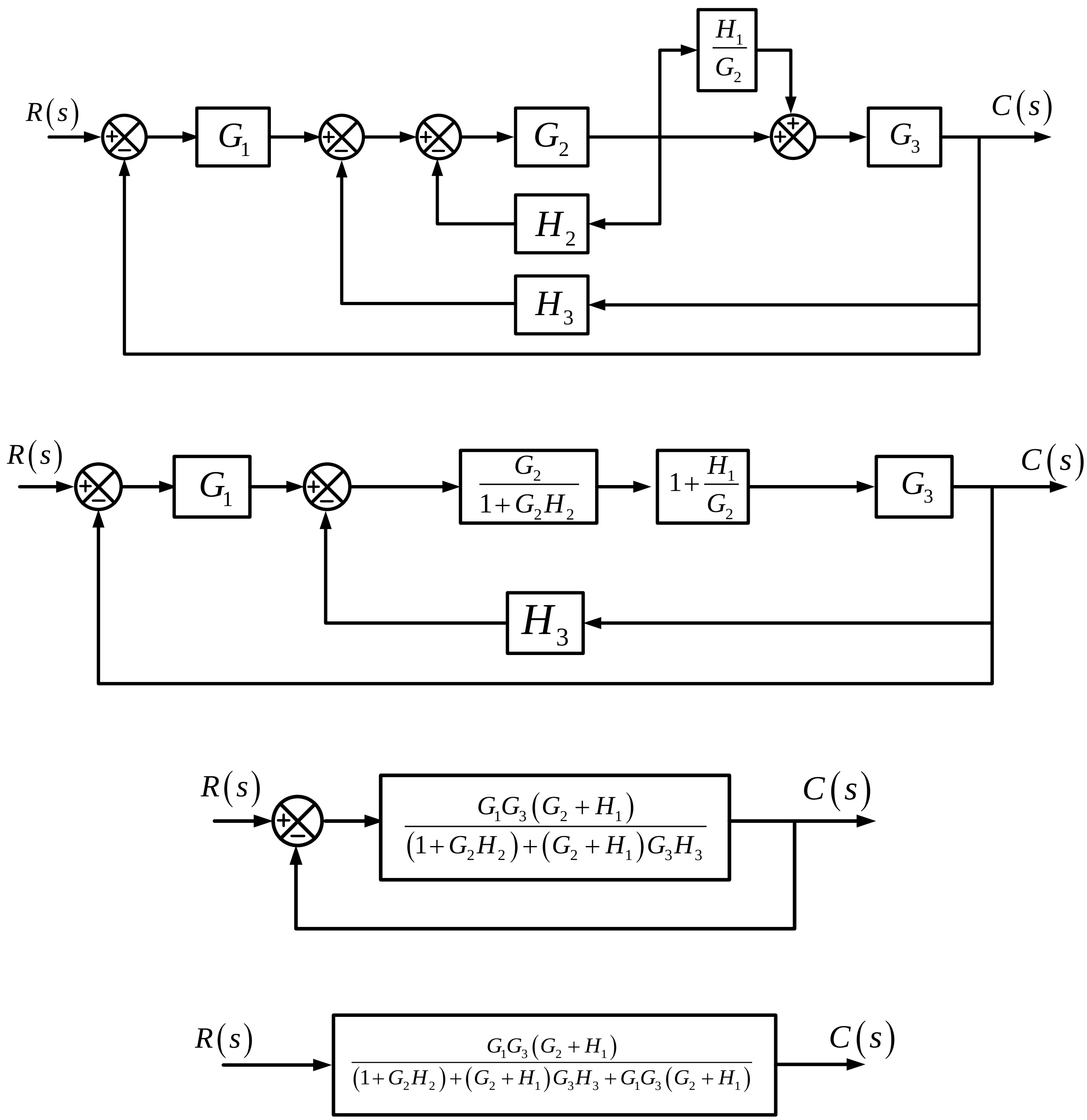
**SEOUL NATIONAL UNIVERSITY**
**SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING**

**SYSTEM CONTROL** **Fall 2014**

**HW#2 Mathematical modeling of control systems** **Out: September 18, 2014 (Th)**
**Due: September 30, 2014 (Th)**

[1]




$$\therefore\ \frac{C(s)}{R(s)}=\frac{G_1G_3\left(G_2+H_1\right)}{1+G_2H_2+G_2G_3H_3+G_3H_1H_3+G_1G_2G_3+G_1G_3G_1}$$

