0} sF(s) - f(0)$$

From which $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$

[6] Solve following differential equation by Laplace Transformation.

$$\ddot{y} + 2\dot{y} + 4y = 1$$
$$y(0) = 0, \dot{y}(0) = 2$$

$$s^2Y(s) - sy(0) - \dot{y}(0) + 2sY(s) - 2y(0) + 4Y(s) = \frac{1}{s}$$
$$\left(s^2 + 2s + 4\right)Y(s) = \frac{1}{s} + 2$$
$$Y(s) = \frac{1}{4s} - \frac{s+2}{4\left((s+1)^2+3\right)} + \frac{2}{(s+1)^2+3} \Rightarrow y(t) = \frac{1}{4} + \left(\frac{1}{12}\cos\sqrt{3}t - \frac{7}{12}\sin\sqrt{3}t\right)e^{-t}$$