[2]

When there is no disturbance, the output is given by

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)}$$

Thus, the error is the difference between R(s) and the actual output $C_R(s)$. The error $E_R(s)$ is given by

$$E_R(s) = R(s) - C_R(s) = R(s)\left[1 - \frac{C_R(s)}{R(s)}\right] = R(s)\left[1 - \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)}\right] = \frac{R(s)}{1+G_1(s)G_2(s)}$$

If the system is stable, the steady-state error $e_{SSR}(t)$ can be given by

$$e_{SSR}(t) = \lim_{t\to\infty} e_R(t) = \lim_{s\to\infty} sE_R(s) = \lim_{s\to\infty} \frac{sR(s)}{1+G_1(s)G_2(s)}$$

When only the disturbance input D(s) is present, the output $C_D(s)$ is given by

$$\frac{C_D(s)}{R(s)} = \frac{G_2(s)}{1+G_1(s)G_2(s)}$$

Since the desired output to the disturbance input D(s) is zero, the error ED(s) can be given by

$$E_D(s) = 0 - C_D(s) = -C_D(s) = -\frac{G_2(s)}{1+G_1(s)G_2(s)}D(s)$$

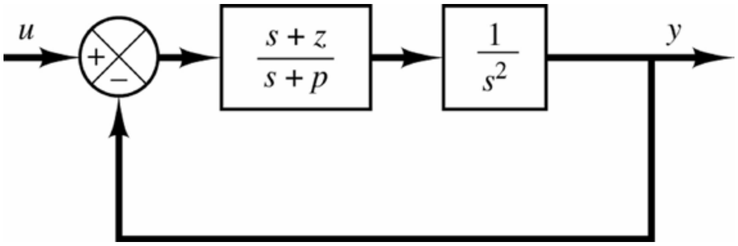
For the stable system, the steady-state error $e_{SSD}(t)$ is given by

$$e_{SSD}(t) = \lim_{t\to\infty} e_D(t) = \lim_{s\to\infty} sE_D(s) = \lim_{s\to\infty} \frac{-sG_2(s)D(s)}{1+G_1(s)G_2(s)}$$

The steady-state error when both the reference input R(s) and disturbance input D(s) are present is the sum of above two error, $e_{SSR}(t)$ and $e_{SSD}(t)$, and is given by

$$\begin{aligned} e_{ss}(t) &= e_{SSR}(t) + e_{SSD}(t) \\ &= \lim_{s\to 0}\left[\frac{sR(s)}{1+G_1(s)G_2(s)} - \frac{sG_2(s)D(s)}{1+G_1(s)G_2(s)}\right] = \lim_{s\to 0}\frac{s}{1+G_1(s)G_2(s)}\left[R(s) - G_2(s)D(s)\right] \end{aligned}$$

[3]




From above figure, following transfer function can be obtained.

$$\frac{Y(s)}{U(s)} = \frac{s+z}{s^3 + ps^2 + s + z}$$

This is equivalent to differential equation

$$\dddot{y} + p\ddot{y} + \dot{y} + zy = \dot{u} + zu$$

Comparing this equation with the standard equation

$$\dddot{y} + a_1\ddot{y} + a_2\dot{y} + a_3 y = b_0\dddot{u} + b_1\ddot{u} + b_2\dot{u} + b_3 u$$

We obtain

$$a_1 = p,\ a_2 = 1,\ a_3 = z,\ b_0 = 0,\ b_1 = 0,\ b_2 = 1,\ b_3 = z$$

Define state

$$\begin{aligned} x_1 &= y - \beta_0 u \\ x_2 &= \dot{y} - \beta_0\dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u \\ x_3 &= \dot{x}_2 - \beta_2 u \end{aligned}$$

where

$$\begin{aligned} \beta_0 &= b_0 = 0 \\ \beta_1 &= b_1 - a_1\beta_0 = 0 \\ \beta_2 &= b_2 - a_1\beta_1 - a_2\beta_0 = 1 \\ \beta_3 &= b_3 - a_1\beta_2 - a_2\beta_0 = z - p \end{aligned}$$

$$\therefore \quad \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -z & -1 & -p \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ z-p \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$